Network design: taxi planning

Angel Marín · Esteve Codina

Abstract The effect of managing aircraft movements on the airport's ground is an important tool that can alleviate the delays of flights, specially in peak hours or congested situations. Although some strategic design decisions regarding aeronautical and safety aspects have a main impact on the airport's topology, there exists a number of other additional factors that must be evaluated according to the on ground operations, i.e. previous to the taking-off or after landing. Among these factors one can consider capacities at waiting points and directions of some corridors. These factors are related to the demand situation of a given period and influence the aircraft's routing on the ground or short term Taxi Planning problem (or `TP-S`). While the `TP-S` problem studies the aircraft routing and scheduling on the airport's ground under a dynamic point of view, this paper presents a Taxi Planning network design model (or `TPND`), attending to these additional factors of the airport's topology and the conflicting movements of the aircraft on them with the same modelling approach used in the `TP-S` problem. The `TPND` model is formulated as a binary multicommodity network flow problem with additional side constraints under a multiobjective approach. The side constraints included are the classical limitations due to capacity and also as a distinctive approach, constraints that restrict the interference of aircraft in order to decrease the intervention of human controllers during the operations or increase their safety margins. The multiobjective approach adopted for the `TPND` model balances conflicting objectives: airport's throughput, travel times, safety of operations and costs. In the paper computational results are included on two test airports solving the `TPND` model by "Branch and Bound" showing the effect of the conflicting objectives in the design decisions.

Keywords Aircraft routing and scheduling · Taxi planning · Airport management · Binary capacitated multicommodity flow network · Branch and Bound

1 Introduction

The airport Terminal Area and the aircraft movements on the ground are in great part responsible for the annual increase in the average delays of flights. The situation is even more complicated during peak hours as a result of irregularities due to congestion or during periods with low visibility conditions or other meteorological interferences. During any of these events the handling of aircraft movements on the ground using the adequate network topology is crucial to maintain the airport capacity.

The Taxi Planning tool is not only needed in the planning of the airport traffic, but also as a basis for any airport network design or maintenance model. In fact, each design option might be evaluated using a Taxi Planning model.

In the current state of practice the evaluation of routing decisions for aircraft on the ground is carried out using Numerical Simulation and we are not aware of previous optimization models dealing with this problem. The limited literature that exists is not explicit about the methods and algorithms used to solve the Taxi Planning approaches considered. Idris et al. (1998) researched the identification of flow constraints that difficult departure operations at major airports. In its work it is concluded that the runway system is the main bottleneck and source of delay. They also proposed strategies for improving the performance of departure operations, and to determine the control points in the airport where the departure sequence at the runway can be improved. Some simplified models for different airports were presented.

Pujet et al. (1999) provided a simple queueing model of busy airport departure operations. The calibration and validation of the model using available runway configurations and traffic data were included. In the works of Gotteland et al. (2000), a model based on the conflict characterization is used in the context of pattern recognition. They used genetic algorithms to solve the model. Anagnostakis et al. (2001) presented several possible formulations of the runway operations planning with objectives and constraints. Some properties of their model in order to be used in the context of Branch and Bound were mentioned, although a detailed methodological proposal was not given and computational experience was not reported. Andersson et al. (2002) proposed two simple queue models to represent the taxi-out and taxi-in processes. Idris et al. (2002) estimated the taxi-out time in terms of factors like: runway/terminal configurations, downstream restrictions and takeoff queues.

The general airport design problem studies the maintenance of the airport facilities or the enlargement of a given airport topology, looking at more strategic design decisions about the airport topology (runway number and orientation, etc.). These decisions are taken considering more crucial aeronautical aspects relative to the aircraft safety and taking into account the Terminal Airport area and its special airspace constraints.

The distinctive approach taken in this paper has been the use of an optimization model for evaluating decisions regarding aspects of the airport topology from a tactical point of view that affect the routing of aircraft on the ground instead to use simulation.

Another distinctive approach has been the use of optimization instead of simulation for evaluating the design alternatives from an operative Taxi Planning point of view. These decisions are relative to aspects of the airport topology that affect the routing and scheduling of aircraft on ground.

Depending on the design problem it will be adequate to adopt a "medium" vision or "short" term planning perspective to analyze the system accordingly to the design alternatives under study. In "medium" planning, it will not be necessary to consider the system variables disaggregated by time, and average flow values for a period of time would be considered in order to study the design alternatives.

In the TP "medium" design problem, the system variables are defined by average flow values and they may be continuous. These medium term planning models have been considered by different authors under the title of "Airport Capacity". Stoica Dragos (2004) provides a good review of these models. However, if the design problem considers decisions about alternatives where the time plays a decisive role, a "short" term vision will be necessary and in this event, the different system alternatives must be evaluated taking into account the temporal detail of the routing decisions.

If the system alternatives are defined using an operative planning or short term planning TP model, TP-S in short, the system variables are disaggregated by time, and the decisions of the system are described by variables associated to the periods that the planning period is divided. Short term planning permits a closer evaluation of the aircraft interferences in the links, or in general, the elements of the system, so that they may be affected more realistically by the design decisions. The system variables are binary and space-temporal. Usually the TP planning period is about 15 to 20 minutes long although larger periods of up to 30 minutes can also be considered.

TP-S may be used to study different airport network design problems, such as:

- How to reduce the bidirectional links choosing the correct orientation of the overall taxiway links. This problem is important as each bidirectional link produces the necessary intervention of controllers, and these interventions must be reduced to a minimum.
- How to reduce the interference between parkings, especially if they must do push back, choosing the correct configuration.
- Where to locate waiting points to warm up engines, or simply to permit that some aircraft could be overtaken by others.
- Where to locate a given waiting point in an area close to the landing exit points or to the access points of a take-off runway.

In all these cases, the dynamic interaction between aircraft will be affected by the design decision, so TP Network Design must be evaluated using a TP-S. These model will be referred to as TPND for short.

First the Taxi Planning model is expressed in its short term formulation, so the operative decisions are considered in its space and temporally disaggregated form. Next, the reduced formulation of that one is used to include the combined operative design Taxi Planning model.

The layout of the paper is as follows: in Sect. 2, the Taxi Planner model is formulated using a space-temporal network, in Sect. 3 the Taxi Planning reduced formulation is presented. The Network Design Taxi Planning problem is modelled in Sect. 4. In Sect. 5 some computational experiments in Taxi Planning design are presented. Finally, in Conclusions the main paper contributions and further research are mentioned.

