Generating method and application of basic probability assignment based on interval number distance and model reliability

Junwei Li
Henan University
Baolin Xie (xbl1212@henu.edu.cn)
Henan University https://orcid.org/0000-0003-0037-1840
Yong Jin
Henan University
Lin Zhou
Henan University https://orcid.org/0000-0002-2700-834X

Research Article

Keywords: Dempster-Shafer evidence theory, Basic probability assignment, Interval number distance, Comprehensive reliability

Posted Date: August 29th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1671429/v1

Generating method and application of basic probability assignment based on interval number distance and model reliability

Junwei Li[1], Baolin Xie[1†], Yong Jin[1†] and Lin Zhou[1*]

[1*]School of Artificial Intelligence, Henan University, Zhengzhou, 450046, Henan Province, China.

*Corresponding author(s). E-mail(s): zhoulin@henu.edu.cn; Contributing authors: lijunwei@henu.edu.cn; xbl1212@henu.edu.cn; jy@henu.edu.cn;
†These authors contributed equally to this work.

Abstract

In the Dempster-Shafer (D-S) evidence theory, how to transform the objective data in reality into the basic probability assignment (BPA) is still an open issue. Based on this problem, a new method of generating BPA based on interval number distance and model reliability is proposed. First, construct the interval number model under each attribute. Secondly, calculate the interval number distance between the test sample and the interval number model and convert it into the initial basic probability assignment (IBPA). Thirdly, the final BPA is obtained by discounting the IBPA by constructing the comprehensive reliability from the static reliability and dynamic reliability of the interval number model. Finally, the Dempster combination rule is used to fuse the final BPA one by one, and the decision is made according to the fusion result. The ten-fold cross-validation results show that the classification accuracy under the three data sets is higher than other methods, and the classification accuracy of the Iris data set is 0.9733. At the same time, it is verified that the proposed method still has good effectiveness and robustness in the incomplete information environment.

Keywords: Dempster-Shafer evidence theory, Basic probability assignment, Interval number distance, Comprehensive reliability

1 Introduction

Multi-sensor information fusion technology fuses information collected by sources, resulting in rational decision-making. Since effectively use information from sources, it may break limitations of decision-making caused by a single sensor, thus can be widely used in multi-attribute decision-making [1–3], fault diagnosis [4–6], target recognition [7, 8], medical diagnosis [9, 10], and so on.

As a branch of multi-sensor information fusion technology, Dempster-Shafer (D-S) evidence theory was first proposed by Dempster [11] in 1967 and further expanded and popularized by his student Shafer [12]. D-S evidence theory is commonly composed of three steps as follows. Firstly, the objective data sampling from the real world is transformed into the basic probability assignment (BPA) function of the D-S evidence theory framework. Then, the Dempster combination rule is used to fuse the generated BPA function. The decision is made according to the fusion results finally. Since using belief function and plausibility function to distinguish the unknowledge and uncertainty of information, D-S evidence theory has shown an excellent ability to deal with uncertain information, leading it widely used to area of multi-sensor information fusion [13–20].

1.1 Related Work

Although has many advantages, D-S evidence theory still has some problems that need to be further studied including how to reasonably and effectively generate the BPA in the framework of D-S evidence theory [21–32] and how to effectively solve the counter-intuitive results induced by combining the high conflicting evidence with Dempster's combination rule [33–40].

Employing D-S evidence theory to solve practical problems, converting objective data of the real world into the BPA in the theoretical framework of D-S evidence theory is the first and most crucial step. Effective methods of BPA generation proposed by researchers are following as.

From the perspective of interval number model construction, Kang et al. [21] employed mathematical statistics method to obtain maximum and minimum values related to each attribute of the training sample firstly, and then to construct interval number model of the each attribute, thus to obtain the BPA by calculating the similarity between the test sample data and the model finally. Similar to Kang's works, Qin and Xiao [22] used the K-means++ method to construct the interval number model and then to obtain the BPA by calculating the similarity between the test sample and model.

In the view of constructing a membership function model, Jiang et al. [23] modeled a triangular membership function of each attribute according to training samples, and then constructed BPA according to the difference between the test sample and the model. Although using the same triangular membership function model as Jiang's method, Zhang and Deng [24] obtained the BPA by calculating the intersection between the test sample and the model. Jiang et al. [25] employed the Gaussian membership function model to generate the

BPA firstly, and then revised it according to the reliability of the Gaussian membership function model to obtain the final BPA. Zhang et al. [26] defined a distance between two triangular membership functions firstly, and then calculated this distance between the triangular membership function of test data and the triangular membership function model to obtain the BPA.

In addition, some researchers have discussed other generating methods of BPA. Fei et al. [27] used the K-means method to determine the range of the model, which contains all training sample firstly, and then obtained the BPA by judging the test sample within the range of the model or not. Xu et al. [28] modeled a normal distribution of each attribute according to training samples and generated the BPA according to the relationship between the test sample and normal distribution modeled. Deng et al. [29] constructed a confusion matrix according to the training data classification firstly, and then obtained the BPA through the confusion matrix processing. Zhang et al. [30] extracted key samples of each attribute from the training samples and then determined the BPA according to the distance between the test sample and the key samples. Hu et al. [31] extracted the characteristics of each pattern underlying in the samples firstly, and then employed neural networks to classify the sample according to extracted characteristics, the output of the classifier may be used to determine the BPA finally. Liu et al. [32] combined fuzzy naive Bayes and nearest mean classifier to generate the BPA according to the fuzzy membership degree and the distance between the centroids of each classes.

1.2 Contributions

The initial basic probability assignment (IBPA) based on interval number distance and discounted of IBPA based on model reliability are proposed. The main contributions of this paper are summarized as follows.

- Construct an interval number model according to objective data coming from the real world, and IBPA according to the distance between the test sample and the model.
- Discounted of IBPA may be achieved by comprehensive considering static reliability and dynamic reliability of the model.
- The effectiveness of the proposed method, namely classification accuracy, is verified by the 10-fold cross-validation experiment in the UCI database. Moreover, the accurately classifying the ability of the proposed method in an incomplete information environment are verified by statistical experiments too.

1.3 Paper Organization

The organization of this paper is presented as follows. Section II briefly introduces some basic theoretical knowledge related to the research of this article. Section III describes the method proposed in this paper in detail. Section IV introduces some simulation experiments carried out by applying the proposed method. Finally, Section V is the conclusions of this paper.

2 Preliminaries

2.1 Interval number theory

Definition 1 [41] $\forall a^+, a^- \in R$ and $a^- \leq a^+$ are defined as $a = [a^-, a^+]$, and a is referred to as an interval number.
Definition 2 [41] For any interval number a, if $a^- = a^+$, then a is a certain real number.
Definition 3 [41] Interval number distance
Let $a_1 = \left[a_1^-, a_1^+\right]$ and $a_2 = \left[a_2^-, a_2^+\right]$ be interval number, the distance between the a_1 and a_2 is defined as follows

$$d(a_1, a_2) = \sqrt{\lambda[m\left(a_1\right) - m\left(a_2\right)]^2 + \mu[l\left(a_1\right) - l\left(a_2\right)]^2} \tag{1}$$

where $m\left(a_1\right) = \frac{a_1^- + a_1^+}{2}$, $m\left(a_2\right) = \frac{a_2^- + a_2^+}{2}$, $l\left(a_1\right) = a_1^+ - a_1^-$, $l\left(a_2\right) = a_2^+ - a_2^-$. λ and μ are respectively called the midpoint influence factor and the width influence factor of the interval number.
Definition 4 [21] Interval number similarity
Let $a_1 = \left[a_1^-, a_1^+\right]$ and $a_2 = \left[a_2^-, a_2^+\right]$ be interval number, the similarity between the a_1 and a_2 is defined as follows

$$S\left(a_1, a_2\right) = \frac{1}{1 + \alpha d(a_1, a_2)} \tag{2}$$

where $\alpha > 0$ is the support coefficient, and $d(a_1, a_2)$ is the distance between the a_1 and a_2.

