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The Effect of Supplier Capacity on the Supply Chain Profit

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Abstract

In this paper, we study the role of capacity on the efficiency of a two-tier supply chain with two suppliers (leaders, first tier) and one retailer (follower, second tier). The suppliers compete via pricing (Bertrand competition) and, as one would expect in practice, are faced with production capacity. We consider a model with differentiated substitutable products where the suppliers are symmetric differing only by their production capacity. We characterize the prices, production amounts and profits in three cases: 1) the suppliers compete in a decentralized Nash equilibrium game, 2) the suppliers "cooperate" to optimize the total suppliers' profit, and 3) the two tiers of the supply chain are centrally coordinated. We show that in a decentralized setting, the supplier with a lower capacity may benefit from restricting her capacity even when additional capacity is available at no cost. We also show that the loss of total profit due to decentralization cannot exceed 25% of the centralized chain profits. Nevertheless, the loss of total profit is not a monotonic function of the "degree of asymmetry" of the suppliers' capacities. Furthermore, we provide an upper bound on the supplier profit loss at equilibrium (compared with the cooperation setting) that depends on the "market power" of the suppliers as well as their market size. We show that there is less supplier profit loss as the asymmetry (in terms of their capacities) increases between the two suppliers. The worst case arises when the two suppliers are completely symmetric.

1 Introduction

1.1 Motivation

As many manufacturing systems, the automotive industry is subject to tight capacity constraints in practice. Quantities produced are limited by existing workforce, facilities, and/or raw material availability. The production capacity is often reached and effectively limits the possible output

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quantities. For instance, in April 2010, "the upgraded Sonata sedan and Tucson SUV led Hyundai's growth, bringing its U.S. plant in Alabama to run nearly at full capacity" (Reuters [39]). In addition, these two car models can be found at the same "retailers" (i.e., Hyundai car dealerships) and may be considered as imperfect substitutes as they compete for similar customer segments. In a cooperative management regime, the car manufacturer Hyundai would make decisions to maximize its overall profits from selling both car models ("brands") through a given external retailer; however, under a decentralized management regime, the two independent brand managers can make quantity and pricing decisions "for each brand separately to maximize profit of their own brand" ([23]) (this can be due to meeting individual targets). A centrally coordinated setting where the car manufacturer has its own internal cardealership and makes all decisions can be used as a performance benchmark. This brings up the question of the role played by the competition among these two products and the potential benefit for the parent company Hyundai should they cooperate to make decisions jointly so as to maximize the car manufacturer's overall profit.

Our aim is to fill a gap in the literature by studying a model of competition in supply chains that capture two important features: the existence of production capacity and competition among suppliers that sell through a common retailer. Production capacity constraints are often ignored in existing research on supply chain management. Federgruen and Zipkin also mention that the "assumption of limitless capacity, while a reasonable approximation in some situations, is a poor one in others"[17]; they note that "finite capacity results in complications, the degree of which depends on the details of the problem". Our model features supplier competition. Competition among manufacturers supplying items to a single retailer has been considered in the literature [12, 29, 8]. Similarly to Choi [12], in our model, a single retailer interacts with two competing suppliers. In a review paper on supply chain management in the presence of multiple suppliers, Minner [31] observes that "the allocation of quantities among several suppliers depends on the respective capacity constraints", hence the relative values of the production capacity has an impact on the nature of the competition among suppliers. Capacity is often the outcome of prior investment or infrastructure decisions, thus by the time prices and production quantities need to be determined, the capacity is already fixed. As a result, there is a need to understand the role of capacitated and competing suppliers in a supply chain.

It is well known that decentralized supply chains, i.e. supply chains where each agent makes decisions in its own interest, lead to inefficiencies and loss of profits. In a supply chain with one supplier and one retailer, Spengler [43] observed that when the supplier and the retailer make decentralized decisions, the aggregate profits are lower than when a central decision-maker imposes price and quantity decisions, an effect called double marginalization. This observation has motivated significant research in recent years; see for example, [7]. Of particular interest is measuring how "inefficient" the decentralized supply chain becomes due to lack of coordination among the tiers of the supply chain as well as due to competition at each tier. Supply chain efficiency has been

studied in a variety of settings and with different focuses, including: one supplier and one retailer [43, 34, 27] two or more suppliers and one retailer [29, 28, 12, 8, 37], one supplier and multiple retailers [4, 5, 9, 37], multiple suppliers and multiple retailers [1], oligopolistic competition [16], complementary vs. substitute products [33], bullwhip effect [11]. To the best of our knowledge, the effect of capacity on supply chain efficiency has not been addressed in the current literature.

