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Reliability intelligence analysis of concrete arch bridge based on Kriging model and PSOSA hybrid algorithm

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Abstract: The traditional probabilistic reliability analysis methods have problems such as poor convergence, low calculation accuracy, and long time-consuming in the reliability calculation of concrete arch bridges. Due to the uncertainty of the parameters of the structure itself, the performance function is highly nonlinear, and other factors. A reliability calculation method for concrete arch bridges based on the Kriging model and particle swarm optimization algorithm (PSOSA) based on a simulated annealing algorithm is proposed. Take advantage of the Kriging model in small samples, nonlinear, high-dimensional data processing capabilities. With the help of the PSO algorithm, it has the advantages of strong global optimization ability and strong robustness. Combined with the SA algorithm self-adaptive, variable probability mutation operation. The ability of the PSO algorithm to get rid of the local minima is enhanced and supplemented, effectively avoiding falling into the local minima and making the result tend to the global optimum, which improves the slow convergence speed and precociousness of the traditional PSO algorithm. A numerical example verifies the method's effectiveness, and a reliability evaluation of an actual concrete arch bridge is carried out. The research results show that the method improves the calculation accuracy, dramatically improves the calculation efficiency, and realizes the rapid and accurate assessment of the reliability of complex bridge structures.

Key words: bridge engineering; reliability analysis; Kriging surrogate model; PSO algorithm; SA algorithm

1 Introduction

The performance function of complex bridge structures is often highly nonlinear, and there is no clear analytical expression, and the value of the performance function generally needs to be obtained through time-consuming numerical calculation (Frangopol and Imai 2000). How to quickly and accurately analyze the reliability of complex bridge structures has become a hot issue in current structural reliability analysis. Although the traditional Monte Carlo simulation method (Ben Seghier et al. 2018) (MCS) is suitable for solving the implicit performance function's reliability problem, the calculation accuracy is high. However, due to the large number of sampling times required, especially when the performance function value needs to be obtained through finite elements, the vast calculation work leads to highly lengthy and time-consuming, and the calculation efficiency is very low. The conventional first-order second-moment method (Arab et al. 2019) (FORM) and second-order second-moment method (Lim et al. 2014) (SORM) have the problem that the derivative of the implicit performance function is challenging to solve. However, the response surface method is currently a powerful tool for solving complex structural reliability problems.

The basic idea of the response surface method is to use an easy-to-handle regression model as the response surface function based on several sample points to solve the implicit or computationally time-consuming performance function problem. Make the implicit performance function explicit and combine it with the conventional reliability analysis method to solve the failure probability (Fang 2020). The key to the response surface method is the fitting effect of the response surface function. When the performance function is strongly nonlinear, the widely used classical response surface method (Luo and Zhu 2012, Gavin and Yau 2008) (RSM) based on quadratic polynomial is difficult to accurately approximate the real performance function, resulting in a significant calculation error. In recent years, some scholars have proposed using a regression model with better regression performance to construct a response surface and achieved good results, such as artificial neural network (Han et al. 2019, Marugan et al. 2019) (ANN), radial basis function (Zhang et al. 2021, Wang and Fang 2018) (RBF), support vector regression (Pan et al. 2020) (SVR). However, in practical applications, the above-mentioned response surface method still has many shortcomings. For example, ANN has problems such as difficulty determining the optimal network topology, poor generalization ability under minor sample conditions, and overfitting (Xu et al. 2020). SVR has the problem that the model's optimal parameters and loss function are difficult to determine (Pepper et al. 2022). In addition, there is a common problem with the above methods: the computational accuracy is overly dependent on the construction of preset training samples. When the preset training sample size is small or the distribution is not ideal, the regression model will generate a large fitting error, leading to a significant error in the reliability calculation result. Conversely, when the preset training sample size is large, the computational accuracy is high, but the computational efficiency is low.

The Kriging surrogate model is a machine-learning method developed in recent years. It has a strict statistical theoretical foundation and good adaptability to deal with complex regression problems such as high dimensions, small samples, and nonlinearity. It has been widely used in many fields (Sundar and Shields 2019, Yan et al. 2020). The Kriging model includes two models: regression model and correlation model. The regression model is a global approximation in space, and the attribute value of the unknown point is estimated by assigning weights to the known points around the unknown point. Correlation models reflect spatial distribution structures or spatial correlation types. At the same time, the range of the spatial correlation is given, and the observed value of the sampling point can be used to estimate the variable value of the unsampled point in the study area. In addition, selecting and determining relevant model parameters in the Kriging model is a multivariate optimization process. The performance of the numerical optimization algorithm directly affects the accuracy and stability of the relevant model parameters, which in turn affects the performance of the Kriging model (Chu et al. 2020, Qin et al. 2019). Most of the current Kriging models use the pattern search method (Liu et al. 2008, Li 2015) to solve the parameters of the related models. Like other traditional numerical optimization methods (such as the fastest descent method and quadratic programming method), these optimization algorithms have the advantage of high efficiency. However, it is very sensitive to the starting point selection, and it is easy to fall into the trap of local optimum, which cannot effectively guarantee convergence to the global optimum. Therefore, in order to improve the shortcomings of the Kriging model, strive to ensure high computational efficiency while obtaining the global optimum. Therefore, it is necessary to combine better optimization algorithms to improve the optimization efficiency of model parameters.

The PSOSA algorithm integrates two algorithms with different optimization mechanisms, the simulated annealing algorithm (Lee and Kim 2020) (SA) and the particle swarm optimization algorithm (Gu and Hao 2020) (PSO). It is beneficial to enrich the search behavior of the optimization process and enhance searchability and efficiency in the global and local sense. The SA algorithm adopts a serial optimization structure (Zhai and Feng 2022), while the PSO adopts a swarm parallel search (Jiang et al. 2021). The combination of the two can make the SA algorithm a parallel SA algorithm and improve its optimization performance. At the same time, the SA algorithm, as an adaptive and variable probability mutation operation, enhances and supplements the PSO algorithm's ability to get rid of local minima, effectively avoids falling into local minima, and makes the algorithm eventually tend to the global optimum. Therefore, the Kriging-PSOSA hybrid algorithm is proposed by combining the Kriging model with the particle swarm optimization algorithm (PSOSA) based on the simulated annealing algorithm. This method takes advantage of the Kriging model's advantages in dealing with reliability problems such as complex structural uncertainty, high dimensions, and small samples. It also takes advantage of the PSOSA hybrid algorithm that can better update the coordinates of the particle swarm to search for the optimal global solution faster. The accuracy and efficiency of reliability calculation of complex structures are effectively improved.

