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ORIGINAL PAPER

Addressing the economic and demographic complexity via a neural network approach: risk measures for reverse mortgages

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Abstract
The study deals with the application of a neural network algorithm for fronting and solving problems connected with the riskiness in financial contexts. We consider a specific contract whose characteristics make it a paradigm of a complex financial transaction, that is the Reverse Mortgage. Reverse Mortgages allow elderly homeowners to get a credit line that will be repaid through the selling of their homes after their deaths, letting them continue to live there. In accordance with regulatory guidelines that direct prudent assessments of future losses to ensure solvency, within the perspective of the risk assessment of Reverse Mortgage portfolios, the paper deals with the estimation of the Conditional Value at Risk. Since the riskiness is affected by nonlinear relationships between risk factors, the Conditional Value at Risk is estimated using Neural Networks, as they are a suitable method for fitting nonlinear functions. The Conditional Value at Risk estimated by means of Neural Network approach is compared with the traditional Value at Risk in a numerical application.

Keywords Neural network quantile regression · VaR · CoVaR · Reverse mortgage · Longevity risk · House price risk

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1 Introduction

The use of Neural Networks (NN from herein on) has been steadily growing in the financial sector, as a means of addressing critical modelling aspects, as well as inefficiencies in forecasting. Scenarios in which multiple risk sources interact, especially in the context of portfolios containing derivatives, structured products and insurance contracts (cf. Cheridito et al. 2020), generate complexity, such as assessments on time horizons that are not short enough. In these cases, the nonlinearity of certain pricing models and, more generally, the inapplicability of modelling proxies leading to closed form, complicate the measurement of the risks taken and the consequent determination of the capital to be allocated to cover those risks (cf. Krah et al. 2020; Richman et al. 2019).

Within this framework, the use of Artificial Intelligence (AI)-based techniques offers new potential, which, if properly regulated and used, may effectively match consumers and financial firms' needs (cf. Capone 2021; Krause 2003). Conversely, ethical and legal profiles related to new techniques are not negligible, but any areas of opacity in the procedures implemented (cf. EIOPA 2020; EC 2019; 2020) require attention, to ensure fairness and non-discrimination, as well as all-round transparency.

In the context of risk measurement, NN are opening new horizons for predictive needs and market-consistent assessments, as required by the "ratio" of Solvency Capital Requirement Calculation guidelines (cf. Richman et al. 2019). A wide portion of literature is devoted to the use of NN for Value at Risk (VaR from herein on) estimation; for example, financial studies have been focusing on predictive architectures able to capture (cf. Locarek-Junge et al. 1998) the impact of multiple risk drivers (cf. Arimond et al. 2020), for two decades already, establishing risk categories and measuring the interplay among several uncertainty sources.

The insurance sector is a fluid macrocosm, characterized by the presence and interaction of different risk components: a scenario in which "the perfect storm" between risk profiles inherent in insurance coverage, and structural systematic risks (such as market risk factors and model risks as the longevity risk) is far from negligible. It is precisely in this context that VaR calculation techniques have been developed via NN (cf. Krah et al. 2020; Cheridito et al 2020).

Over the past twenty years, the financial literature has consolidated the use of artificial intelligence algorithms to evaluate the combined effect of multiple risk sources, adopting a methodological approach already broadly validated in other areas of science and technology. Kaastra and Boyd (1996) provided a guideline for building a NN procedure to forecast economic time series data. Galeshchuk (2016) dealt with economic time-series prediction by means of NNs, deepening the algorithm performance in exchange rate forecast. Predictive accuracy of different NN algorithm can be tested using MSE or related measures, and related tests, such as Diebold and Mariano's (Diebold and Mariano 2002). Giudici and Raffinetti (2021), proposed the Shapley Lorenz Approach to identify the relative importance of different risk factors and provided a a normalised measure of predictive accuracy, extending the AUROC concept to all types of variables.

Kristjanpoller et al. (2014) applied a hybrid NN model, performing the volatility trend of stock exchange indexes. Baştürk et al. (2022) developed a NN procedure, viable in risk management, that incorporates the prediction of both volatility and VaR. Wutricht (2019) provided new insights on how NNs can be used in pricing models. NN techniques have been applied in insurance context; optimal control models in insurance field, where multiple economic factors interact, has been approached by Kremsner et al. (2020) through a novel deep NN algorithm. Insurance policies have been valued by several authors, as Doyle and Groendyke (2019) and Gan (2013), by means of NN-based techniques.

In this study we use the Reverse Mortgage (RM from herein on) contract as a paradigm of insurance scenarios, since contracts of this kind are affected by various risk factors, such as longevity risk, house-price risk and financial risk.

RM contracts allow elderly homeowners to get a credit line (as a lump sum or as a periodic cash flow for a defined period or the borrower residual life) that will be repaid through the selling of their homes after their deaths.

They have two main implications: the homeowners will retain the right to live in their own home until they die (cf. Di Lorenzo et al. 2021a, b, c; Di Lorenzo et al. 2021b; Merton et al. 2016a; 2016b) if the loan exceeds the proceeds of sale of the house, the amount due to the lender consists of the second quantity (Non-Equity Guarantee, say NNEG) (cf. D'Amato et al. 2019).

In a nutshell, the NNEG ensures that the debt never exceeds the value of the property. The impact of macroeconomic variables on the evolution of house prices is significant in the valuation of NNEG, as Badescu et al. (2022) argue; in particular, they consider, as main variables, the gross domestic product and the industrial production.

