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Scheduling jobs with simultaneous considerations of controllable processing times and learning effect

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Abstract

This paper considers a scheduling problem with general job-dependent learning curves and controllable processing times on a single machine. The objective is to determine the optimal compressions of the processing times and the optimal sequence of jobs so as to minimize some total cost functions, which consist of regular and non-regular functions and the processing time compressions. It shows that the problem can be solved by an assignment problem, and thus can be solved in polynomial time. Some extensions of the problem are also given.

Keywords: Scheduling; Single machine; Controllable processing times; job-dependent learning curves

1 Introduction

In many branches of industry engineering, logistics and supply chains management, there arise scheduling problems. Pinedo [1] introduced scheduling problems and solving algorithms. Shen et al. [2] reviewed multi-objective dynamic job shop scheduling problems. Khalid and Yusof [3] discussed scheduling problem in flexible manufacturing system. Traditional scheduling models and problems assumed that the processing times of jobs were fixed constant. However, in many realistic problems, the processing times of jobs may be subject to change due to learning effect and/or controllable processing times (resource allocation) phenomena. In some practical cases, the jobs' processing times are controllable by allocating resources, such as additional energy, manpower, money, catalysts, or overtime, to the job operations (Shabtay and Steiner [4]). Reviews of research on scheduling models and problems with learning effects could be found in Biskup [5] and Janiak et al. [6]. Wang and Wang [7] discussed single machine multiple common due dates scheduling problem with learning effects. Their objective function was to minimize the total penalty for all jobs. They showed that

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with the consideration of learning effect to jobs' processing times the problem was polynomially solvable. Wang and Wang [8] studied single machine common due-window assignment scheduling problem with learning effect and deteriorating jobs. Their objective function was to minimize costs that associated with the window location, window size, earliness and tardiness. They proved that this problem was polynomially solvable under the proposed model. Wang et al. [9] addressed single machine scheduling and due window assignment problem with learning effect. The objective function was to minimize costs for the window size, window location, earliness, tardiness, and makespan. They showed that the problem was polynomially solvable under the proposed model for the slack (SLK) and unrestricted (DIF) due date assignment methods. Wang and Zhang [10] discussed the permutation flowshop problems. Jobs' processing times considered position-weighted learning effects. The objective function was to minimize the weighted sum of makespan and total completion time. They proposed heuristic algorithms to solve them and analyzed the worst-case error bound. Shabtay and Steiner [4] and Janiak et al. [11] reviewed research on scheduling models and problems with controllable processing times (resource allocation). Yang et al. [12] considered multiple due windows assignment scheduling problems and controllable processing times on a single machine. The objective function was to minimize a total cost function, which consists of the processing time compressions, the due windows related costs, the earliness, and the tardiness. They proved that for the case when the number of jobs assigned to each due window was given in advance, the problem can be solved in polynomial time. Yin et al. [13] investigated single machine due window assignment and scheduling with a common flow allowance and controllable processing times (resource allocation). They considered five versions of the problem that differ in terms of the objective function and processing time function being used and pointed out structural properties of the optimal schedules. They also proposed polynomial time solution algorithms for these problems. Chang et al. [14] studied unrelated parallel machine scheduling problems considering controllable processing times (resource allocation) and rate-modifying activities. They examined the linear resource allocation model and the convex resource allocation model. The objective was to minimize the cost function containing the resource allocation plus the total completion time and the cost function containing the resource allocation plus the total machine load, respectively. They formulated these problems as assignment problems and thus can be solved in a polynomial time algorithm. More recent papers considered scheduling jobs with learning effects and controllable processing times. Wang et al. [15] considered the following models with learning effect and controllable processing times:

$$p_j = t_j r^a - b_j u_j$$

and

$$p_j = \left(\frac{t_j r^a}{u_j}\right)^k$$

, where $a \leq 0$ is the job-dependent learning factor of job J_j, $k \geq 0$, and u_j is the amount of resource that can be allocated to job J_j. For two cost functions (containing makespan, total completion (waiting) time, total absolute differences in completion (waiting) times and total resource cost) minimization, they provided a polynomial time algorithm respectively. Yin and Wang [16] considered the learning effect and controllable processing times model, i.e., the actual processing time of job J_j scheduled in the rth position is $p_j = (t_j - x_j)r^a$, where t_j is the normal processing time of J_j, x_j is the compression of the processing time of job J_j. For two goals, namely minimizing a cost function containing makespan, total completion time, total absolute differences in completion times, and total compression cost and minimizing a cost function containing makespan, total waiting time, total absolute differences in waiting times, and total compression cost, they proved that the problem can be solved in polynomial time. Lu et al. [17] considered the general models with Wang et al. [15], i.e.,

