A Framework for Self-Tuning Optimization Algorithm

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Abstract

The performance of any algorithm will largely depend on the setting of its algorithm-dependent parameters. The optimal setting should allow the algorithm to achieve the best performance for solving a range of optimization problems. However, such parameter-tuning itself is a tough optimization problem. In this paper, we present a framework for self-tuning algorithms so that an algorithm to be tuned can be used to tune the algorithm itself. Using the firefly algorithm as an example, we show that this framework works well. It is also found that different parameters may have different sensitivities, and thus require different degrees of tuning. Parameters with high sensitivities require fine-tuning to achieve optimality.

Keywords: Algorithm, firefly algorithm, parameter tuning, optimization, metaheuristic, nature-inspired algorithm.

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1 Introduction

Optimization is paramount in many applications such as engineering and industrial designs. Obviously, the aims of optimization can be anything – to minimize the energy consumption, to maximize the profit, output, performance and efficiency [14, 9, 13, 17]. As most real-world applications are often highly nonlinear, it requires sophisticated optimization tools to tackle. There are many algorithms that use swarm intelligence to solve optimization problems, and algorithms such as particle swarm optimization, cuckoo search and firefly algorithm have received a lot of interests. These nature-inspired algorithms have been proved very efficient.

Metaheuristic algorithms are often nature-inspired, and they are now among the most widely used algorithms for optimization. They have many advantages over conventional algorithms [9, 13, 5]. Metaheuristic algorithms are very diverse, including genetic algorithms, simulated annealing, differential evolution, and and bee algorithms, bat algorithm, particle swarm optimization, harmony search, firefly algorithm, cuckoo search and others [8, 15, 18, 6].

Since all algorithms have algorithm-dependent parameters, the performance of an algorithm largely depends on the values or setting of these parameters. Ideally, there should be a good way to tune these parameters so that the performance of the algorithm can be

optimal in the sense that the algorithm can find the optimal solution of a problem using the minimal number of iterations and with the highest accuracy. However, such tuning of algorithm-dependent parameters is itself a very tough optimization problem. In essence, it is a hyper-optimization problem, that is the optimization of optimization. In fact, how to find the best parameter setting of an algorithm is still an open problem.

There are studies on parameter tuning. For example, Eiben provided a comprehensive summary of existing studies [3]. However, these studies are still very preliminary. There is no method of self-tuning in algorithms. Therefore, the main objective of this paper is to provide a framework for self-tuning algorithms so that an algorithm can be used to tune its own parameters automatically. As far as we are concerned, this is the first of its kind in parameter tuning. The paper is thus organized as follows: Section 2 first analyzes the essence of parameter tuning and Section 3 provides a framework for automatic parameter tuning. Section 4 uses the firefly algorithm to show how the self-tuning framework works. Then, Section 5 presents a case study of a gearbox design problem to further test the tuning procedure. Finally, we draw conclusions briefly in Section 6.

2 Algorithm Analysis and Parameter Tuning

An optimization algorithm is essentially an iterative procedure, starting with some initial guess point/solution with an aim to reach a better solution or ideally the optimal solution to a problem of interest. This process of search for optimality is generic, though the details of the process can vary from algorithm to algorithm. Traditional algorithms such as Newton-Raphson methods use a deterministic trajectory-based method, while modern nature-inspired algorithms often are population-based algorithms that use multiple agents. In essence, these multiple agents form an iterative, dynamic system which should have some attractors or stable states. On the other hand, the same system can be considered as a set of Markov chains so that they will converge towards some stable probability distribution.

2.1 An Optimization Algorithm

Whatever the perspective may be, the aim of such an iterative process is to let the evolve system and converge into some stable optimality. In this case, it has strong similarity to a self-organizing system. Such an iterative, self-organizing system can evolve, according to a set of rules or mathematical equations. As a result, such a complex system can interact and self-organize into certain converged states, showing some emergent characteristics of self-organization. In this sense, the proper design of an efficient optimization algorithm is equivalent to finding efficient ways to mimic the evolution of a self-organizing system [1, 7].