2 Short term taxi planning as a space-time network

TP-S was presented in Marín (2006) and it is concerned with the routing and scheduling problem of aircraft on the ground for a fixed topology of the airport dependencies.

The Taxi Planning model is defined on a space time network built up by temporarily replicating a spatial network, represented originally by means of a directed graph $\mathcal{G} = (N, A)$ and using a set of time subintervals that divide a fixed short term planning period, (PP). The set of time subperiods shall be denoted by $T = \{0, 1, 2, \ldots, |T|\}$ where the elements of T are of equal length Δ_t. The resulting space time graph shall be denoted by $\mathcal{G}^* = (N^*, A^*)$, where N^* is the replicated node set, and A^* is the replicated links set. The characteristics of nodes and links in N^* and A^* are inherited from those in N and A respectively and define additional constraints on the time expanded network.

The following subsets of N need to be defined:

- N^W—Subset of nodes where one or more aircraft can stay waiting. The model assumes that the maximum number of aircraft that can stay at $i \in N^P$ at a given instant is related to the dimensions e^w of the aircraft $w \in W$ and the area $c_i > 0$ of the node. A particular subset $N^P \subseteq N^W$ is made up by the parking nodes.
- N^{ER}—Subset of nodes where an aircraft enters in the airport after leaving the landing runways.
- N^{AR}—Subset of nodes with access to runways in order to immediately initiate take-off operations.
- All other nodes in the network not in the above sets are called ordinary nodes N^O. Sometimes it may be necessary to partition a long corridor introducing artificial nodes. These nodes are grouped in the set N^F.

The time used by any aircraft to move along each link $(i, j) \in A$ is assumed constant in the model and it is rounded to an integer number $t_{i,j}$ of time subperiods. The TP-S variables are associated to the space-temporal nodes $N^* = \{(i, t), \forall i \in N, \forall t \in T\}$ and space temporal links $A^* = \{(i, t), (j, t'), \forall (i, j) \in A, \forall t \in T, t' = \min\{t + t_{i,j}, |T|\}\}$. In Fig. 1, the right hand side provides a visual help using a simple case of a time expanded network (N^*, A^*).

The TP-S model determines the routes on the airport's ground for a set W of aircraft during the PP. For each aircraft $w \in W$, we define an origin node, $o(w) \in N$, a destination node, $d(w) \in N$, and a starting time for its route, $t(w) \in T$. In the space time network the origin is a single node, but the destination on the space-time network consists of a set of space-temporal nodes defined by the nodes $(i, t) \in N^*$ with $d(w) = i$ for the different time subperiods if the aircraft arrives at its destination during the PP or a sink node at the end of the PP. The reader is directed again at the left side of Fig. 1.

The binary variables used to define the TP problem are the following ones:

$E^w_{i,t} = 1$, if aircraft "w" waits in node "i" at period "t"; and 0, otherwise.

$X^w_{i,j,t} = 1$, if aircraft "w" is routed from node "i" to node "j" at period "t"; and 0, otherwise.

The flows on the space time network are disaggregated for each aircraft $w \in W$ and are made up with the previous binary variables $E^w_{i,t}$, $X^w_{i,j,t}$. Thus the TP-S model has a multicommodity flow structure. The feasible set of the TP-S problem is defined by the multicommodity flow conservation constraints, the flow capacity constraints, and other side constraints.

The flow conservation constraints at every node are:

$$E^w_{i,t} + \sum_{j \in T^*(i)} X^w_{j,i,t+1-t_{i,j}} = E^w_{i,t+1} + \sum_{j \in F^*(i)} X^w_{i,j,t+1}, \quad \forall t \in T,\ \forall w \in W,\ \forall i \in N, \qquad (1)$$

where the sets "From", F^*, and "To", T^* are defined as:

$$F^*(i) = \{j \,|\, (i, j) \in A\}, \qquad T^*(i) = \{j \,|\, (j, i) \in A\}, \quad \forall i \in N. \tag{2}$$

The flow node conservation constraints need to take into account the aircraft at origin node, $o(w)$. Each aircraft "w" may wait or move at $(o(w), t(w))$:

$$E^w_{o(w),t(w)} + \sum_{j \in F^*(o(w))} X^w_{o(w),j,t(w)} = 1, \quad \forall w \in W. \tag{3}$$

The same applies at $t = |T|$, the aircraft must end waiting in some node (including the air node, if the aircraft can take off during the PP).

$$\sum_{i \in N} E^w_{i,|T|} = 1, \quad \forall w \in W. \tag{4}$$

A visual explanation of the role played by variables E, X in the previous flow conservation constraints (1), (3), (4) for aircraft $w \in W$, may be viewed at Fig. 1 below at the left.

The flow capacity constraints at nodes "i" are defined as follows:

- Capacity constraints at wait nodes $i \in N^W$:

$$\sum_{w \in W} e_w E^w_{i,t} \le c_i, \quad \forall t \in T, \ \forall i \in N^W, \tag{5}$$

  where c_i is the capacity (in surface units) of the node "i", and "e_w" is the surface needed for aircraft "w" when it is waiting in a wait node.
- An aircraft cannot stay waiting at ordinary nodes $i \in N^O$ and exit runway nodes, $i \in N^{ER}$.

$$E^w_{i,t} = 0, \quad \forall t < |T|, \ \forall i \in N^O \cup N^{ER}. \tag{6}$$

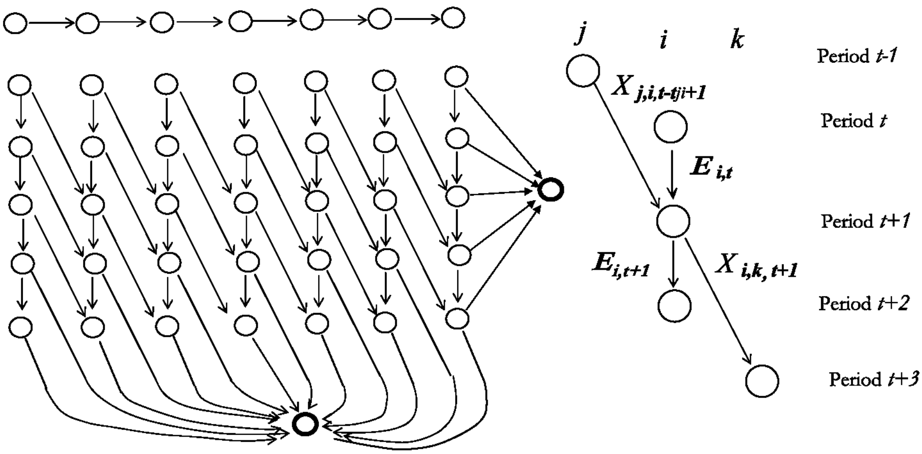


Fig. 1 Structure for the time-space graph of a single runway with $T = 5$, $t_{i,j} = 2$ showing the decision variables $\mathbf{X}, \mathbf{E}$