The other notable feature of these contracts is their characterization as "complementary or supplementary pension savings" (cf. EIOPA 2016), that is financial products designed to support public pension systems and the aging population in developed countries, whose working life expectancy is generally lower than their life expectancy. Recent literature is addressing risk measurement in RM portfolio management, although the application of an appropriate regulatory framework is very complex, as posed by Kenny et al. (2018) following recent regulatory developments on Matching Adjustment to discount the valuation of the long-term liabilities under Solvency II at a more favorable discount rate than the usual risk-free rate.

VaR and related risk measures play a decisive role, not only because they are self-consistent, due to their inherent nature of risk indices, but also because they are suitable for various regulatory context. de la Fuente et al. (2023), considering the importance of RM contracts as financial source to increase elderly people's incomes, highlight the importance to build a risk measurement procedure together with a proper management system.

This paper is focused on the concept of Conditional Value at Risk (say CoVaR) in accordance with the specific contractual lines of the RMs and implement the numerical procedure for estimating this risk measure through the Quantile Regression Neural Network (NN).

The layout of the paper is the following: Sect. 2 is devoted to present the framework description and the basic notations, together with an overview on the Quantile

Regression NN. Section 3 frames the risk measure issue within the regulatory perspective, inserting in this context our innovative proposal concerning the CoVaR calculated by means of a Quantile Regression NN algorithm. The Reverse Mortgage contract is introduced and mathematically described in Sect. 4; in Sect. 5 CoVaR is deepened in the specific case of a RM portfolio. Numerical applications are presented in Sect. 6 and Sect. 7 closes the paper.

2 Financial complexity: description framework

2.1 Notations and recalls

In 1996 J.P. Morgan's famous Risk Metrics Technical Document (cf. Morgan and Reuters 1996; Benninga et al. 1998), defined the Value-at-Risk under normal market conditions as follows:

> *"Value at Risk is a measure of the maximum potential change in value of a portfolio of financial instruments over a pre-set horizon. VaR answers the question: how much can I lose with x% probability over a given time horizon".*

So, VaR estimates the worst expected loss over a given time interval at a given confidence level.

It is well known how important VaR is in risk management and Basel-Accord-implementation, with particular reference to the minimum capital requirements (cf. Basel Committee 2019) and portfolio pricing and risk assessment (cf. Luciano and Regis 2014; Cocozza et al. 2008).

The Value at Risk with confidence level α is expressed as follows:

$$VaR^{\alpha}(t+T) = \inf\{l \in \mathbb{R} : \wp(L(t+T) > l) \leq 1-\alpha\}, \tag{1}$$

where $L\,(t+T)$ is the *loss variable* of the financial activity under consideration from time t to time $t+T$ and $0 < \alpha < 1$.

However, the disadvantage and criticalities of VaR soon became evident, highlighting this risk measure's dependence on the specific calculation methodology, and on the set of axioms representative of the market, which may prove extremely stringent.

Nevertheless, discrepancies between VaR-based results, and those obtained by means of the portfolio selection theory, are evident. The dual name, "seductive but dangerous", contained in the title of a paper in 1995 (cf. Beder 1995), far from creating an oxymoron, effectively synthesized merits and limits of this risk parameter, which is not always sub-additive, and presents major challenges when the portfolio's assets are not normally distributed.

This involves the introduction of the Conditional Tail Expectation, known also as TVaR (cf. Olivieri and Pitacco 2011; Rockafellar et al. 2000):

$$TVaR^{\alpha} = E\left[L(t+T)|L(t+T)\rangle VaR^{\alpha}\right]. \tag{2}$$

Differently from this approach, according with further prudential regulatory guidance, computational needs linked to complex scenarios have enhanced the implementation of risk measures based on NN techniques. In particular we will focus on the Conditional Value at Risk (CoVaR) using NN quantile regression (cf. Keilbar and Wang 2021).

The Conditional Value at Risk is defined as in Definition 1 in Adrian and Brunnermeier 2016 and in the following Eq. 11 of our Sect. 3.

The literature on the use and utility of NN techniques in the risk measuring is very extensive and in the following we report only some reference entries.

Several categories of NN or other machine learning methodologies have been used to calculate VaR, as Sermpinis et al. (2015), Mostafa et al. (2017), Arian et al. (2022) showed; in particular, Arimond et al. (2020) analyzed how NNs can improve the process of estimating VaR, also comparing different structures of NNs.

Explored different aspects of a cryptocurrency fluctuations; within option valuation's framework, for instance in the case of Bermudan style derivatives, NN approaches provide meaningful results, as Lokeshvar et al. (2020) proved.

Adrian and Brunnermeier (2016) proposed the concept of CoVaR, to take into account the impact of systemic risk on the measure of the VaR. They analysed the influence of systemic risk in a linear and bivariate context, focusing on the risk contribution of a financial firm subjected to many macroeconomic influencing variables. Chao et al. (2015) highlighted the nonlinear dependence between the risk of a financial institution and the impact of macroeconomic factors.

Following this observation, introduced the estimation of the CoVar through NN, exploiting their predictive ability in the case of complex nonlinear relationships between input and output variables. Quantile Regression Neural Network (QRNN) defines a nonlinear form of quantile regression. These are computational architectures aimed at the estimation of conditional quantiles of variables, whose dependence on covariate is somewhat explicable by regression equations (cf. Cannon 2011; 2018; Chen 2007). QRNN approaches are, therefore, particularly useful in nonlinear and multivariate contexts, typical of the financial framework, where classic predictive and computational models fail. In fact, recent financial literature has shown that the results obtained with the use of neural networks are meaningful and accurate. Ince (2006) pointed out that the nature of financial time series presents continuous interchange of stochastic and deterministic components and proposed some innovative techniques, as NNs, and supported vector regression to forecast stock returns.

