



# Using interpolated implied volatility for analysing exogenous market changes

Matúš Maciak<sup>1</sup> · Sebastiano Vitali<sup>2</sup>

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## Abstract

This paper focuses on market changes due to exogenous effects. The standard implied volatility is shown to be insufficient for a proper detection and analysis of this type of risk. This is mainly because such changes are usually dominated by endogenous effects coming from a specific trading mechanism or natural market dynamics. A methodologically unique approach based on artificial options that always have a constant time to maturity is proposed and explicitly defined. The key principle is to use interpolated volatilities, which can effectively eliminate instabilities due to the natural market dynamics while the changes caused by the exogenous causes are preserved. Formal statistical tests for distinguishing significant effects are proposed under different theoretical and practical scenarios. Statistical theory, computational and algorithmic details, and comprehensive empirical comparisons together with a real data illustration are all presented.

**Keywords** Implied volatility · Exogenous effects · Artificial options · Constant time to maturity · Panel data · Change point detection

## 1 Introduction

It is a well-known fact that the analysis of financial markets relies on the ability to detect all kinds of sudden changes—*change points*—which randomly and repeatedly occur in the stock market over time. Some changes are caused by the market itself, its natural dynamics, or various trading mechanisms. For instance, considering some specific financial contract, certain changes may occur due to the specific

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✉ Sebastiano Vitali  
sebastiano.vitali@unibg.it

Matúš Maciak  
maciak@karlin.mff.cuni.cz

<sup>1</sup> Department of Probability and Mathematical Statistics, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague, Czech Republic

<sup>2</sup> Department of Economics, University of Bergamo, Via dei Caniana 2, 24127 Bergamo, Italy

payoff structure of the contract, either because of a fixed maturity date or because the payoff structure is highly discontinuous, or both. This is also the case when studying the price of a fixed maturity bond that always converges to a nominal value when approaching its maturity. On the other hand, more important changes for practitioners and financial agents are those that are caused by different impulses due to human interactions (such as the recent outbreak of COVID-19, President Biden's canceling the permit for the Keystone XL pipeline, or the Russian attack on Ukraine). Unfortunately, these two types of changes can not be easily distinguished when using common market data and standard theoretical/methodological principles for the statistical analysis. Additional steps are needed in order to separate specific risk—the natural dynamics of the market—and systemic risk (mainly the changes caused by various external causes). Moreover, among these types of risk, we distinguish between those intrinsically connected with the structure of the financial instrument—so called *endogenous effects*—and those determined by some external cause—referred to as *exogenous effects*.

Focusing on the options market and bearing in mind the *exogenous effects* that are our main interest, we introduce a unique market analysis approach based on *artificial options* with a *constant time to maturity* over time. In general, the price of an option contract follows a specific dynamic when approaching the exercise date, and this dynamic is typically assumed to follow the hypothesis of the Black and Scholes model proposed in Black and Scholes (1973). However, that model postulates normally distributed returns and also restricts the volatility to be constant with respect to strike values. Some modifications to take into account non-normally distributed returns are proposed, for instance, in Corrado and Su (1997). In practice, however, the market agents adjust the former assumption by changing the latter and, as a result, an increasing volatility is quoted for strikes far from the current value of the underlying asset. This quoted volatility is known as the *implied volatility* (IV) and its characteristic convex shape is called the *volatility smile*. The scientific literature related to this topic is quite important from both—the statistical and the financial viewpoints. Some characteristic mean-reverting behaviour is investigated in Ielpo and Guillaume (2010) and various smoothing approaches targeting optimal trading strategies are proposed and discussed in Appel (2003), Chong and Ng (2008), and Chio (2022). Advanced panel data approaches are adopted in Maciak (2019) while the most recent ideas based on neural network models and machine learning techniques are used in, e.g., Jang and Lee (2019). In all these works, as in many others, some pre-analysis is performed on the raw observations in order to (a) smooth the discrete data or (b) remove the endogenous effects while studying the exogenous ones. As far as (a) is concerned, several methods have been proposed to generate a continuous IV curve with respect to the quoted strikes (see, for instance, Kahlé (2004), Fengler (2005), Benko et al. (2007), or Homescu (2011) for a complete review). Some recent applications of these smoothing techniques can be found in Fengler (2012), Glaser and Heider (2012), Fengler and Hin (2015), Ludwig (2015), and Kopa et al. (2017), but all of them assume some underlying smoothness of the unknown IV surface—which is not the case in our approach.

On the other hand, there are not so many studies regarding (b). Usually, the objective of the research is to capture and explain the endogenous dynamics of the

IV smiles while removing the exogenous effects. However, it is still important for many practical applications to do the opposite: to remove the endogenous features that naturally affect the evolution of the IV and are strictly related to the financial structure of the option itself, and to rather capture and model the exogenous effects contained in the data as unnatural jumps or sudden breaks—*changepoints* in general. The main feature of the evolution of the IV over time is its increasing convexity when approaching the maturity date. When the option contract is close to its maturity, this adjustment becomes more important because even small fluctuations in the underlying value can cause the option to move suddenly from an in-the-money (positive payoff) position to an out-of-the-money (zero payoff) position, having huge effects on the price of the option. Therefore, the market agents further adjust the model, quoting a more convex IV smile. Alternatively, one could use the jump-diffusion process proposed by Jiang and Tian (2005) or the so-called model-free implied volatilities suggested in Britten-Jones and Neuberger (2000), but the corresponding theoretical framework is rather more restrictive and empirical calculations tend to be more extensive and less intuitive. Nevertheless, in order to study the dynamics of the option price one needs to study the dynamics of the IV, and to make inference on the *external causes* affecting the dynamics of the IV one must first (somehow) remove the *natural dynamics* of the market since it is evident that the convexity of the IV increases when the time progresses towards the maturity date. One could argue that to remove the *maturity effect* it would be enough to consider options with longer maturity. Unfortunately, this is difficult for several reasons: (1) For most of the companies (excluding American blue chips) long maturity options are not quoted; (2) Even if they are quoted, the amount of different strikes is very limited and most strikes start to be available only at times close to maturity; (3) In any case, the liquidity of the options is not enough for the options far from maturity.

There are some rather exploratory approaches discussed in Guhathakurta et al. (2010) and Marcaccioli et al. (2022) to distinguish between *exogenous effects* and *endogenous effects* as they are generally considered to be of different nature. However, unlike the aforementioned papers, we propose a formal statistical/inferential tool based on removing the *endogenous effects* caused by the market itself while allowing us to focus directly on the *exogenous effects*—which has not been, to the best of our knowledge, done so far. There are some nonparametric tests for detecting jumps within a (discrete or continuous) stochastic process of asset prices proposed in Lee and Mykland (2008), Ait-Sahalia and Jacod (2009), or Fan and Fan (2011) or a simple location model for volatility changes based on a segmentation (see Brigida and Pratt 2017) but they all focus on a detection of any changepoints in general. Other alternatives are very popular volatility indexes, like DAX Volatility Index and CBOE VIX, (see CBOE (2003) or Kuepper (2022) for details) that estimate the implied volatility of options with an average expiration of 30 days, but they aggregate multiple put/call options (over both the maturities and the strikes), they are computationally more complex, and they mainly serve as exploratory tools to assess the overall market sentiment. Nevertheless, there can be also used some nonparametric detection approaches as suggested in Nystrup et al. (2016) or Füss et al. (2011). However, our primary focus—embedded within an option and strike specific interpolation instead—is

to provide market agents with a valid stochastic tool to be able to correctly make inferences about some rather specific market based on the significance of changes occurring due to some particular (well recognized) external stimuli. The corresponding market reaction can be either uncertain and can be explained just by some random fluctuation, but some other market changes are more essential and typically statistically significant. The method proposed in this paper can properly and consistently distinguish between these two possibilities.