In order to solve the above problems and propose applying the Kriging-PSOSA method to solving reliability problems. In this paper, the Kriging model, based on the theoretical basis of statistics and has analytical uncertainties, high dimensions, and nonlinear problems, is used to construct the response surface of the implicit performance function. The Kriging model is combined with the particle swarm optimization algorithm (PSOSA) based on a simulated annealing algorithm to improve the optimization efficiency of model parameters. The constructed implicit function can truly simulate the limit state function of the structure, and then a Kriging-PSOSA hybrid algorithm suitable for the reliability assessment of such complex structures is proposed. The correctness and feasibility of the method proposed in this paper in reliability calculation are verified by numerical example analysis and practical engineering application of a concrete arch bridge. The research results show that the method overcomes the limitation of the classical response surface method on the reliability of highly nonlinear structures. It solves the problems of the low computational efficiency of the MCS method, and the computational accuracy of the existing response surface method is overly dependent on the scale and distribution of preset samples and achieves the purpose of rapid and accurate assessment of the reliability of complex bridge structures.

2 Basic theory of Kriging-PSOSA hybrid algorithm

2.1 Theory of Kriging Model

The Kriging model is an interpolation model formed by superimposing a non-parametric stochastic process with a parametric linear regression model **(Yang *et al.* 2022)**. The expression for the model is:

$$G(x) = \Gamma(\beta, x) + z(x) = f^T(x)\beta + z(x) \tag{1}$$

In the formula: $\Gamma(\beta, x)$ is the polynomial regression model. β is the regression coefficient vector, $\beta = [\beta_1, ...\beta_p]^T$. f^T is the polynomial of variable x , $f^T = [f_1(x), f_2(x), ..., f_p(x)]^T$. $z(x)$ is a random Gaussian process with zero mean and variance σ^2. At different locations in the design space, the correlation between these random variables is expressed by covariance as:

$$Cov[z(x_i), z(x_j)] = \sigma^2 R(x_i, x_j; \theta) \tag{2}$$

In the formula: $R(x_i, x_j; \theta)$ is the correlation function between x_i and x_j. In the classic Kriging model, common correlation function models include exponential model, Gaussian model, linear model, spline function model, etc. At present, the commonly used function model is the Gaussian model **(Wang 2021)**:

$$R(x_i, x_j; \theta) = \prod_{m=1}^{M} \exp[-\theta_m (x_i^m - x_j^m)^2] \tag{3}$$

In the formula: θ is the parameter vector, $\theta = [\theta_1, \theta_2, ..., \theta_m]^T$. m is the m -th dimension element of the input vector; M is the total dimension of the input vector.

Define the correlation matrix $R[(x_i, x_j; \theta)]_{N_0 \times N_0}$, then the estimated values of β and σ^2 are:

$$\beta = (F^T R^{-1} F) F^T R^{-1} G \tag{4}$$

$$\sigma^2 = \frac{1}{N_0}(G - \beta F)^T R^{-1}(G - \beta F) \tag{5}$$

In the formula: F is the identity matrix of $N_0 \times 1$. N_0 represents the number of training sample points. From equations (1) to (3), it can be known that a Kriging model can be completely defined by the regression coefficient vector β, the variance σ^2 of the random process, and the parameter vector θ. From equations (4) and (5), it can be known that the regression coefficient vector β and the variance σ^2 of the random process depend on the parameter vector θ. Therefore, when constructing the Kriging model, the parameter vector θ should be obtained first according to the sample points, and this process can be realized by maximum likelihood estimation. which is:

$$\theta = \arg\max_{\theta>0}(-N_0 \ln(\sigma^2) - \ln(|R|)) \tag{6}$$

The mean and variance of the predicted value $G(x)$ for the predicted point x is expressed as:

$$\mu_G(x) = \beta + r(x)R^{-1}(G - \beta F) \tag{7}$$

$$\sigma_G{}^2(x) = \sigma^2(x)(1 + \mu^T(x)(F^T R^{-1} F)^{-1} \mu(x) - r^T(x)R^{-1}r(x)) \tag{8}$$

In the formula: $r(x) = [\square(x, x_1; \theta), \square(x, x_2; \theta), ..., \square(x, x_{N_0}; \theta)]$, $\mu(x) = F^T R^{-1} r(x) - f(x)$. And $\mu_G(x)$ is taken as the predicted value of point x.

2.2 The basic theory of particle swarm optimization algorithm (PSOSA) based on simulated annealing algorithm

Particle swarm optimization (PSO) is a global random search algorithm based on swarm intelligence, which is proposed by simulating birds' migration and flocking behavior during foraging. Based on the concepts of "population" and "evolution", it realizes the search for the optimal solution in complex space through cooperation and competition among individuals (Pawan et al. 2022). Suppose that at time t, in an n-dimensional search space $S \in R^n$ and a population consisting of m particles, the position of the i-th particle is represented by an n-dimensional vector, namely $X_i = [X_{i1}, X_{i2}, ..., X_{in}]$. Each particle represents a candidate solution of the problem to be sought, and the fitness value is calculated by substituting X_i into the fitness function. The quality of each solution is determined by its corresponding fitness value. The better the fitness value, the better the corresponding solution. The closer to the true solution, the moving speed of the particle is also an n-dimensional vector, namely $V_i = [V_{i1}, V_{i2}, ..., V_{in}]$. The optimal position searched by the i-th particle so far in the S space is called the extreme individual value, denoted as $P_i = [P_{i1}, P_{i2}, ..., P_{in}]$. The optimal position searched by the particle swarm so far is the global extremum, marked as $g_{best} = [P_{q1}, P_{q2}, ..., P_{qn}]$, to represent the position of the best particle in the swarm. Each particle updates its velocity and position during the optimization process according to the following formula.