2.2 Dempster-Shafer evidence theory

In Dempster-Shafer evidence theory [11, 12], the frame of discernment (FOD) $\Theta = \{\theta_1, \theta_2, \cdots, \theta_N\}$ is composed of N exhaustive and mutually exclusive elements, and 2^Θ is the power set of the FOD, expressed as $2^\Theta = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \cdots, \{\theta_N\}, \{\theta_1, \theta_2\}, \{\theta_1, \theta_3\}, \cdots, \Theta\}$.
Definition 5 [12] Assume that the FOD is Θ. If the function $m : 2^\Theta \to [0, 1]$ satisfy

$$\begin{cases} m\left(\emptyset\right) = 0 \\ \sum\limits_{A \subseteq \Theta} m\left(A\right) = 1 \end{cases} \tag{3}$$

Then m is called the BPA, if $m(A) > 0$, the proposition A is a focal element.
Definition 6 [12] For the given evidence m, the belief function $Bel : 2^\Theta \to [0, 1]$ is defined as

$$Bel\left(A\right) = \sum_{B \subseteq A} m\left(B\right), \forall A \subseteq \Theta \tag{4}$$

Definition 7 [12] For the given evidence m, the plausibility function $Pl : 2^\Theta \to [0, 1]$ is defined as

$$Pl\left(A\right)=\sum_{B\cap A\neq\emptyset}m\left(B\right),\forall A\subseteq\Theta \tag{5}$$

Definition 8 [12] Let m_1 and m_2 be two BPAs on the same FOD Θ, $m_1 \oplus m_2$ is used to represent the new BPA according to the combination of m_1 and m_2 and the Dempster combination rule is defined as

$$m_1\oplus m_2\left(A\right)=\begin{cases}0, A=\emptyset\\ \frac{\sum\limits_{B\cap C=A} m_1(B)m_2(C)}{1-k}, A\neq\emptyset\end{cases} \tag{6}$$

where $k=\sum\limits_{B\cap C=\emptyset}m_1\left(B\right)m_2\left(C\right)$ is called the conflict coefficient.

Definition 9 [42] Let m be a BPA on the FOD Θ, the pignistic probability transformation function $BetP_m:\Theta\rightarrow[0,1]$ is defined as

$$BetP_m\left(\theta_i\right)=\sum_{A\subseteq\Theta,\theta_i\in A}\frac{1}{|A|}\frac{m\left(A\right)}{1-m\left(\emptyset\right)} \tag{7}$$

where $|A|$ represents the number of elements in A.

3 The new generation BPA method

In the D-S evidence theory, BPA should contain as much original information as possible from the data source. However, some methods only consider the singleton subset proposition and full set proposition at the BPA generating while ignore the intersection of singleton subset proposition, resulting in information loss [29]. Therefore, it is necessary to consider the proposition including the singleton subset proposition, full set propositions and the multi-subset propositions at the BPA generating. In addition, in a multi-sensor system, sensors may hard to accurately measure the data related to each attribute of the target due to the uncertainty of objective things. Interval number with the characteristic of simplicity and flexibility can describe the uncertainty of objective things well, and the close connection between interval numbers and membership functions, which can be easily converted to membership functions, makes interval numbers useful in more areas. Based on the above considerations, this section combines interval number and D-S evidence theory to propose a method for generating BPA based on interval number distance and model reliability.

3.1 The framework of the proposed method

Fig. 1 gives the process for determining BPA using interval number distance and model reliability. It mainly includes the five parts as follows: (1). constructing the interval number model; (2). generating the IBPA; (3). comprehensive reliability of the calculation model; (4). determining the final BPA; (5). fusion and decision making.

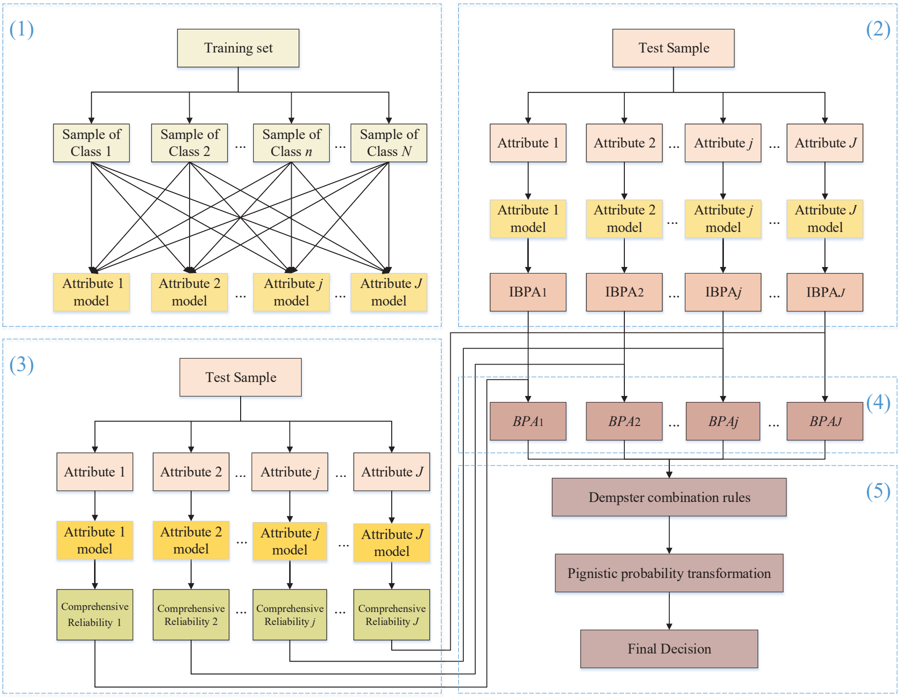

Fig. 1 BPA determination process

3.2 The specific steps of the proposed method

Assuming that there are S samples, N classes, and J attributes. The data set and class are expressed as $\mathbf{Y} = (\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_s, \cdots, \mathbf{y}_S)^T$ and $\{1, 2 \cdots, n, \cdots, N\}$, respectively. Where $\mathbf{y}_s = (y_{s1}, y_{s2}, \cdots, y_{sj}, \cdots, y_{sJ})$, $s = 1, 2, \cdots, S$, $n = 1, 2, \cdots, N$, $j = 1, 2, \cdots, J$.