$$p_j = t_j r^{a_j} - b_j u_j$$

and

$$p_j = \left(\frac{t_j r^{a_j}}{u_j}\right)^k,$$

where $a_j \leq 0$ is the job-dependent learning factor of job J_j, $k \geq 0$, and u_j is the amount of resource that can be allocated to job J_j. For two due date assignment methods (i.e., the common (CON) due date, and the slack (SLK) due date), they proved that these problems can be solved in polynomial time respectively. Liu and Feng [18] considered two-machine no-wait flowshop scheduling with learning effect and convex resource-dependent processing times. Li et al. [19] considered the same models with Lu et al. [17], they proved that a single-machine slack due window assignment scheduling problem (i.e. all jobs have slack due window (SLKW)) can be solved in polynomial time. Li et al. [20] considered the following model with learning effect, deterioration effect and controllable processing times: $p_j = \left(\left(\frac{t_j}{u_j}\right)^k + bt\right) r^a$, where $a \leq 0$ is the job-dependent learning factor of job J_j, $k \geq 0$, $t \geq 0$ is the start time of job J_j, $b \geq 0$ is the common deterioration rate and u_j is the amount of resource that can be allocated to job J_j. For two cost functions (containing makespan, total completion (waiting) time, total absolute differences in completion (waiting) times and total resource cost) minimization, they provided a polynomial time algorithm respectively.

In the real life, the phenomena of learning effects and resource allocation may happen simultaneously. For example, during the production of a chemical compound, the workers will be more familiar with operating the machines through experience accumulation, which reflects learning effects. On the other hand, the processing time of a chemical compound may be changed by increasing the amount of catalysts, which involves extra cost. Compressing jobs may be rational and possible when the additional costs is compensated by the gains from job completion at an earlier time(Wang et al. [15], Yin and Wang [16]). To minimize the total cost, the scheduler need to consider learning effect and controllable processing times and determine the optimal job sequence and resource allocation. Considering these factors, this paper extends the results of [16], by considering a more general learning effect and controllable processing times model that includes the one given in [16] as a special case. Notations and assumptions of the problem are given in section 2. Optimal algorithms for several regular and non-regular objective functions are given in section 3. Some extensions are presented in section 4. A test example is given in section 5. Section 6 gives the conclusions.

2 Notations and problem assumptions

The following notations will be used throughout the paper:

n: The number of jobs;

$[r]$: The job scheduled in the rth position;

J_j: The job j, $j = 1, 2, \ldots, n$;

t_j: The normal (basic/non-compressed) processing time of J_j;

t'_j: The compressed processing time of J_j, which considers learning effect and/or resource allocation;

d_j: The due date of job J_j;

v_j: The per time unit cost associated with the compression below t_j of the processing time of job J_j;

$m_j = t_j - t'_j$: The maximum reduction in processing time of job J_j;

x_j: The compression of the processing time of job J_j, which can take any value in $[0, m_j]$;

p_j: the actual processing time of job J_j in position r in a sequence, i.e., $p_j = (t_j - x_j)r^{a_j}$, where $a_j \leq 0$ is a learning factor for job J_j;

C_j : The completion time of job J_j;

W_j : The waiting time of job J_j, where $W_j = C_j - p_j$;

E_j : The earliness time of job J_j, where

$E_j = \max\{0, d_j - C_j\}$;

T_j : The tardiness time of job J_j, where $T_j = \max\{0, C_j - d_j\}$;

$C_{\max}$: The makespan of all jobs, that is,

$C_{\max} = \max\{C_j | j = 1, 2, \ldots, n\}$;

$\sum_{j=1}^{n} C_j$: The total completion times;

$\sum_{j=1}^{n} W_j$: The total waiting times;