Contributions: The overall goal of this research is to provide a better understanding of the strategic role of capacity and how it affects the overall loss of profit in a supply chain due to lack of coordination and price competition. In particular, we consider a two-tier supply chain with two competing suppliers in the first tier and one retailer in the second tier. In our model, the suppliers are Stackelberg leaders, and the retailer plays the role of a follower. Our model thus involves both horizontal competition (between the two suppliers) and vertical competition (between the suppliers and the retailer). We compare the decisions and profits i) when each entity in the supply chain acts independently (Nash equilibrium), ii) when the suppliers collaborate (cooperative setting) (but the retailer still acts independently) iii) when the whole supply chain is centrally coordinated (see Figure 1). Furthermore, we quantify how much is lost in terms of total profit in the supply chain due to lack of coordination. We ask questions of the following nature: How much do suppliers lose in terms of profit due to the fact that they are competing and not cooperating? What is the effect of vertical vs. horizontal competition? How do capacities affect the equilibrium profits? How do capacities affect the overall supply chain profit loss due to competition and lack of coordination? Can the maximum profit loss be very large? In order to isolate the capacity effects, we consider two suppliers A and B who differ only through their production capacities. We refer to the supplier with lower capacity as supplier B.

We find several **novel insights**, described next. (1) We show what might at first seem like a "counter intuitive" result, namely that supplier B's profit at the equilibrium is *not* monotonically increasing with her capacity level, i.e. increasing her capacity level (even at no cost) may yield lower profits at equilibrium, and hence it may be beneficial for this supplier to lower its capacity. We give in Section 3 an explanation for this finding. (2) We also show that the total decentralized profit loss (as compared with the centrally coordinated optimum) can be no greater than 25%. This 25% bound (which appears often in the literature) is tight when the suppliers are symmetric with a large enough capacity level and either there is no production cost or the products are perfect substitutes. The managerial implication of this result is that competing suppliers have the most to gain from coordination among themselves and the retailer when they are symmetric in terms of their capacity level, capacity is large, and either production cost is low or products are perfect substitutes. (3) We find that the total profit loss is in general not monotonic with the degree of asymmetry of the two capacities. However, we show that (4) the loss of supplier profit decreases with the degree of asymmetry in the system, and (5) we provide an upper bound on the loss of supplier profit that depends only on the price sensitivities and the market base (i.e., demand when prices are zero).

This implies that competing suppliers have the less and less to gain from coordination between themselves when they become less symmetric in terms of their capacity level. In particular, we find that the loss of supplier profit is worst when the capacity is sufficiently large and either there is no production cost or the products are perfect substitutes, which are thus conditions when competing suppliers have the most to gain from coordination among themselves.

1.2 Relation to the literature

This work is related to the Economics literature on price competition. The books by Vives [46] and Tirole [44] survey major results on Bertrand competition. A large stream of Economics literature on price competition in the presence of capacity constraints assumes that firms are competing to supply a non-differentiated product, and hence the consumers select the firm with the lowest price. This type of demand allocation implies that products are not differentiated. Kreps and Scheinkman [25] consider a two-stage duopoly setting where the two agents make quantity pre-commitments, then compete on prices: demand is allocated in priority to the supplier with a lower price, the other supplier is allocated any unmet demand, in the limit of their pre-committed quantities. They show that under certain assumptions this type of Bertrand competition with quantity pre-commitment yields the same unique equilibrium as a Cournot game. Our paper differs from these models in a variety of ways: production is introduced, production capacity is exogenous, products are differentiated, and the demand depends on the prices of both players and does not go in priority to the lowest-priced firm.

Our paper is also related to the literature on capacitated production management. Van Mieghem [45] provides a literature review on strategic capacity management and Kapuscinski and Tayur [22] review the literature on capacitated supply chains. Carr, Duenyas and Lovejoy [10] consider price competition in a capacitated setting under demand and capacity uncertainty. They show that improvements in firms' production processes, such as reduction of production variability, may actually lead to reduced profits for the firm. Nevertheless, to the best of our knowledge, quantifying the effect of capacity on the loss of profit in a supply chain with two suppliers and one retailer has not been studied.

Our work also complements the "price of anarchy" literature first introduced by Papadimitriou and Koutsoupias [24]. This stream of literature introduced in recent years has been trying to quantify the inefficiency of equilibria in non-cooperative games, compared with a centrally managed setting. The book by Roughgarden [40] provides an extensive coverage of results on the price of anarchy literature. The term price of anarchy refers to the worst-case ratio between the value of the system objective (in our setting, overall supplier profits or total supply chain profits) under user optimum (when competing agents act unilaterally in a Nash game, i.e. decentralized setting) versus that of the system optimum (when a single central decision-maker optimizes the entire system, or centralized coordination setting). Chen et al. [11] quantify the inefficiency that occurs

in competitive supply chains due to the variance of orders versus the variance of demand. Perakis and Roels [37] analyze different supply chain configurations and compute the price of anarchy between the integrated supply chain and the decentralized supply chain for price-only contracts. Authors found in a variety of contexts and models that the loss of efficiency due to competition cannot exceed 25%, including Martínez-de-Albéniz and Simchi-Levi [29] in a procurement game with option contracts, Farahat and Perakis [16, 15] in multi-product oligopolistic price competition, Roughgarden and Tardos [41] in traffic routing with congestion, and Johari and Tsitsiklis [21] in a network resource allocation model with congestion.