3.2.1 Constructing the interval number model

The samples in the data set Y are divided into the training set $\boldsymbol{Y}_1 = (\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_T)^T$ and the test set $\boldsymbol{Y}_2 = \left(\mathbf{y}_{T+1}, \mathbf{y}_{T+2}, \cdots, \mathbf{y}_S\right)^T$ firstly, where $\boldsymbol{Y}_1$ is a $T \times J$ matrix, $\boldsymbol{Y}_2$ is a $(S–T) \times J$ matrix, J is the number of attributes. Then, interval number a_n^j with respect to $j-$th attribute of n-th class is determined by maximum and minimize of all samples with $j-$th attribute in n-th class, respectively. Similarly, we may obtain the interval numbers set $\left\{a_1^j, a_2^j, \cdots, a_n^j, \cdots, a_N^j\right\}$ of $j-$th attribute of N classes . Since samples with $j-$th attribute may belong to any sets of classes, interval number set $\left\{a_{N+1}^j, a_{N+2}^j, \cdots, a_{2^N-1}^j\right\}$ of intersections between the N classes should be obtained by using set $\left\{a_1^j, a_2^j, \cdots, a_n^j, \cdots, a_N^j\right\}$. Note, the interval number of the empty set $\emptyset$ is defined as $a_{2^N}^j = [0, 0]$ since FOD is constructed in a closed world. Thus, the interval number model of the $j-$th attribute is defined as η_j=$\left\{a^j_1, a^j_2, \cdots, a^j_i, \cdots, a^j_{2^N}\right\}$, where $i = 1, 2, \cdots, 2^N$.

3.2.2 Generating the IBPA

The interval number distance of the $j-$th attribute $d(\xi_{lj}, \eta_j)$ between the interval number ξ_{lj} of the test sample $\xi_l = (\xi_{l1}, \xi_{l2}, \cdots, \xi_{lj}, \cdots, \xi_{lJ})$ and the interval number model $\eta_j = \left\{a^j_1, a^j_2, \cdots, a^j_p, \cdots, a^j_{2^N}\right\}$ are described in Eq. (8). Using Eq. (2), we can transform $d(\xi_{lj}, \eta_j)$ into $S(\xi_{lj}, \eta_j)$ described in Eq. (9). IBPA m^j of the test sample ξ_l described in Eq. (10) is obtained by normalizing Eq. (9).

$$d(\xi_{lj}, \eta_j) = \left\{d\left(\xi_{lj}, a^j_1\right), d\left(\xi_{lj}, a^j_2\right), \cdots, d\left(\xi_{lj}, a^j_i\right), \cdots, d\left(\xi_{lj}, a^j_{2^N}\right)\right\} \tag{8}$$

$$S(\xi_{lj}, \eta_j) = \left\{S\left(\xi_{lj}, a^j_1\right), S\left(\xi_{lj}, a^j_2\right), \cdots, S\left(\xi_{lj}, a^j_i\right), \cdots, S\left(\xi_{lj}, a^j_{2^N}\right)\right\} \tag{9}$$

$$m^j = \left\{m^j(\{1\}), \cdots, m^j(\{N\}), m^j(\{1,2\}), m^j(\{1,3\}), \cdots, m^j(\Theta), m^j(\emptyset)\right\} \tag{10}$$

3.2.3 Comprehensive reliability of the calculation model

Inspired by Jiang's work [25], we combine static reliability which describes the stability of the model and dynamic reliability which describes the real-time correlation between the test sample and the mode to construct the comprehensive reliability of the model. The final BPA is determined by discounting the IBPA according to the comprehensive reliability of the model.

1.Static reliability

The static reliability is employed to describe discrimination among classes according to the scale of intersection among classes. Specifically, the larger intersection among classes is, the lower static reliability of the model.

Fig. 2 shows the relationship between classes with $j-$th attribute in the interval number model. The intersection of class φ and class ψ are described in subgraphs (a) and (b) of Fig. 2. Inclusion relationships between class φ and class ψ are described in subgraphs (c) and (d) of Fig. 2. Neither intersect nor contain between class φ and class ψ are described in subgraphs (e) and (f) of Fig. 2. Where $sim^j_{\varphi\psi}$ indicates the similarity of class φ and class ψ under the j-th attribute, $\varphi, \psi \in \{1, 2, \cdots, N\}$, $a^j_\varphi = \left[a^{j-}_\varphi, a^{j+}_\varphi\right]$, $a^j_\psi = \left[a^{j-}_\psi, a^{j+}_\psi\right]$.

According to the subgraph (a) and the subgraph (b), we can obtain

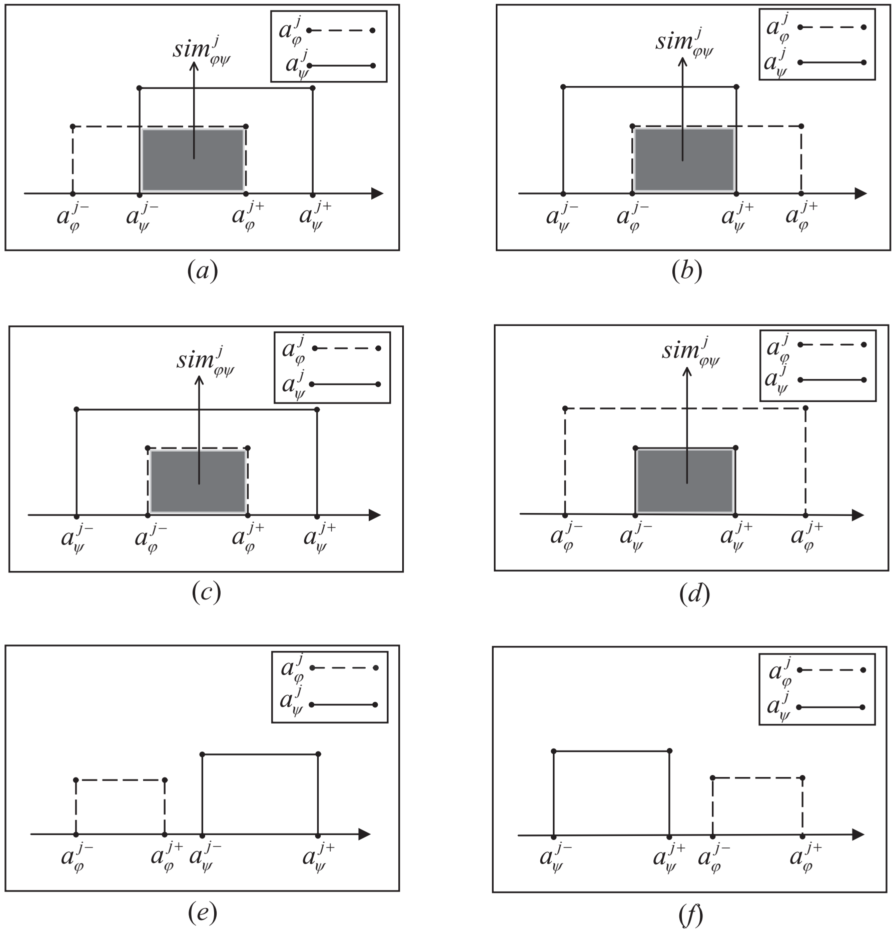

Fig. 2 Several cases of static reliability

$$sim^j_{\varphi\psi} = \begin{cases} \frac{a^{j+}_{\varphi} - a^{j-}_{\psi}}{a^{j+}_{\psi} - a^{j-}_{\varphi}}, & a^{j-}_{\varphi} \leq a^{j-}_{\psi} \leq a^{j+}_{\varphi} \leq a^{j+}_{\psi} \\ \frac{a^{j+}_{\psi} - a^{j-}_{\varphi}}{a^{j+}_{\varphi} - a^{j-}_{\psi}}, & a^{j-}_{\psi} \leq a^{j-}_{\varphi} \leq a^{j+}_{\psi} \leq a^{j+}_{\varphi} \end{cases} \tag{11}$$