Taylor (2000) suggested a QRNN to estimate the conditional probability distribution of multiperiod financial returns, and Gu et al. (2020) considered NN forecasts into portfolio management strategies. Chronopoulos et al. (2021) provided a "deep quantile estimator" by means of NN techniques apt to treat a nonlinear association between the conditional quantiles of dependent variables; this approach is particularly useful for forecasting Value-at-Risk. Moreover, Jantre et al. (2022) deepened the statistical framework for Bayesian quantile regression neural network, thus achieving results with a broad application spectrum, including finance. Xu et al. (2016) proposed a novel quantile autoregression neural network structure, designed to capture nonlinear relationships among quantiles in time series data. This technique allows to forecast, among other things, different real stock indices.

In addition, QRNNs have also been implemented in the insurance field, for example in relation to the claim amount of an insurance policy (cf. Laporta et al. 2021, 2023; Heras et al. 2018).

We start from the idea that the concept of CoVaR can be extended to all cases in which the measure of the riskiness of a financial or insurance product depends on relationships among risk factors, whether they are dependent or not. In particular, we propose to implement the measure of CoVaR in the risk valuation of RM contracts using a NN approach. In the following, we briefly describe the estimation of CoVaR through the Linear Quantile Regression and the NN, framing them in the actuarial design of the RM.

2.2 An overview of linear quantile regression and neural network quantile regression

We consider the variable Y_t dependent on X_t. We are interested in studying its probability distribution at a given quantile τ. If X_t is a K-dimensional vector of independent variables which influence Y_t, Koenker and Basset (1982) describe the linear quantile regression by the following equation:

$$Y_t = \beta X_t +_t, \quad t = 1, \dots, n \tag{3}$$

with $Q^\tau = \left(\varepsilon_t | X_t\right) = 0$.

The linear quantile estimator is found by minimizing the following quantity:

$$\min_\beta \sum_{t=1}^{n} \rho_\tau\left(Y_t - X_t\right), \tag{4}$$

where the loss function is:

$$\rho_\tau\left(Y_t - X_t\right) = |\left(Y_t - X_t\right)| * |\tau - I(\left(Y_t - X_t\right) < 0) \tag{5}$$

and I(.) is the indicator function.

The value of the conditional quantile in the case of a nonlinear relation between Y_t and X_t is offered by the algorithm of sieve NN. The relation can be expressed as in Chen (2007):

$$Y_t = h_\theta(X_t) + \varepsilon_t = \sum_{m=1}^{M_n} \omega_m^0 \varphi\left(\sum_{k=1}^{K} \omega_{k,m}^h X_{k,t} + b_m^k\right) + b_0 + \varepsilon_t, \quad t = 1, ..., n \tag{6}$$

where $\varphi(.)$ is a nonlinear activation function, typically a sigmoid one, M_n is the number of bases functions equal to that one of the hidden layers, $\omega_{k,m}^h$ and b_m^k are hidden layer parameters, ω_m^0 and b_0 are output parameters. For more datails see Grenander (1981). Cybenko (1989) proves that the single layer neural networks are universal approximators for sigmoid function. The estimator can be found solving the following non-convex optimization problem:

$$\min_{\theta} \sum_{t=1}^{n} \rho_{\tau}\left(Y_t - h_{\theta}\left(X_t\right)\right) \tag{7}$$

due to the property of consistency of non-parametric sieve single hidden layer NN estimator for conditional quantile (see White 1992); it is interesting to highlight that this property holds for both independent and dependent data and this makes it useful for a broad spectrum of applications. The solution is calculated through the gradient based back propagation algorithm proposed by Rumelhart et al. (1988).

3 The financial and regulatory framework

3.1 The RM contract as a resource for an ageing society

RMs can be considered as income generating products for an ageing society.

The progressive ageing in industrialized countries, together with the alarming phenomenon of denatality, has undermined the financial sustainability of public pension schemes, which have initiated reform processes involving changes in retirement age and in the labour-law rules. Nevertheless, employer pensions and personal pension products suffer from relevant weaknesses (cf. D'Amato et al. 2019; Guerin 2016). In short, the need to meet the demand for protection in old age together with the long-term sustainability of a system characterized by an increasing number of elderly people, has encouraged the study of new financial/insurance instruments. Against this backdrop, reverse mortgages are included: such contracts allow elderly homeowners to liquidate their real estate, making an illiquid asset liquid, and obtaining access to a credit line.

There has been rapid growth in those contracts in Canada and the US, where they are also known as Home Equity Conversion Mortgage, largely managed by the Federal Housing Administration (cf. Merton et al. 2016a; 2016b). Reverse Mortgages are growing in Australia (cf. D'Amato et al. 2019), in Asian economies and spread widely in the UK, where they are known as the Equity Release. According to a welfare perspective and life cycle investing, RMs are a technical solution to satisfy the demand for protection in old age, when, compared to the increased life expectancy, the elderly, while owning a home, does not have adequate pension income.

3.2 *A few more details about TVaR$^{\alpha}$*

The topic of profitability is of great interest in the management of RM portfolios and has recently found some interesting contributions in the literature. For example, in Lee et al. 2018, the profitability and the risk profile for such portfolios are analyzed using stochastic dominance methods, applied specifically to a market with an external insurer and to one without an external insurer. Criteria related to the calculation of quantiles of distributions are already used in Cho et al. 2013, where the Authors calculate CoVaR for Reverse Mortgage portfolios, framing it within the theme of risk analysis and profitability. In this seminal paper, the Authors highlight the purposes of risk measures such as VaR and $TVaR^{\alpha}$, in the context of RM portfolios. They point out that VaR and $TVaR^{\alpha}$ are calculated to define the risk-based capital to be set aside by the lender. Specifically, $TVaR^{\alpha}$ is the expected loss referred to the $1-\alpha$ percent VaR. The traditional procedure for calculating $TVaR^{\alpha}$ is to determine the expected value of the simulated probability distribution of the present value of the lender's net payoff, provided that this present value is at or below the α-th percentile level (Cho et al. 2014).