From a mathematical point of view, an algorithm A tends to generate a new and better solution \mathbf{x}^{t+1} to a given problem from the current solution \mathbf{x}^t at iteration or time t. For example, the Newton-Raphson method to find the optimal solution of $f(\mathbf{x})$ is equivalent to finding the critical points or roots of $f'(\mathbf{x}) = 0$ in a d-dimensional space. That is,

$$x^{t+1} = x^t - \frac{f'(x^t)}{f''(x^t)} = A(x^t).$$
 (1)

Obviously, the convergence rate may become very slow near the optimal point where $f'(x) \to 0$. In general, this Newton-Raphson method has a quadratic convergence rate [12]. Sometimes, the true convergence rate may not be as quick as it should be, it may have

non-quadratic convergence property. A way to improve the convergence in this case is to modify the above formula slightly by introducing a parameter p so that

$$\boldsymbol{x}^{t+1} = \boldsymbol{x}^t - p \frac{f'(\boldsymbol{x}^t)}{f''(\boldsymbol{x}^t)}.$$
 (2)

If the optimal solution, i.e., the fixed point of the iterations [11], is x_* , then we can take p as

$$p = \frac{1}{1 - A'(\boldsymbol{x}_*)}. (3)$$

The above iterative equation can be written as

$$\boldsymbol{x}^{t+1} = A(\boldsymbol{x}^t, p). \tag{4}$$

It is worth pointing out that the optimal convergence of Newton-Raphson's method leads to an optimal parameter setting p which depends on the iterative formula and the optimality x_* of the objective f(x) to be optimized.

This above formula is valid for a deterministic method; however, in modern metaheuristic algorithms, randomization is often used in an algorithm, and in many cases, randomization appears in the form of a set of m random variables $\boldsymbol{\varepsilon} = (\varepsilon_1, ..., \varepsilon_m)$ in an algorithm. For example, in simulated annealing, there is one random variable, while in particle swarm optimization [8], there are two random variables. In addition, there are often a set of k parameters in an algorithm. For example, in particle swarm optimization, there are 4 parameters (two learning parameters, one inertia weight, and the population size). In general, we can have a vector of parameters $\boldsymbol{p} = (p_1, ..., p_k)$. Mathematically speaking, we can write an algorithm with k parameters and m random variables as

$$\boldsymbol{x}^{t+1} = \boldsymbol{A}(\boldsymbol{x}^t, \boldsymbol{p}(t), \boldsymbol{\varepsilon}(t)), \tag{5}$$

where A is a nonlinear mapping from a given solution (a d-dimensional vector x^t) to a new solution vector x^{t+1} .

2.2 Type of Optimality

Representation (5) gives rise to two types of optimality: optimality of a problem and optimality of an algorithm. For an optimization problem such as $\min f(x)$, there is a global optimal solution whatever the algorithmic tool we may use to find this optimality. This is the optimality for the optimization problem. On the other hand, for a given problem Φ with an objective function f(x), there are many algorithms that can solve it. Some algorithms may require less computational effort than others. There may be the best algorithm with the least computing cost, though this may not be unique. However, this is not our concern here. Once we have chosen an algorithm A to solve a problem Φ , there is an optimal parameter setting for this algorithm so that it can achieve the best performance. This optimality depends on both the algorithm itself and the problem it solves. In the rest of this paper, we will focus on this type of optimality.

That is, the optimality to be achieved is

Maximize the performance of
$$\xi = A(\Phi, \mathbf{p}, \boldsymbol{\varepsilon}),$$
 (6)

for a given problem Φ and a chosen algorithm $A(., \boldsymbol{p}, \boldsymbol{\varepsilon})$. We will denote this optimality as $\xi_* = A_*(\Phi, \boldsymbol{p}_*) = \xi(\Phi, \boldsymbol{p}_*)$ where \boldsymbol{p}_* is the optimal parameter setting for this algorithm so

that its performance is the best. Here, we have used a fact that ε is a random vector can be drawn from some known probability distributions, thus the randomness vector should not be related to the algorithm optimality.

It is worth pointing out that there is another potential optimality. That is, for a given problem, a chosen algorithm with the best parameter setting p_* , we can still use different random numbers drawn from various probability distributions and even chaotic maps, so that the performance can achieve even better performance. Strictly speaking, if an algorithm $A(.,.,\varepsilon)$ has a random vector ε that is drawn from a uniform distribution $\varepsilon_1 \sim U(0,1)$ or from a Gaussian $\varepsilon_2 \sim N(0,1)$, it becomes two algorithms $A_1 = A(.,.,\varepsilon_1)$ and $A_2 = A(.,.,\varepsilon_2)$. Technically speaking, we should treat them as different algorithms. Since our emphasis here is about parameter tuning so as to find the optimal setting of parameters, we will omit effect of the randomness vector, and thus focus on

Maximize
$$\xi = A(\Phi, \mathbf{p}).$$
 (7)

In essence, tuning algorithm involves in tuning its algorithm-dependent parameters. Therefore, parameter tuning is equivalent to algorithm tuning in the present context.