From the subgraph (c) and the subgraph (d), we can obtain

$$sim^j_{\varphi\psi} = \begin{cases} \frac{a^{j+}_{\varphi} - a^{j-}_{\varphi}}{a^{j+}_{\psi} - a^{j-}_{\psi}}, & a^{j-}_{\psi} \leq a^{j-}_{\varphi} \leq a^{j+}_{\varphi} \leq a^{j+}_{\psi} \\ \frac{a^{j+}_{\psi} - a^{j-}_{\psi}}{a^{j+}_{\varphi} - a^{j-}_{\varphi}}, & a^{j-}_{\varphi} \leq a^{j-}_{\psi} \leq a^{j+}_{\psi} \leq a^{j+}_{\varphi} \end{cases} \tag{12}$$

From the subgraph (e) and the subgraph (f), we can obtain

$$sim^j_{\varphi\psi} = 0, a^{j+}_{\varphi} \leq a^{j-}_{\psi} \; or \; a^{j+}_{\psi} \leq a^{j-}_{\varphi} \tag{13}$$

Thus, the similarity matrix $\boldsymbol{SMM}_j$ with $j-$th attribute of the interval number model can be obtained in Eq. (14)

$$\mathbf{SMM}_j = \begin{bmatrix} 1 & \cdots & sim_{1n}^j & \cdots & sim_{1N}^j \\ \vdots & & \vdots & & \vdots \\ sim_{n1}^j & \cdots & 1 & \cdots & sim_{nN}^j \\ \vdots & & \vdots & & \vdots \\ sim_{N1}^j & \cdots & sim_{Nn}^j & \cdots & 1 \end{bmatrix} \tag{14}$$

The static reliability R_j^p related to $\mathbf{SMM}_j$ can be calculated by the Eq. (15) and Eq. (16).

$$r_j = \sum_{\varphi<\psi} 1 - sim_{\varphi\psi}^j \tag{15}$$

$$R_j^p = e^{r_j^p} \tag{16}$$

2.Dynamic reliability

Dynamic reliability represents time varying correlation between the test sample and the model. As shown in Fig. 3, dynamic reliability $d_{\varphi\psi}^j$ indicates the distance between the midpoint of the test sample and the midpoint of the intersection of class φ and class ψ. It is clearly that the larger $d_{\varphi\psi}^j$ means the longer distance between ξ_{lj} and intersection of class φ and class ψ, thus indicating that the less likely the test sample is to be misclassified. Where $\xi_{lj} = \left[b_j^-, b_j^+\right]$, $a_\varphi^j = \left[a_\varphi^{j-}, a_\varphi^{j+}\right]$, $a_\psi^j = \left[a_\psi^{j-}, a_\psi^{j+}\right]$.

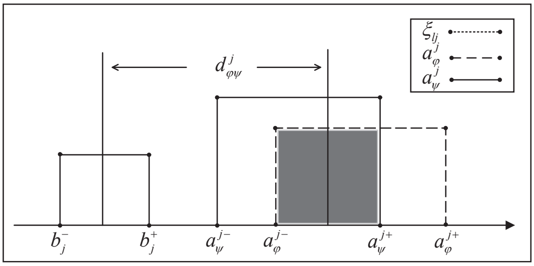

Fig. 3 Dynamic reliability based on test samples and models

Several situations of dynamic reliability with j-th attribute determined by test sample and the interval number model are described in Fig. 4.

Assuming classes in the interval number model intersecting each other, the distance between the test sample and the model is described in Eq. (17) according to subplots (a) and (b) of the Fig. 4.

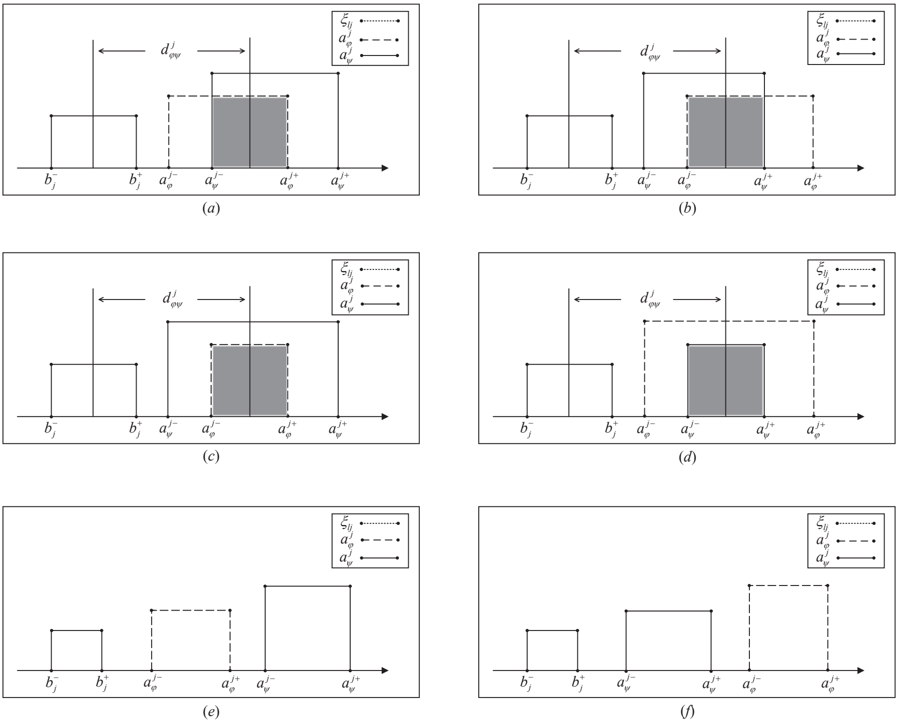

Fig. 4 Several cases of dynamic reliability

$$d_{\varphi\psi}^{j}=\begin{cases}\dfrac{\left|\frac{a_{\varphi}^{j+}+a_{\psi}^{j-}}{2}-\frac{b_{j}^{-}+b_{j}^{+}}{2}\right|}{a_{\psi}^{j+}-a_{\varphi}^{j-}}, a_{\varphi}^{j-}\leq a_{\psi}^{j-}\leq a_{\varphi}^{j+}\leq a_{\psi}^{j+}\\ \dfrac{\left|\frac{a_{\psi}^{j+}+a_{\varphi}^{j-}}{2}-\frac{b_{j}^{-}+b_{j}^{+}}{2}\right|}{a_{\varphi}^{j+}-a_{\psi}^{j-}}, a_{\psi}^{j-}\leq a_{\varphi}^{j-}\leq a_{\psi}^{j+}\leq a_{\varphi}^{j+}\end{cases} \tag{17}$$