$\sum_{i=1}^{n} \sum_{j=i}^{n} |C_j - C_i|$: The total absolute differences in completion times;

$\sum_{i=1}^{n} \sum_{j=i}^{n} |W_j - W_i|$: The total absolute differences in waiting times;

α: The per unit time earliness penalty, $\alpha > 0$;

β: The per unit time tardiness penalty, $\beta > 0$;

γ: The per unit time due date penalty, $\gamma > 0$;

Consider a set of n independent jobs $\mathcal{J} = \{J_1, J_2, \ldots, J_n\}$ to be processed on a single machine. All jobs are ready for processing at time zero and no job preemption (splitting) is allowed. The machine is available at time zero and can handle at most one job at a time. Biskup [21] considered the learning effect model $p_j = t_j r^a$, where $a \leq 0$ is a learning factor for all jobs. Mosheiov and Sidney [22] considered the job-dependent learning effect model $p_j = t_j r^{a_j}$, where $a_j \leq 0$ is a learning factor for job J_j. Yin and Wang [16] considered the learning effect and controllable processing times model $p_j = (t_j - x_j) r^a$. This paper considers a general job-dependent learning effect and controllable processing times model

$$p_j = (t_j - x_j) r^{a_j}, 0 \leq x_j \leq m_j = t_j - t'_j; j, r = 1, 2, \ldots, n. \tag{1}$$

For a given schedule $\pi = (J_1, J_2, \ldots, J_n)$, let $C_j = C_j(\pi)$ denote the completion time of job J_j. The objective is to determine the optimal compressions of the processing times $x = (x_1, x_2, \ldots, x_n)$ and the optimal sequence of jobs π so as to minimize the following cost functions:

$$Z(\pi, x) = \delta\rho + (1 - \delta) \sum_{j=1}^{n} v_j x_j, \tag{2}$$

where $\rho \in \{C_{\max}, \sum C_j, \sum W_j, \sum_{i=1}^{n} \sum_{j=i}^{n} |C_j - C_i|, \sum_{i=1}^{n} \sum_{j=i}^{n} |W_j - W_i|, \sum_{j=1}^{n} (\alpha E_j + \beta T_j + \gamma d_j)\}$ and $0 \leq \delta \leq 1$. In the remaining part of the paper, all the problems considered will be denoted by using the three-field notation scheme (Graham et al. [23]).

3 A unified analysis for single machine scheduling problems

Panwalkar et al. [24] introduced the common (CON) due date assignment method, for which all jobs are assigned the same due date, i.e., $d_j = d$ for $j = 1, 2, \ldots, n$.

Adamopoulos and Pappis [25] considered the slack (SLK) due date assignment method, for which all jobs are given an equal flow allowance according to the following equation, $d_j = p_j + q$ for $j = 1, 2, \ldots, n$, where $q \geq 0$ is a decision variable.

Seidmann et al. [26] studied the unrestricted (DIF) due date assignment method, for which each job can be assigned a different due date with no restrictions.

Lemma 1 *(Panwalker et al. [24], Adamopoulos and Pappis [25] and Seidmann et al. [26]) For the problem $1|p_j = (t_j - x_j)r^{a_j}|\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d_j)$, there holds the following properties:*

1) There exists an optimal schedule π^* without any machine idle time between the starting time of the first job and the completion time of the last job. Furthermore, the first job in the schedule starts at time zero.

2) For the CON model, there exists an optimal schedule with the property that d equal to $C_{[k^*]}$, where

$$k^* = \max\left\{\left\lceil\frac{n(\beta-\gamma)}{\alpha+\beta}\right\rceil, 0\right\} \tag{3;}$$

For the SLK model, there exists an optimal schedule with the property that q coincide with the completion times of the $(k^*-1)th$, where k^* is given by Eq. (3);

For the DIF model, the optimal due date assignment strategy is defined as follows: if $\gamma \geq \beta$ then set $d^*_{[j]} = 0$; otherwise, set $d^*_{[j]} = C_{[j]}$.

3) The optimal total costs can be written as: $\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d_j) = \sum_{j=1}^{n}\omega_j p_{[j]}$, where the positional weight of position j in the schedule is given by $\omega_j = \min\{n\gamma + (j-1)\alpha, (n+1-j)\beta\}$ for the CON method; by $\omega_j = \min\{n\gamma + j\alpha, \beta(n-j)\}$ for the SLK method; by $\omega_j = \min\{\beta,\gamma\}(n+1-j)$ for the DIF method.