2.3 Parameter Tuning

In order to tune $A(\Phi, \mathbf{p})$ so as to achieve its best performance, a parameter-tuning tool, i.e., a tuner, is needed. Like tuning a high-precision machinery, sophisticated tools are required. For tuning parameters in an algorithm, what tool can we use? One way is to use a better, existing tool (say, algorithm B) to tune an algorithm A. Now the question may become: how do you know B is better? Is B well-tuned? If yes, how do you tune B in the first place? Naively, if we say, we use another tool (say, algorithm C) to tune B. Now again the question becomes how algorithm C has been tuned? This can go on and on, until the end of a long chain, say, algorithm Q. In the end, we need some tool/algorithm to tune this Q, which again come back to the original question: how to tune an algorithm A so that it can perform best.

It is worth pointing out that even if we have good tools to tune an algorithm, the best parameter setting and thus performance all depend on the performance measures used in the tuning. Ideally, the parameters should be robust enough to minor parameter changes, random seeds, and even problem instance [3]. However, in practice, they may not be achievable. According to Eiben [3], parameter tuning can be divided into iterative and non-iterative tuners, single-stage and multi-stage tuners. The meaning of these terminologies is self-explanatory. In terms of the actual tuning methods, existing methods include sampling methods, screening methods, model-based methods, and metaheuristic methods. Their success and effectiveness can vary, and thus there are no well-established methods for universal parameter tuning.

3 Framework for Self-Tuning Algorithms

3.1 Hyper-optimization

From our earlier observations and discussions, it is clear that parameter tuning is the process of optimizing the optimization algorithm, therefore, it is a hyper-optimization problem. In essence, a tuner is a meta-optimization tool for tuning algorithms.

For a standard unconstrained optimization problem, the aim is to find the global minimum f_* of a function f(x) in a d-dimensional space. That is,

Minimize
$$f(x)$$
, $x = (x_1, x_2, ..., x_d)$. (8)

Once we choose an algorithm A to solve this optimization problem, the algorithm will find a minimum solution f_{\min} which may be close to the true global minimum f_* . For a given tolerance δ , this may requires t_{δ} iterations to achieve $|f_{\min} - f_*| \leq \delta$. Obviously, the actual t_{δ} will largely depend on both the problem objective $f(\boldsymbol{x})$ and the parameters \boldsymbol{p} of the algorithm used.

The main aim of algorithm-tuning is to find the best parameter setting p_* so that the computational cost or the number of iterations t_{δ} is the minimum. Thus, parameter tuning as a hyper-optimization problem can be written as

Minimize
$$t_{\delta} = A(f(\boldsymbol{x}), \boldsymbol{p}),$$
 (9)

whose optimality is p_* .

Ideally, the parameter vector p_* should be sufficiently robust. For different types of problems, any slight variation in p_* should not affect the performance of A much, which means that p_* should lie in a flat range, rather than at a sharp peak in the parameter landscape.

3.2 Multi-Objective View

If we look the algorithm tuning process from a different perspective, it is possible to construct it as a multi-objective optimization problem with two objectives: one objective f(x) for the problem Φ and one objective t_{δ} for the algorithm. That is

Minimize
$$f(\mathbf{x})$$
 and Minimize $t_{\delta} = A(f(\mathbf{x}), \mathbf{p}),$ (10)

where t_{δ} is the (average) number of iterations needed to achieve a given tolerance δ so that the found minimum f_{\min} is close enough to the true global minimum f_* , satisfying $|f_{\min} - f_*| \leq \delta$.

This means that for a given tolerance δ , there will be a set of best parameter settings with a minimum t_{δ} . As a result, the bi-objectives will form a Pareto front. In principle, this bi-objective optimization problem (10) can be solved by any methods that are suitable for multiobjective optimization. But as δ is usually given, a natural way to solve this problem is to use the so-called ϵ -constraint or δ -constraint methods. The naming may be dependent on the notations; however, we will use δ -constraints.