Assuming there are contained the relationship of classes in the interval number model, the distance between the test sample and the model is described in Eq. (18) according to subplots (c) and (d) of the Fig. 4.

$$d_{\varphi\psi}^{j}=\begin{cases}\dfrac{\left|\frac{a_{\varphi}^{j+}+a_{\varphi}^{j-}}{2}-\frac{b_{j}^{-}+b_{j}^{+}}{2}\right|}{a_{\psi}^{j+}-a_{\psi}^{j-}}, a_{\psi}^{j-}\leq a_{\varphi}^{j-}\leq a_{\varphi}^{j+}\leq a_{\psi}^{j+}\\ \dfrac{\left|\frac{a_{\psi}^{j+}+a_{\psi}^{j-}}{2}-\frac{b_{j}^{-}+b_{j}^{+}}{2}\right|}{a_{\varphi}^{j+}-a_{\varphi}^{j-}}, a_{\varphi}^{j-}\leq a_{\psi}^{j-}\leq a_{\psi}^{j+}\leq a_{\varphi}^{j+}\end{cases} \tag{18}$$

Assuming there are neither intersection nor containment relationships of classes in the interval number model, the distance between the test sample and the model is described in Eq. (19) according to subplots (e) and (f) of the Fig. 4.

$$d_{\varphi\psi}^{j}=1, a_{\varphi}^{j+}\leq a_{\psi}^{j-}\, or\; a_{\psi}^{j+}\leq a_{\varphi}^{j-} \tag{19}$$

Following above descriptions, the j-th attribute distance matrix $\boldsymbol{D}^{j}$ related to the test sample and the interval number model can be obtained as

$$\boldsymbol{D}^j = \begin{bmatrix} 0 & \cdots & d_{1n}^j & \cdots & d_{1N}^j \\ \vdots & & \vdots & & \vdots \\ d_{n1}^j & \cdots & 0 & \cdots & d_{nN}^j \\ \vdots & & \vdots & & \vdots \\ d_{N1}^j & \cdots & d_{Nn}^j & \cdots & 0 \end{bmatrix} \tag{20}$$

The average distance d_j is obtained by using Eq. (21) and dynamic reliability R_j^q is achieved according to Eq. (22).

$$d_j = \frac{\sum\limits_{\varphi<\psi} d_{\varphi\psi}^j}{N(N-1)/2} \tag{21}$$

$$R_j^q = e^{d_j} \tag{22}$$

Thus, comprehensive reliability R_j of j-th attribute with respect to (w.r.t) interval number model is obtained by the normalized product of static reliability and dynamic reliability.

$$R_j = \frac{R_j^p R_j^q}{\max(R_j^p R_j^q)} \tag{23}$$

3.2.4 Determining the final BPA

According to R_j, the final BPA of the j-th attribute w.r.t the test sample ξ_l is achieved by discounting elements of set m^j described in Eq. (10).

$$m_j : \begin{cases} m_j(A_l) = R_j m^j(A_l), A_l \neq \Theta \\ m_j(\Theta) = R_j m^j(\Theta) + (1 - R_j) \end{cases} \tag{24}$$

3.2.5 Fusion and decision making

The BPAs are fused according to the Dempster combination rule described in Eq. (6) firstly. Then, the subset with the highest belief is the final decision result, which is achieved by using Eq. (7) to process fused BPA.

3.3 Numerical example

Iris data set [43] which contains a total of 150 samples with three classes such as Setosa(Se), Versicolour(Ve), Virginica (Vi) and four attributes including Sepal Length (SL), Sepal Width (SW), Petal Length (PL) and Petal Width (PW) are used to demonstrate the performance of the proposed method.

Step1. Constructing interval number models for each attribute of the Iris data set.

Random sampling of 40 samples from each class of the Iris data set was used to construct the interval number model of the Iris data set shown in Fig. 5.

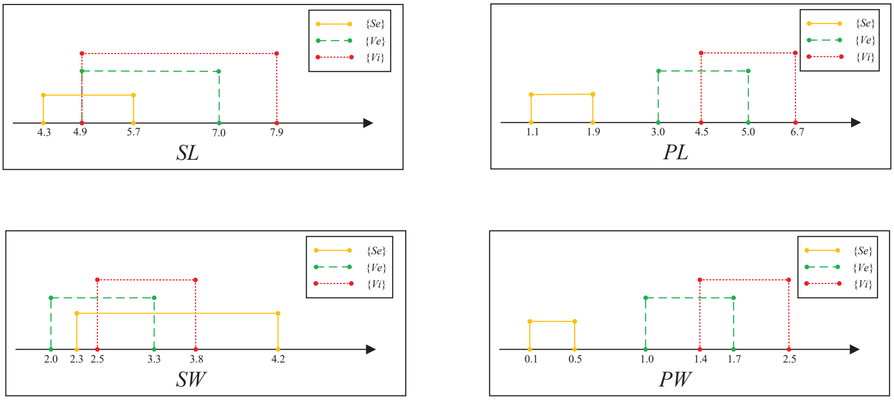

Fig. 5 Interval number model for the four attributes in the Iris data set

Step2. Generate the IBPA of the test sample.

Step2-1. Assume that sample $\xi_l = (4.6, 3.1, 1.5, 0.2)$ of Setosa class is randomly selected from the test set. PL attribute of ξ_l in PL attribute model is shown in Fig. 6.

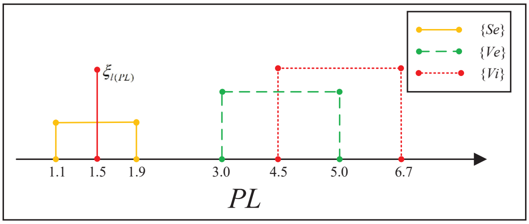

Fig. 6 Relationship between the test sample and attribute PL model

Step2-2. Suppose λ, μ, α take 0.9, 0.1 and 5, respectively, interval number distance and interval number similarity between PL attribute of ξ_l and proposition are described in Table 1. Where IND, INS are abbreviations for interval number distance and interval number similarity respectively.

Table 1 Interval number distance and interval number similarity of PL attribute of ξ_l

Proposition	$\{Se\}$	$\{Ve\}$	$\{Vi\}$	$\{Se, Ve\}$	$\{Se, Vi\}$	$\{Ve, Vi\}$	$\{Se, Ve, Vi\}$
IND	0.2530	2.4546	3.9513	0	0	3.0873	0
INS	0.4415	0.0753	0.0482	0	0	0.0608	0

Step2-3. Normalizing interval number similarity listed in Table 1, we get IBPA of PL attribute of ξ_l which are shown in Table 2.

Table 2 IBPA of PL attribute of ξ_l

Belief of Proposition	IBPA (PL)
$m(\{Se\})$	0.7054
$m(\{Ve\})$	0.1204
$m(\{Vi\})$	0.0770
$m(\{Se, Ve\})$	0
$m(\{Se, Vi\})$	0
$m(\{Ve, Vi\})$	0.0972
$m(\{Se, Ve, Vi\})$	0

Step3. Calculate the comprehensive reliability of each attribute model in the Iris data set.

In Table3, Static reliability R_j^p, dynamic reliability R_j^q and comprehensive reliability R_j of each attribute model can be obtained by Eq. (16), Eq. (22) and Eq. (23), respectively.

