



Correction to: Multi-valued neural networks I: a multi-valued associative memory

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1 Introduction

Following the publication of our article [1], we have found out that expression (7) for $TS_{ij}^G((\mathbf{W}^0, \mathbf{d}); Y)$ and the proof of Theorem 2 should be as follows:

$$TS_{ij}^G((\mathbf{W}^0, \mathbf{d}); Y) = \{k \in P : | x_i^k \wedge y_j^k \leq w_{ij}^0\}, \quad (7)$$

Theorem 2 The set $M^{wcd} \neq \emptyset$ and $(\mathbf{W}^0, \mathbf{c}, \mathbf{d}) \in M^{wcd}$, if $\forall j \in M, i \in N, c_j \leq d_j^0 = \bigwedge_{k \in P} y_j^k, \bigcup_{i \in N} TS_{ij}^G((\mathbf{W}^0, \mathbf{d}); Y) = P$ and $\forall k \in P \bigvee_{i: k \in TS_{ij}^G} x_i^k \geq y_j^k$.

Proof Let us $\forall j \in M, k \in P, \exists n \in N : k \in TS_{nj}^G((\mathbf{W}^0, \mathbf{d}); Y)$. Then,

$$w_{nj}^0 \geq x_n^k \wedge y_j^k, \quad (9)$$

and

$$\bigvee_{n: k \in TS_{nj}^G} x_n^k \wedge y_j^k = y_j^k. \quad (10)$$

Let us consider:

$$\begin{aligned} & \bigvee_{i \in N} ((x_i^k \vee c_j \vee d_j) \wedge (w_{ij}^0 \vee d_j)) \geq \\ & \geq \bigvee_{i \in N} (x_i^k \wedge w_{ij}^0) \wedge y_j^k \geq \\ & \geq \bigvee_{n: k \in TS_{nj}^G} (x_n^k \wedge w_{nj}^0 \wedge y_j^k) = y_j^k, \end{aligned}$$

since 9 and 10.

Hence,

$$\begin{aligned} & \bigvee_{i \in N} \{(x_i^k \vee c_j \vee d_j) \wedge (w_{ij}^0 \vee d_j)\} \\ & \in \{x_i^l \in L \mid l \in GE_{ij}(X, Y)\}. \end{aligned} \quad (11)$$

On the other hand, we get by the definitions of implication, \mathbf{d}^0 , and \mathbf{W}^0 and Theorem 1 that $\forall i \in N, j \in M, k \in P, (x_i^k \vee c_j \vee d_j) \wedge (w_{ij}^0 \vee d_j) \in \{x_i^l \in L \mid l \in LE_{ij}(X, Y)\}$, since $w_{ij}^0 \geq d_j^0 \geq d_j, x_i^k \wedge w_{ij}^0 \leq y_j^k$ (since $x_i^k \wedge \bigwedge_l [x_i^l \Rightarrow y_j^l] \leq x_i^k \wedge [x_i^k \Rightarrow y_j^k] \leq y_j^k$), and $(c_j \vee d_j) \leq y_j^k$. Hence,

$$\begin{aligned} & \bigvee_{i \in N} \{(x_i^k \vee c_j \vee d_j) \wedge (w_{ij}^0 \vee d_j)\} \\ & \in \{x_i^l \in L \mid l \in LE_{ij}(X, Y)\}. \end{aligned} \quad (12)$$

Combining (11) and (12), we get $(\mathbf{W}^0, \mathbf{c}, \mathbf{d}) \in M^{wcd}$, i.e., $M^{wcd} \neq \emptyset$. However, if there is a set of lattice elements that satisfy the condition of the theorem: $\{(y')_j^k \mid y_j^k < (y')_j^k \leq w_{ij}^0\}$ with the same matrix w_{ij}^0 , the matrix is the solution only for

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y , since $(x_i^k \vee c_j \vee d_j) \wedge (w_{ij}^0 \vee d_j) \leq y_j^k$ and this expression does not change for such y'^1 . Hence, the proof is valid only for y_j^k . \square

Declarations

Conflict of interest The author declare that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Reference

1. Maximov D, Goncharenko VI, Legovich YS (2021) Multi-valued neural networks i: a multi-valued associative memory. *Neur. Comp. and App.* 33(16):10189–10198. <https://doi.org/10.1007/s00521-021-05781-6>

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¹ $x_i^k \wedge \bigwedge_l [x_i^l \Rightarrow y_j^l] \leq x_i^k \wedge [x_i^k \Rightarrow y_j^k] = x_i^k \wedge y_j^k$.