Similarly, for $\rho \in \{C_{\max}, \sum C_j, \sum W_j, \sum_{i=1}^{n}\sum_{j=i}^{n}|C_j - C_i|, \sum_{i=1}^{n}\sum_{j=i}^{n}|W_j - W_i|\}$, we have

$$\rho = \sum_{j=1}^{n}\omega_j p_{[j]}, \tag{4}$$

where $\omega_j = 1$ for $C_{\max}$, $\omega_j = (n-j+1)$ for $\sum_{j=1}^{n} C_j$, $\omega_j = (n-j)$ for $\sum_{j=1}^{n} W_j$, $\omega_j = (j-1)(n-j+1)$ for $\sum_{i=1}^{n}\sum_{j=i}^{n}|C_j - C_i|$ (Kanet [27]), $\omega_j = j(n-j)$ for $\sum_{i=1}^{n}\sum_{j=i}^{n}|W_j - W_i|$ (Bagchi [28]),

Substituting Eqs (1) and (4) into (2), we have

$$\begin{aligned} Z(\pi, x) &= \delta\sum_{j=1}^{n}\omega_j p_{[j]} + (1-\delta)\sum_{j=1}^{n} v_{[j]}x_{[j]} \\ &= \sum_{j=1}^{n}\delta\omega_j t_{[j]} j^{a_{[j]}} + \sum_{j=1}^{n}[(1-\delta)v_{[j]} - \delta\omega_j j^{a_{[j]}}]x_{[j]}. \end{aligned} \tag{5}$$

From (5), for any sequence, the optimal resource allocation of a job in a position with a negative $(1-\delta)v_{[j]} - \delta\omega_j j^{a_{[j]}}$ should be its upper bound on the amount of resource $m_{[j]}$, and the optimal resource allocation of a job in a position with a positive $(1-\delta)v_{[j]} - \delta\omega_j j^{a_{[j]}}$ should be 0. If

$(1-\delta)v_{[j]} - \delta\omega_j j^{a_{[j]}} = 0$, then the optimal resource allocation of the job in this position may be any value between 0 and $m_{[j]}$. These can be written in the notational form as follows:

$$x^*_{[j]} = \begin{cases} m_{[j]}, & \text{if} \quad (1-\delta)v_{[j]} - \delta\omega_j j^{a_{[j]}} < 0, \\ x_{[j]} \in [0, \bar{u}_{[j]}], & \text{if} \quad (1-\delta)v_{[j]} - \delta\omega_j j^{a_{[j]}} = 0, \\ 0, & \text{if} \quad (1-\delta)v_{[j]} - \delta\omega_j j^{a_{[j]}} > 0, \end{cases} \tag{6}$$

From (6), we can obtain the optimal resource allocation for any given optimal sequence. In order to determine the optimal sequence for the problem $1|p_j = (t_j - x_j)r^{a_j}|\delta\rho + (1-\delta)\sum_{j=1}^n v_j x_j$, where $\rho \in \{C_{\max}, \sum C_j, \sum W_j, \sum_{i=1}^n \sum_{j=i}^n |C_j - C_i|, \sum_{i=1}^n \sum_{j=i}^n |W_j - W_i|, \sum_{j=1}^n (\alpha E_j + \beta T_j + \gamma d_j)\}$, we formulate the problem $1|p_j = (t_j - x_j)r^{a_j}|\delta\rho + (1-\delta)\sum_{j=1}^n v_j x_j$, $\rho \in \{C_{\max}, \sum C_j, \sum W_j, \sum_{i=1}^n \sum_{j=i}^n |C_j - C_i|, \sum_{i=1}^n \sum_{j=i}^n |W_j - W_i|, \sum_{j=1}^n (\alpha E_j + \beta T_j + \gamma d_j)\}$ as an assignment problem.