Table 3 Reliability of the model

Reliability	SL	SW	PL	PW
R_j^p	1.7815	1.1469	2.8649	2.8000
R_j^q	1.3515	1.5760	2.6104	2.6293
R_j	0.3219	0.2417	1.0000	0.9844

Step4. Determine the final BPA of the test sample.

Discounting the IBPA of PL attribute of ξ_l in Table 2 according to R_j of the PL attribute in Table 3, final BPA of PL attribute of ξ_l is achieved in Table 4.

Table 4 Final BPA of test samples under attribute PL

Belief of Proposition	BPA (PL)
$m(\{Se\})$	0.7054
$m(\{Ve\})$	0.1204
$m(\{Vi\})$	0.0770
$m(\{Se, Ve\})$	0
$m(\{Se, Vi\})$	0
$m(\{Ve, Vi\})$	0.0972
$m(\{Se, Ve, Vi\})$	0

Step5. Fusion and decision making.

Step5-1. Similarly, the final BPAs of the ξ_l with other attributes can be obtained, and then final fused BPAs shown in Table 5 are obtained by using the Dempster combination rule described in Eq. (6).

Table 5 Final fusion result

Belief of Proposition	BPA
$m_1 \oplus m_2 \oplus m_3 \oplus m_4 (\{Se\})$	0.8140
$m_1 \oplus m_2 \oplus m_3 \oplus m_4 (\{Ve\})$	0.1016
$m_1 \oplus m_2 \oplus m_3 \oplus m_4 (\{Vi\})$	0.0613
$m_1 \oplus m_2 \oplus m_3 \oplus m_4 (\{Se, Ve\})$	0
$m_1 \oplus m_2 \oplus m_3 \oplus m_4 (\{Se, Vi\})$	0
$m_1 \oplus m_2 \oplus m_3 \oplus m_4 (\{Ve, Vi\})$	0.0231
$m_1 \oplus m_2 \oplus m_3 \oplus m_4 (\{Se, Ve, Vi\})$	0

Step 5-2. Using pignistic probability to transform fusion of BPA described in Table 5, we obtain

$$\begin{aligned} &BetP(\{Se\}) = 0.8140, BetP(\{Ve\}) = 0.1131, \\ &BetP(\{Vi\}) = 0.0729 \end{aligned}$$

Since $BetP(\{Se\})$ has the highest belief, it indicates ξ_l belongs to Setosa class which is coinciding with fact.

4 Experiments and analysis

In this section, some classic data sets in the UCI database [43] are used to verify the performance of the proposed BPA generation method.

4.1 Data set information

Brief background on the dataset used is described as follows. Basic information of the Iris data set was presented earlier. The Wine data set contains 13 chemical composition of three different wine varieties produced in the same area of Italy. The Seed data set collects information on seven attributes of 210 samples from three different wheat varieties. Table 6 provides basic information about the three datasets.

Table 6 Basic information of the data set

Data set	class	Number of attributes	Number of instances
Iris	3	4	150
Wine	3	13	178
Seed	3	7	210

4.2 Experiments

4.2.1 Ten-fold cross-validation experiment

Ten-fold cross-validation experiment is the most commonly used verification method in classification experiments. The main process is as follows. The original data is randomly divided into ten parts, one part of the data is used for testing, and the remaining nine parts of the data are used to build the model, cycling in turn, until each part of the data has done for the test set. Finally, the average classification accuracy of 10 experiments is taken as the overall classification accuracy. Fig. 7 shows the average classification accuracy of ten-fold cross-validation experiments on the three data sets.

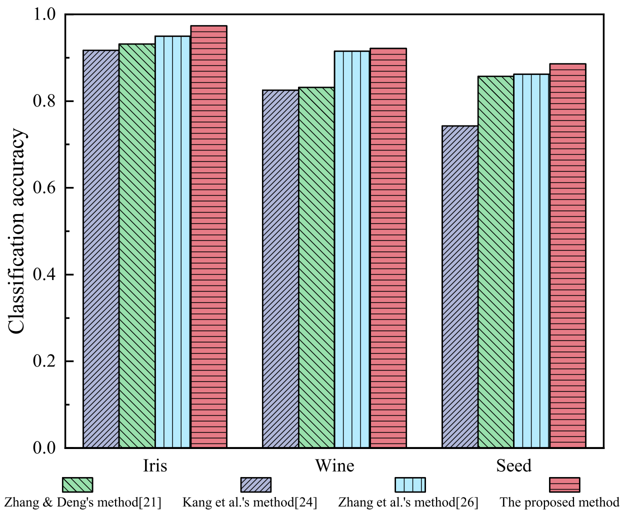

Fig. 7 Results of ten-fold cross-validation experiment

Fig. 7 shows that the proposed method achieves the highest classification accuracy among all methods. This is due to the fact that the proposed method uses the comprehensive reliability to discount the IBPA during the BPA generating. Since the comprehensive reliability is completely driven by data, it can avoid the influence of artificial subjective factors, leading to more reasonable results.

4.2.2 Experiments in incomplete information environments

Harsh environments such as high temperature, extreme cold and humidity may deteriorate the accuracy of the sensors, resulting in data collected by the sensors being inaccurate. Thus, research on the effectiveness and robustness of the proposed method in an incomplete information environment is necessary.

1. Incomplete data experiments with missing attribute information

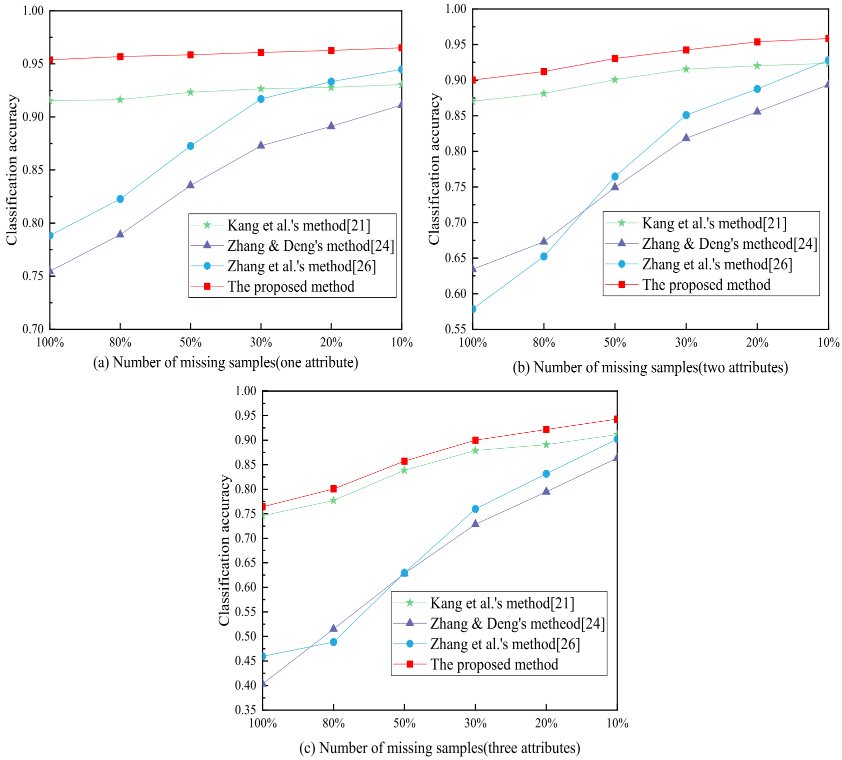

Fig. 8 Incomplete data experiment with missing attribute information

Taking the Iris data set as an example, three groups of experiments are set up to erase one attribute, two and three attributes information of partial samples, respectively. The proportion of samples with missing attribute information randomly erased in each group of experiments to the overall sample was 10%, 20%, 30%, 80% and 100%. 1000 Monte Carlo experiments were performed for each group of experiments and the results are as follows.

