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PREFACE

Series B

Special Issue: Continuous Optimization and Stability Analysis

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Continuous optimization is a vital research area with many active branches such as linear and nonlinear optimization, stochastic optimization, vector optimization, semi-infinite optimization, etc. Stability analysis of optimization problems is of crucial interest in applications, and makes optimization an ideal testing bench for many disciplines such as convex analysis, set-valued analysis, variational analysis, and well-posedness**,** among others.

The present special issue offers some recent developments in this wide area and aims to honor Prof. Marco Antonio López Cerdá on the occasion of his 70th birthday, as well as to show recognition by his colleagues to his contribution to this field. His achievements include some monographs, several surveys and more than 150 scientific articles on optimality, duality, stability and algorithms in semi-infinite programming, Lipschitz-type properties, error bounds, subdifferential calculus, and others such as game theory, robustness, stationarity and regularity.

Marco A. López was born in Alcoy (Alicante, Spain) in 1949 and has spent most of his academic career (since 1985) as a full professor of statistics and operations research at the University of Alicante, where nowadays he is Emeritus Professor. Marco is a corresponding fellow of the Spanish Royal Academy of Sciences, an Honorary Research Fellow of the Federation University (Australia), and a Doctor Honoris Causa at the University of Limoges (France). He has also led important research projects, theoretical and applied. From 2008 to 2012 he was the coodinator of i-MATH Consolider

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2006-2012, which gathered more than 300 Spanish research groups in different areas of mathematics, with a budget of 7.5 million euros. His book (with M.A. Goberna) Linear Semi-Infinite Optimization (Wiley, 1998) is a benchmark in this subject. But aside from his honors and awards, Marco has been and is a teacher, mentor and friend for several generations of colleagues.

The guest editors would like to thank the Editor-in-Chief of Math. Program. Series B, Sven Leyffer, for his help and support throughout the process. We would also like to express our gratitude to all referees, whose professionality has essentially contributed to the success of this project. All papers have been subject to the strict standard reviewing procedure of Mathematical Programming. And, of course, we sincerely thank all authors who decided to submit their work to the special issue.

This volume contains 20 excellent articles, that are briefly described below.

- Adly et al. [1] establish new metric properties of prox-regular sets, which are based on the Legendre–Fenchel transform and complements of balls. In particular, the authors prove that the distance of a point to a prox-regular set is the maximum of the distances of the point from boundaries of all such complements separating the set and the point. Overall, the obtained results show that the class of prox-regular sets exhibit good properties known for convex sets in convex analysis.
- Adly and Rockafellar [2] deal with the study of sensitivity to perturbation of parameterized variational inclusions involving maximally monotone operators in a Hilbert space. Using the concept of proto-differentiability of a multifunction and the notion of semi-differentiability of a single-valued map, the authors establish the differentiability of the solution of a parameterized monotone inclusion.
- Bauschke et al. [3] introduce the notion of conically nonexpansive operators which generalize nonexpansive mappings. Averaged operators are characterized as resolvents of comonotone operators under appropriate scaling. As a consequence, the proximal point mappings associated with hypoconvex functions are characterized as cocoercive operators. Several examples illustrate the analysis and demonstrate the tightness of the results.
- Beer et al. [4] analyze the Lipschitz behavior of the feasible set mapping associated with linear and convex inequality systems in the n-dimensional Euclidean space. Linear system are identified with their set of coefficients and the Hausdorff distance is used to measure data perturbations. Convex systems are then analyzed through linearization techniques.
- Bivas et al. [5] investigate the possibility to represent a multifunction in the form of the so-called Fillipov regularization generated by a single-valued measurable function. The authors obtain a characterization of this property, which also yields an elegant description of those mappings that are Clarke generalized gradients of Lipschitz functions.
- Borges et al. [6] develop a new approach to regularize and approximate solution mappings of parametric convex optimization problems that combines interior penalty (log-barrier) solutions with Tikhonov regularization. In particular, the authors show that their approach, being computationally implementable, provides locally bounded upper bounds for the subdifferential of the value function of qualified convex problems.