This paper contributes to the supply chain efficiency literature. In the model introduced by Netessine and Zhang [33], the retail price is exogenous and retailers face a stochastic demand that depends on the order quantities of all retailers. They compare the double marginalization effect in the case of substitute products as well as complement products. Martínez-de-Albéniz and Simchi-Levi [29] consider competition between multiple suppliers that offer option contracts to one buyer facing a stochastic demand. They find that the loss of profit due to competition cannot exceed 25%. Martínez-de-Albéniz and Roels [28] consider a supply chain with multiple competing suppliers and a single retailer, where suppliers compete both in price and shelf space. They show that the loss of efficiency due to suboptimal shelf space allocations is small, but the loss of efficiency due to suboptimal retail pricing may be as large as 27%. Two papers consider a setting similar to ours. Choi [12] studies a supply chain with two manufacturers and one retailer under both linear and nonlinear demand. Nevertheless, our paper differs from this model due to the presence of capacity constraints, as well as in terms of focus. We are concerned with the loss of profit due to lack of coordination in the supply chain with and without supplier collusion while Choi [12] investigates the effect of power structure, cost asymmetry, and product differentiation on the equilibrium. Cachon and Kök [8] also consider two manufacturers that compete for a single retailer's business. Their objective is to understand which kind of contracts (wholesale-price contract, quantity-discount contract, and two-part tariff) aimed at coordinating the supply chain would be preferable from the point of view of respectively the retailer and the manufacturers. While we also consider two suppliers and one retailer, we are not concerned with the impact of different contractual forms but with the effect of capacity constraints on the loss of profits due to the lack of coordination.

Outline: The paper is structured as follows. In Section 2 we describe three supply chain settings. In particular, in Subsection 2.2, we consider a two-tier supply chain where the suppliers act in a decentralized manner, we present the suppliers' problem formulation in the decentralized setting and provide the Nash equilibrium solution. In Subsection 2.3, we consider a two-tier supply chain where in the first tier the two suppliers cooperate, that is, the suppliers' decisions are made jointly to optimize their overall profits. In Subsection 2.4, we formulate and solve the centrally coordinated problem that considers the total profits of the retailer and the two suppliers. In Section 3, we investigate the effect of capacity asymmetry on the suppliers' profits at equilibrium.

In Section 4, we provide a bound on the total supply chain profit loss due to lack of coordination. We consider symmetric and asymmetric suppliers in terms of their capacities. In Section 5, we focus on the supplier profit loss due to competition and give two bounds. We show that it decreases with the degree of asymmetry (in terms of the capacities of the two suppliers).

2 Three Two-Tier Supply Chain Settings

In this section we lay the foundations (notations, formulations and optimal policies) for the three supply chain settings we consider in this paper. That is, we formulate and analyze three two-tier supply chain settings: (a) when suppliers act in a decentralized manner, (b) when suppliers belong to the same company and hence they are cooperating (horizontal integration), (c) when the two tiers of the supply chain are centrally coordinated (see Figure 1).

Figure 1: Three settings considered: (a) Nash equilibrium (decentralized setting) (b) Cooperative setting (c) Centralized coordination. The circled entities jointly make decisions in each setting.

In what follows we describe the assumptions we impose throughout this paper. Furthermore, we refer the reader to Appendix A for the notation we will use throughout this paper.

We consider a single-period, single-product problem. The suppliers, acting as leaders and competing among each other via Nash competition, decide the wholesale price for their respective product. The two products are gross substitutes. The retailer is the follower: for given wholesale price decisions of the suppliers, the retailer decides the quantity of each product to order. The retailer's prices are set to clear the retail market.

First tier: suppliers. In the decentralized setting two suppliers $k = A, B$ compete by adjusting their respective wholesale prices $p^k,\ k = A, B$ in order to maximize their profit. Without loss of generality, we assume that $K^B \leq K^A$, where K^k denotes supplier k's capacity level. To study the effect of capacity, it is sufficient with no loss of generality to fix one capacity level (here, K^A) and vary the other (K^B). The products they are selling to the retailer are differentiated and substitutable. The wholesale price vector p is denoted by (p^A, p^B). Since the model is single period, deterministic and the suppliers have no initial inventory, the production quantities are determined to exactly match the quantity ordered by the retailer, so that there will be no remaining inventory. Our goal is to focus on the effect of production capacity, therefore, we consider a problem where the suppliers are completely symmetric in terms of cost and demand parameters; they differ only

by their production capacities K^A and K^B. We assume in this paper that the production cost is quadratic: γu^2, where u is the production quantity (equal to the quantity ordered by the retailer). This type of cost has been used often in the literature on production and inventory control (see [3, 14, 20, 38, 42, 2] for example). As noted by Ha et al. [19], this cost function means that production has a diseconomy of scale, that is, increasingly more expensive production capacity. Empirical evidence supports this assumption in several industries, such as petroleum refining [18] and auto-making [32].