Assuming one attribute,two and three attributes information erased of randomly partial samples, respectively, the classification accuracy of some methods on are plotted in subgraphs (a), (b), and (c) of Fig. 8. It is can be find that the proposed method achieves the best classification accuracy among all methods. Moreover, the classification accuracy of the proposed method and Kang et al.'s method experience slightly decrease, while classification accuracy of other methods shows apparent decrease, with the proportion of samples with missing attribute information to the overall sample increasing.

Classification accuracy changing according to the ratio of samples with the missing attributes is plotted in Fig. 9. We find that the classification accuracy of all methods trends to decrease with the number of missing attributes increasing at the ratio of samples with missing attributes fixed. Moreover, the classification accuracy of the proposed method is the highest in all methods

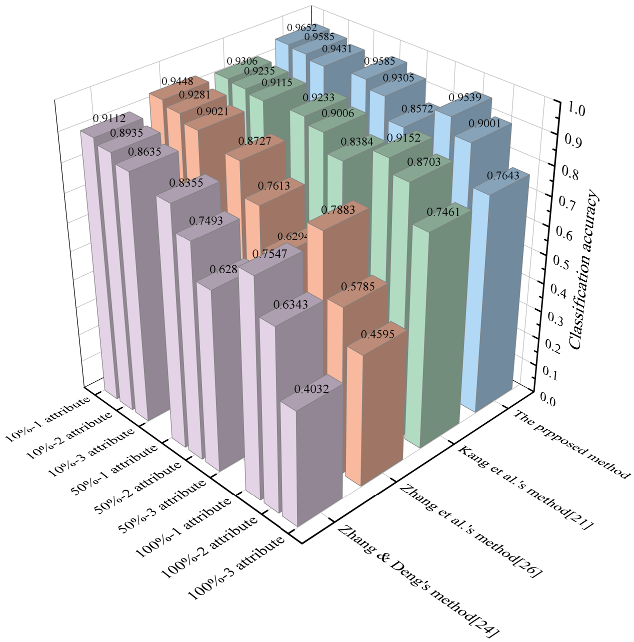

Fig. 9 Effect of the number of missing attributes on classification accuracy when the proportion of samples with missing attribute information to the overall sample is fixed

and is always higher than 0.85 at 50% of samples with missing attributes. Even 100% samples with two missing attributes, the classification accuracy of the proposed method is still higher than 0.90. It is indicate that proposed method has better robustness than other methods.

2. Imprecise data experiments with added noise

Taking the Iris data set as an example, Gaussian noise with different signal-to-noise ratios (SNR) was added to the data set. Where the SNR is taken to be in the range 0dB-20dB. The classification performance of the proposed method in an imprecise information environment was analyzed by 100 Monte Carlo experiments for each SNR environment and the results are shown below.

Fig. 10 shows the classification accuracy comparison between the proposed method and other methods in the Iris dataset when the SNR of the added noise varies from 0 dB to 20 dB. We find that the proposed method has similar classification accuracy to that of Zhang & Deng's method when the SNR is 0 dB, and is significantly higher than that of Kang et al.'s method and Zhang et al.'s method. With the gradual increase of the SNR, the proposed method has a higher classification accuracy than other methods with the same SNR. This is due to the proposed method secondary correcting the BPA through model reliability at the BPA generating, which gives the generated BPA better resistance to interference. Experiments on imprecise data with added noise demonstrate that the proposed method shows better classification ability and resistance to interference in an environment of imprecise information.

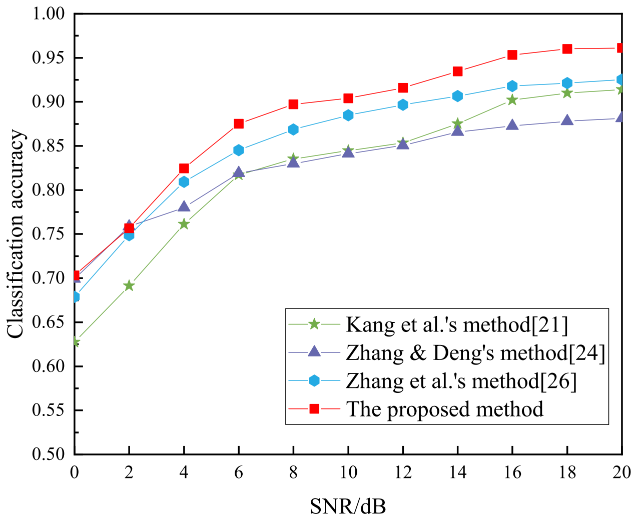

Fig. 10 Imprecise data experiments with added noise

5 Conclusion

In this paper, a new BPA generation method based on interval number distance and model reliability is proposed to solve how to construct the BPA in the framework of D-S evidence theory. Since the proposed method is data-driven from constructing the model to determine the IBPA to calculating the model reliability to determine the final BPA, the algorithm can avoid the influence of human subjectivity on the generated BPA and make the generated BPA more reasonable.

In simulation experiments, the proposed method has higher classification accuracy than other methods in ten-fold cross-validation experiments, incomplete data experiments with missing attribute information and imprecise data experiments with added noise. This shows that the proposed method is valid and reasonable. At the same time, the performance of the proposed method in incomplete data experiments with missing attribute information and imprecise data experiments with added noise shows that the proposed method is more robust in the incomplete information environment.

In future research, we will explore how to combine the proposed BPA generation method with previous work on conflicting evidence fusion and apply the combined method to the field of multi-sensor information fusion.

Acknowledgments. The work is partially supported by National Natural Science Foundation of China (Grant No. 61771006), Programs for Science and Technology Development in Henan Province of China(Grant Nos.222102210002, 222102210004) Key Research Projects of University in Henan Province of China (Grant Nos. 20B510001, 21A413002), Innovation and Quality Improvement Program Project for Graduate Education of Henan University(Grant No. SYL20060143).