Let

$$\lambda_{jr} = \begin{cases} \delta\omega_r t_j r^{a_j}, & \text{if} \quad (1-\delta)v_j - \delta\omega_r r^{a_j} \geq 0, \\ \delta\omega_r t_j r^{a_j} + [(1-\delta)v_j - \delta\omega_r r^{a_j}]m_j, & \text{if} \quad (1-\delta)v_j - \delta\omega_r r^{a_j} < 0. \end{cases} \tag{7}$$

Furthermore, let x_{jr} be a 0/1 variable such that $x_{jr} = 1$ if job J_j is scheduled in position r, and $x_{jr} = 0$, otherwise. As in Biskup [21], the problem $1|p_j = (t_j - x_j)r^{a_j}|\delta\rho + (1-\delta)\sum_{j=1}^n v_j x_j$ can be formulated as the following assignment problem:

$$\min \sum_{j=1}^n \sum_{r=1}^n \lambda_{jr} x_{jr} \tag{8}$$

subject to

$$\sum_{j=1}^n x_{jr} = 1, \quad r = 1, 2, \ldots, n,$$

$$\sum_{r=1}^n x_{jr} = 1, \quad j = 1, 2, \ldots, n,$$

$$x_{jr} = 0 \text{ or } 1, \quad j, r = 1, 2, \ldots, n.$$

For any given r (r=1,2,...,n), $\sum_{j=1}^n x_{jr} = 1$ means only one job can be processed in the rth position. For any given j (j=1,2,...,n), $\sum_{r=1}^n x_{jr} = 1$ means job J_j can be processed in only one position.

Recall that solving an assignment problem of size n requires an effort of $O(n^3)$ time (Papadimitriou and Steiglitz [29]).

From Lemma 1 and the above analysis, for the problem $1|p_j = (t_j - x_j)r^{a_j}|\delta\rho + (1-\delta)\sum_{j=1}^n v_j x_j$, where $\rho \in \{\sum_{j=1}^n (\alpha E_j + \beta T_j + \gamma d)$ (the CON method), $\sum_{j=1}^n (\alpha E_j + \beta T_j + \gamma q)$ (the SLK method), $\sum_{j=1}^n (\alpha E_j + \beta T_j + \gamma d_j)$ (the DIF method)$\}$, we can propose the following optimization algorithm:

Algorithm 1.

Step 1. Calculate $k^* = \max\left\{\left\lceil \frac{n(\beta-\gamma)}{\alpha+\beta} \right\rceil, 0\right\}$.

Step 2. For the CON method, calculate the λ_{jr} by using $\omega_r = \min\{n\gamma + (r-1)\alpha, (n+1-r)\beta\}$;

For the SLK method, calculate the λ_{jr} by using $\omega_r = \min\{n\gamma + r\alpha, \beta(n-r)\}$;

For the DIF method, calculate the λ_{jr} by using $\omega_r = \min\{\beta, \gamma\}(n+1-r)$.

Step 3. Solve the assignment problem (8) to determine the optimal job sequence.

Step 4. Compute the optimal compression $x^*_{[j]}$ by (6).

Step 5. Compute the optimal processing times $p^*_{[j]}$ by (1).

Step 6. For the CON method, set the optimal due date $d^* = C_{[k^*]}$. For the SLK method, set the optimal slack $q^* = C_{[k^*-1]}$. For the DIF method, if $\gamma \geq \beta$ then set $d^*_{[j]} = 0$; otherwise, set $d^*_{[j]} = C_{[j]}$.

Theorem 1 *The problem* $1|p_j = (t_j - x_j)r^{a_j}|\delta\rho + (1-\delta)\sum_{j=1}^n v_j x_j$, *where* $\rho \in \{C_{\max}, \sum C_j, \sum W_j, \sum_{i=1}^n \sum_{j=i}^n |C_j - C_i|, \sum_{i=1}^n \sum_{j=i}^n |W_j - W_i|, \sum_{j=1}^n (\alpha E_j + \beta T_j + \gamma d_j)\}$ *can be solved in* $O(n^3)$ *time by Algorithm 1.*

Proof The correctness of the Algorithm 1 follows from above analysis. Steps 1, 2, 4, 5 and 6 require $O(n)$ time and Step 3 requires $O(n^3)$ time. Thus the total computational complexity of Algorithm 1 is $O(n^3)$. □