The detection of the *exogenous effects* within the dynamics of the IV smile is very important for understanding the sentiment of the market agents since the IV captures the expectations of the market about the evolution of the underlying in the near future. As the *change-points* caused by the *exogenous effects* have become more and more frequent in the last couple of years, a rigorous statistical analysis is needed here. The proposed idea moves the market analysis from the hypothesis that the implied volatility of the options of a given asset is able to capture the feeling and the view of the financial agents about the future changes of the asset itself. The focus on the volatility of the *artificial options* also reduces the inter-day bias, which is further diminished using a carefully calibrated interpolation between the implied volatilities of the options having consecutive maturities.

The rest of this paper is organized as follows: The principal idea of the paper—the *interpolated volatility* of the *artificial options*—is introduced in Sect. 2. A detailed description of the corresponding interpolation algorithm can be found there as well. Some formal mathematical and statistical theory regarding the detection of change-points is provided in Sect. 3. Some sensitivity analysis, practical illustrations on real data, and empirical comparisons based on simulations are discussed in Sect. 4. Final remarks and conclusions are summarized in Sect. 5.

## 2 Interpolated volatility of artificial options

Let  $\{z_{itm}; i = 1, \dots, N; t = 1, \dots, T; m \in 1, \dots, M\}$  represent the values of the implied volatility for some underlying asset. Here,  $i$  stands for the option's strike label,  $t$  is the observing day from the follow-up period, and  $m$  is the maturity dataset index. Such implied volatilities are well known for being non-stationary over time mainly due to the specific payoff structure of the market. Therefore, when focusing on the *exogenous effects* in particular, one needs to firstly deal with this non-stationarity induced by the trading mechanism. For this purpose, we construct a new artificial dataset  $\{Y_{it}; i = 1, \dots, N; t = 1, \dots, T\}$  such that the new data values will report, for each strike  $i$  and each observing day  $t$ , an (artificial) implied volatility value  $Y_{it}$  of an *artificial option* which always has a constant (over time) time to maturity of  $T_m$  days. The construction of the artificial option is based on a simple (weighted) linear interpolation across different IV datasets  $m \in \{1, \dots, M\}$ . For each day  $t \in \{1, \dots, T\}$ , the observed implied volatilities of the two options having the same strike and the corresponding maturities immediately before and immediately after the given day  $t \in \{1, \dots, T\}$  are interpolated together. The analytic formula can be expressed as

**Algorithm** IVintAO

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**Input:**

IVpanel           ▷ 3-dim (dates/strikes/months) panel of observed implied volatilities  
M                   ▷ Number of IV panels, one for each month  
dates               ▷ Follow-up period  
strikes             ▷ Set of strikes  
 $\mathcal{T}_m$                ▷ Artificial time to maturity (in days)

**Output:** interpolatedIVpanel           ▷ Interpolated implied volatilities

**Using:**

length( $x$ )                   ▷ Returns the number of elements of a given vector  $x$   
lookFor( $r, c, X$ ) ▷ Look for the value  $r$  in first column of panel  $X$  and for the value  $c$  in first row of panel  $X$ , finally returns the value of panel  $X$  in the corresponding position  
isEmpty( $x$ )                   ▷ Returns 1 if  $x$  is empty, otherwise returns 0  
maturity( $X$ )                 ▷ Returns the maturity date of options in a given panel  $X$   
closest2zero( $x$ ) ▷ Returns the bigger negative value and the smallest positive value of vector  $x$ , both in absolute value  
extract( $v, x, y$ )           ▷ Finds the position of  $v$  in vector  $x$  and returns the value of  $y$  in that position

$D \leftarrow \text{length}(\text{dates})$                    ▷ Number of days in the follow-up period  
 $S \leftarrow \text{length}(\text{strikes})$                ▷ Number of given strikes  
**for**  $d = 1$  **to**  $D$  **do**                       ▷ Compute the artificial IV for each day  
  **for**  $s = 1$  **to**  $S$  **do**                     ▷ Compute the artificial IV for each strike  
    artMat  $\leftarrow \text{dates}(d) + \mathcal{T}_m$            ▷ Compute the artificial time to maturity  
     $i \leftarrow 0$   
    **for**  $m = 1$  **to**  $M$  **do**           ▷ Look for observed IV in each of the available IV panels  
      **if** isEmpty(lookFor(artMat, strikes( $s$ ), IVpanel(:, :,  $m$ ))) = 0 **then**  
         $i \leftarrow i + 1$   
        IV( $i$ )  $\leftarrow$  lookFor(artMat, strikes( $s$ ), IVpanel(:, :,  $m$ ))  
        TTM( $i$ )  $\leftarrow$  maturity(IVpanel(:, :,  $m$ )) – artMat  
      **end if**  
    **end for**  
    **if**  $i \geq 2$  and min(TTM) < 0 and max(TTM) > 0 **then**  
      (TTMneg, TTMpos)  $\leftarrow$  closest2zero(TTM)  
      **if** TTMneg > 2 and TTMneg < 35 and TTMpos > 2 and TTMpos < 35 **then**  
        IVneg  $\leftarrow$  extract(TTMneg, TTM, IV)  
        IVpos  $\leftarrow$  extract(TTMpos, TTM, IV)  
        Wneg  $\leftarrow 1/\text{TTMneg}$   
        Wpos  $\leftarrow 1/\text{TTMpos}$   
        interpolatedIVpanel( $d, s$ )  $\leftarrow [Wneg * \text{IVneg} + Wpos * \text{IVpos}] / [Wneg + Wpos]$   
      **end if**  
    **end if**  
  **end for**  
**end for**

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$$Y_{it} = \frac{\frac{1}{(t+\mathcal{T}_m)-t_b} z_{itm_b} + \frac{1}{t_a-(t+\mathcal{T}_m)} z_{itm_a}}{\frac{1}{(t+\mathcal{T}_m)-t_b} + \frac{1}{t_a-(t+\mathcal{T}_m)}}, \quad (1)$$