VaR and $TVaR^{\alpha}$ are closely related to each other. This is clearly seen through the formulas for their calculation. It follows that the assumptions made underlying the calculation of VaR (on the distribution of returns, the periodicity of the data used to calibrate the processes and their volatility, etc.) severely limit its efficiency as a measure of risk. Although they also affect $TVaR^{\alpha}$, the possible damage is limited being the $TVaR^{\alpha}$ a more conservative measure. $TVaR^{\alpha}$ measures the risk of expected losses that go beyond the threshold value inherent in VaR. This is why $TVaR^{\alpha}$ has been preferred over VaR by the regulator.

Risk measures such as VaR and $TVaR^{\alpha}$ involve the capital requirements that one must set aside to cover oneself against possible losses. As in Olivieri and Pitacco 2011, setting aside such sums allow extreme happenings in their negativity not to be compromising to the business being considered.

As reported in Olivieri and Pitacco 2011, the Value at Risk of a generic quantity Z is the negative quantity that verifies the following equation:

$$\mathbb{P}\left[Z \leq VaR_{\alpha}(Z)\right] = \alpha \tag{8}$$

where α is given and represents a generally low probability (e.g., 5 percent) and 1- α is the confidence level attributed to this measure (e.g., 95%). Formula 8 simplifies definition in formula 1.

It is immediate to understand the relationship that links VaR to $TVaR^{\alpha}$. $TVaR^{\alpha}$ is defined as follows:

$$TVaR^{\alpha}[Z] = \mathbb{E}\left[Z/Z \leq VaR_{\alpha}(Z)\right] \tag{9}$$

and it derives that:

$$-TVaR_{\alpha}[Z] > -VaR_{\alpha}(Z) \tag{10}$$

whereby it can be said that the use of $TVaR^{\alpha}$ favors a more conservative and efficient approach to risk, in the sense that it originates the hedging of more losses due to extreme events.

The difference $TVaR^{\alpha}$-VaR can be interpreted as the average value of capital to be paid into the company's coffers in order to prevent it from incurring bankruptcy if a particularly unfavorable scenario occurs, albeit one with a low probability.

Exactly as in the case of VaR, investors will be attracted to investments with low $TVaR^{\alpha}$ s, but of course potentially more desirable investments show higher $TVaR^{\alpha}$ s: in this, $TVaR^{\alpha}$ manifests itself as a classic measure of risk.

3.3 Why CoVaR and why use neural networks for its determination.

As noted in Laforêt 2018, despite in Solvency 2 the indicated risk measure is VaR, back in the day $TVaR^{\alpha}$ was already a widely used measure by insurers in their internal models, as they are heavy-tailed economic actors. The focus on more prudential risk measures is very well observed in the evolution of the Basel Accords. In Basel II, regulatory capital requirements are assessed through VaR, but in Basel III, in particular after the 2008 subprime crisis, there is already a clear recommendation on whether regulatory capital requirements should be assessed by measuring the amounts at risk in a more prudential way, through the calculation of $TVaR^{\alpha}$ (see Basel III). In fact, in the chapter on forward looking, it is explicitly stated that the Committee is oriented toward indicating procedures based on expected losses and clear reference is made to $TVaR^{\alpha}$. Guidance is provided for moving from the VaR approach to the $TVaR^{\alpha}$ approach in order to enable banks to set aside larger sums for the benefit of stakeholder protection. The increased degree of prudence in determining these risk measures is mainly due to the increased instability of the markets in which the business is implemented. In our case, the risk drivers impacting RM portfolios (longevity risk, financial risk, and real estate market-related risk, as deepened in the following) are manifestly characterized by increasing instability.

This paper is set in the risk management scenario of a financial intermediary playing the role of a lender in a portfolio of RM, in line with the prudential view of the Basel III accords. We will implement a CoVaR risk measure and will graphically compare it with VaR values at the same confidence level, for the very purpose of bringing out the differences between the two risk measures represented in (10), with the aim of verifying that the proposed risk measure is more prudential than the Var measure.

There are many works in the context of RM portfolios that offer VaR and $TVaR^{\alpha}$ values. See for example Cho et al. 2015, de la Fuente et al. 2023. In this paper we propose the calculation of the CoVaR using artificial intelligence.

Artificial Intelligence procedures are particularly well suited to capturing nonlinearities involving various risk components, and we have found such methods to be appropriate for dealing with the calculation of a risk measure such as CoVaR. Numerous are the uses of Artificial Intelligence in a variety of scientific fields. For a look at this output see Keilbar and Wang 2021.

Our work proposes the determination of $\mathrm{CoVaR}^{\mathrm{q}}$ given by the following formula:

$$P\left[L(t+T)|\mathbb{C}(L(t)) \geq CoVaR^{q}\right] = q\% \tag{11}$$

where $\mathbb{C}(L(t))$ denotes some event of the financial activity under consideration. We will use a computational procedure that succeeds in capturing nonlinearities between interacting risk processes in a general framework of high process instability.

4 The financial case study: the product design and the mathematical model

4.1 Contractual design

The Reverse Mortgage contract we will consider is issued at time $t=0$ on a homeowner aged x, generally referring to retirement age. Insurer-lender and insured-homeowner-borrower are engaged in the following roles, viewed from a financial perspective: schematically, the insured-borrower, the owner of the house, will use part or all the whole value of the property itself to obtain a sum (Lump Sum) or a regular stream of installments, paid by the lender provided the borrower is alive.