For a given $\delta \geq 0$, we change one of the objectives (i.e., f(x)) into a constraint, and thus the above problem (10) becomes a single-objective optimization problem with a constraint. That is

Minimize
$$t_{\delta} = A(f(\boldsymbol{x}), \boldsymbol{p}),$$
 (11)

subject to

$$f(x) \le \delta. \tag{12}$$

In the rest of this paper, we will set $\delta = 10^{-5}$.

The important thing is that we still need an algorithm to solve this optimization problem. However, the main difference from a common single objective problem is that the present problem contains an algorithm A. Ideally, an algorithm should be independent of the

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Implement an algorithm A(., \boldsymbol{p}, \boldsymbol{\varepsilon}) with \boldsymbol{p} = [p_1, ..., p_K], \boldsymbol{\varepsilon} = [\varepsilon_1, ..., \varepsilon_m];
Define a tolerance (e.g., \delta = 10^{-5});
Algorithm objective t_{\delta}(f(\boldsymbol{x}), \boldsymbol{p}, \boldsymbol{\varepsilon});
Problem objective function f(\boldsymbol{x});
Find the optimality solution f_{\min} within \delta;
Output the number of iterations t_{\delta} needed to find f_{\min};
Solve \min t_{\delta}(f(\boldsymbol{x}), \boldsymbol{p}) using A(., \boldsymbol{p}, \boldsymbol{\varepsilon}) to get the best parameters;
Output the tuned algorithm with the best parameter setting \boldsymbol{p}_*.
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Figure 1: A Framework for a Self-Tuning Algorithm.

problem, which treats the objective to be solved as a black box. Thus we have $A(., \mathbf{p}, \varepsilon)$, however, in reality, an algorithm will be used to solve a particular problem Φ with an objective $f(\mathbf{x})$. Therefore, both notations $A(., \mathbf{p})$ and $A(f(\mathbf{x}), \mathbf{p})$ will be used in this paper.

3.3 Self-Tuning Framework

In principle, we can solve (11) by any efficient or well-tuned algorithm. Now a natural question is: Can we solve this algorithm-tuning problem by the algorithm A itself? There is no reason we cannot. In fact, if we solve (11) by using A, we have a self-tuning algorithm. That is, the algorithm automatically tunes itself for a given problem objective to be optimized. This essentially provides a framework for a self-tuning algorithm as shown in Fig. 1.

This framework is generic in the sense that any algorithm can be tuned this way, and any problem can be solved within this framework. This essentially achieves two goals simultaneously: parameter tuning and optimality finding.

In the rest of this paper, we will use firefly algorithm (FA) as a case study to self-tune FA for a set of function optimization problems.

4 Self-Tuning Firefly Algorithm

4.1 Firefly Algorithm

Firefly Algorithm (FA) was developed by Xin-She Yang in 2008 [14, 15, 16], which was based on the flashing patterns and behaviour of tropical fireflies. In essence, FA uses the following three idealized rules:

- Fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex.
- The attractiveness is proportional to the brightness and they both decrease as their distance increases. Thus for any two flashing fireflies, the less brighter one will move towards the brighter one. If there is no brighter one than a particular firefly, it will move randomly.
- The brightness of a firefly is determined by the landscape of the objective function.

As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, we can now define the variation of attractiveness β with the distance r by

$$\beta = \beta_0 e^{-\gamma r^2},\tag{13}$$

where β_0 is the attractiveness at r=0.

The movement of a firefly i is attracted to another more attractive (brighter) firefly j is determined by

$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \beta_0 e^{-\gamma r_{ij}^2} (\boldsymbol{x}_j^t - \boldsymbol{x}_i^t) + \alpha \, \boldsymbol{\epsilon}_i^t, \tag{14}$$

where the second term is due to the attraction. The third term is randomization with α being the randomization parameter, and ϵ_i^t is a vector of random numbers drawn from a Gaussian distribution at time t. Other studies also use the randomization in terms of ϵ_i^t that can easily be extended to other distributions such as Lévy flights [15, 16].

For simplicity for parameter tuning, we assume that $\beta_0 = 1$, and therefore the two parameters to be tuned are: $\gamma > 0$ and $\alpha > 0$. It is worth pointing out that γ controls the scaling, while α controls the randomness. For this algorithm to convergence properly, randomness should be gradually reduced, and one way to achieve such randomness reduction is to use

$$\alpha = \alpha_0 \theta^t, \quad \theta \in (0, 1), \tag{15}$$

where t is the index of iterations/generations. Here α_0 is the initial randomness factor, and we can set $\alpha_0 = 1$ without losing generality. Therefore, the two parameters to be tuned become γ and θ .