$$V_i^{t+1} = \omega \times V_i^t + c_1 r_1 (P_i^t - X_i^t) + c_2 r_2 (g_{best}{}^t - X_i^t) \tag{9}$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{10}$$

In the formula: particle number $i = 1, 2, ..., m$. t is the current iteration number. ω is the inertia weight, representing the influence coefficient of the last speed on the particle. c_1 and c_2 are learning factors, and c_1 represents the cognitive ability of the particle's experience, which is used to adjust the progress of the particle flying toward its best position. c_2 represents the cognitive ability of the particle to learn social experience and adjusts the step size of the particle to the optimal global position. r_1 and r_2 are random numbers uniformly distributed in the interval [0, 1]. The purpose is to allow the particle to fly to the best position of the particle itself and the global best position of the particle with an equal probability of acceleration.

The simulated annealing algorithm is an extension of the local search method. However, it differs from local search by selecting the state with the largest cost value in the neighborhood with a certain probability of jumping out of the local extreme point. The acceptance criterion allows the objective function to deteriorate within a limited range, accepting new solutions with a certain probability (Gao et al. 2022). In reference (Aslett et al.

2017), the acceptance criterion allows the objective function to deteriorate without choosing according to probability, but directly according to $\Delta E < e$, where ΔE is the change in fitness value caused by two positions, and e is the allowable target function deterioration range. Therefore, this paper combines the core steps of the two and proposes a particle swarm optimization algorithm (PSOSA) based on simulated annealing algorithm. Initialize each particle, set the number of particles n, randomly generate n initial solutions or give n initial solutions, and randomly generate n initial velocities. According to the current position and speed, the new position of each particle is generated, and the fitness value of each particle's new position is calculated. For each particle, if the fitness value of the particle is better than the original individual extreme value p_i, set the current fitness value to the individual extreme value p_i. According to each particle's individual extreme value p_i, find the global extreme value p_g. Update itself speed according to formula (9), update the current position according to formula (10), and calculate the amount of adaptation value change ΔE caused by the two positions. If $\Delta E < e$, accept the new value; otherwise, reject. If the conditions are not met, or the maximum number of iterations is not reached, go to step 3-otherwise, end.

3 Design and verification of calculation reliability based on Kriging-PSOSA hybrid algorithm

3.1 Design of Kriging-PSOSA calculation method

Based on the finite element model and the MATLAB calculation program, the Kriging-PSOSA hybrid response surface method is proposed to calculate and analyze the structural reliability. The process is as follows: (1) Determine the statistical characteristics and probability distribution of random variables in the operating state of the bridge structure, and use the uniform design method to generate input sample points. (2) Establish a structural finite element model based on the bridge design data and operating conditions, calculate the target variables corresponding to each input sample, and obtain an output sample. And then form a training sample with the input sample. (3) Normalize the sample points, and use the DACE toolbox (Soltani-Mohammadi 2016) to establish the basic Kriging model. Through the unsupervised training and parameter optimization process of the basic model by inputting sample points, the structural Kriging model is obtained. (4) Standard normalization of random variables, using penalty function to transform the constrained optimization problem into an unconstrained optimization problem, and construct a fitness equation suitable for the PSOSA algorithm to solve. Furthermore, the PSOSA algorithm updates the optimal position of search particles and particle swarms. Iteratively obtains the optimal weights of random variables to support the unsupervised learning process of the Kriging model. (5) A mathematical model for solving the structural reliability index is established through the prediction results of the Kriging model. In this process, it is necessary to update and optimize the samples of each Kriging prediction model so that the Kriging prediction model can well approximate the sample points; until the model builds a sufficiently accurate response surface, it can realistically simulate the structural limit state function. The specific flow of the Kriging-PSOSA hybrid algorithm for bridge structural reliability analysis is shown in Figure 1.

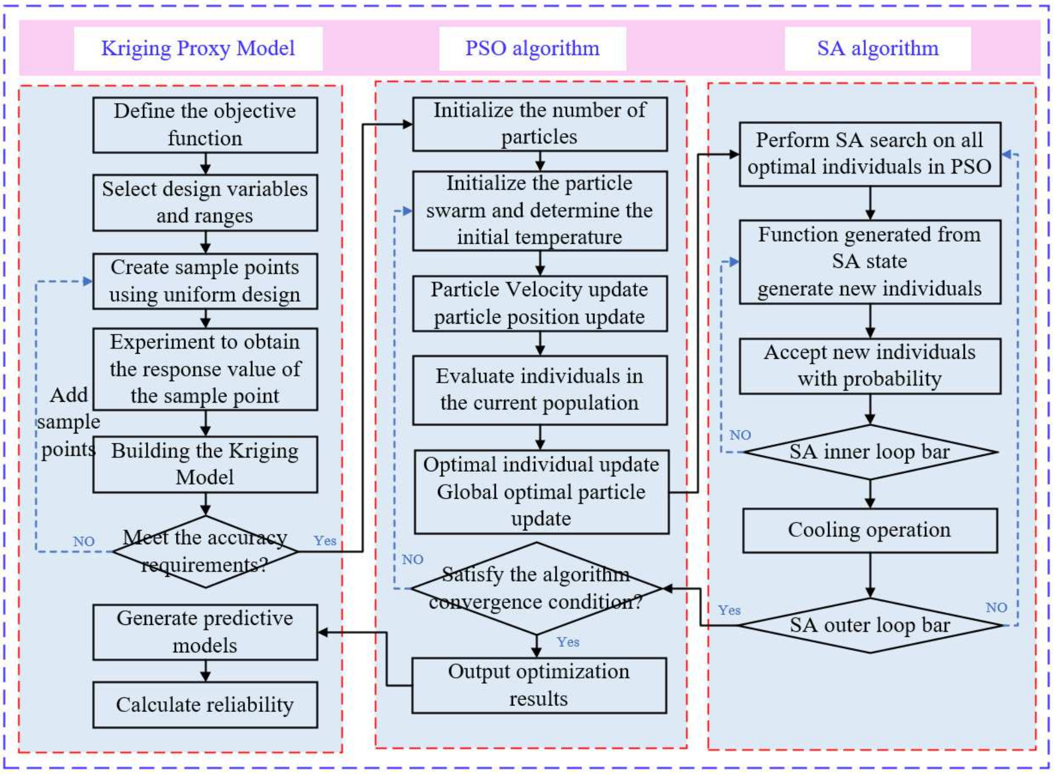


Figure 1 Kriging-PSOSA hybrid algorithm reliability calculation design and process