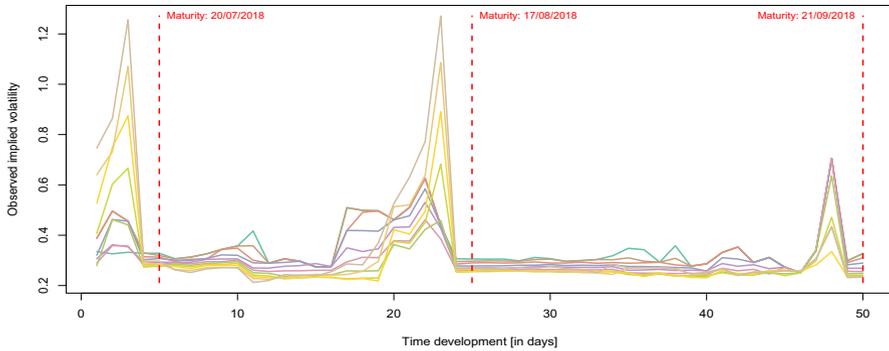
where  $m_b$  is the maturity dataset index of the first option expiring before the time  $t + \mathcal{T}_m$  (at the day  $t_b$ ) and  $m_a$  is the maturity dataset index of the first option expiring after the time  $t + \mathcal{T}_m$  (at the day  $t_a$ ). Computational details are described in the Implied Volatility interpolation/Artificial Options—IVintAO Algorithm below. Note

that unlike the popular VIX index (CBOE 2003), the interpolation in (1) is: (i) so-called strike specific; (ii) it does not merge put and call options together; (iii) it still provides the typical IV smile for every trading day  $t \in \{1, \dots, T\}$ ; (iv) and, finally, it is computationally simpler and, therefore, easier to interpret. The proposed approach interpolates the IVs of the options that have the same strikes. For this reason, it is needed to consider a follow-up window in which the number of the quoted strikes does not change substantially or, at least, the amount of the quoted strikes that are available for the whole period is sufficiently large. If there is an interest to analyze a longer period, it is possible to split the whole follow-up window into a sequence of shorter sub-periods and to run the analysis in each period separately.

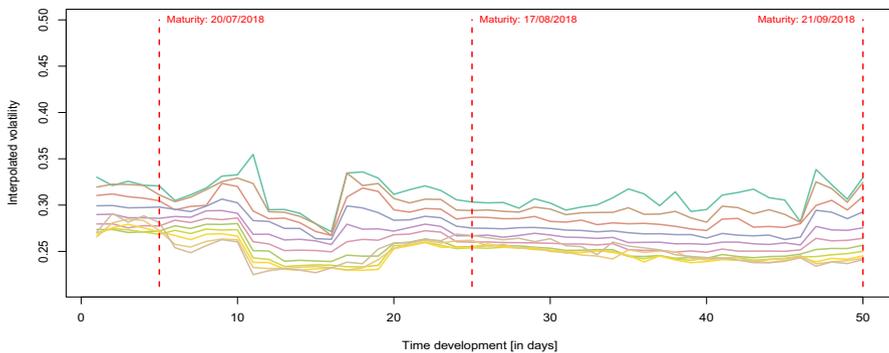
For a brief illustration of the main principle of the interpolation defined by (1), using  $\mathcal{T}_m = 35$  (i.e., artificial options with a constant time to maturity of 35 days), we use the call options data of Erste Group (see Sect. 4 for more details). The first day of the observation period ( $t = 1$ ) is July 16th, 2018. Thus, the artificial option will expire at  $t + 35$ , i.e., August 20th, 2018, and the two real options that must be considered for the interpolation in (1) are the options with the maturity August 17th, 2018 (denoted by  $m_b$ ) and the maturity September 21st, 2018 (denoted by  $m_a$ ). The distance (in days) between the artificial maturity (August 20th, 2018) and the maturity of the first real option is  $(t + 35) - t_b = 3$  days and the distance between the artificial time to maturity and the maturity of the second real option is  $t_a - (t + 35) = 32$  days. Therefore, the equation in (1) becomes

$$Y_{it} = \frac{\frac{1}{3}z_{im_b} + \frac{1}{32}z_{im_a}}{\frac{1}{3} + \frac{1}{32}}, \quad (2)$$

where  $z_{im_b}$  and  $z_{im_a}$  are the corresponding values of the raw implied volatility obtained from the market ( $m_b$  is the index for the maturity dataset expiring on August 17th, 2018 and  $m_a$  is the index for the maturity dataset expiring on September 21st, 2018). The interpolation procedure is repeated for all available strikes  $i \in \{1, \dots, N\}$  and all trading days  $t \in \{1, \dots, T\}$  from the given observation period. The specific choice of  $\mathcal{T}_m = 35$  takes into account the fact that a reasonable value of  $\mathcal{T}_m$  should be in between 30 and 40 days. Any  $\mathcal{T}_m < 30$  (and  $t$  being immediately after the expiration of the options of the given month) would make  $t + \mathcal{T}_m$  smaller than the first consecutive maturity and, therefore,  $t_b$  would obviously not exist. On the other hand, any  $\mathcal{T}_m > 40$  would generate a proxy too far away from the current day, not representing the sentiment of the market effectively while also considering  $t_a$  for which several strikes are not quoted yet. However, unlike the VIX index which—by default—produces the options with an average 30-day expiration, there is some flexibility in (1). Different choices of  $\mathcal{T}_m$  are addressed in Sect. 4. For illustration, Fig. 1 shows the Black–Scholes implied volatilities of Erste Group and the corresponding interpolated volatilities of the artificial options over the same follow-up period around the days of a possible external stimulus—the tribunal trial between Erste Group and Croatia. One effect of the proposed interpolation is obvious: While the original IV structure (Fig. 1a) is clearly non-stationary over time, mainly due to the natural market dynamics driven by the trading mechanism and the



(a) Implied volatility of the Erste Group call options



(b) Interpolated volatility of the artificial options

**Fig. 1** **a** Strike specific panels of the values of the implied volatility of the Erste Group call options and **b** the corresponding interpolated volatility of the artificial options with a constant (over time) time to maturity of  $\mathcal{T}_m = 35$  days. Both panels are given over the same follow-up period of 50 trading days (from Monday, July 16th, 2018 to Friday, September 21st, 2018). Three maturities (July 20th, August 17th, and September 21st), present within the follow-up period, are represented by the red vertical (dashed) lines

option payoff structure (e.g., three spikes corresponding with the maturity dates—July 20th, August 17th, and September 21st), the IV structure of the artificial options visualized in Fig. 1b seems to be relatively stationary, with only minor fluctuations. If some fluctuation is systematic at some specific time—which can be particularly observed, for instance, between the trading days no. 10 and no. 18—it is believed to be, very likely, a result of an *exogenous effect*.

An analogous pattern can also be recognized in Fig. 1a, but the magnitudes of the changes are rather negligible when compared with the overall magnitudes of the three main spikes related to the expiration dates. Thus, it is clearly unavoidable that one must first somehow smooth the natural market dynamics when a statistical analysis of the market changes due to the *exogenous effects* is of some interest. The *artificial options* constructed in terms of the proposed interpolation in (1) filter the natural market dynamics and the related (rather non-essential) effects are effectively

removed. The remaining volatility—which is supposed to reflect the *exogenous effects*—is, however, not affected and, moreover, the magnitudes of the changes become stronger when compared with the overall variability of the volatility of the artificial options. Therefore, the statistical analysis performed in terms of a formal statistical test will have much more power to detect significant changes related to the *external causes*.

### 3 Changepoint tests for exogenous effects

Many different statistical approaches have been proposed in the literature to deal with the so-called *changepoint problem* when detecting significant changes in some underlying probabilistic model—see Csörgö and Horváth (1997) for an overview. The approach discussed in this paper is based on a nice property of the proposed *Implied Volatility interpolation/Artificial Options—IVintAO Algorithm*, which takes the original non-stationary (and typically skewed) values of the IV and interpolates them into a set of volatilities of artificial options which already seem to form a stationary structure. This is crucial from the theoretical point of view when formulating the underlying stochastic model. On the other hand, considering the applicational viewpoint and also bearing in mind different arguments of different practitioners, it may be appropriate to distinguish three theoretically alternative (but practically very similar) scenarios when performing the formal statistical test. We briefly address all of them. However, specific details are only given for the last one, which is, as we believe, the most appropriate one (explicit arguments to justify this statement will be provided later).