- At access to runway nodes $i \in N^{AR}$ and artificial nodes $i \in N^F$ only one aircraft is allowed at a time:

$$\sum_{w \in W} E^w_{i,t} \leq 1, \quad \forall t \in T, \ \forall i \in N^{AR} \cup N^F. \tag{7}$$

- Other constraints are also related to node capacities, as it is the case for some nodes, where the total number of aircraft arriving at them during each time subperiod is limited to only one if they are not occupied or zero if they are already occupied.

$$\sum_{w \in W} E^w_{i,t} + \sum_{w \in W} \sum_{j \in T^*(i)} X^w_{j,i,t+1-t_{j,i}} - 1 \leq 0, \quad \forall t \in T, \ \forall i \in N^O \cup N^{AR} \cup N^F. \tag{8}$$

TP-S minimizes total routing time for all the flights, so the objective function τ is defined as a total weighted time expressed in number of time subperiods and spent to route all aircraft:

$$\tau(X, E) = \sum_{w \in W} \sum_{t \geq t(w)} \lambda^w \left(\sum_{i,j \in \mathcal{A}} t_{i,j} X^w_{i,j,t} + \sum_{i \in N^W} E^w_{i,t} \right) + \sum_{i \in N} \sum_{w \in W} r^w_i E^w_{i,|T|}, \tag{9}$$

where λ^w is the priority for aircraft "$w \in W$". r^w_i is the estimated time outside the PP measured in number of time subperiods and that is necessary for aircraft "w" to arrive at its destination from the node "i" in the PP. r^w_i is obtained using a shortest path algorithm.

3 TP-S reduced formulation

A reduced TP-S formulation may be obtained with the flow vectors $U^w = (U^w_a, \forall a \in A^*)$, $w \in W$, where for all $a \in \mathcal{A}^*$: $U^w_a = (X^w_{i,j,t}, \forall i, j \in A, \forall t \in T; E_{i,t}, \forall i \in N, \forall t \in T)$.

The previous node flow conservation constraints (1) to (4) can be expressed in compact form by means of proper node-link incidence matrices B^w and origin-destination vector b^w of each aircraft $w \in W$, with components $1, -1$ or 0 depending on the associated node being the aircraft origin, destination or other respectively.

$$B^w U^w = b^w, \quad w \in W. \tag{10}$$

The capacity constraints (5) to (8) that model the capacity limitations can be expressed as generic link capacity constraint such as,

$$\sum_{w \in W} m_1^{a,w} U^w_a \leq q_a, \quad a \in A_1^* \tag{11}$$

or in compact form $M_1^a \mathbf{U}^a \leq q^a, \forall a \in A_1^*$, where $\mathbf{U}^a = (U^w_a, \forall w \in W)$, M_1^a is the vector of coefficients $m_1^{a,w}$, and A_1^* is the set of links associated to nodes with capacity. A_1^* is defined by $A_1^* = \{((i,t),(i,t+1)) \in A^* : i \in N^W, 1 \leq t \leq |T|-1\}$.

The capacity constraint (8) that model the node access conflicts can be also expressed as generic node capacity constraints such as,

$$\sum_{w \in W} m_2^{i,w} U^w_a \leq q_i, \quad i \in N_1 \tag{12}$$

or in compact form $M_2^i \mathbf{U}^i \leq q^i, \forall i \in N_1$, where M_2^a is the vector of coefficients $m_2^{i,w}$, and $N_1 = N^O \cup N^{AR} \cup N^F$.

Then, TP-S can be expressed as a binary multicommodity network flow problem with capacity constraints,

$$\begin{array}{lll} \mathrm{Min}_{\mathbf{U}} & \tau(U) & \\ \text{s.t.:} & B^w U^w = b^w, & w \in W, \\ & M_1^a \mathbf{U}^a \leq q^a, & a \in A_1^*, \\ & M_2^i \mathbf{U}^i \leq q^i, & i \in N_1, \\ & \mathbf{U} \text{ binary}. & \end{array} \tag{13}$$

4 Network design based on TP model

The previous TP-S is concerned with the routing problem of aircraft on the ground for a fixed topology of the airport dependencies. In this section we introduce several additional constraints and objectives as well as decision variables into the Taxi Planning model in order to take into account the possibility of considering certain types of short-to-medium term operational decisions such as temporarily closing or opening or reversing a runway, a terminal, etc. We will formulate this model in the form of a network design problem where the objective function will be a weighted sum of conflicting objectives. As a distinctive characteristic, the model takes into account in the design decisions the factor of interventions of the controllers that monitor the evolution of the aircraft. In this sense, the model makes decisions on topology and/or aircraft routes on the ground that do not force to increase the number of interventions thus trying to minimize situations that would require a mandatory preventive action. The resulting model shall be referred to as a Taxi Planning Network Design for Short Term operations or TPND in short.

4.1 Incorporating short-to-medium term operational decisions into the model

In practice, the following (conflicting) objectives are to be minimized under low to medium congestion situations during the PP, from higher to lower priority:

1. Number of interventions of controllers in order to solve possible conflicts.
2. Total delay of outgoing traffic.
3. Total time until take-off or arrival at a final parking.
4. Total delay of incoming traffic.

In situations of high congestion an objective that takes priority is the total number of arrivals at the final destination on the ground (parking or terminal) plus the total number of taking-offs. We will now present the formulation of the objectives mentioned above.

4.1.1 Number of controller interventions

The design decisions given by the TPND model must take into account the practical feasibility of their solutions. Airport Management Authorities usually consider that one of the most important operational tasks of the controllers is their surveillance and capacity to prevent any kind of conflicts. For normal operational conditions, design solutions regarding aspects of the airport topology must be considered so that one of the aspects that enforce the security of the operations is to make the task of the airport controllers as easy as possible. In this model a conflict arises when two or more aircraft approach at a point and at least one of them is going to cross it and the crossing trajectories either coincide or are separated by a short time. Remember that constraints (8) ensure that two trajectories do not coincide in time at a single node of the network model. This will cause the intervention of a controller in order to ensure that some aircraft stop, thus guaranteeing that the conflict point is crossed safely by only one of them at a time. A set of constraints modelling the proximity in time of two trajectories at a conflict point and an objective function that provides the number of times during the PP that controllers enter into action will be developed in this section.