In order to reflect the accuracy and efficiency of the method, this paper uses the response surface method based on the Kriging surrogate model, the support vector machine response surface method, and other types of machine learning methods to analyze the reliability of the same structure. Moreover, through the accuracy analysis, the number of iteration steps, and other aspects, different methods are compared and analyzed to verify the feasibility and efficiency of the structural reliability calculation based on the Kriging-PSOSA hybrid response surface method.

3.2 Example 1 Verification

As shown in Figure 2, for the ten-bar truss structure, let the length of the member be L, the cross-sectional area of the member be A_s, the elastic modulus is E, and the external loads are P_1, P_2, and P_3. All random variables are normally distributed, and their distribution characteristics are shown in Table 1. Taking the vertical displacement limit $V(x)$

of the No. 2 node as the control variable, the allowable displacement is 0.004m, and the performance function $g = 0.004 - V(x)$ is established.

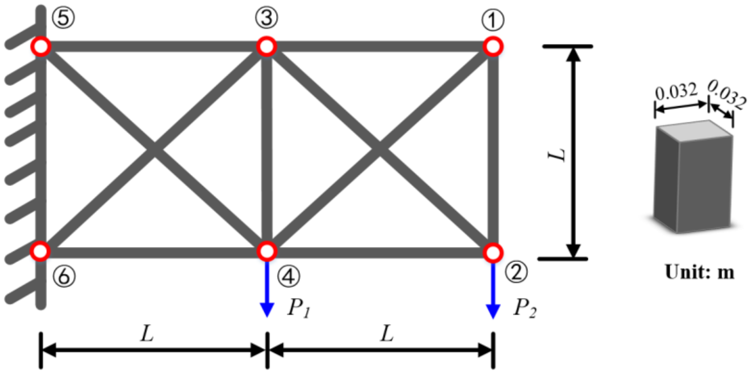
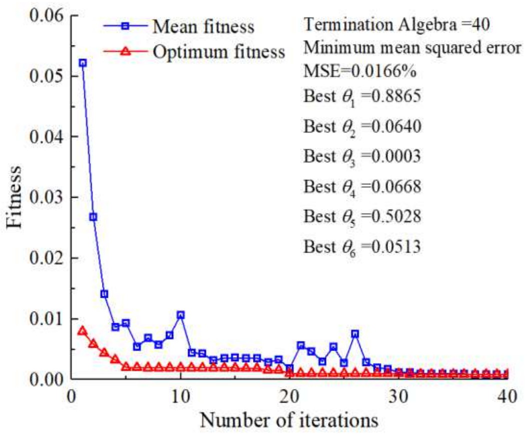


Figure 2 Calculation diagram of ten-bar truss

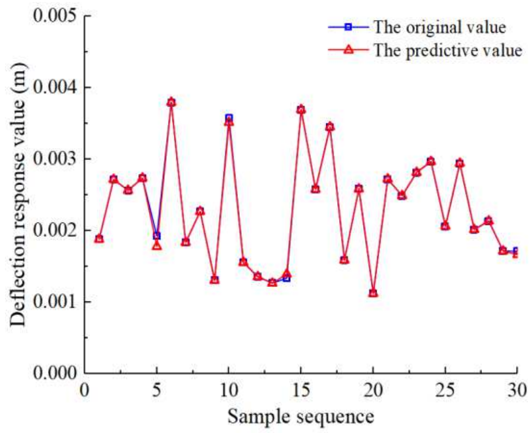
Table 1 Statistical parameters of random variables of cross truss structure

Random Variables	Mean μ	Standard deviation σ	Distribution type
L/m	1.0	0.05	normal distribution
A/m^2	0.001	0.00015	normal distribution
E/GP_a	100.0	5.0	normal distribution
P_1/kN	80.0	4.0	normal distribution
P_2/kN	10.0	0.5	normal distribution
P_3/kN	10.0	0.5	normal distribution

It can be seen from the above performance functions that it is a structural reliability problem with high-dimensional nonlinear implicit performance functions. The vertical displacement in the performance function is obtained using the ANSYS commercial finite element analysis program. The uniform design sampling method was used to generate 30 groups of samples within the range of $[\mu-3\sigma, \mu+3\sigma]$, and the training samples were normalized and entered into the Kriging model for training, and the PSOSA algorithm was used to optimize the parameters. The optimization process to obtain the optimal weight parameter B of the model is shown in Figure 3(a). The optimal weight parameter of the model can be obtained by searching the PSOSA algorithm as $[\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6] = [0.8865, 0.0640, 0.0003, 0.0668, 0.5028, 0.0513]$.



(a) Parameter optimization process of PSOSA algorithm



(b) Kriging model sample regression prediction results

Figure 3 The calculation process of Kriging-PSOSA mixed response surface method

Figure 3(b) shows the prediction result of the sample regression based on the Kriging model. It can be seen that the Kriging model constructed by the sample learning can accurately reflect the actual response surface of the performance function. Table 2 shows the calculation results of this example by the method in this paper and different methods in reference (Su et al. 2013). Taking the calculation result of Monte Carlo sampling 200,000 times as the exact solution, its failure probability is 5.2253×10^{-3}, and the corresponding reliability index β is 2.6013. The failure probability calculated by the method in this paper is 5.1087×10^{-3}, the corresponding reliability index β is 2.6133, and the relative error is only 0.461%. Its iteration times and calculation accuracy are better than the Kriging model response surface method and support vector machine method (SVM). The calculation results of the example can fully demonstrate that the Kriging-PSOSA hybrid response surface method can well solve the structural reliability problem of high-dimensional nonlinear implicit performance functions.