Assuming some form of stationarity, the volatility of the artificial options can be represented by a formal underlying probabilistic model

$$Y_{it} = \mu_i + \delta_i \mathbb{1}\{t > \tau\} + \varepsilon_{it}, \quad (3)$$

for the set of strikes  $i = 1, \dots, N$  observed within the follow-up period  $t \in \{1, \dots, T\}$ . Such model is also known as a *panel data model*. Parameters  $\mu_i \in \mathbb{R}$  for  $i = 1, \dots, N$  in (3) are the panel specific mean parameters (i.e., the unknown true strike specific volatilities) which may change at some unknown time point  $\tau \in \{1, \dots, T\}$ . The location of the changepoint is common for all panels (i.e., the same *external cause* affects the market), however, the panel specific magnitudes of the changepoint,  $\delta_i \in \mathbb{R}$ , may differ (i.e., the resulting effect of the change depends on the strike value) while also allowing for a situation where only some proportion of the panels is subjected to the change (meaning that  $\delta_i = 0$  for some  $i \in \{1, \dots, N\}$ ). Each panel specific mean parameter (for some fixed  $i \in \{1, \dots, N\}$ ) equals  $\mu_i$  before the change and it becomes  $\mu_i + \delta_i$  after the change. The errors  $\varepsilon_i = [\varepsilon_{i1}, \dots, \varepsilon_{iT}]^\top$  can be seen as panel-specific disturbances (explicit theoretical details are provided later).

Statistically speaking, the whole problem of detecting the changepoint within the given panels (the panel data model respectively) can be formulated in terms of a statistical test of the null hypothesis

$$H_0 : \tau = T, \quad (4)$$

against a general alternative

$$H_1 : \tau < T \quad \exists k \in \{1, \dots, N\} \quad \text{such that } \delta_k \neq 0. \quad (5)$$

While the formulation of the null and alternative hypothesis is rather simple, particular details regarding the statistical test itself may differ a lot—depending mainly on the theoretical assumptions imposed on the number of the panels  $N \in \mathbb{N}$  and the length of the follow-up period  $T \in \mathbb{N}$ .

### 3.1 Scenario 1: $T \rightarrow \infty$ and $N$ is fixed

A rather simple and straightforward method can be applied when treating the whole problem within the context of multivariate time series. The dimensionality is determined by the number of panels and the follow-up period is assumed to tend to infinity. An asymptotically consistent statistical test is proposed, for instance, in Horváth et al. (1999). The test can properly take into account the spatial dependence between the panels and, also, the dependence structure over time. On the other hand, when analysing market changes due to specific exogenous effects, practitioners usually focus on short periods before and after the event, and close to the event, that is assumed to trigger the change. Therefore, the assumption of  $T \rightarrow \infty$  is slightly impractical. Nevertheless, the test can be still applied and some empirical comparisons are also provided in Sect. 4.

### 3.2 Scenario 2: $T \rightarrow \infty$ and $N \rightarrow \infty$

A more complex methodological framework is elaborated, for instance, in Horváth and Hušková (2012) and Chan et al. (2013), where both the panels number  $N \in \mathbb{N}$  and the observational period length  $T \in \mathbb{N}$  are assumed to tend to infinity (so that  $N/T^2 \rightarrow c \neq 0$ ). However, the results rely on the assumption that the individual panels are independent observations while the error terms within the panels form causal linear processes. Some limited dependent structure in terms of a common stochastic factor among the panels may be assumed but it is, in our opinion, not that realistic for the market scenarios considered in practice. Some finite sample drawbacks of this approach can also be seen in the empirical comparisons in Sect. 4. Nevertheless, the formal statistical test is very analogous to the third scenario below, which effectively detects *exogenous effects* assuming the given underlying artificial IV structure.

### 3.3 Scenario 3: $T$ is fixed and $N \rightarrow \infty$

Bearing in mind the true character of the financial markets, it is reasonable to assume that the error vectors  $\varepsilon_i = [\varepsilon_{i1}, \dots, \varepsilon_{iT}]^T$  are neither independent nor identically distributed. A strong mixing condition among the panels is postulated to reflect the fact that more distant strikes have less dependent implied volatility than the volatilities of two strikes close to each other (see Sect. 4.2 for some empirical justification). There

is also no specific form of stationarity being assumed within the panels and some heteroscedasticity across the panels is also allowed. This accounts for situations where the strikes close to the at-the-money position are expected to have smaller volatility. Different approaches under various theoretical assumptions can be used (see, for instance, Andrews (1993), Csörgö and Horváth (1997), Horváth et al. (2008), Shao and Zhang (2010), and Peřtová and Peřta (2018)) but we rely on the approach presented in Maciak et al. (2020) where two competitive self-normalized test statistics are defined by

$$Q_N(T) = \max_{t=1, \dots, T-1} \frac{|\mathcal{L}_N(t, T)|}{\max_{s=1, \dots, t} |\mathcal{L}_N(s, t)| + \max_{s=t, \dots, T-1} |\mathcal{R}_N(s, t)|} \tag{6}$$

and

$$S_N(T) = \sum_{t=1}^{T-1} \frac{\mathcal{L}_N^2(t, T)}{\sum_{s=1}^t \mathcal{L}_N^2(s, t) + \sum_{s=t}^{T-1} \mathcal{R}_N^2(s, t)} \tag{7}$$

where  $\mathcal{L}_N(s, t) := \sum_{i=1}^N \sum_{r=1}^s (Y_{ir} - \bar{Y}_{it})$  and  $\mathcal{R}_N(s, t) := \sum_{i=1}^N \sum_{r=s+1}^T (Y_{ir} - \tilde{Y}_{it})$ . Moreover,  $\bar{Y}_{it}$  denotes the average of the first  $t$  observations in panel  $i$  and  $\tilde{Y}_{it}$  is the average of the last  $T - t$  observations in panel  $i$ , i.e.,  $\bar{Y}_{it} = \frac{1}{t} \sum_{s=1}^t Y_{is}$  and  $\tilde{Y}_{it} = \frac{1}{T-t} \sum_{s=t+1}^T Y_{is}$ .

Under some standard regularity conditions and the assumptions listed below, the statistical test based on the test statistics in (6) or (7) can be proved to be consistent. The distribution of the test statistics under the null hypothesis in (4) is given by the next theorem. For further theoretical and technical details, we refer to Maciak et al. (2020).

**Assumption A** The random error vectors  $\epsilon_i = [\epsilon_{i1}, \dots, \epsilon_{iT}]^\top$ , for  $i = 1, \dots, N$  form a zero mean  $\alpha$ -mixing sequence such that the mixing coefficients  $\alpha(i)$  satisfy  $\sum_{i=1}^\infty (\alpha(i))^{\gamma/(2+\gamma)} < \infty$  for some  $\gamma > 0$  and also  $\sum_{i \in \mathbb{N}} \mathbb{E}|\epsilon_{i,t}|^{2+\gamma} < \infty$  for all  $t \in \{1, \dots, T\}$ .