Clearly, (8) prevents more than one aircraft from accessing node i at a time. Consequently, trajectories cross each other separated by a time subperiod of Δ_t seconds. Also, as the right hand side of (8) can be considered the excess of aircraft at node i during time subperiod $t \in T$, (8) imposes that no excess of aircraft may exist at node i otherwise, solutions provided by the model would permit crashing of two trajectories. The concept of excess aircraft at a node i and during time $t \in T$ can be extended to a set of nodes of the airport network model for $\nu + 1$ time subperiods. The set of nodes where a zero excess would be desirable due to safety of operations will be referred to as a conflicting point and, on these points, in case of positive excess aircraft, the intervention of controllers would be required. In other words, at conflicting points only one aircraft is allowed each ν time subperiods or also, trajectories might be separated ν subperiods in time whenever possible.

Let $\mathcal{K}$ be the set of conflicting points on the airport. Let $K_A(K)$ and $K_N(K)$ be the set of incoming arcs to the conflicting point $K \in \mathcal{K}$ and the set of nodes inside the conflict point $K \in \mathcal{K}$ respectively, where there can be aircraft waiting.

Let $x_{i,j,t} = \sum_{w \in W} X^w_{i,j,t}$ be the total number of aircraft traversing link (i, j) at time $t \in T$ and let $e_{i,t} = \sum_{w \in W} E^w_{i,t}$ be the total number of aircraft staying at node $i \in N$ during time subinterval $t \in T$. At a conflicting point $K \in \mathcal{K}$ and as a generalization of (8), the excess aircraft at time $t \in T$ for a preventive horizon of ν time subperiods is denoted by $C_{K,t}$ and is given by:

$$C_{K,t} = \sum_{\ell=\max\{t-\nu,2\}}^{t} \left(\sum_{\substack{(i,j)\in K_A(K) \\ \ell - t_{i,j} \geq 1}} x_{i,j,\ell-t_{i,j}} + \sum_{i \in K_N(K)} e_{i,\ell-1} \right) - 1,$$

$$K \in \mathcal{K}, \; 2 \leq t \leq |T|. \tag{14}$$

Note that the excess aircraft expressed in (14) is defined only for $t \geq 2$ because it is implicitly assumed that the PP starts without excess at $K \in \mathcal{K}$, or equivalently that the initial distribution of aircraft on the airport is such that $\sum_{i \in K_N(K)} e_{i,1} \leq 1$ at $K \in \mathcal{K}$.

Under the previous definition in (14) of $C_{K,t}$, the intervention of a controller will occur at time $t \in T$ for $K \in \mathcal{K}$ if $C_{K,t} > 0$. Then a convenient objective can be the minimization of the total number of interventions. For this purpose the binary variables $\gamma_{K,t}$ need to be defined:

$$\begin{aligned} C_{K,t} > 0 \quad &\Rightarrow \quad \gamma_{K,t} = 1, \\ C_{K,t} \leq 0 \quad &\Rightarrow \quad \gamma_{K,t} = 0. \end{aligned} \tag{15}$$

Logical conditions (15) can be readily expressed as linear constraints and the following sharp bound for $C_{K,t}$ can be useful. It must be noticed that, due to (8) no more than one aircraft can be present on a link at a time and also no more than one aircraft can be staying at a node at a time. Because the excess aircraft $C_{K,t}$ is computed for ν previous additional time subperiods, then $C_{K,t}$ must be bounded by:

$$C_{K,t} \leq \nu \cdot (|K_A(K)| + |K_N(K)|). \tag{16}$$

Using the previous bound (16) on $C_{K,t}$, the following linear constraints at a conflicting point $K \in \mathcal{K}$ and time $2 \leq t \leq |T|$,

$$C_{K,t} \leq \nu \cdot (|K_A(K)| + |K_N(K)|) \cdot \gamma_{K,t}, \quad \gamma_{K,t} \in \{0, 1\}, \tag{17}$$

are equivalent to the following logical relationships between $C_{K,t}$ and $\gamma_{K,t}$:

$$\begin{aligned} C_{K,t} > 0 \quad &\Rightarrow \quad \gamma_{K,t} = 1, \\ C_{K,t} \leq 0 \quad &\Rightarrow \quad \gamma_{K,t} \in \{0, 1\} \\ &\text{(i.e. no additional constraints on } \gamma_{K,t}) \end{aligned} \tag{18}$$

Then, if the total number of controller interventions I_C on all the conflicting points during the PP is expressed as:

$$I_C = \sum_{K \in \mathcal{K}} \sum_{2 \leq t \leq |T|} \gamma_{K,t} \tag{19}$$

and this objective is to be minimized, then, if for some $K \in \mathcal{K}$, $2 \leq t \leq |T|$ it happens that $C_{K,t} \leq 0$ this implies that $\gamma_{K,t} = 0$. Consequently, linear constraints (17) together with the minimization of a linear objective that includes I_C as a positive term is equivalent to the logical conditions (15).

4.1.2 Worst routing times and delays

Let the routing time τ^w for aircraft $w \in W$ be defined in terms of the flow vectors U^w as:

$$\tau^w(U^w) = \sum_{t=1}^{|T|-1} \left(\sum_{(i,j) \in A} t_{i,j} X^w_{i,j,t} + \sum_{i \in N^W} E^w_{i,t} \right) + \sum_{i \in N^W} r^w_i E^w_{i,|T|}. \tag{20}$$

If λ^w is a coefficient for the priority of aircraft $w \in W$, then the weighted total routing time τ can be expressed as in (9).

In addition to the weighted routing times τ given by (9) it is convenient to take into account the worst routing time $\hat{\tau}$ given by:

$$\hat{\tau} = \mathrm{Max}_{w \in W}\{\tau^w\}. \tag{21}$$

The worst travel time can be included by means of the following constraints,

$$\tau^w(U^w) \leq \hat{\tau}, \quad w \in W \tag{22}$$

with τ^w defined in terms of the flow variables $X^w_{i,j,t}$ and $E^w_{i,t}$ by (20).

Delays D^w_{IN}, D^w_{OUT} of incoming and outgoing traffic are given by:

$$D^w_{\mathrm{IN}} = \sum_{t \in T} \sum_{i \in N^W} E^w_{i,t}, \quad \forall w \in W^A. \tag{23}$$

where W^A is the set of aircraft arriving at the airport during the PP.

$$D^w_{\mathrm{OUT}} = \sum_{t \in T} \sum_{i \in N^W} E^w_{i,t}, \quad \forall w \in W^D, \tag{24}$$

where W^D is the set of departing aircraft during the PP.