4.2 Tuning the Firefly Algorithm

Now we will use the framework outlined earlier in this paper to tune FA for a set of five test functions. The Ackley function can be written as

$$f_1(\mathbf{x}) = -20 \exp\left[-\frac{1}{5}\sqrt{\frac{1}{d}\sum_{i=1}^d x_i^2}\right] - \exp\left[\frac{1}{d}\sum_{i=1}^d \cos(2\pi x_i)\right] + 20 + e,\tag{16}$$

which has a global minimum $f_* = 0$ at (0, 0, ..., 0).

The simplest of De Jong's functions is the so-called sphere function

$$f_2(\mathbf{x}) = \sum_{i=1}^d x_i^2, \quad -5.12 \le x_i \le 5.12,$$
 (17)

whose global minimum is obviously $f_* = 0$ at (0, 0, ..., 0). This function is unimodal and convex.

Yang's forest function [16]

$$f_3(\mathbf{x}) = \left(\sum_{i=1}^d |x_i|\right) \exp\left[-\sum_{i=1}^d \sin(x_i^2)\right], -2\pi \le x_i \le 2\pi,$$
 (18)

is highly multimodal and has a global minimum $f_* = 0$ at (0, 0, ..., 0).

Rastrigin's function

$$f_4(\mathbf{x}) = 10d + \sum_{i=1}^{d} \left[x_i^2 - 10\cos(2\pi x_i) \right], \quad -5.12 \le x_i \le 5.12,$$
 (19)

Table 1: Results of	parameter	tuning for	the firef	ly algorithm.

Function	Mean $t_{\delta} \pm \sigma_t$	Mean $\gamma \pm \sigma_{\gamma}$	Mean $\theta \pm \sigma_{\theta}$
f_1	589.7 ± 182.1	0.5344 ± 0.2926	0.9561 ± 0.0076
f_2	514.4 ± 178.5	0.5985 ± 0.2554	0.9540 ± 0.0072
f_3	958.1 ± 339.0	1.0229 ± 0.5762	0.9749 ± 0.0047
f_4	724.1 ± 217.6	0.4684 ± 0.3064	0.9652 ± 0.0065
f_5	957.2 ± 563.6	0.8933 ± 0.4251	0.9742 ± 0.0052

whose global minimum is $f_* = 0$ at (0, 0, ..., 0). This function is highly multimodal.

Zakharov's function [13]

$$f_5(\mathbf{x}) = \sum_{i=1}^d x_i^2 + \left(\frac{1}{2}\sum_{i=1}^d ix_i\right)^2 + \left(\frac{1}{2}\sum_{i=1}^d ix_i\right)^4,\tag{20}$$

has a global minimum $f_* = 0$ at (0, 0, ..., 0).

For each objective function, we run the FA to tune itself 50 times so as to calculated meaningful statistics. The population size n = 20 is used for all the runs. The means and standard deviations are summarized in Table 1 where d = 8 is used for all functions.

From this table, we can see that the variations of γ is large, while θ has a narrow range. The best settings for parameters are problem-dependent. These results imply the following:

- The optimal setting of parameters in an algorithm largely depends on the problem, and there is no unique best setting for all problems.
- The relatively large standard deviation of γ means that the actual setting of γ is not important to a given problem, and therefore, there is no need to fine tune γ . That is to say, a typical value of $\gamma = 1$ should work for most problems.
- Some parameters are more sensitive than others. In the present case, θ needs more fine-tuning, due to its smaller standard deviations.

These findings confirm the earlier observations in the literature that $\gamma = O(1)$ can be used for most applications [14, 15], while α needs to reduce gradually in terms of θ . That is probably why other forms of probability distributions such as Lévy flights may lead to better performance then the random numbers drawn from the Gaussian normal distribution [16].

5 Applications

From the results for the test functions, we know that the tuning of γ is not important, while θ needs more fine-tuning. Let us see if this conclusion is true for a real-world application. In the rest of the paper, let us focus on a gearbox design problem.