**Assumption B** Suppose there is a positive definite matrix  $\Lambda$  such that

$$\Lambda = \lim_{N \rightarrow \infty} \frac{1}{N} \text{Var} \left\{ \sum_{i=1}^N \left[ \sum_{s=1}^1 \epsilon_{is}, \dots, \sum_{s=1}^T \epsilon_{is} \right]^\top \right\}.$$

**Theorem 1** *Suppose that Assumptions (A) and (B) hold. Then, under the null hypothesis in (4),*

$$\mathcal{Q}_N(T) \xrightarrow[N \rightarrow \infty]{\mathcal{D}} \max_{t=1, \dots, T-1} \frac{|X_t - \frac{t}{T}X_T|}{\max_{s=1, \dots, t} |X_s - \frac{s}{t}X_t| + \max_{s=t, \dots, T-1} |Z_s - \frac{T-s}{T-t}Z_t|}$$

and

$$\mathcal{S}_N(T) \xrightarrow[N \rightarrow \infty]{\mathcal{D}} \sum_{i=1}^{T-1} \frac{(X_t - \frac{t}{T}X_T)^2}{\sum_{s=1}^t (X_s - \frac{s}{t}X_t)^2 + \sum_{s=t}^{T-1} (Z_s - \frac{T-s}{T-t}Z_t)^2},$$

for  $Z_t = X_T - X_t$ , where  $[X_1, \dots, X_T]^T$  is a multivariate normal random vector with zero mean and covariance matrix  $\Lambda$ .

Practically speaking, the test can be easily performed using the asymptotic distribution stated in the theorem above. Monte Carlo simulations or bootstrap approaches can be used as an alternative to mimic the distribution of interest and obtain the corresponding quantiles. Once the null hypothesis in (4) is rejected, practitioners and financial agents are usually interested in having a consistent estimate of the changepoint's location  $\tau \in \{1, \dots, T-1\}$ . A simple and straightforward estimator is proposed in Pešta et al. (2020) which, unlike many other changepoint estimators suggested in the literature, does not suffer from any of the usual boundary issues when the true changepoint is located at the beginning or at the end of the follow-up period. The estimator for  $\tau \in \{1, \dots, T\}$  is defined by

$$\hat{\tau}_N = \arg \max_{t=1, \dots, T} \mathcal{U}_N(t), \quad (8)$$

where

$$\mathcal{U}_N(t) = \begin{cases} \frac{1}{t(T-t)} \sum_{i=1}^N \sum_{u=1}^t \sum_{v=t+1}^T (Y_{iu} - Y_{iv})^2 & t < T; \\ \frac{2}{(T-1)^2} \sum_{i=1}^N \sum_{v=2}^T \sum_{u=1}^{v-1} (Y_{iu} - Y_{iv})^2 & t = T. \end{cases}$$

Further theoretical details can be found in Pešta et al. (2020). One could argue that the underlying assumption of just one changepoint within the given follow-up period in the model in (3) is not realistic, as market changes usually occur frequently and multiple “shocks” are typically observed within a consecutive series. However, the proposed statistical test in Scenario 3 does not require that  $T \rightarrow \infty$  and, moreover, the follow-up period  $T \in \mathbb{N}$  can even be arbitrarily short (as short as 2–3 days). Therefore, with regard to the practical applicability of the proposed method, the market agents can just focus on any follow-up period around some specific external cause and if some significant change is detected by (6) or (7), the follow-up period can be split into two parts—before and after the change, and the whole mechanism is applied to both disjoint intervals again. On the other hand, if no changepoint is detected in some particular follow-up period, no further splitting is needed.

## 4 Empirical investigations

In this section we tackle some finite sample particularities which may be considered important for practitioners and, also, we illustrate the proposed method using an example of real data. The empirical performance of various changepoint scenarios is investigated via an extensive simulation study.

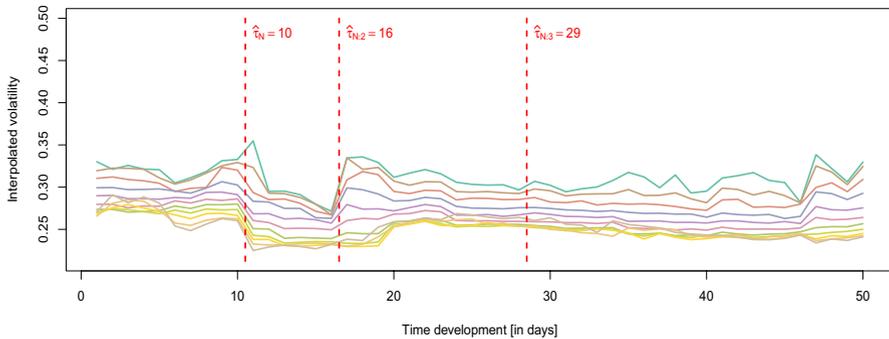
### 4.1 Erste Group call options

The real data—the Black–Scholes model implied volatilities for the call options of Erste Group quoted in the EUREX Deutschland market—were downloaded from Thomson Reuters Datastream. A follow-up period of 50 trading days is considered<sup>1</sup> starting on Monday, July 16th, 2018, ending on Friday, September 21st, 2018. The follow-up period is long enough to implicitly include some changes due to the natural market dynamics when approaching any of the three maturity dates (there are three consecutive maturities explicitly included in the follow-up period—July 20th, August 17th, and September 21st—to demonstrate the role of the proposed interpolation) and, also, possibly some changes caused by *external causes* because the company underwent a tribunal trial (Erste Group against Croatia, ARB 17/49), which is still a pending dispute at that time. The first tribunal meeting took place on August 10th, 2018, and the first procedural order was issued on August 20th, 2018. Both events are typically considered to be important enough to have a serious impact on the market of the underlying asset and both dates are, therefore, intentionally included in the considered follow-up period. There are 11 strike specific panels equidistantly spanning from 30 Euros to 40 Euros (with a step of 1 Euro). The number of panels is limited in this example by the fact that the strikes must be quoted in all three maturity datasets but we rather consider this to be a technical issue that could be overcome in practice.

The raw IV values for the call options of Erste Group and the corresponding interpolated volatilities of the artificial options (for  $T_m = 35$ ) are both presented in Fig. 1. Three maturity dates (July 20th, August 17th, and September 21st) implicitly present within the given observation period are visualized by the red vertical lines. The changes in the IV caused by the natural market dynamics are clearly visible in the top panel (Fig. 1a), while the panel below (Fig. 1b) provides more insight about the IV smile as it adapts to miscellaneous *exogenous effects* after the so called “getting-close-to-maturity” effect is removed by the proposed interpolation.

Applying the changepoint test on the raw IV values (from Fig. 1a), both test statistics, (6) and (7), detect a significant changepoint (max-type test statistics  $Q_N(T) = 1.0897$  with the corresponding critical value 0.8204 and the  $p$ -value 0.0059; sum-type test statistic  $S_N(T) = 3.0476$ , critical value 2.1747,  $p$ -value 0.0144). The changepoint estimated in terms of (8) yields  $\hat{\tau}_N = 4$  which is July 20th,

<sup>1</sup> Various options markets of different companies over a whole range of follow-up periods were considered with more-or-less analogous results and conclusions. The Erste Group case is taken as an illustrative example.