4 Extensions

4.1 Extensions 1

Similar to the proof of Section 3, the proposed model is extended by a large set of scheduling problems where the objective function can be expressed by using positional penalties, i.e.,

$$\rho = \sum_{j=1}^n \omega_j p_{[j]}, \tag{9}$$

where ω_j is a position, job-dependent penalty for any job schedule in the jth position. For examples:

The multiple common due date assignment scheduling

Let $D_1 \leq D_2 \leq \ldots \leq D_m$ denote the m due dates and let I_i denote the set of jobs assigned to due date D_i for $i = 1, 2, \ldots, m$. The multiple common due date assignment problem (Chand and Chhajed [30], Dickman et al. [31], Wang and Wang [7]) is to determine the optimal $D = \{D_1, D_2, \ldots, D_m\}$, $I = \{I_1, I_2, \ldots, I_m\}$ and a schedule π to minimize

$$Z(D, I, \pi) = \sum_{i=1}^m \sum_{j \in I_i} (\alpha E_j + \beta T_j + \gamma D_i) \tag{10}$$

and $d_j = D_i$ for $j \in I_i$.

Three due window assignment problems

Let $[d_j^1, d_j^2]$ be the due-window of job J_j such that $d_j^1 \leq d_j^2$, where d_j^1 and $S_j = d_j^2 - d_j^1$ is the starting time and the due window size of job J_j, respectively. The due window assignment problem is to determine the optimal starting time of due dates $d^1 = (d_1^1, d_2^1, \ldots, d_n^1)$, the due window sizes $S = (S_1, S_2, \ldots, S_n)$, the optimal compressions of the processing times $x = (x_1, x_2, \ldots, x_n)$ and a schedule π to minimize

$$Z(d^1, S, x, \pi) = \sum_{j=1}^{n} (\alpha E_j + \beta T_j + \gamma d_j^1 + \delta S_j) \tag{11}$$

where $E_j = \max\{0, d_j^1 - C_j\}$ and $T_j = \max\{0, C_j - d_j^2\}$ are the earliness and the tardiness of job J_j, $j = 1, 2, \ldots, n$. Three more commonly used methods are as follows:

(a) The common due window assignment method (CONW), i.e., $d_j^1 = d$, $D = d_j^2 - d_j^1$ (Liman et al. [32], and Wang and Wang [8]).

(b) The slack due window assignment method (SLKW), i.e., $d_j^1 = p_j + q_1$, $d_j^2 = p_j + q_2$ (Mosheiov and Oron [33], Mor and Mosheiov [34], and Wang et al. [9]).

(c) The unrestricted due window assignment method (DIFW), in which each job can be assigned a different due window with no restrictions (Seidmann et al. [26] and Wang et al. [9]).

The multiple common due window assignment scheduling

Let d_i and w_i $(i = 1, 2, \ldots, m)$ denote the due window starting time and the due window finishing time of the ith due window and I_i denote the set of jobs assigned to the ith due window, S_i be the size of the ith due window, $S_i = w_i - d_i$. The multiple common due window assignment problem (Yang et al. [12]) is to determine the optimal $d = (d_1, d_2, \ldots, d_m\}$, $S = (S_1, S_2, \ldots, S_m)$, $I = \{I_1, I_2, \ldots, I_m\}$ and a schedule π to minimize

$$Z(d, S, I, \pi) = \sum_{i=1}^{m} \sum_{j \in I_i} (\alpha E_j + \beta T_j + \gamma d_i + \delta S_i). \tag{12}$$

These extensive problems can be formulated as assignment problems and thus can be solved in a polynomial time.

4.2 Extensions 2

In the real chemical compound production process, learning effect may reduce the jobs' processing time. However, the processing time can not be reduced without limitation. We need a bound to limit the processing time. Wang et al. [35] considered the following truncated job-dependent learning effect model, i.e., $p_j = t_j \max\{r^{a_j}, B\}$, where $0 < B < 1$ is a truncation parameter for all jobs. Similar to the proof of Section 3, the proposed model can be extended by the following model:

$$p_j = (t_j - x_j)\max\{r^{a_j}, B\}, \tag{13}$$

and all the results can be obtained for the model $p_j = (t_j - x_j)\max\{r^{a_j}, B\}$.