Table 2 Comparison of results of different reliability calculation methods for ten-bar truss

Calculation method	Number of samples	Number of iterations	Failure probability	Reliability index β	Relative error
Monte Carlo method	200000	/	5.2253e-3	2.6013	0

Krging Model Response Surface Method	30	44	5.3993e-3	2.5834	0.688%
Support Vector Machines (SVM)	30	29	4.9736e-3	2.6272	0.996%
Kriging-PSOSA	30	20	5.1087e-3	2.6133	0.461%

3.3 Example 2 Verification

As shown in Figure 4, a plane truss has a calculated span and height are 9.0m and 1.5m, respectively. The cross-sectional area of the member is a random variable A with a mean of $1.6\times10^{-3}m^2$. Moreover, it is assumed that the elastic modulus E and the concentrated loads P_0, P_1, and P_2 are random variables, and the mean values are $2.0\times10^7 kN/m^2$, $30kN$, $50kN$, and $20kN$, respectively. And the coefficient of variation of each random variable is 0.1. The node's vertical displacement limit at the lower chord's midspan (node number 4) is 0.12m. The Kriging-PSOSA hybrid response surface method is used to calculate the reliability index of the plane truss. The statistical parameters of each variable of the structure are shown in Table 3.

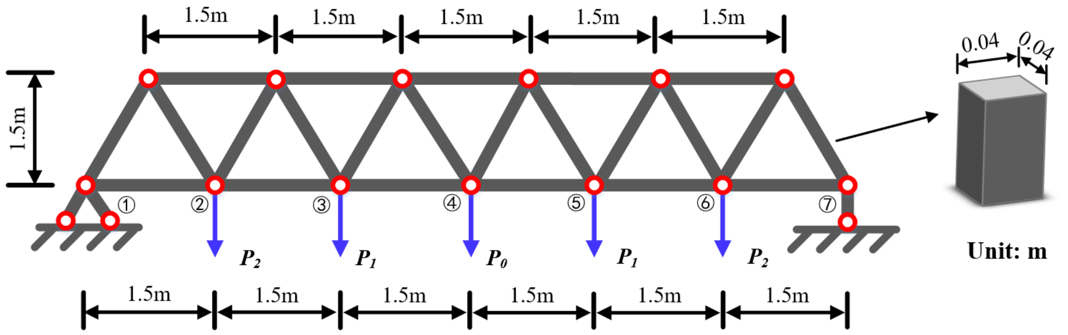


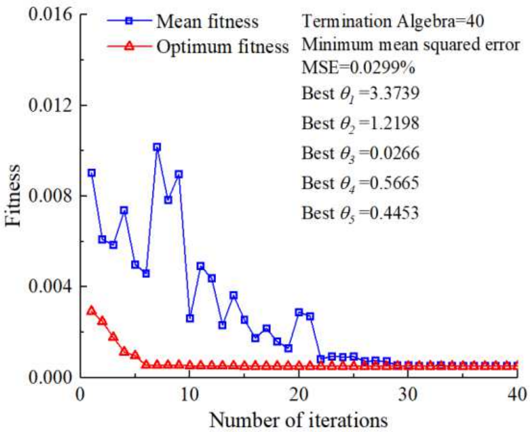
Figure 4 Reliability calculation diagram of plane truss

Table 3 Statistical parameters of random variables of plane truss structure

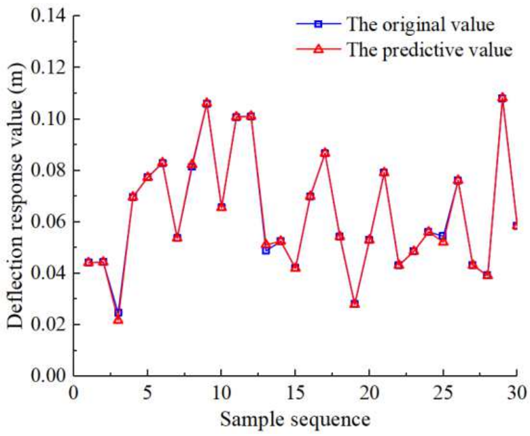
Random Variables	Mean μ	Standard deviation σ	Distribution type
A/m^2	0.0016	0.00016	lognormal
$E/(10^7 kN\cdot m^{-2})$	2	0.2	lognormal
P_0/kN	30	3	Extreme value type I
P_1/kN	50	5	Extreme value type I

P_2 / kN	20	2	Extreme value type I

According to the limit state space function containing five random variables, a uniform design experiment was used to generate 30 groups of samples within the range of $[\mu-3\sigma,\mu+3\sigma]$. At this time, the interval contains 99.73% of the points, which meets the requirements of reliability calculation. Substitute the input sample into the ANSYS finite element model to calculate the corresponding response value (the vertical displacement value of the midspan node of the lower chord). The sample input and output points are combined to form a training sample, and the sample points are normalized and fed into the Kriging model for training. The PSOSA algorithm is used to optimize the parameters, and the optimization process of the optimal weight parameter $\theta_1 \sim \theta_5$ of the model is obtained, as shown in Figure 5(a). It can be seen from Figure 5(b) that the response surface function established based on the Kriging model can truly simulate the structural limit state function with good accuracy.



(a) PSOSA parameter optimization process



(b) Kriging model sample regression prediction results

Figure 5 Process diagram of Kriging-PSOSA mixed response surface method

The failure probability calculated by the method in this paper is 9.1046×10^{-4}, and the corresponding reliability index β is 3.1182. In addition, according to reference (Yang et al. 2014), the failure probability of the plane truss structure obtained by 200,000 Monte Carlo important sampling simulations is 9.3322×10^{-3}, and the corresponding reliability index is 3.1107. Considering the Monte Carlo calculation result as the exact value, the relative error of the Kriging-PSOSA algorithm is only 0.241%, which is similar to the reference results. In addition, the calculation methods and processes of different reliability of the plane truss are shown in Table 4.