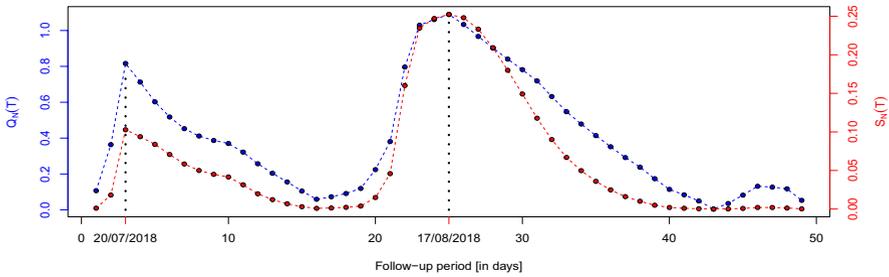
**Fig. 2** Strike specific panels of the interpolated volatilities of artificial options for the Erste Group call options considering the same follow-up period as in Fig. 1. Three significant changepoints detected by the proposed testing procedure are indicated by dashed (red) vertical lines. The estimated changepoints correspond to July 30th, August 8th, and August 23rd, 2018

2018. This perfectly corresponds with the first maturity date. Thus, the detected changepoint is clearly related with the natural stock market dynamics. Applying the same testing procedure on both sides of the estimated location of the changepoint, another significant change (on the right side of  $\hat{\tau}_N$ ) is detected by both test statistics. The estimated location of the changepoint is August 17th, 2018, which again corresponds with the maturity date. No other changes are found to be significant. Thus, both detected (significant) changes are clearly due to the natural market dynamics (see also Fig. 3a for an illustration of the actual performance of the test statistics).

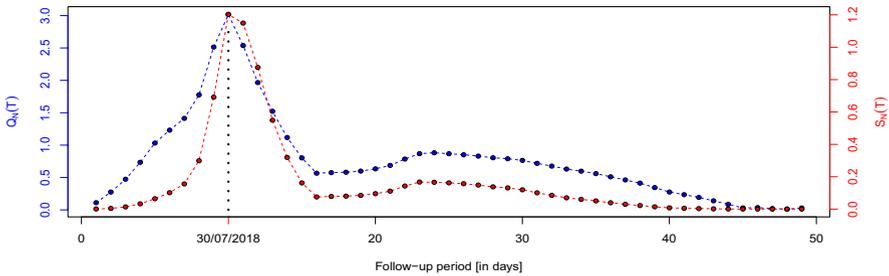
On the other hand, using the interpolated volatilities of the artificial options instead, the results become different and more informative. Again, both test statistics detect a significant changepoint (max-type test statistic  $Q_N(T) = 3.0171$  with the corresponding critical value 1.6541 and the  $p$ -value 0.0001; sum-type test statistic  $S_N(T) = 7.9775$ , critical value 7.4855,  $p$ -value 0.0230) but the location of the changepoint is different:  $\hat{\tau}_N = 10$  (July 30th, 2018). This changepoint is mostly caused by some external event.

This becomes even more evident when repeating the whole testing procedure again considering either the side on the right of the previously detected changepoint or the left side. Another significant changepoint is detected by both test statistics (max-type test statistics  $Q_N(T) = 0.5820$ , critical value 0.4932,  $p$ -value 0.0016; sum-type test statistics  $S_N(T) = 4.9291$ , critical value 4.8702,  $p$ -value 0.0494) while the estimated location of the changepoint is  $\hat{\tau}_{N:2} = 16$  (August 8th, 2018). Finally, one more significant changepoint can be detected (with the test based on  $Q_N(T)$ , with the corresponding  $p$ -value 0.0470) on August 23rd, 2018 ( $\hat{\tau}_{N:3} = 29$ ) and no other significant changepoints are detected any more (see Fig. 2).

Looking back at important dates of the ongoing dispute between Erste Group and Croatia (ARB17/49), two out of three detected changepoints are immediately linked with specific events of the tribunal trial. While the first changepoint ( $\hat{\tau}_N = 10$ , July 30th, 2018) is most likely also somehow related to the dispute, there seem to be no doubts regarding the other two changepoints. One ( $\hat{\tau}_{N:2} = 17$ , August 8th,



(a) Performance of the proposed test statistics based on the raw IV values



(b) Performance of the proposed test statistics applied to the interpolated volatility

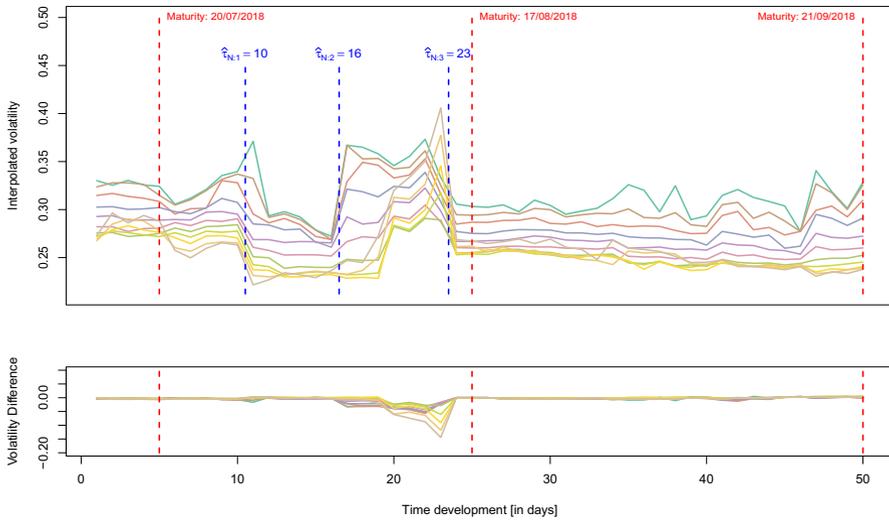
**Fig. 3** Daily quantitative contributions to the test statistics defined in (6) and (7). The overall test statistic  $Q_N(T)$  is defined as the maximum attained by the blue curve while  $S_N(T)$  is given as the overall sum of all daily contributions. Black dotted lines correspond with the estimated locations of the changepoint defined in terms of the estimator in ()

2018) reflects a sudden increase of uncertainty just before the first tribunal meeting and the other ( $\hat{\tau}_{N:3} = 29$ , August 20th, 2013) occurs right after the first procedural order when the situation at the market stabilized due to the positive outcome of the meeting.

Thus, unlike the raw IV values, where the only significant changepoints are related to the natural market dynamics—i.e., some volatility spikes when approaching the maturity dates—the interpolated volatility allows detecting significant changes which are clearly related to some more interesting *exogenous effects* instead. The *interpolated volatility* of the *artificial options* with a constant (over time) time to maturity is indeed a useful tool for the analysis of the *exogenous effects* due to *external causes* which are otherwise practically undetectable, as they are hidden by the more marked changes created by the natural market dynamics.