5 An example

For the common (CON) due date assignment problem $1|p_j = (t_j - x_j)r^{a_j}|Z(\pi, x) = \delta\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d) + (1-\delta)\sum_{j=1}^{n} v_j x_j$, the computational process is illustrated by the following example.

Example 1. Let $\alpha = 8, \beta = 10, \gamma = 5, \delta = 0.6$. Consider a problem containing n =6 jobs. The parameters for each job as given in Table 1. Now we can solve the CON due date assignment problem $1|p_j = (t_j - x_j)r^{a_j}|\delta\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d)\} + (1-\delta)\sum_{j=1}^{n} v_j x_j$ as follows:

Step 1. Calculate $k^* = \max\left\{\left\lceil\frac{n(\beta-\gamma)}{\alpha+\beta}\right\rceil, 0\right\} = \max\left\{\left\lceil\frac{6(10-5)}{8+10}\right\rceil, 0\right\} = 2$.

Step 2. For the CON method, $\omega_1 = 30, \omega_2 = 38, \omega_3 = 40, \omega_4 = 30, \omega_5 = 20, \omega_6 = 0$, the λ_{jr} values can be calculated by using (7) and are given in Table 2.

Step 3. The costs of solution for the assignment problem (8) are highlighted in bold in Table 2 and the optimal schedule is $\pi^* = (J_5, J_2, J_4, J_1, J_3, J_6)$.

Step 4. The optimal compression $x^*_{[j]}$ obtained by (6) are $x^*_5 = 15, x^*_2 = 9, x^*_4 = 14, x^*_1 = 0, x^*_3 = 0, x^*_6 = 0$.

Step 5. The optimal processing times $p^*_{[j]}$ obtained by (1) are

$p^*_5 = (24 - 15) * 1^{-0.1} = 9, p^*_2 = (16 - 9) * 2^{-0.15} = 6.3088,$

$p^*_4 = (20 - 14) * 3^{-0.3} = 4.3153, p^*_1 = 14 * 4^{-0.25} = 9.8995,$

$p^*_3 = 18 * 5^{-0.2} = 13.0460, p^*_6 = 25 * 6^{-0.35} = 13.3533.$

Step 6. For the CON method, $d^* = C_{[k^*]} = C_{[2]}$. The optimal solution is $Z(\pi^*, x^*) = 285.0000 + 254.8663 + 212.3068 + 148.4924 + 130.4603 + 66.76626 = 1097.89206$.

Table 1. The data of Example 1

J_j	1	2	3	4	5	6
t_j	14	16	18	20	24	25
t'_j	8	7	10	6	9	10
m_j	6	9	8	14	15	15
a_j	-0.25	-0.15	-0.2	-0.3	-0.1	-0.35
v_j	28	30	25	18	20	28

Table 2. The λ_{jr} of Example 1

$j \setminus r$	1	2	3	4	5	6
1	204.0000	211.8163	205.5737	**148.4924**	93.62364	44.7260
2	240.0000	**254.8663**	253.7298	194.9406	125.6824	61.14589
3	250.0000	265.4046	260.5483	204.6217	**130.4603**	62.89444
4	216.0000	218.5968	**212.3068**	185.3779	123.4068	58.41907
5	**285.0000**	309.5486	311.2725	267.5243	204.3216	100.3151
6	360.0000	359.0710	340.3906	230.8396	142.3313	**66.76626**

6 Conclusions

This paper studied single machine scheduling problems considering a more general learning effect and controllable processing times model, that is, $1|p_j = (t_j - x_j)r^{a_j}|\delta\rho + (1-\delta)\sum_{j=1}^{n} v_j x_j$. $\rho \in \{C_{\max}, \sum C_j, \sum W_j, \sum_{i=1}^{n}\sum_{j=i}^{n}|C_j - C_i|, \sum_{i=1}^{n}\sum_{j=i}^{n}|W_j - W_i|, \sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d_j)\}$. It also gives some extensions of these problems. These problems can be formulated as assignment problem and thus can be solved in a polynomial time. The problems and the algorithms' complexity are listed in Table 3. In the future, we may consider scheduling problem involving controllable processing times and deteriorating effect. We may extend the corresponding results to the case $a_j > 0$ (i.e., aging effect, see Janiak et al. [6]) and discuss whether the proposed algorithms can be applied to this case. We plan to extend the problems to multiple machine scheduling problems.