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] Zhang R, Xu Z, Gou X (2021) An integrated method for multi-criteria decision-making based on the best-worst method and Dempster-Shafer evidence theory under double hierarchy hesitant fuzzy linguistic environment. Appl Intell 51(2): 713-735

[2] Zhang Y, Hu S, Zhou W (2020) Multiple attribute group decision making using J-divergence and evidential reasoning theory under intuitionistic fuzzy environment. Neural Comput Appl 32(10): 6311-6326

[3] Ma W, Liu W, Luo X, McAreavey K, Jiang Y, Ma J (2019) A DempsterShafer theory and uninormbased framework of reasoning and multiattribute decisionmaking for surveillance system. Int J Intell Syst 34(11): 3077-3104

[4] Zhang H, Deng Y (2020) Weighted belief function of sensor data fusion in engine fault diagnosis. Soft Comput, 24(3):2329-2339

[5] Wang S, Zhao D (2019) A hierarchical power grid fault diagnosis method using multi-source information. IEEE Trans Smart Grid 11(3): 2067-2079

[6] Qiang C, Deng Y (2021) A new correlation coefficient of mass function in evidence theory and its application in fault diagnosis. Appl Intell https://doi.org/10.1007/s10489-021-02797-2

[7] Wang S, Jiang F, Zhang B, Ma R, Hao Q (2019) Development of UAV-based target tracking and recognition systems. IEEE Trans Intell Transp Syst 21(8): 3409-3422

[8] Wei P, Wang B (2020) Multi-sensor detection and control network technology based on parallel computing model in robot target detection and recognition. Comput Commun 159: 215-221

[9] Si A, Das S, Kar S (2021) Picture fuzzy set-based decision-making approach using DempsterShafer theory of evidence and grey relation analysis and its application in COVID-19 medicine selection[J]. Soft Comput, DOI:10.1007/s00500-021-05909-9

[10] Tobore I, Kandwal A, Li J, Yan Y, Omisore O, Enitan E, Li S, Liu Y, Wang L, Nie Z (2020) Towards adequate prediction of prediabetes using spatiotemporal ECG and EEG feature analysis and weight-based multi-model approach. Knowl-Based Syst 209: 106464

[11] Dempster A P (1967) Upper and lower probabilities induced by a multivalued mapping. Ann Math Statist 38(2): 325-339

[12] Shafer G (1976) A mathematical theory of evidence. Princeton, NJ, USA: Princeton Univ. Press

[13] Xiao F, Cao Z, Jolfaei A (2020) A novel conflict measurement in decision-making and its application in fault diagnosis. IEEE Trans Fuzzy Syst 29(1): 186-197

[14] Zhang H, Deng Y (2020) Weighted belief function of sensor data fusion in engine fault diagnosis. Soft Comput 24(3): 2329-2339

[15] Jiang W, Hu W, Xie C (2017) A new engine fault diagnosis method based on multi-sensor data fusion. Appl Sci 7(3): 280

[16] Zhou R, Fang W, Wu J (2020) A risk assessment model of a sewer pipeline in an underground utility tunnel based on a Bayesian network. Tun Undergr Space Technol 103: 103473

[17] Xu X, Zhang D, Bai Y, Chang L, Li J (2020) Evidence reasoning rule-based classifier with uncertainty quantification. Inf Sci 516: 192-204

[18] Khalaj F, Khalaj M (2020) Developed cosine similarity measure on belief function theory: An application in medical diagnosis. Commun Stat Theory Methods https://doi.org/10.1080/03610926.2020.1782935

[19] Du Y, Chen Q, Sun Y, Li C, (2021) Knowledge structure-based consensus-reaching method for large-scale multiattribute group decision-making. Knowl-Based Syst 219: 106885

[20] Liu Z, Pan Q, Dezert J, Martin A (2017) Combination of classifiers with optimal weight based on evidential reasoning. IEEE Trans Fuzzy Syst 26(3): 1217-1230

[21] Kang B, Li Y, Deng Y, Zhang J, Deng X (2012) Determination of basic probability assignment based on interval numbers and its application. Acta Electronica Sinica 40(6): 1092-1096

[22] Qin B, Xiao F (2019) An improved method to determine basic probability assignment with interval number and its application in classification. Int J Distrib Sens Netw 15(1): 1550147718820524

[23] Jiang W, Wei B, Qin X, Zhan J, Tang Y (2016) Sensor data fusion based on a new conflict measure. Math Probl Eng https://doi.org/10.1155/2016/5769061

[24] Zhang J, Deng Y (2017) A method to determine basic probability assignment in the open world and its application in data fusion and classification. Appl Intell 46(4): 934-951

[25] Jiang W, Zhuang M, Xie C (2017) A reliability-based method to sensor data fusion. Sensors 17(7): 1575

[26] Zhang Z, Han D, Dezert J, Yang Y (2017) Determination of basic belief assignment using fuzzy numbers. 2017 20th International Conference on Information Fusion (Fusion) IEEE, PP 1-6

[27] Fei L, Xia J, Feng Y, LIu L (2019) A novel method to determine basic probability assignment in DempsterShafer theory and its application in multi-sensor information fusion. Intl J Distrib Sensor Netw 15(7): 1550147719865876

[28] Xu P, Deng Y, Su X, Mahadevan S (2013) A new method to determine basic probability assignment from training data. Knowl-Based Syst 46(1): 69-80

[29] Deng X, Liu Q, Deng Y, Mahadevan S (2016) An improved method to construct basic probability assignment based on the confusion matrix for classification problem. Inf Sci 340: 250-261

[30] Zhang C, Hu Y, Chan F, Sadiq R, Deng Y (2014) A new method to determine basic probability assignment using core samples. Knowl-Based Syst 69: 140-149

[31] Hu Y, Gao J, Hu L, Dong H (2003) A new method of determining the basic belief assignment in DS evidence theory. Proceedings of the 2003 International Conference on Machine Learning and Cybernetics (IEEE Cat. No. 03EX693). IEEE 5: 3208-3211

[32] Liu Y, Pal N, Marathe A, Lin C (2017) Weighted fuzzy DempsterShafer framework for multimodal information integration. IEEE Trans Fuzzy Syst 26(1): 338-352

[33] Jiang W (2018) A correlation coefficient for belief functions. Int J Approx Reason 103: 94-106

[34] Song Y, Wang X, Zhu J, Lei L (2018) Sensor dynamic reliability evaluation based on evidence theory and intuitionistic fuzzy sets. Appl Intell 48(11): 3950-3962

[35] Liu Z, Liu Y, Dezert J, Cuzzolin F (2019) Evidence combination based on credal belief redistribution for pattern classification. IEEE Trans Fuzzy Syst 28(4): 618-631

[36] Li H, Xiao F (2020) A method for combining conflicting evidences with improved distance function and Tsallis entropy. Int J Intell Syst 35(11): 1814-1830

[37] Li J, Xie B, Jin Y, Hu Z, Zhou L (2020) Weighted conflict evidence combination method based on Hellinger distance and the belief entropy. IEEE Access 8: 225507-225521

[38] Li R, Chen Z, Li H, Tang Y (2021) A new distance-based total uncertainty measure in Dempster-Shafer evidence theory. Appl Intell https://doi.org/10.1007/s10489-021-02378-3

[39] Ren Z, Liao H (2021) Combining conflicting evidence by constructing evidence's angle-distance ordered weighted averaging pairs. Int J Fuzzy Syst 23(2): 494-505

[40] Xiao F (2020) A new divergence measure for belief functions in DS evidence theory for multisensor data fusion. Inf Sci 514: 462-483

[41] Tu L (2015) Pattern recognition and comprehensive evaluation thodbased on interval numbers. Chengdu: Southwest Jiaotong University

[42] Smets P (2005) Decision making in the TBM: the necessity of the pignistic transformation. Int J Approx Reason 38(2): 133-147

[43] Dua D, Graff C (2019) UCI machine learning repository [http://archive.ics.uci.edu/ml]. Irvine, CA: University of California, School of Information and Computer Science