### 4.2 Sensitivity analysis & residual inspection

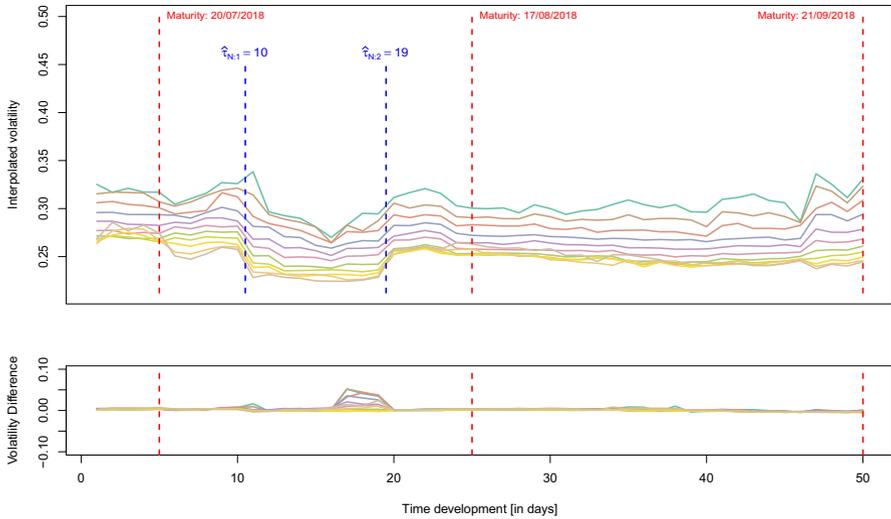
Firstly, we briefly address some sensitivity issues related to different choices of the artificial time to maturity,  $T_m \in \mathbb{N}$ . As already mentioned, the specific choice of  $T_m = 35$  takes into account the fact that any reasonable value should be in



**Fig. 4** Artificial call option volatilities of Erste Group for a constant (over time) time to maturity of  $\mathcal{T}_m = 30$  days (top panel) and strike specific differences when compared with the artificial call options with the reference time to maturity of  $\mathcal{T}_m = 35$  days (bottom panel). The maturities are given by red dashed lines and three detected changepoints in blue

between 30 and 40 days. Moreover, the choice of  $\mathcal{T}_m = 35$  reduces the number of situations where no interpolation can be performed. For example, considering August 13th, 2018 and  $\mathcal{T}_m = 35$ , the artificial option will expire on September 17th, 2018. Thus, the option expiring right before this artificial time to maturity is the one expiring on August 16th, 2018 and the option expiring right after expires on September 20th, 2018. Both dates are sufficiently far from the artificial time to maturity to allow very effective interpolation. On the one hand, if  $\mathcal{T}_m = 39$ , the artificial option would expire on September 21st, 2018 and the option expiring before has the maturity September 20th, 2018, and the one expiring after has the maturity October 18th; in this way, we do not use the front contract expiring on August 16th which would still be very informative and we include the contract expiring on October 18th that is too far from the current date. On the other hand, if  $\mathcal{T}_m \leq 30$ , and the current day is immediately after the expiration of an option, there is a high probability that the artificial option would expire before the maturity of the front contract making impossible to perform any interpolation.

The three specific choices  $\mathcal{T}_m \in \{30, 35, 40\}$  have been particularly chosen for illustrative purposes. However, in general, small values of  $\mathcal{T}_m$  seem to undersmooth the natural market dynamics (see Fig. 4), but the most important structural breaks are still nicely preserved (significant changepoints are detected for trading days no. 10, 16, and 23). Larger values of  $\mathcal{T}_m$  seem to oversmooth the natural market dynamics (see Fig. 5) with only two significant changepoints being detected (trading days nos. 10 and 19). However, the overall conclusions made with respect to the *exogenous effects* are very similar for all three choices of  $\mathcal{T}_m \in \{30, 35, 40\}$ . The first changepoint is always detected at the same location—no matter what is the



**Fig. 5** Artificial call option volatilities of Erste Group for a constant (over time) time to maturity of  $\mathcal{T}_m = 40$  days (top panel) and strike specific differences when compared with the artificial call options with the reference time to maturity of  $\mathcal{T}_m = 35$  days (bottom panel). The maturities are given by red dashed lines, two detected changepoints in blue

underlying time to maturity of the artificial options. The other changepoints depend on the amount of smoothness introduced by the interpolation algorithm. Thus, there is a typical statistical trade-off when determining the value of  $\mathcal{T}_m$  for constructing the artificial options and the value of  $\mathcal{T}_m = 35$  seems to provide the best empirical results.

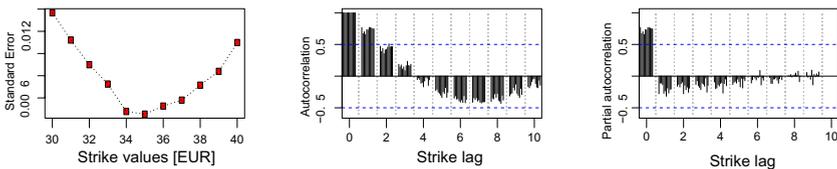
In the second part, we briefly provide some empirical investigation of the model based residuals for the Erste Group call options—used for the practical illustration in the previous section—in order to justify the theoretical assumptions postulated for Scenario 3 in Sect. 3. Crucial assumptions involve non-independent and non-identically distributed error terms which form a specific string mixing sequence—which can also be concluded from the estimated correlations in Table 1 and the partial auto-correlation plot in Fig. 6c. On the other hand, some heteroscedasticity among the strike specific panels is obvious again from Table 1 or, alternatively, from Fig. 6a). The assumptions postulated in Scenario 3 indeed seem to reflect the underlying stochastic nature of the market (interpolated) volatilities.

### 4.3 Simulation study

Finally, we empirically compare different  $\mathcal{T}_m$  testing approaches explicitly mentioned in Sect. 3. Three specific scenarios are considered: a) The follow-up period  $T \in \mathbb{N}$  tends to infinity but the number of panels  $N \in \mathbb{N}$  is kept fixed and the statistical test proposed in Horváth et al. (1999) is applied; b) Both the follow-up period as well as

**Table 1** Residual based empirical estimates for the strike specific standard errors (over time) and the corresponding volatility correlations between two given strikes. The volatility of two distant strikes is clearly less dependent than the volatility of two strikes close to each other

Strike(s) (in EUR)	Standard Errors	Correlations									
		31	32	33	34	35	36	37	38	39	40
30	0.0136	0.90	0.76	0.71	0.58	0.51	0.41	0.30	0.25	0.22	0.18
31	0.0112		0.93	0.90	0.80	0.71	0.59	0.45	0.39	0.33	0.28
32	0.0089			0.95	0.86	0.79	0.67	0.53	0.47	0.41	0.38
33	0.0072				0.95	0.89	0.78	0.64	0.56	0.46	0.39
34	0.0047					0.97	0.91	0.81	0.73	0.61	0.52
35	0.0045						0.97	0.89	0.81	0.68	0.58
36	0.0052							0.97	0.91	0.78	0.67
37	0.0058								0.98	0.88	0.77
38	0.0071									0.94	0.86
39	0.0083										0.94
40	0.0109										



(a) Strikes pecific std.errors (b) Autocorrelation function (c) Autocorrelation function

**Fig. 6** Strike specific standard error estimates with the minimum attained around the at-the-money position (left panel) and the corresponding auto-correlation and partial auto-correlation functions for the time points  $t \in \{1, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$  (middle and right panels)

the number of panels tend to infinity and the test proposed in Horváth and Hušková (2012) is used; c) Eventually, the number of panels is assumed to tend to infinity while the follow-up period is fixed. Either the statistical test based on the test statistic in (6) or that based on the test statistic in (7) is performed and all the results are compared.

In order to closely mimic the example with real data discussed above, relatively small values are considered for the length of the follow-up period and the number of panels (namely,  $T, N \in \{10, 20, 50\}$ ). The error terms in the underlying model (3) are either independent and normally distributed with zero mean and unit variance (a benchmark setup denoted by  $\mathcal{D}_1$ ), or they form a dependent auto-regressive time series of order one (with the dependence coefficient  $\rho = 0.8$  and a burn out period of length 100) denoted by  $\mathcal{D}_2$ , and, finally, block resampled residuals (with a block length of four) taken from the Erste Group data are used to mimic the true (unknown) dependence structure—denoted by  $\mathcal{D}_3$ .