We will consider this second case, which is more complex but at the same time more oriented toward the aim of protecting the weaker segments of homeowners and ensuring a standard of living as close as possible to that usually held by the borrower. The latter, who continues to live in her/his house, will pay off the loan received over the years with the sale of the house upon his death (NMRLA 2018). Specifically, when the borrower dies, the only case we will consider as a contractual termination cause, the loan consisting of the installments received will be returned with the liquidation value of the house along with the roll-up interest. We will place the value of the property equal to a percentage of the current value of the property itself, meaning that the value of the stream of installments should be proportional to the value obtainable from the sale of the house: if H_0 is the current value of the property, the actuarial valuation of the borrower and lender commitments will have to comply with the following relationship:

$$R\ddot{a}_x < \alpha H_0 \tag{12}$$

with $0<\alpha<1$. In formula (6), R represents the annual installment due to the borrower provided he is alive and $\ddot{a}_x$ the average value, calculated at the contract issue time, of the stream of unitary installments the lender is going to pay to the borrower at the beginning of each year. In the case of $\alpha = 1$, at the time of the borrower's death, the heirs will have to pay off in full the stream of installments, valued at that instant, through the sale of the asset. On the other hand, the case that we will consider ($\alpha<1$) is based setting the valuation of the property equal to a percentage of its current value, and thus the installment that the lender will pay, valued accordingly, will be less than that calculated with $\alpha=1$: this will result in the heirs having to pay a percentage of the proceeds from the sale. The important aspect, highlighted already in Merton (2016a) and (2016b), lies in the meaning of the difference:

$$\alpha H_0 - R\ddot{a}_x, \tag{13}$$

representing the cost of the guarantee borne by the borrower (cf. Di Lorenzo et al. 2021c). By borrower's guarantee we mean the protection, due to the lender, from the circumstance that the value obtained from the sale of the property is less than the loan paid up to that point. As stated in Merton, the borrower, although receiving a smaller discounted sum, lends the sum αH_0 and pays interest on this sum at a rate that also includes profit for the lender. In this sense, the difference in (13) is understood as the cost of collateral borne by the borrower.

The above contract, of great social value, is subjected to numerous sources of risk (cf. D'Amato et al. 2019):

the longevity risk, consisting of the underestimation of the residual life of the homeowner, resulting in capital loss. It comes from underestimating the life expectancy of the policyholder, who remains in his own home for a longer period than expected;

the financial risk (cf. D'Amato et al. 2019) arises from two main uncertainty sources: possible underperformance of the real estate market (house price risk) and possible increases in the interest rate that the insurer will pay until the contract's liquidation (interest rate risk).

In an ex-ante evaluation, the financial risk components are increasingly insidious in time horizons of random magnitude (the future lifetime of the homeowner); moreover, the performance of the house price alone depends on many geographical, political, macroeconomic and accidental factors, which make the formulation of appropriate quantitative models difficult. In fact, just to better capture all the components that contribute to the evolution over time of the house price, NN algorithms have been used (cf. Xu and Zhang 2021; Mora-Garcia et al. 2022), obtaining good accuracy in forecasting comparing to empirical evidence (cf. Xu and Zhang 2021).

This risk influences the amount of the interest accrued on the loan and the value of the loan. The profitability of the loan depends on the difference between the loan interest rate (which may be fixed or variable) and the rate the lender pays to finance himself on financial markets. This variability determines a relevant further consequence: the more the interest accrued in the loan, the higher the risk that the house selling will be made at a liquidation value not sufficient to cover the loan itself.

The risk that the property market value is lower than the value of the mortgage and, consequently, the reimbursement capacity of the borrower is inadequate, has been defined by Wang et al. (2008) the crossover risk.

In this contractual context, our goal is to measure the risk for the insurer-lender from a forward-looking perspective, placing the risk assessment made at the issue time, proceeding year by year in a conditional analysis of the amounts at risk.

4.2 The mathematical model

Our study objective is the function of losses $L_t^{(P)}$ related to year $(t-1,\ t)$, $t=1,2,\ldots$, with t extended to the insured's entire life span from the issue time on; we will consider a portfolio P of N contracts of RM, issued in $t=0$ on homogeneous individuals aged x.

With the following formula we calculate the VaR of $L_t^{(P)}$:

$$VaR_t^{\tau}\left[L_t^{(P)}\right] = VaR_t^{\tau}\left\{\left[(N-\sum_{h=0}^{t-1} D_h\right]R - E\left[D_{t-1}\right]\left\{\alpha H_t\right\}\right\} \tag{14}$$

in which we denote by D_h the random number of deaths that occurred among the initial N subscribers in the interval *(h – 1,h)*.

The amount R to be paid at the beginning of each year provided that the insured is alive, turns out to be calculable after financially estimating at the instant of the insured's death, the amount payable by the heirs (cf. Di Lorenzo et al. 2021c): this amount arises from the option embedded in the contract. Denoting the instant of the borrower's death with the random variable K_x, we can write that this amount is given by:

$$V_{K_x} = \min(A_{K_x}, H_{K_x}) \tag{15}$$

where:

$$A_{K_x} = \alpha H_0 r^{RM}\left(0, K_x\right) \tag{16}$$

represents the valuation of αH_0 at the insured's death, valued at that time by means of the capitalization factor $r^{RM}(0, K_x)$ at the interest rate i^{RM} applied in the actuarial RM operation, while:

$$H_{K_x} = \alpha H_0 r^{HV}\left(0, K_x\right) \tag{17}$$

represents the value of the house at the time of death, valued by means of the capitalization factor $r^{HV}(0, K_x)$ at the rate i^{HV} of house value appreciation.