- Boţ et al. [7] investigate the asymptotic properties of the trajectories generated by a second-order dynamical system with Hessian driven damping and a Tikhonov regularization term in connection with the minimization of a smooth convex function in Hilbert spaces. The Tikhonov regularization term enables the derivation of strong convergence results of the trajectory to the minimizer of the objective function of minimum norm.
- Burachik and Kaya [8] present an algorithm which constructs, via an ODE involving Steklov function, a convexification of a continuous function f. For a family of quartic polynomials, an estimate for the size of a ball that contains all respective global minimizers is provided. Finally, the method is illustrated by means of numerous computational examples.
- Correa et al. [9] establish general formulas for the subdifferential of the pointwise supremum of convex functions covering both the compact continuous and the noncompact non-continuous setting. Moreover the authors provide two applications: to nonconvex Fenchel duality, and to Fritz-John and KKT conditions in convex semi-infinite programming.
- Dentcheva and Ruszczyński [10] consider nonlinear multistage stochastic optimization problems in the spaces of integrable functions. Causal operators describing the dynamics of the system are investigated. The authors introduce the concept of subregular recourse in nonlinear multistage stochastic optimization and establish subregularity of the resulting systems in two formulations: with built-in nonanticipativity and with explicit nonanticipativity constraints. Finally, optimality conditions for both formulations are studied.
- Dinh et al. [11] provide zero duality gap and strong duality theorems for the minimization of a special class of constrained robust sum optimization problems, whose objective function is given as the supremum of an infinite amount of finite sums of functions. Closedness and convex criteria for the formulas on the subdifferential of the sup-function are also provided.
- Dutta and Martínez-Legaz [12] develop an approach to explicitly calculate the constant in the Hoffman's error bound for (not necessarily convex) inequality systems defining convex sets. The authors give a constructive proof of the Hoffman's error bound and show that their method can calculate the constant at least in simple cases.
- Flores-Bazán et al. [13] show the existing relationship between two well-known statements saying that the sum and the minimum of two quasiconvex functions is not quasiconvex in general. To do that, the authors introduce and characterize the notion of quasiconvex family. Three applications in quasiconvex optimization are presented, one of this dealing with a class of problems having zero duality gap.
- Hernández Escobar and Rückmann [14] consider the class of mathematical programs with complementarity constraints (MPCC). Under an appropriate constraint qualification of Mangasarian-Fromovitz type, the authors present a topological and an equivalent algebraic characterization of a strongly stable C-stationary point for MPCC. This refers to the local uniqueness, existence and continuous dependence of a solution for each sufficiently small perturbed problem, where perturbations up to second order are allowed.

- Huerga et al. [15] introduce new notions of quasi efficiency and quasi proper efficiency for multiobjective optimization problems. Characterization and main properties for these solutions are provided by linear and nonlinear scalarizations. With the help of quasi efficient solutions, a generalized subdifferential of a vector mapping is introduced and an application to optimality conditions for quasi efficient solutions is also given.
- Luc and Volle [16] analyze strong duality of extended monotropic optimization problems with possibly infinite sum of separable functions. The results are applied to a minimization problem of the infinite sum of proper convex functions. An application to minimum cost flow problems in infinite networks is also discussed.
- Martínez-Legaz and Pintea [17] characterize the closed convex subsets of $\mathbb{R}^n$ which have open or closed Gauss ranges. They also characterize those lower semicontinuous proper convex functions whose epigraphs have open and closed Gauss ranges, respectively.
- Meng et al. [18] establish some necessary condition and sufficient condition respectively for a set-valued mapping to have the Lipschitz-like property relative to a closed set by employing regular normal cone and limiting normal cone of a restricted graph of the set-valued mapping. The authors obtain a complete characterization for a set-valued mapping to have the Lipschitz-property relative to a closed and convex set by virtue of the projection of the coderivative onto a tangent cone.
- Mohammad-Nezhad and Terlaky [19] provide the parametric analysis of a second-order conic optimization problem, where the objective function is perturbed along a fixed direction. The authors characterize the notions of so-called invariancy set and nonlinearity interval, which serve as stability regions of the optimal partition. They also develop an iterative procedure to compute a nonlinearity interval of the optimal partition and present results of numerical experiments.
- Mordukhovich and Pérez-Aros [20] develop new extremal principles of variational analysis that concern measurable multifunctions. These results are used to derive integral representations and upper estimates of regular and limiting normals cones to essential intersections of sets defined by measurable multifunctions, which are in turn crucial for novel applications to stochastic and semi-infinite programming.

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