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Table 3. Summary of main results

Problem	Complexity	Ref.
$1 \mid p_j=(t_j-x_j)r^a \mid \delta_1 C_{\max}+\delta_2\sum C_j+\delta_3\sum_{i=1}^n\sum_{j=i}^n \lvert C_j-C_i\rvert+\delta_4\sum_{j=1}^n v_jx_j$	$O(n^3)$	Yin and Wang [24]
$1 \mid p_j=(t_j-x_j)r^a \mid \delta_1 C_{\max}+\delta_2\sum W_j+\delta_3\sum_{i=1}^n\sum_{j=i}^n \lvert W_j-W_i\rvert+\delta_4\sum_{j=1}^n v_jx_j$	$O(n^3)$	Yin and Wang [24]
$1 \mid p_j=(t_j-x_j)r^{a_j} \mid \delta C_{\max}+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Theorem 1
$1 \mid p_j=(t_j-x_j)r^{a_j} \mid \delta\sum C_j+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Theorem 1
$1 \mid p_j=(t_j-x_j)r^{a_j} \mid \delta\sum W_j+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Theorem 1
$1 \mid p_j=(t_j-x_j)r^{a_j} \mid \delta\sum_{i=1}^n\sum_{j=i}^n \lvert C_j-C_i\rvert+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Theorem 1
$1 \mid p_j=(t_j-x_j)r^{a_j} \mid \delta\sum_{i=1}^n\sum_{j=i}^n \lvert W_j-W_i\rvert+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Theorem 1
$1 \mid p_j=(t_j-x_j)r^{a_j} \mid \delta\sum_{j=1}^n(\alpha E_j+\beta T_j+\gamma d_j)+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Theorem 1
$1 \mid p_j=(t_j-x_j)r^{a_j} \mid \delta\sum_{i=1}^m\sum_{j\in I_i}(\alpha E_j+\beta T_j+\gamma D_i)+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 1
$1 \mid p_j=(t_j-x_j)r^{a_j} \mid \delta\sum_{j=1}^n(\alpha E_j+\beta T_j+\gamma d_j^1+\delta S_j)+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 1
$1 \mid p_j=(t_j-x_j)r^{a_j} \mid \delta\sum_{i=1}^m\sum_{j\in I_i}(\alpha E_j+\beta T_j+\gamma d_i+\delta S_i)+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 1
$1 \mid p_j=(t_j-x_j)\max\{r^{a_j},B\} \mid \delta C_{\max}+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 2
$1 \mid p_j=(t_j-x_j)\max\{r^{a_j},B\} \mid \delta\sum C_j+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 2
$1 \mid p_j=(t_j-x_j)\max\{r^{a_j},B\} \mid \delta\sum W_j+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 2
$1 \mid p_j=(t_j-x_j)\max\{r^{a_j},B\} \mid \delta\sum_{i=1}^n\sum_{j=i}^n \lvert C_j-C_i\rvert+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 2
$1 \mid p_j=(t_j-x_j)\max\{r^{a_j},B\} \mid \delta\sum_{i=1}^n\sum_{j=i}^n \lvert W_j-W_i\rvert+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 2
$1 \mid p_j=(t_j-x_j)\max\{r^{a_j},B\} \mid \delta\sum_{j=1}^n(\alpha E_j+\beta T_j+\gamma d_j)+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 2
$1 \mid p_j=(t_j-x_j)\max\{r^{a_j},B\} \mid \delta\sum_{i=1}^m\sum_{j\in I_i}(\alpha E_j+\beta T_j+\gamma D_i)+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 2
$1 \mid p_j=(t_j-x_j)\max\{r^{a_j},B\} \mid \delta\sum_{j=1}^n(\alpha E_j+\beta T_j+\gamma d_j^1+\delta S_j)+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 2
$1 \mid p_j=(t_j-x_j)\max\{r^{a_j},B\} \mid \delta\sum_{i=1}^m\sum_{j\in I_i}(\alpha E_j+\beta T_j+\gamma d_i+\delta S_i)+(1-\delta)\sum_{j=1}^n v_jx_j$	$O(n^3)$	Extensions 2

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