If T_x is the continuous random variable representing the future lifetime of the insured of age x, we can now determine the amount of the installment to be paid in advance to the borrower over his lifetime:

$$R = \frac{E\left\{E\left[V_{K_x} v^S\left(0, K_x\right)\right] / T_x\right\}}{\ddot{a}_x} \tag{18}$$

In Eq. (18) above, the expected value in the numerator is calculated under a real probability measure and $v^S(0, K_x)$ represents the discount factor calculated at the risk-free rate i^S.

5 The CoVaR of a portfolio of RMs

We consider formula (8) with the following position:

$$\begin{aligned} VaR_t^\tau\left[L_t^{(P)}\right] &= VaR_t^\tau\left\{\left[(N - \sum_{h=0}^{t-1} D_h\right] R - E\left[D_{t-1}\right]\left\{\alpha H_t\right\}\right\} \\ &= f(D_{t-1}, H_{t-1}) \end{aligned} \tag{19}$$

and define the VaR_t^τ of the portfolio of N RM contracts, as above described, at the end of year *(t – 1,t)* as follows:

$$P\left[\left(f(D_{t-1}, H_{t-1} = VaR_t^{\tau}\right)\right] = \tau. \tag{20}$$

We could estimate the VaR through conditional linear quantile regression using a set of explanatory variables. While in the explanatory variable that impacts on the financial systematic risk at time t are macroeconomic factors observed at *(t − 1)*, in our proposal the variables that influence the amounts under consideration are the random number of deaths and the evolution of the house price, both valued year by year:

$$f(D_{t-1}, H_{t-1}) = \alpha + \gamma_1 D_{t-1} + \gamma_2 H_{t-1} + \varepsilon_t \tag{21}$$

Posing that $Q^{\tau} = \left(\varepsilon_t | D_{t-1}, H_{t-1}\right) = 0$, we can write:

$$VaR_t^{\tau} = \hat{\alpha} + \hat{\gamma}_1 D_{t-1} + \hat{\gamma}_2 H_{t-1}. \tag{22}$$

However, in the Quantile NN framework we relax the linearity hypothesis and define:

$$P\left[f\left(D_{t-1}, H_{t-1}\right) \leq CoVaR_t^{\tau} | f\left(D_{t-2}, H_{t-2}\right)\right] = \tau \tag{23}$$

In this case the conditional quantile of the portfolio cash flow at t is regressed on the cash flows at time *(t − 1)*:

$$\begin{aligned} f\left(D_{t-1}, H_{t-1}\right) &= h_\theta\left[f\left(D_{t-2}, H_{t-2}\right)\right] + \varepsilon_t \\ &= \sum_{m=1}^{M_n} \omega_m^0 \varphi\left(\sum_{k=1}^{K} \omega_{k,m}^h \left[f\left(D_{k,t-2}, H_{k,t-2}\right)\right] + b_m^k\right) + b_0 + \varepsilon_t, \quad t = 1, \ldots, n \end{aligned}$$

6 Numerical evidence: the case study

Aim of this section is to apply the Quantile Neural Network techniques to measure the CoVaR of a portfolio of Reverse Mortgages issued in the Italian market. The first step consists in the VaR estimate year by year until the expiration of the RM contracts, to perform an empirical risk analysis extended to the entire portfolio of homogeneous contracts.

In this application we will focus on two main risk drivers, specifically the longevity and house price risk; the financial risk is synthetized in a constant interest rate.

Our goal is to calculate the portfolio CoVaR through the Quantile Neural Network.

To simulate the number of deaths occurred year by year, we consider the life table of male Italian population of the year 2019, downloaded from the Human Mortality Database. We calculate $t-1/q_x$, the probability that the borrower aged x at the issue time dies between the ages $x+t-1$ and $x+t$, necessary for the calculation of the installment R. The installment is paid annually in advance starting from $t=0$ until the borrower is alive.

Knowing R, the analysis goes on dealing with risk assessment of the portfolio of N contracts of RMs.

Let $E_{x,t}$, $t=1,\ldots,T$, be the exposure to risk, that is the number of survivors aged x at time t; in order to outline the loss or gain of the lender as function of time, we simulate a number B of different paths of $E_{x,t}$ where:

$$E_{x,0} = N$$

$$E_{x+t,t} \sim binomial\left(E_{x+t-1,t-1};t-1/q_x\right)$$

As regards the house price risk, the evolution trend is modelled according to a GARCH(1,1) model calibrated on the quarterly variations of the Italian Real Estate Index IPAB diffused by ISTAT (Italian National Institute of Statistics) between 2010 and 2019. The adoption of this specific representation of the house price risk factor is not relevant to our main purpose, which is to test the model's ability to provide consistent and interpretable results in the context of risk assessment.

Let us consider a RM contract offered to a retired Italian male aged $x=80$; we consider a final age equal to 100 and a maximum residual life equal to 20. The borrower owns a house that, at the issue time $t = 0$, is valued 200000€ and takes out a RM on the α% of his home, $\alpha = 0.50, 0.60, 0.70, 0.80, 0.90, 1$. We assume that $i^{RM}=5\%$ and $i^{s}=2\%$. Given these parameters, the instalments *R's* are reported in Table 1.

The installment values shown in Table 1, highlight the sensitivity of the installment amount due to the borrower for the considered values of α. As α increases, the constant value of the installment received by the borrower increases, and thus the value of the loan that the heirs will have to pay through the sale of the house.

As an example, if $\alpha = 0.70$, the present value in $t=0$ of the payment stream due to the man aged *80* is 114,167.7 €, less than the amount $\alpha H_0 = 140000$ €, on which the loan was written, as in the difference in formula (12) is represented. This happens because the borrower is also buying a portfolio of protective long put options, that gives his heirs at the random time of his death the possibility to repay the smaller amount between the accumulated interest i^{RM} on the loan and the agreed percentage α of the realized value of the home at that date. Depending on the financing needs, the borrower can decide to write the loan choosing the appropriate value for α.