Clearly, the worst routing times for aircraft either $\hat{\tau}$ or the related delay $D^w_{\mathtt{IN}}$, $D^w_{\mathtt{OUT}}$ may be objectives that conflict with I_C, the number of controller interventions defined in the previous subsection in case of congestion, because reducing the number of times that space-time trajectories of two or more aircraft are close to each other generally must result in delays for waiting or adopting routes with greater travel times.

4.1.3 Number of arrivals and take-offs

A magnitude that can be also monitored after solving the `TNDP-S` model is the total throughput $\mathcal{T} = \mathcal{T}^- + \mathcal{T}^+$ comprised of the total number of take-offs, $\mathcal{T}^-$, and arrivals at destinations on ground, $\mathcal{T}^+$, within the `PP`.

The number of arrivals on the ground is:

$$\mathcal{T}^+ = \sum_{t \in T} \sum_{w \in W^A} \sum_{i \in N^{ER}} \sum_{j \in F^*(i)} X^w_{i,j,t}. \tag{25}$$

The number of take-offs during the `PP` is:

$$\mathcal{T}^- = \sum_{t \in T} \sum_{w \in W^D} \sum_{i \in N^{AR}} \sum_{j \in F^*(i)} X^w_{i,j,t}. \tag{26}$$

As with the aircraft travel times of previous subsection, the total airport throughput $\mathcal{T}$ may be in conflict with I_C as the effect in minimizing I_C may lead to a reduction in the airport's capacity.

4.2 Design decision variables

Design variables are related to elements of the `TP-S` network. Let y be a vector of binary variables that will be associated to open/close during the whole `PP` links within a reduced subset $\hat{A}$ of links with capacity A^*_1. If $y_{i,j}$ is the decision variable associated to link $(i, j) \in \hat{A}$ with capacity $q_{i,j}$, then, clearly the constraints linking the flow variables in $\mathbf{U}$ to the decision variables will be:

$$\sum_{t \in T} \sum_{w \in W} X^w_{i,j,t} \leq q_{i,j} y_{i,j}, \quad \forall (i, j) \in \hat{A} \tag{27}$$

or in compact form $H^a \mathbf{U}^a \leq q_a y^a$, $\forall a \in \hat{A}$.

The design variables may belong to a design feasible set Y, that could be defined by means of budget or environmental constraints.

4.3 TPND formulation

Let $g(y)$ be the link location costs associated with the decisions y_{ij}. It will be assumed that it is a linear function $g(y) = \sum_{(i,j) \in \hat{A}} c_{i,j} y_{i,j}$ with nonnegative coefficients $c_{i,j}$. The `TPND` model can be stated in the form of a binary optimization problem that minimizes a multiobjective function of the decision variables $U \in \{0, 1\}$, $y \in Y$ during the `PP` while ensuring that the airport constraints can be satisfied. Along with them, constraints (17) that model controllers intervention and the ones that model the worst travel time (22) must also be taken into account. This set of constraints will be expressed simply as $\Gamma_1 \mathbf{U} \leq \Gamma_2 y$, where

$\gamma = (\cdots \gamma_{K,t} \cdots; t \in T, K \in \mathcal{K})$, are defined in (15). Adding the constraints between flow variables and decision variables, the TPND problem can be stated as:

$$\begin{array}{ll} \mathrm{Min}_{\mathbf{U},y,\gamma,\hat{\tau}} & \phi \\ \text{s.t.:} & B^w U^w = b^w, \quad \forall w \in W, \\ & M_1^a \mathbf{U}^a \leq q^a, \quad a \in A_1^*, \\ & M_2^i \mathbf{U}^i \leq q^i, \quad i \in N_1, \\ & \tau^w(U^w) \leq \hat{\tau}, \quad \forall w \in W, \\ & H^a \mathbf{U}^a \leq q_a y^a, \quad \forall a \in \hat{A}, \\ & \Gamma_1 \mathbf{U} \leq \Gamma_2 \gamma, \\ & \mathbf{U}, \quad \gamma \text{ binary}, \quad y \in Y, \end{array} \tag{28}$$

where ϕ is a weighted sum of objectives given by (9), (19), (23), (24), (25), and (26), taking into account the worst travel time $\hat{\tau}$ and the cost g associated to the binary variables y:

$$\phi = \alpha_\tau \tau + \alpha_L g(y) + \alpha_{\mathrm{IC}} I_c + \alpha_{\mathrm{OUT}} \sum_{w \in W^D} D_{\mathrm{OUT}}^w + \alpha_{\mathrm{IN}} \sum_{w \in W^A} D_{\mathrm{IN}}^w + \alpha_{\hat{\tau}} \hat{\tau} + \alpha_{\mathcal{T}} \mathcal{T}, \tag{29}$$

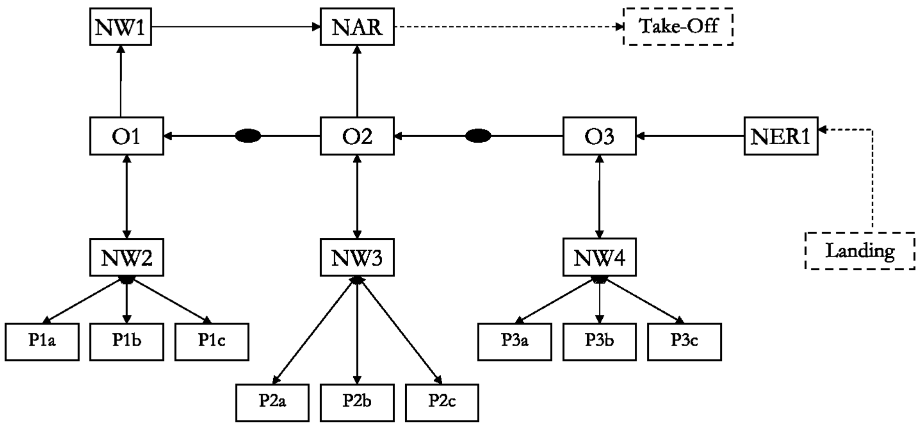
where $\alpha_\tau, \alpha_L, \alpha_{\mathrm{IC}}, \alpha_{\mathrm{OUT}}, \alpha_{\mathrm{IN}}, \alpha_{\hat{\tau}}, \alpha_{\mathcal{T}} \geq 0$ and $\alpha_\tau + \alpha_L + \alpha_{\mathrm{IC}} + \alpha_{\mathrm{OUT}} + \alpha_{\mathrm{IN}} + \alpha_{\hat{\tau}} + \alpha_{\mathcal{T}} = 1$.