The optimal design of a speed reducer or a gearbox is a well-known design benchmark with seven design variables [2, 5], including the face width (b), module of the teeth (h), the number of teeth on pinion (z), the length (L_1) of the first shaft between bearing, the length (L_2) of the second shaft between between bearings, the diameter (d_1) of the first shaft, and

the diameter (d_2) of the second shaft. The main objective is to minimize the total weight of the speed reducer, subject to 11 constraints such as bending stress, deflection and various limits on stresses in shafts. This optimization problem can be written as

$$f(b, h, z, L_1, L_2, d_1, d_2) = 0.7854bh^2(3.3333z^2 + 14.9334z - 43.0934)$$
$$-1.508b(d_1^2 + d_2^2) + 7.4777(d_1^3 + d_2^3) + 0.7854(L_1d_1^2 + L_2d_2^2), \tag{21}$$

subject to

$$g_{1} = \frac{27}{bh^{2}z} - 1 \leq 0,$$

$$g_{2} = \frac{397.5}{bh^{2}z^{2}} - 1 \leq 0,$$

$$g_{3} = \frac{1.93L_{1}^{3}}{hzd_{1}^{4}} - 1 \leq 0,$$

$$g_{4} = \frac{1.93L_{2}^{3}}{hzd_{2}^{4}} - 1 \leq 0,$$

$$g_{5} = \frac{1}{110d_{1}^{3}} \sqrt{(\frac{745L_{1}}{hz})^{2} + 16.9 \times 10^{6}} - 1 \leq 0,$$

$$g_{6} = \frac{1}{85d_{2}^{3}} \sqrt{(\frac{745L_{2}}{hz})^{2} + 157.5 \times 10^{6}} - 1 \leq 0,$$

$$g_{7} = \frac{hz}{40} - 1 \leq 0,$$

$$g_{9} = \frac{b}{12h} - 1 \leq 0,$$

$$g_{10} = \frac{1.5d_{1} + 1.9}{L_{1}} - 1 \leq 0,$$

$$g_{11} = \frac{1.1d_{2} + 1.9}{L_{2}} - 1 \leq 0.$$

$$(22)$$

In addition, the simple bounds are $2.6 \le b \le 3.6, 0.7 \le h \le 0.8, 17 \le z \le 28, 7.3 \le L_1 \le 8.3, 7.8 \le L_2 \le 8.3, 2.9 \le d_1 \le 3.9, \text{ and } 5.0 \le d_2 \le 5.5.$ z must be integers.

By using the self-tuning framework via the firefly algorithm with n=20, the following best solutions have been obtained:

$$b = 3.5, h = 0.7, z = 17, L_1 = 7.3, L_2 = 7.8,$$

 $d_1 = 3.34336445, d_2 = 5.285350625, fmin = 2993.7495888,$ (23)

which are better than $f_* = 2996.348165$ obtained by others [2, 5].

The best parameters obtained after tuning are $\gamma = 1.0279 \pm 0.4937$ and $\theta = 0.9812 \pm 0.0071$, which are indeed consistent with the results in Table 1.

6 Discussion

Parameter tuning is the process of tuning an algorithm to find the best parameter settings so that an algorithm can perform the best for a given set of problems. However, such parameter tuning is a very tough optimization problem. In fact, such hyper-optimization is the optimization of an optimization algorithm, which requires special care because the optimality depends on both the algorithm to be tuned and the problem to be solved. Though it is possible to view this parameter-tuning process as a bi-objective optimization problem; however, the objectives involve an algorithm and thus this bi-objective problem is different from the multiobjective problem in the normal sense.

In this paper, we have successfully developed a framework for self-tuning algorithms in the sense that the algorithm to be tuned is used to tune itself. We have used the firefly algorithm and a set of test functions to test the proposed self-tuning algorithm framework. Results have shown that it can indeed work well. We also found that some parameters require fine-tuning, while others do not need to be tuned carefully. This is because different parameters may have different sensitivities, and thus may affect the performance of an algorithm in different ways. Only parameters with high sensitivities need careful tuning.

Though successful, the present framework requires further extensive testing with a variety of test functions and many different algorithms. It may also be possible to see how probability distributions can affect the tuned parameters and even the parameter tuning process. In addition, it can be expected that this present framework is also useful for parameter control, so a more generalized framework for both parameter tuning and control can be used for a wide range of applications. Furthermore, our current framework may be extended to multiobjective problems so that algorithms for multiobjective optimization can be tuned in a similar way.

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