Table 4 Comparison of results of different reliability calculation methods for plane trusses

Calculation method	Number of samples	Number of iterations	Failure probability	Reliability index β	Relative error
Monte Carlo method	200000	/	9.3322e-4	3.1107	0
Krging Model Response Surface Method	30	43	8.9204e-4	3.1243	0.437%
Support Vector Machines (SVM)	30	29	8.8824e-4	3.1255	0.476%
Kriging-PSOSA	30	22	9.1046e-4	3.1182	0.241%

It can be seen from Table 4 that compared with the Monte Carlo method, Kriging model response surface method, and support vector machine method (SVM). The relative error of the Kriging-PSOSA method proposed in this paper is only 0.241%, and the convergence condition is reached when the number of sample iterations is 22. It can be seen that the method in this paper has certain advantages in the calculation accuracy and calculation efficiency, and it has the characteristics of high precision and high efficiency.

In addition, the structural model of the above two examples is relatively simple. Compared with the traditional reliability calculation method, it is difficult to reflect the advantages of the Kriging-PSOSA hybrid response surface method in the reliability calculation of complex bridge structures. In order to further illustrate the application effect and advantages of this method in practical complex bridge structures and high-dimensional nonlinear implicit performance functions. In this paper, the reliability analysis and calculation of the deflection system of an actual concrete tied arch bridge is carried out, and the application of the Kriging-PSOSA hybrid response surface method in the actual complex bridge structure is further verified.

4 Engineering application examples of reliability analysis of concrete arch bridges

4.1 Project overview and establishment of bridge finite element model

A concrete arch bridge is a bottom-bearing reinforced concrete tied arch bridge with a span of 70m. The sag-span ratio is 1/5, the sag height is 14m, the arch axis is a quadratic parabola, and the bridge deck width is 10.0m. The main arch and longitudinal beams are made of C50 concrete, the wind bracing is made of C40 concrete, and the suspenders are flexible. The site photo of the concrete tied arch bridge is shown in Figure 6. According to the design data, ANSYS commercial finite element software is used to establish the bridge's initial finite element analysis model. Arch ribs and longitudinal beams are simulated by the BEAM44 element. The suspender is simulated by a three-dimensional unidirectional force LINK10 element, and the tensile force of the suspender is applied in the form of initial strain. The bridge deck is simulated with SHELL181 elements. The entire bridge has a total of 2049 nodes and 2360 units. The finite element model of the entire bridge is shown in Figure 7.

Figure 6 Actual bridge scene diagram

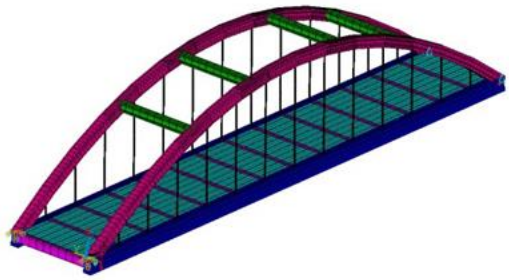


Figure 7 Finite element model of the entire bridge

4.2 Analysis and calculation of bridge deflection reliability

According to "General Design Specification for Highway Bridges and Culverts" (JTG D60-2015) and reference (Chen et al. 2007). The maximum allowable vertical deflection of the main arch under the action of vehicle load (excluding impact) is $\delta = L/1000 = 0.07m$. The maximum allowable vertical deflection of the main girder and bridge deck under the action of vehicle load (excluding impact) is $\delta = L/800 = 0.0875m$. A performance function for establishing the limit state of regular use, namely:

$$\mu_1 = 0.07 - \mu_{v1}(x) \quad (11)$$

$$\mu_2 = 0.0875 - \mu_{v2}(x) \quad (12)$$

Where: $\mu_{v1}(x)$ is the maximum vertical deflection of the main arch. $\mu_{v2}(x)$ is the maximum vertical deflection of the main girder and deck.

During the service process of bridges, there are uncertainties due to the changes in parameters such as material properties, structural geometric dimensions, and external loads with environmental changes. As a result, there is a big difference between the actual structural and theoretical design states. Many factors affect the safety of concrete arch bridges during operation. In this paper, the geometric deformation of the structure is used as the control index, and the random factors shown in Table 5 are selected for analysis mainly from the factors that significantly influence the bridge-forming state of the structure. The parameters of random variables are obtained through design data and actual measurement, as shown in Table 5.

Table 5 Statistical characteristics of random variables of concrete arch bridges

Random Variables	Type	Mean	Coefficient of variation	Distribution form
E_1 / MPa	Elastic modulus of main beam	3.45e4	0.1	Normal distribution
E_2 / MPa	Arch rib elastic modulus	3.45e4	0.1	Normal distribution
E_3 / MPa	Tie beam modulus of elasticity	3.45e4	0.1	Normal distribution
E_4 / MPa	Wind brace elastic modulus	3.25e4	0.1	Normal distribution
E_5 / MPa	Suspender modulus of elasticity	6.10e4	0.1	Normal distribution
$r_1 / (kN \cdot m^{-3})$	Main beam bulk density	24.0	0.1	Normal distribution

$r_2 / (kN \cdot m^{-3})$	Arch rib bulk density	24.0	0.1	Normal distribution
$r_3 / (kN \cdot m^{-3})$	Tie beam bulk density	24.0	0.1	Normal distribution
$r_4 / (kN \cdot m^{-3})$	Wind brace bulk density	24.0	0.1	Normal distribution
$r_5 / (kN \cdot m^{-3})$	Suspender bulk density	78.0	0.1	Normal distribution
A_1 / m^2	Main beam cross-sectional area	1.98	0.05	Lognormal
A_2 / m^2	Arch rib cross-sectional area	1.43	0.05	Lognormal
I_1 / m^4	Bending moment of inertia of main beam	0.535	0.05	Lognormal
I_2 / m^4	Arch rib bending moment of inertia	0.201	0.05	Lognormal
$q_1 / (N \cdot m^{-1})$	Vehicle load	25.5	0.13	Extreme value type I