The new TPND defined with this objective function is a multiobjective problem and different methodologies would be applied. A simple but effective technique is shown in the next section in order to manage the computational difficulties that may arise in balancing the components of the objective function regarding especially the inclusion of the terms related to the number of controller interventions.

5 Running the TPND

The test airports are those depicted in Figs. 2 and 3 which shall be referred to as airport J1 and J2 respectively. For each airport a PP of 30 minutes has been considered using time subintervals of 30 seconds.

Airport J1 is comprised of two separate runways one of them for take off and the other one for landing. Airport J2 has a single runway for taking off/landing purposes. The use for taking-off or landing is conditioned by meteorological conditions and not for the demand during the PP and is therefore outside of decisions that can be considered by the TPND model. Airport J2 can be considered as an extension of J1 with additional corridors for access/exit to/from the taking-off/landing runways and with a more complex mix runway. Both airports have three parking platforms, each parking platform consisting of three parking locations with capacity for five aircraft. In the computational tests all the aircraft being considered have equal characteristics, requiring an extension of nearly 1700 m^2 for parking purposes. In the diagrams shown in Figs. 2 and 3 links joining two nodes model corridor sections which can be bidirectional accordingly to the double/single arrow for the link and have equal length (309 m), excluding those with a black circle in between which are of double length. It is allowed that an aircraft stays waiting on these double length links. This is captured in the graph model splitting the double length links with an intermediate node for modelling the possibility of a wait. Aircraft are supposed to move at a constant speed of almost 40 km/h through the corridors and thus movements through a corridor section are done in one subperiod (i.e., $t_{i,j} = 1$). Waits can occur at any of the nodes in the diagrams for airports J1 and J2, excluding those that are exit from landing runways (NER nodes) or nodes that are runway headers (NAR nodes).



Parking:	P1a, P1b, P1c, P2a, P2b, P2c, P3a, P3b, P3c
Output from Parking:	NW2, NW3, NW4
Regular:	O1, O2, O3
Wait:	NW1
Runway Header:	NAR
Output from Runway:	NER1

Fig. 2 Test airport J1

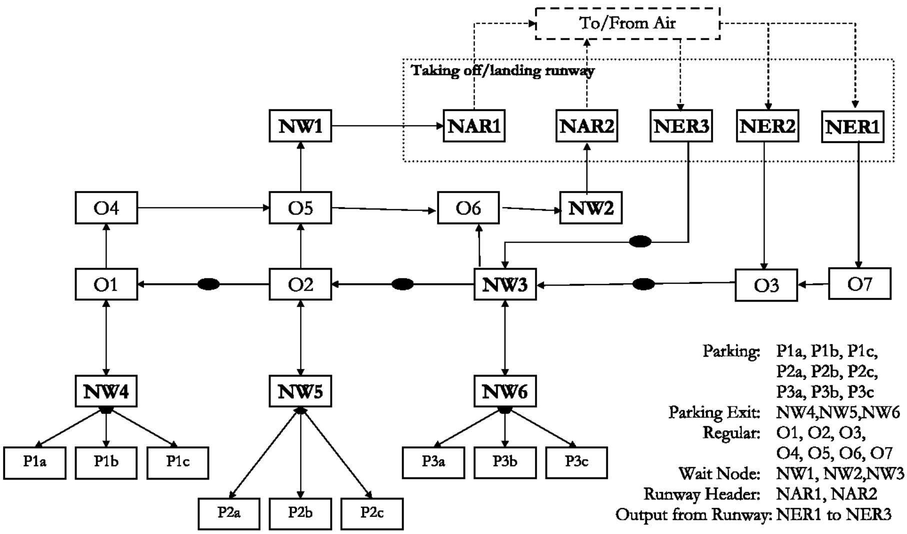


Fig. 3 Test airport J2

For airport J1, the points where it is possible that interventions of controllers may occur are NAR, O1, O2, O3, NW2, NW3 and NW4, whereas in airport J2 the points are O5, O6, O1, O2, NW3, O7, NW4, NW5 and NW6. A period ν of three time subintervals has been considered for the time that may trigger an intervention of a controller.

Table 1 Problem size

	vars.	N.C.	I.C.	E.C.
J1	6057	2155	5362	2167
J2	15674	6089	10346	5981

vars = N. of variables

N.C. = Network constraints

I.C., E.C. = (In)equality constraints

For airport J1 the decision being considered is the capacity at node NW1 in terms of the number of aircraft that can stay waiting. For airport J2 the decisions taken into account by the TPND model are:

1. The capacities at nodes NW1, NW2, NW3 also expressed in number of aircraft that can stay waiting at each of these nodes. The range of possible capacities is set from 0 to an upper bound of 5 aircraft for NW1, NW2 and a range of 1 to 5 for NW3. Simultaneously to the previous decisions, the possibility of opening/closing exit from runways NER1, NER2, NER3 is also considered.
2. As before, capacities within an equal set of ranges for waiting nodes NW1, NW2, NW3 and simultaneously, for the exit from runways NER1, NER2, NER3 the possibility of being opened only two out of three at most.

The coefficients adopted for the linear function $g(y)$ that models the decision costs associated to decisions y were set to one in all cases. In all the computational tests the results showed that, for airport J1 a capacity of one aircraft suffices at node NW1 and also at nodes NW1, NW2, NW3 in airport J2. With regard to exit from runways NER1, NER2, NER3 for airport J2 all three exits should be opened if possible and in case of choosing a maximum of two of them, exits NER2 and NER3 are preferable.

The tests of the model have been performed on a Pentium IV, 1.7 GHz with 256 MB of memory using the AMPL-CPLEX System. The size of the problems solved for the tests is given in Table 1.

The layout of Tables 2, 3 and 4 is as follows. Each table is comprised of a set of runs and for each subset of runs the set of weighting parameters in the objective function is given in the initial record. The total number of aircraft operating on the airport is expressed in column $|W|$, whereas the number of aircraft that complete their operation within the PP is in column $\mathcal{T}$. The number of aircraft arrival aircraft is given in column $|W_A|$, whereas the number of departing aircraft is given in column $|W_D|$. From these, aircraft that arrive at its parking destination within the PP is given by $\mathcal{T}_+$ and the total number of aircraft that take-off within the PP is given in $\mathcal{T}_-$. I_C is the total number of controller's interventions, D_{IN} and D_{OUT} are the total delay time for aircraft entering/leaving the airport respectively, $\hat{\tau}$ is the worst travel time amongst all the aircraft operating on the airport, τ is the total weighted time. Finally column B&B is the number of nodes of the tree expanded during the execution of the Branch and Bound algorithm in the run.