Table 1 The annual installment *R* due to the borrower aged *80* in case of life, as α varies

α	R
0.50	10,306.66
0.60	12,367.99
0.70	14,429.23
0.80	16,490.66
0.90	18,551.99
1	20,163.32

Let us consider that the lender issues N identical RM contracts in $t = 0$. In order to evaluate the CoVar on the whole portfolio, we simulate B paths of the evolution of the number of deaths and the house value at each t. We set $N = 100$ and $B = 10000$ simulations. Figure 1 shows the cash flows as function of time and the estimation of the Value at Risk of the simulated cash flow in each year, at the confidence level $\tau = 0.05$ with Monte Carlo Simulation (dashed red line).

In Fig. 1, the simulated paths of cash flows are understood as the difference between the amount paid in the form of installments to all borrowers who survive at the date of its payment, and the amount of proceeds from sales relative to all those who have passed away in that year. The paths are reported from the issue time of the contract until its maximum term (20 years). The flows thus calculated and simulated are compared with the dashed red curve representing the VaR at 5% year by year, i.e. the percentile of the simulated distribution of cash flows at the selected confidence level. The figure allows a quick comparison of the flows with the VaR of the flows themselves: it can be seen from the figure that the VaR tends to settle around values slightly less than zero from the twelfth year of the contract until the end. It can be seen that the VaR, in the early years of the contract, signals how much the initial imbalance between payments and receipts, in net favor of the former, is reflected in a large amount of potential loss up to 5 percent; this imbalance is destined to be reabsorbed as the contract term lengthens.

Our next focus is centered in the analysis of CoVaR. It will also be simulated year by year, as the previous analysis, considering the variables "number of deaths" and "house prices" as input variables for the quantile neural network training. The output variable is the simulated CoVaR at 5% in reference to the following year. The analysis is implemented with the “QRNN”, that fits a quantile regression neural network

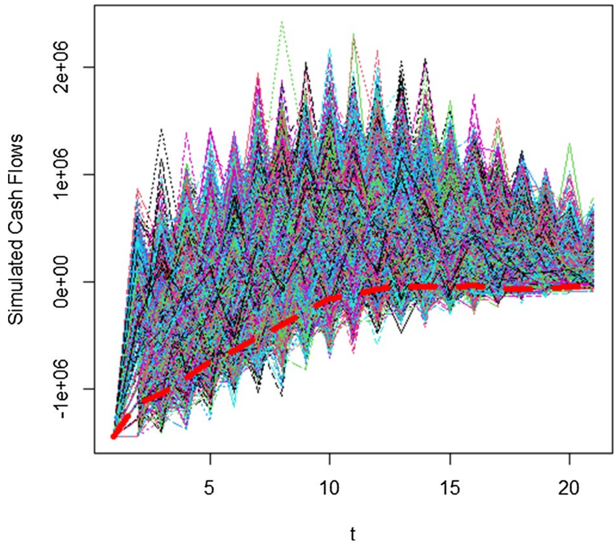

Fig. 1 The simulated cash flows and related percentile at $\tau = 0.05$ (the dashed red line) (colour figure online)

model for the τ-quantile by minimizing a cost function based on smooth Huber-norm approximations. The activation function selected is the nonlinear sigmoid one; the NN architecture used is a one hidden layer network according to Cybenko (1989), that proves that the single layer neural networks are universal approximators for sigmoid function. Figure 2 shows the CoVar for the selected portfolio on RMM estimated with NN.

In Fig. 2 we report the simulated CoVaR and the VaR at 5%, this last with the dashed red line, with respect with the contract time. The conditioning implicit in the calculation of CoVaR at 5% manifests itself with simulated outputs systematically higher than VaR at 5%, and the former indicator tends to stabilize around values close to 0 already around the eighth year of the contract.

The decreasing trend of the maximum potential annual loss with the 5% confidence interval as the contract term increases is evidenced by the growth of both risk indicators in Figs. 1 and 2.

Figure 2 shows that the extent of the loss simulated using the CoVar risk indicator is reduced compared to the VaR estimate. The dependence that the model allows to capture in the calculation of the CoVaR, and therefore the increase in information from which the neural network benefits in determining this risk indicator with respect to the VaR, has a positive effect on the evaluation of the business riskiness. Consequently, the proposed tool allows for a better allocation of resources by reducing the estimate of the maximum potential loss at the same confidence interval. We highlight that the conditional risk measure is different from the classical one (VaR) because the percentile calculation is updated due to conditioning with respect to the last observed variables. In other words, at a given probability level, the VaR offers the maximum potential loss while the CoVaR, calculated with the NN, updates from

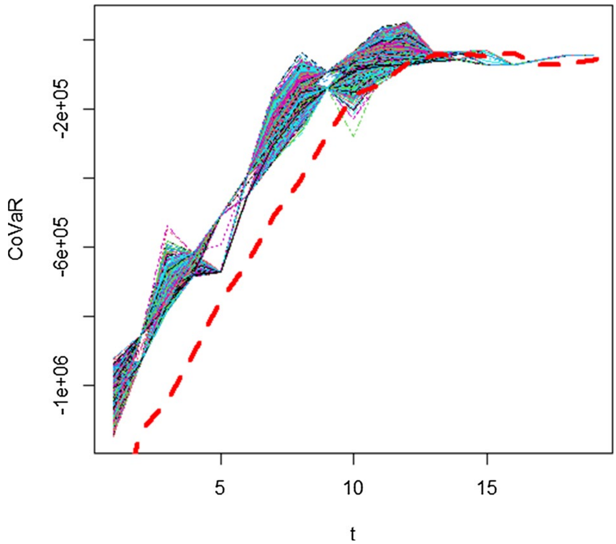

Fig. 2 The NN CoVaR simulations vs the VaR of the simulated cash flows at $\tau = 0.05$ (the dashed red line) (colour figure online)

time to time this calculation, offering the maximum potential loss given what has happened up to that moment. To show this, we randomly select a simulated loss path and show the loss percentile calculation conditioned on it. In Fig. 3 one of the simulated paths of the cash flow, randomly chosen, is compared with the CoVaR (red line), calculated conditioned on the evolution of mortality and house prices, and with the VaR (blue line). The conditioning with respect to the path information allows to define a loss percentile closest to the path losses. Table 2 shows the Mean Difference over the B simulated paths between the simulated loss and the estimated percentile with CoVaR and VaR.