According to the "Unified Standard for Reliability Design of Engineering Structures" (GB 50153-2008) and reference (Yang and Qin 2008), it can be known that the engineering reliability requirements can be met when the engineering structure reliability index β is greater than 4.0. A 15-dimensional space is formed with random parameter variable $E_1, E_2, E_3, E_4, E_5, r_1, r_2, r_3, r_4, r_5, A_1, A_2, I_1, I_2, q_1$. The uniform design sampling method was used to generate 50 groups of random samples within the range of $[\mu - 3\sigma, \mu + 3\sigma]$ to form random input samples, as shown in Table 6 (due to space limitations, only the first three groups are listed). Based on the finite element model of the concrete arch bridge, the above samples are substituted into the calculated response values in turn, and 50 sets of corresponding vertical deflection response values of the main arch and vertical deflection of the main beam are obtained. Finally, two sets of training samples of random parameter-deflection of main arch and random parameter-deflection of main beam are generated. The two groups of sample points were normalized and substituted into the Kriging model for training, and the PSOSA algorithm was used for parameter optimization.

Table 6 Training samples for uniform design generation variables

No	E_1	E_2	E_3	E_4	E_5	r_1	r_2
1	2.7107e4	4.0625e4	2.7529e4	3.0486e4	5.0169e4	23.559	27.673
2	2.7529e4	4.3582e4	3.7245e4	2.8374e4	7.5565e4	30.906	24.147
3	4.2315e4	3.0909e4	4.4850e4	3.0064e4	4.2700e4	22.971	24.441

Table 6 (Continued) Training samples for uniform design generation variables

No	r_3	r_4	r_5	A_1	A_2	I_1	I_2	q_1
1	24.441	30.612	98.535	1.919	1.754	0.5055	0.1604	20.660
2	19.739	20.914	68.927	2.235	1.614	0.4727	0.1702	28.466
3	18.857	24.440	80.388	2.574	1.544	0.6365	0.2219	30.964

After the two sets of training samples of the main arch and the main beam are optimized by the PSOSA algorithm, the optimal weight parameter $\theta_1 \sim \theta_{15}$ of the model is obtained, respectively. The optimization process and results are shown in Figure 8-9. It can be seen from Figure 8 and Figure 9 that the minimum mean square error of the weight parameters optimized by the PSOSA algorithm is as low as 0.0174% and 0.0224%, respectively. It is sufficient to ensure the construction of high-precision response surfaces.

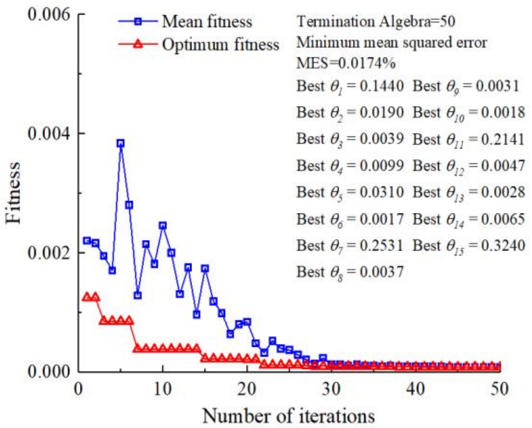


Figure 8 Parameter optimization of PSOSA algorithm for deflection response of main arch

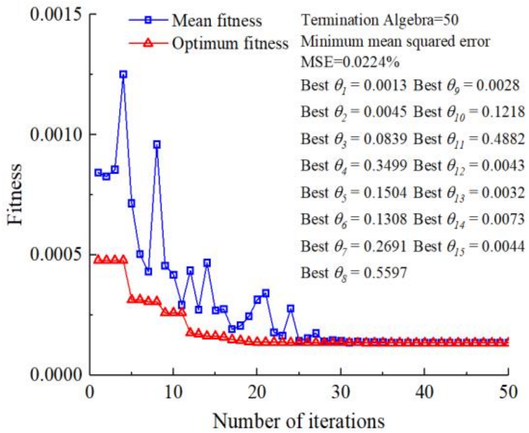
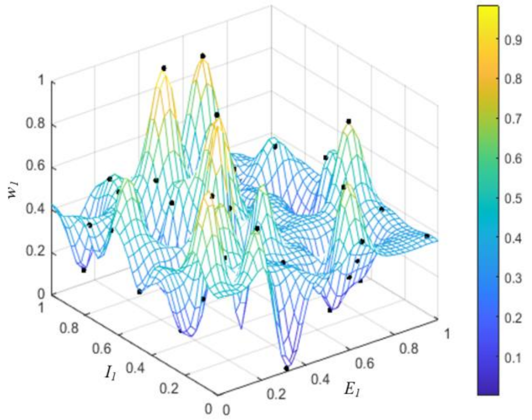
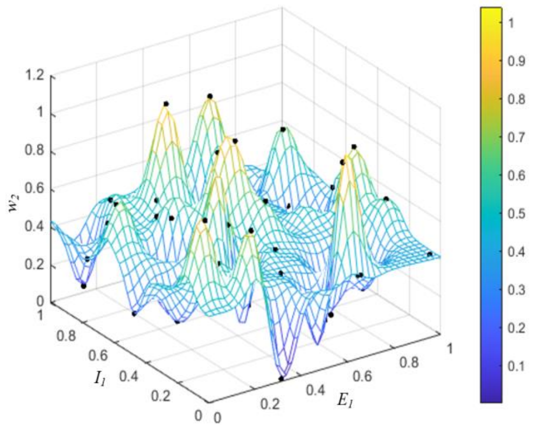


Figure 9 Parameter optimization of PSOSA algorithm for deflection response of main beam

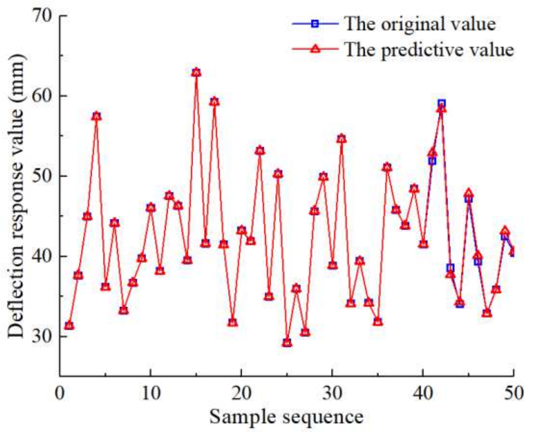
Based on the Kriging model, the deflection response surface functions of the main arch and the main beam were constructed, respectively. This paper gives the response surface diagrams of parameters E_1, I_1, and main arch and main beam, respectively, as shown in Figure 10(a)-(b). The response surface plot can reflect the fluctuation of the deflection response value within the sample threshold. In addition, based on the Kriging model, the training samples are used for regression prediction, and the prediction results are shown in Figure 10(c)-(d). It can be seen that the response surface function constructed by the Kriging model can truly simulate the structural limit state functions of the main arch and the bridge deck and has good accuracy.