The first set of tests have were performed on Airport J1 in order to show the difficulty of dealing with the modeling of the number of interventions of the Airport controller's, i.e. constraints (17). A total of 14 runs have been performed for airport J1. Runs 1 to 6 were done with $\alpha_{\mathrm{IC}} = 0.5$ with an increasing number of aircraft entering/leaving J1. The computer requirements (column $\mathrm{T}_{\mathrm{CPU}}$ in seconds) increase drastically as $|W|$ increases and a run with $|W| = 8$ was abandoned. Runs 7 and 8 have adopted as objective function just the

Table 2 Tests on airport J1

	ϕ	T_{CPU}	$\lvert W\rvert$	$\mathcal{T}$	$\lvert W_A\rvert$	$\mathcal{T}_+$	$\lvert W_D\rvert$	$\mathcal{T}_-$	I_C	D_{IN}	D_{OUT}	$\hat{\tau}$	τ	B&B
$\alpha_{IC} = \mathbf{0.5}$, $\alpha_\tau = 0.2$, $\alpha_{IN} = 0.15$, $\alpha_{OUT} = 0.1$, $\alpha_L = 0.0499$, $\alpha_{\mathcal{T}} = 0$, $\alpha_{\hat{\tau}} = 0.0001$														
1	3.09	0.06	1	1	0	0	1	1	0	0	0	4	4	0
2	4.84	0.21	2	2	1	1	1	1	0	1	0	8	12	0
3	5.84	0.28	3	3	1	1	2	2	0	1	0	8	17	0
4	8.84	6.57	4	4	2	2	2	2	1	3	0	11	28	0
5	11.89	38.73	5	5	2	2	3	3	1	6	0	14	41	1
6	17.39	303.6	6	6	3	3	3	3	8	6	0	15	51	2
$\alpha_{IC} = \mathbf{0}$, $\alpha_\tau = 1.0$, $\alpha_{IN} = \alpha_{OUT} = \alpha_L = \alpha_{\mathcal{T}} = \alpha_{\hat{\tau}} = 0$														
7	29.0	0.46	6	6	3	3	3	3	187	2	0	7	29	0
8	105.0	59.26	16	16	7	7	9	9	187	3	0	14	105	0
$\alpha_{IC} = \mathbf{0}$, $\alpha_\tau = 0.9$, $\alpha_{IN} = \alpha_{OUT} = 0$, $\alpha_L = 0.1$, $\alpha_{\mathcal{T}} = \alpha_{\hat{\tau}} = 0$														
9	99.10	83.39	16	16	7	7	9	9	187	3	0	14	105	0
10	30.70	0.57	6	6	3	3	3	3	187	0	0	7	29	0
$\alpha_{IC} = \mathbf{0.001}$, $\alpha_\tau = 0.899$, $\alpha_{IN} = \alpha_{OUT} = 0$, $\alpha_L = 0.1$, $\alpha_{\mathcal{T}} = \alpha_{\hat{\tau}} = 0$														
11	99.06	191.6	16	16	7	7	9	9	71	3	0	12	105	620
$\alpha_{IC} = \mathbf{0.01}$, $\alpha_\tau = 0.89$, $\alpha_{IN} = \alpha_{OUT} = 0$, $\alpha_L = 0.1$, $\alpha_{\mathcal{T}} = \alpha_{\hat{\tau}} = 0$														
12	98.74	265.1	16	16	7	7	9	9	69	3	0	9	105	1496
$\alpha_{IC} = \mathbf{0.05}$, $\alpha_\tau = 0.85$, $\alpha_{IN} = \alpha_{OUT} = 0$, $\alpha_L = 0.1$, $\alpha_{\mathcal{T}} = \alpha_{\hat{\tau}} = 0$														
13	97.30	110.2	16	16	7	7	9	9	69	3	0	9	105	338
$\alpha_{IC} = \mathbf{0.1}$, $\alpha_\tau = 0.8$, $\alpha_{IN} = \alpha_{OUT} = 0$, $\alpha_L = 0.1$, $\alpha_{\mathcal{T}} = \alpha_{\hat{\tau}} = 0$														
14	95.50	147.0	16	16	7	7	9	9	69	3	0	9	105	508

total weighted time τ and thus the constraints on the number of controller interventions are inactive. Clearly, run number 8 with 16 aircraft is solved very swiftly without the need of expanding a Branch and Bound tree (i.e. the initial linear relaxation of the problem provides the solution). In this case column I_C is meaningless as $\alpha_{IC} = 0$. These runs are included in order to show how the constraints on the number of controller interventions affect the computation times. Runs 9 and 10 show that the inclusion of the component with decision costs can be handled also very well. Runs 11 to 14 show that in order to take into account in the solutions the I_C factor, different runs with increasing values for α_{IC} should be done until no enhancement is observed for $I_C (= 69)$.

For airport J2 the same procedure followed in the previous example is applied. Runs 1 to 5 in Table 3 have been made with $|W| = 11$ aircraft. Starting from the objective with coefficients $\alpha_\tau = 0.9$ and $\alpha_L = 0.1$, the presence of the factor I_C is increased obtaining solutions with bigger I_C. This time the total weighted time τ does not degrade as the factor I_C can not be lowered from 46. Notice that the computer requirements are also increasing

Table 3 Tests on airport J2

	ϕ	T_{CPU}	$\lvert W\rvert$	$\mathcal{T}$	$\lvert W_A\rvert$	$\mathcal{T}_+$	$\lvert W_D\rvert$	$\mathcal{T}_-$	I_C	D_{IN}	D_{OUT}	$\hat{\tau}$	τ	B&B
$\alpha_{IC}=0$, $\alpha_\tau=0.9$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
1	78.60	8.59	11	11	4	4	7	7	235	3	0	13	82	0
$\alpha_{IC}=0.01$, $\alpha_\tau=0.89$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
2	78.24	19.28	11	11	4	4	7	7	46	6	0	13	82	87
$\alpha_{IC}=0.05$, $\alpha_\tau=0.85$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
3	76.80	40.65	11	11	4	4	7	7	46	3	0	12	82	269
$\alpha_{IC}=0.1$, $\alpha_\tau=0.8$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
4	75.00	72.34	11	11	4	4	7	7	46	3	0	12	82	509
$\alpha_{IC}=0.2$, $\alpha_\tau=0.7$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
5	71.39	377.71	11	11	4	4	7	7	46	6	0	12	82	3637
6	99.8	1943.5	16	16	7	7	9	9	69	6	0	12	116	11059[a]
$\alpha_{IC}=0.01$, $\alpha_\tau=0.89$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
7	108.7	56.31	16	16	7	7	9	9	68	7	0	12	116	264
$\alpha_{IC}=0.05$, $\alpha_\tau=0.85$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
8	106.8	82.48	16	16	7	7	9	9	68	7	0	13	116	340