**Table 2** Empirical levels of four statistical tests (for  $\alpha = 0.05$ ) described in Sect. 3 applied to three different scenarios with respect to the length of the follow-up period  $T \in \mathbb{N}$  and the number of panels  $N \in \mathbb{N}$ . The results are collected over 1000 Monte Carlo runs

Simulation setup			$T \rightarrow \infty$	$T \rightarrow \infty$	$T$ fixed $\wedge N \rightarrow \infty$	
$\mathcal{D}$	$T$	$N$	$N$ fixed	$N \rightarrow \infty$	$\mathcal{Q}_N(T)$	$\mathcal{S}_N(T)$
$\mathcal{D}_1$	10/20/50	10	0.20 0.12 0.10	0.06 0.06 0.01	0.10 0.10 0.08	0.10 0.09 0.07
		20	0.09 0.08 0.05	0.06 0.02 0.00	0.08 0.07 0.07	0.07 0.07 0.06
		50	0.10 0.09 0.05	0.05 0.03 0.01	0.07 0.06 0.05	0.07 0.06 0.05
$\mathcal{D}_2$	10/20/50	10	0.03 0.00 0.01	0.07 0.07 0.03	0.21 0.19 0.19	0.20 0.19 0.17
		20	0.08 0.01 0.01	0.08 0.07 0.06	0.13 0.13 0.12	0.14 0.13 0.11
		50	0.10 0.07 0.03	0.08 0.06 0.09	0.10 0.09 0.07	0.09 0.08 0.06
$\mathcal{D}_3$	10/20/50	10	0.06 0.04 0.04	0.07 0.02 0.01	0.18 0.15 0.15	0.17 0.15 0.15
		20	0.08 0.08 0.07	0.06 0.02 0.01	0.12 0.10 0.10	0.10 0.10 0.09
		50	0.09 0.09 0.07	0.04 0.03 0.01	0.09 0.07 0.06	0.07 0.06 0.05

**Table 3** Empirical powers of four statistical tests described in Sect. 3 applied to three different scenarios with respect to the length of the follow-up period  $T \in \mathbb{N}$  and the number of panels  $N \in \mathbb{N}$ . The results are collected over 1000 Monte Carlo runs

Simulation setup			$T \rightarrow \infty$	$T \rightarrow \infty$	$T$ fixed $\wedge N \rightarrow \infty$	
$\mathcal{D}$	$T$	$N$	$N$ fixed	$N \rightarrow \infty$	$\mathcal{Q}_N(T)$	$\mathcal{S}_N(T)$
$\mathcal{D}_1$	10/20/50	10	0.24 0.65 1.00	0.94 0.90 0.81	0.97 0.99 0.99	0.96 0.98 0.98
		20	0.95 0.99 1.00	0.99 0.97 0.58	0.99 1.00 1.00	0.99 1.00 1.00
		50	0.99 1.00 1.00	1.00 1.00 0.97	1.00 1.00 1.00	1.00 1.00 1.00
$\mathcal{D}_2$	10/20/50	10	0.40 0.60 1.00	0.99 0.99 0.99	0.48 0.58 0.61	0.47 0.56 0.56
		20	0.73 0.73 1.00	0.99 1.00 1.00	0.70 0.75 0.79	0.71 0.72 0.72
		50	0.75 0.86 1.00	1.00 1.00 1.00	0.91 0.98 0.99	0.89 0.94 0.97
$\mathcal{D}_3$	10/20/50	10	0.70 0.88 1.00	0.60 0.52 0.36	0.77 0.86 0.88	0.78 0.83 0.83
		20	0.72 0.82 1.00	0.76 0.54 0.20	0.91 0.97 0.99	0.90 0.94 0.97
		50	0.87 1.00 1.00	0.96 0.89 0.29	0.99 1.00 1.00	0.99 1.00 1.00

The empirical levels of the statistical tests performing under the null hypothesis of no changepoint in the model (3) are collected over 1000 Monte Carlo runs in Table 2 (considering the theoretical level  $\alpha = 0.05$ ). Regarding the empirical power of the tests, the following alternative hypothesis is considered: a common changepoint location is placed after the first third of the follow-up period and the corresponding panel-specific changepoint magnitudes are generated randomly from the uniform distribution over the interval  $(0, \theta)$ , for  $\theta > 0$ , so that the signal-to-noise ratio equals one. The empirical performance of the tests under the alternative hypothesis (again over 1000 Monte Carlo runs) is presented in Table 3.

Recall that all statistical tests used in this section (and also mentioned in Sect. 3) are asymptotic tests. The theoretical critical level  $\alpha = 0.05$  should be achieved asymptotically (either for  $T \rightarrow \infty$ , or  $N \rightarrow \infty$ , or both). Considering rather small

values for  $T \in \mathbb{N}$  and  $N \in \mathbb{N}$ , the probability of a type one error is usually larger than expected but it seems to properly converge to the nominal level (at different rates in different scenarios). On the other hand, the empirical powers of the tests seem to be the best for the test statistics defined in (6) and (7), especially when considering rather short follow-up periods, small numbers of panels, and some dependence within the panels.

## 5 Conclusion

The implied volatility serves as a very common and popular tool for analysing options markets but there are also some obvious limitations. For instance, the natural behaviour of the IV smiles usually generates changes of considerably higher magnitudes than those being observed when the market adapts to some sudden external impulses. Therefore, when analysing the IV smiles automatically, most of the detected changes are very likely to be only related to the underlying market dynamics and any detection of exogenous effects is almost impossible. However, practitioners and financial agents are typically interested in all kinds of external events that may or may not affect the riskiness (or the price) of the underlying asset. Therefore, we have proposed a whole methodological approach aiming explicitly and exclusively at the analysis of the market reactions to external, *exogenous effects*. The key contribution of our paper is four-fold: First, the *artificial options* with a *constant time to maturity* over time are introduced as the key tool for the analysis of *exogenous effects*; Second, the standard implied volatility is shown to be insufficient for a proper detection of the exogenous effects and the implied volatility of the artificial options is empirically proved to be an underappreciated surrogate able to eliminate the natural market dynamics while conveniently preserving all exogenous effects; Third, the implied volatility of the artificial options is constructed in a simple and straightforward way using a (weighted) linear interpolation of the raw implied volatilities while introducing only a very mild aggregation of the existing information (almost no information loss); Finally, as a direct consequence, the changes due to the *exogenous causes* are emphasized and a formal consistent statistical test is proposed as a valid inferential tool to detect statistically significant market changes which can be usually directly linked by market experts to some specific external (usually man-made and well-recognized) event. Thus, the proposed method allows an effective and efficient analysis of market behaviour when focusing on the changes caused by the external causes rather than the natural market dynamics itself. The presented approach is simple and the analysis can be performed within a fully automatic (data-driven) procedure. The artificial options are easily constructed using the standard IV values available in various financial databases and the [IVintAO Algorithm](#) explicitly described in this paper.

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## Declarations

**Conflict of interest** The authors have no conflicts of interest to declare that are relevant to the content of this article.

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