In this example, it is evident how CoVaR, taking advantage of the periodic updating of the impact of the risk sources, allows, unlike VaR, the refinement of the calculation of the capital to be allocated for solvency purposes. Differently, the nature of the VaR calculation, lacking such conditioning, does not provide the same accuracy in the assessment and, indeed, may burden the amount of resources to be set aside to solvency aims.

7 Conclusions

In this paper we propose an extension of the CoVaR index to a portfolio of Reverse Mortgages by means of a Quantile Neural Network procedure. This is a paradigmatic case study involving a complex financial scenario, in which multiple risk sources interact. The main risk drivers, on which we focus our analysis and train the net system, are the longevity risk, due to the underestimation of homeowners' life expectancy, and the financial risk, in the component due to the volatility of the real

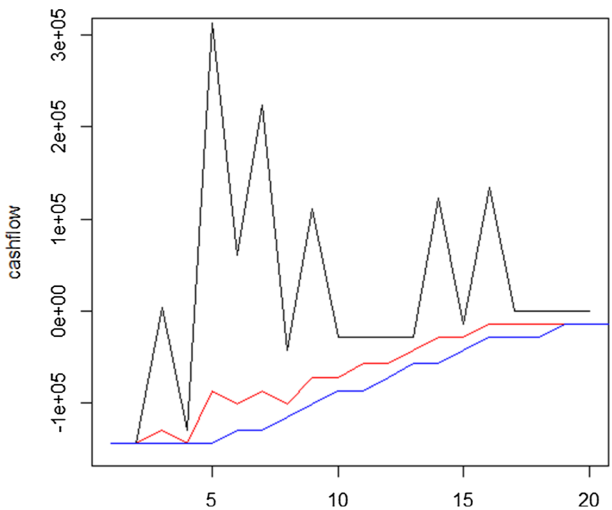

Fig. 3 A simulated cash flow (black line) vs the CoVaR (red line) and the VaR (red line) (colour figure online)

Table 2 Mean Difference over the $B = 10000$ simulated paths between the simulated loss and the estimated percentile with CoVaR and VaR

t	CoVaR	VaR
1	84,672.1	84,672.1
2	88,755.3	96,664.1
3	103,193.9	111,483.1
4	119,844.4	128,739.2
5	132,713.4	128,705.1
6	129,965.9	139,570.1
7	130,884.6	137,592.8
8	132,609.2	135,102.9
9	134,763.2	128,861.3
10	127,545.2	134,518.9
11	119,285.21	134,518.9
12	106,252.18	106,252.1
13	96,230.28	101,873.2
14	81,727.1	81,727.1
15	66,686.2	64,716.2
16	57,946.3	57,946.2
17	49,023.6	52,205.6
19	32,408.3	32,408.2
19	27,210.9	27,210.9
20	23,662.1	23,662.1

estate market. Achieving CoVaR by means of NNs for RM portfolios is a novel finding in the literature.

In the numerical illustration we estimate the VaR of a RM portfolio considering, year by year, both the random number of deaths and the random house price until the contracts' expiration. The proposed approach can be framed in the Design of Experiment branch: we have built a model and made hypotheses to implement it with the aim of offering a conceptual framework to determine a percentile of the losses according to the regulator's requirements. In this way, the proposal is flexible and easily adaptable to the different market contexts where RM contracts can be placed.

The aim of our study is the quantification of risk through self-consistent measures such as VaR and CoVaR, in this paper implemented through the use of novel methodologies such as NN. In particular, our contribute consists in achieving the CoVaR by means of a QRNN algorithm, applied to a portfolio of identical RMs issued in the Italian market. After simulating the number of deaths on the basis of the male Italian population of the year 2019 and the house price evolution according to a GARCH (1,1) model, we calculate the CoVaR via the QRNN method. The model offers fast and intuitive results: observing from year to year the realization of the two variables number of deaths and house prices on which income and expenses of the portfolio essentially depend, the network offers us an estimate conditioned by these risk drivers of the maximum potential loss at a certain level of probability that will occur next year (CoVaR). As expected, the riskiness of the portfolio and the potential loss is highest in the first few years, when the number of survivors is still high, and decreases towards zero as one approaches the expiration date of the contract, when by now all the contracts will have expired.

The main advantage of the proposed model, in addition to the flexibility of the framework suitable to different geographical context and different markets, lies in the fact that the NN framework allows us to model the complexity of the risks in a nonlinear setting Any entity proposing itself as a guarantor must be aware of the measure of risk it is taking on. As measures of risk, and therefore quantitative indicators of the uncertainty inherent in the Reverse Mortgage transaction, VaR and CoVaR also constitute an indispensable tool for all institutions, public or private, wishing to hedge the risk associated with the transaction.

Further research could deepen the quantile network scheme by introducing other sources of risk as those deriving from the fluctuation of interest rates applied to the loan.

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Declarations

Conflict of interest The authors declare no Conflict of interest.

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