[a]Execution interrupted

Table 4 Tests on airport J2; Select two out of three exits from runway

	ϕ	T_{CPU}	$\lvert W\rvert$	$\mathcal{T}$	$\lvert W_A\rvert$	$\mathcal{T}_+$	$\lvert W_D\rvert$	$\mathcal{T}_-$	I_C	D_{IN}	D_{OUT}	$\hat{\tau}$	τ	B&B
$\alpha_{IC}=0.01$, $\alpha_\tau=0.89$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
1	108.7	65.17	16	16	7	7	9	9	69	7	0	11	116	541
$\alpha_{IC}=0.05$, $\alpha_\tau=0.85$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
2	106.8	104.28	16	16	7	7	9	9	69	5	0	12	116	769
$\alpha_{IC}=0.1$, $\alpha_\tau=0.8$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
3	104.5	142.15	16	16	7	7	9	9	69	6	0	12	116	555
$\alpha_{IC}=0.15$, $\alpha_\tau=0.75$, $\alpha_{IN}=\alpha_{OUT}=0$, $\alpha_L=0.1$, $\alpha_{\mathcal{T}}=\alpha_{\hat{\tau}}=0$														
4	102.1	1468.1	16	16	7	7	9	9	69	7	0	12	116	10523

as $\alpha_{\mathtt{IC}}$ increases. Runs 6 to 8 correspond to a demand with $|W| = 16$ aircraft and the same effect on the CPU time is observed. Run numbers 7 and 8 is for $\alpha_{\mathtt{IC}} = 0.01$ and $\alpha_{\mathtt{IC}} = 0.05$ and also $|W| = 16$. It must be noticed that this time trying to optimize I_C has some effect on the travel times as shown by the worst travel time $\hat{\tau}$. Run number 6 for $\alpha_{\mathtt{IC}} = 0.2$ had to be interrupted after more than half an hour running, probably at a solution very close to the optimum because it is obtained a solution very similar to the ones obtained in runs 7 and 8.

Table 4 shows the results obtained by the `TNDP-S` model with airport `J2` with the additional constraint of choosing two out of the three exits from the landing runway. Runs 1 to 4 are for increasing values of $\alpha_{\mathtt{IC}} = 0.01, 0.05, 0.1, 0.15$. As before, increasing values of $\mathtt{T}_{\mathtt{CPU}}$ with small impact on the travel times of aircraft can be observed.

6 Conclusions and extensions

In this paper a network design model for the ground airport's Taxi Planning operations (`TPND` for short) has been presented. The `TPND` model has been formulated as a binary multicommodity network flow problem with side constraints developed on a time-space network under the same approach on which the short term Taxi Planning model in Marín (2006) was developed. The model adopts a multiobjective approach by balancing several objectives that are the most relevant aspects usually taken into account in real practice by the Spanish airport management authorities: total weighting and routing time on the ground, input and output airport's throughput, worst routing time and number of interventions of controllers at a preselected set of conflicting points. The `TPND` model has been shown to be an adequate tool to analyze the optimal airport configuration considering aircraft congestion and airport facilities, when at the design level there are decisions closely related to the dynamic aspects of the routing of aircraft on the ground.

The computational experiments have been run using Branch and Bound, and they have been carried out with two simplified airport networks, taken from actual data of Madrid-Barajas Airport, supplied by Aeropuertos Españoles y Navegación Aérea, the Spanish airport management corporation in order to better illustrate the key aspects of the model. Computational tests on these networks have been run using different weights of the objective function in order to show the most relevant aspects of the implicit multicriteria decision process on which the model relies as well as to show the sources of computational difficulty that may arise in solving the optimization problems. It has been shown how the inclusion of factors relative to the number of interventions of the controllers is the source of the major computational difficulties in the model and also how these difficulties can be avoided analyzing the model using different runs.

As extensions of the current work a complete multiobjective analysis may be performed and the good computational results shown on the test networks permit to consider further extensions of this design model. In the algorithmic field it is appealing and necessary to develop of more efficient computational methods in order to solve larger problems. In this sense the multicommodity network flow structure suggests the dualization of the side constraints making the Lagrangean relaxation a clear candidate.

References

Anagnostakis, I., Clarke, J.-P., Böhme, D., & Völckers, U. (2001). Runway operations planning and control: sequences and scheduling. *Journal of Aircraft*, *38*(6), 988–996.

Andersson, K., Carr, F., Feron, E., & Hall, W. D. (2002). *Analysis and modeling of ground operations at hub airports*. 3nd USA/Europe Air Traffic Management R & D Seminar, Napoli, 13–167 June 2002.

Gotteland, J. B., Durán, N., Alliot, J. M., & Page, E. (2000). *Aircraft ground traffic optimisation*. Paper of Ecole Nationale de l'Aviation Civile.

Idris, H. R., Delcaire, B., Anagnostakis, I., Feron, E. Clarke, J.-P. & Odoni, A.R. (1998). *Identification of flow constraint and control points in departure operations at airport systems*. Paper AIAA-98.4291 American Institute of Aeronautics and Astronautics.

Idris, H., Clarke, J.-P., Bhuva, R., & Kang, L. (2002). Queueing model for taxi-out time estimation. *Air Traffic Control Quarterly*, *10*(1), 1–22.

Marín, A. (2006). Airport management: taxi planning. *Annals of Operation Research*, *143*, 189–200.

Pujet, N., Delcaire, B., & Feron, E. (1999). *Input-output modeling and control of the departure process of busy airports*. Report No. ICAT-99-3. MIT International Center for Air Transportation.

Stoica Dragos, C.M. (2004). *Analyse, represéntation et optimisation de la circulation des avions sur une plate-forme aéroportuaire*. Thesis Laboratoire d'Analyse et d'Architecture des Systèmes du CNRS, Toulouse, France.