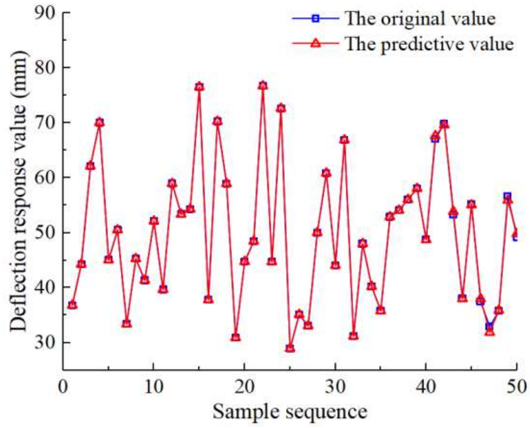
(a) Deflection response surface of main arch E_1-I_1



(b) Deflection response surface diagram of main beam E_1-I_1



(c) Sample regression prediction of main arch deflection response surface



(d) Sample regression prediction of main beam deflection response surface

Figure 10 Comparison of sample regression predictions of response surface functions

Using the response surface model obtained by the Kriging-PSOSA hybrid algorithm for iterative calculation, the failure probability of the main arch of the concrete arch bridge based on the vertical deflection index is 2.4253×10^{-5}, and the corresponding main arch reliability index β is 4.1316. The failure probability of the main beam based on the vertical deflection index is 5.2891×10^{-6}, and the corresponding main beam reliability index β is 4.4667. All meet the requirements of engineering structure reliability index $\beta > 4.0$.

In addition, the Kriging model response surface method and the support vector machine method (SVM) were used to calculate the reliability of the main arch and main beam of the concrete arch bridge. The calculation results of different methods are shown in Table 7-8. It can be seen from the calculation results that the Kriging-PSOSA hybrid algorithm proposed in this paper achieves convergence with 24 and 28 iterations of the calculation of the main arch and the main girder in the reliability calculation process of the concrete arch bridge. Compared with the other two methods, the number of iterations is relatively small, and the calculation accuracy can meet the requirements. It can be seen that the Kriging-PSOSA hybrid algorithm proposed in this paper can be well applied to the reliability calculation and analysis of actual complex bridge structures, with high calculation efficiency and reliable calculation accuracy.

Table 7 Comparison of reliability results of different calculation methods for main arch

Calculation method	Number of samples	Number of iterations	Failure probability	Reliability index β
Krging Model Response Surface Method	50	90	2.8168e-5	4.0624
Support Vector Machines (SVM)	50	39	1.7325e-5	4.2539
Kriging-PSOSA	50	24	2.4253e-5	4.1316

Table 8 Comparison of reliability results of different calculation methods for main beams

Calculation method	Number of samples	Number of iterations	Failure probability	Reliability index β
Krging Model Response Surface Method	50	87	1.2562e-5	4.3881
Support Vector Machines (SVM)	50	29	3.6379e-6	4.4958

Kriging-PSOSA	50	28	5.2891e-6	4.4667

5 Conclusion

For the traditional response surface method in the reliability calculation of concrete arch bridges, due to the complex structure, highly nonlinear performance functions, and no explicit analytical expressions, the calculation accuracy is low, and it is not easy to converge. The Kriging model is proposed to establish a small sample, nonlinear, high-dimensional implicit performance function response surface model. It is combined with the particle swarm optimization algorithm (PSOSA) based on the simulated annealing algorithm. A Kriging-PSOSA hybrid algorithm for fast calculation of bridge failure probability is proposed. Through the verification of numerical examples and the engineering application of actual concrete arch bridges, the research results show that the method has obvious advantages in the calculation accuracy and calculation efficiency. Moreover, it is easy to combine with the existing general finite element analysis software to realize a fast and accurate analysis of bridge reliability. The main conclusions are as follows:

(1) A Kriging-PSOSA hybrid algorithm for the reliability calculation of concrete arch bridges is proposed, which combines the Kriging model with the particle swarm optimization algorithm (PSOSA) based on the simulated annealing algorithm. Using the Kriging model establishes a small sample, nonlinear, high-dimensional implicit performance function response surface model. Combined with the SA algorithm's self-adaptive and variable probability mutation operation, the problems of slow convergence and premature maturity of the traditional PSO algorithm are improved. It effectively avoids falling into the local minimum problem and makes the calculation result tend to the global optimum.

(2) Numerical examples and actual concrete arch bridge engineering case analysis and research results show that the Kriging-PSOSA hybrid algorithm for bridge reliability calculation proposed in this paper is correct, and it effectively improves the accuracy and

efficiency of reliability calculation for complex structures. It overcomes the limitations of the traditional response surface method on the reliability of highly nonlinear structures. It solves the problem of the low computational efficiency of the traditional Monte Carlo method and the excessive dependence of the calculation accuracy of other response surface methods on the preset sample size and distribution.

(3) Compared with the traditional bridge reliability analysis method, the Kriging-PSOSA hybrid response surface method has the advantages of high calculation accuracy, fast iteration speed, and ease of combining with general finite element analysis software. It is convenient for practical engineering applications, especially for solving complex structural reliability problems with high structural analysis costs and highly nonlinear and implicit performance functions. It provides a new idea for the research on the reliability calculation method of large and complex bridges.

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