Second tier: retailer. The relationship between the retail prices and demand of the two products at the retailer is modeled using a linear inverse demand model:

$$\bar{\mathbf{p}}(\mathbf{D}) = \bar{\mathbf{a}} - \bar{B}\mathbf{D},$$

where $\mathbf{D} = (d^A, d^B)$ is the vector of quantities of the two products sold by the retailer, $\bar{\mathbf{p}}(\mathbf{D}) = (\bar{p}^A(\mathbf{D}), \bar{p}^B(\mathbf{D}))$ is the vector of retail prices, $\bar{\mathbf{a}} = (\bar{\alpha}, \bar{\alpha})$ is a constant vector corresponding to the prices should the quantities be equal to zero, and $\bar{B}$ is the matrix of inverse retail price–demand sensitivities. We assume that $\bar{B}$ is given by:

$$\bar{B} = \begin{pmatrix} \bar{\beta} & \bar{\beta}' \\ \bar{\beta}' & \bar{\beta} \end{pmatrix}$$

so that

$$\bar{p}^A(\mathbf{D}) = \bar{\alpha} - \bar{\beta}d^A - \bar{\beta}'d^B, \qquad \bar{p}^B(\mathbf{D}) = \bar{\alpha} - \bar{\beta}d^B - \bar{\beta}'d^A.$$

The assumption of a linear inverse demand function of prices is common in the revenue management and pricing literature (see for example [10, 16, 26, 30, 35, 47, 48]). In this model, $\bar{\beta}$ denotes the retail price sensitivity of a product with respect to its own demand and $\bar{\beta}'$ denotes the retail price sensitivity of a product with respect to its competitor's product's demand.

Assumption 1.
1. $\bar{\beta} > 0$. A product's retail price is a strictly decreasing function of its own demand.
2. $\bar{\beta}' \geq 0$. A product's retail price is a decreasing function of its competitor's product's demand (that is, the products that the two suppliers offer are gross substitutes).
3. $\bar{\beta}' < \bar{\beta}$. This suggests that the price of a product is more sensitive to that product's demand rather than to the competitor's product's demand.

This assumption is standard in price–demand models (see Vives [46]). Note that under Assumption 1, matrix $\bar{B}$ is invertible.

2.1 The retailer's problem

The retailer is a follower and reacts to the wholesale price decision $\mathbf{p} = (p^A, p^B)$ made by the suppliers. Proceeding by backward induction, we consider the optimization problem faced by the retailer for a given vector of wholesale prices.

The retailer must select the quantity vector $\mathbf{D} = (d^A, d^B)$ of the two products to order from the suppliers (since the model is deterministic, there is no mismatch between the quantity ordered by the retailer and the quantity sold to the consumer) in order to maximize her net profits.

$$\max_{\mathbf{D}} J^R = \bar{\mathbf{p}}(\mathbf{D})^T\mathbf{D} - \mathbf{p}^T\mathbf{D} = (\bar{\mathbf{a}} - \bar{B}\mathbf{D})^T\mathbf{D} - \mathbf{p}^T\mathbf{D}.$$

Note that the retailer's order quantity is not subject to an upper limit as the supplier adjusts its wholesale price in order to be able to fulfill the retailer's order in its entirety. It is easy to show that maximizing the retailer profit J^R for a fixed price vector p over quantity vectors $\mathbf{D}$ leads to optimal order quantities $\mathbf{D}(\mathbf{p}) = \frac{1}{2}\bar{B}^{-1}(\bar{\mathbf{a}} - \mathbf{p})$. We denote

$$\beta \equiv \frac{1}{2}\frac{\bar{\beta}}{\bar{\beta}^2 - \bar{\beta}'^2}, \quad \beta' \equiv \frac{1}{2}\frac{\bar{\beta}'}{\bar{\beta}^2 - \bar{\beta}'^2}, \quad B \equiv \frac{1}{2}\bar{B}^{-1} = \begin{pmatrix} \beta & -\beta' \\ -\beta' & \beta \end{pmatrix}, \quad \alpha \equiv \bar{\alpha}(\beta - \beta'), \quad \mathbf{a} \equiv \frac{1}{2}\bar{B}^{-1}\bar{\mathbf{a}} = (\alpha, \alpha)$$

The retailer's optimal order quantity decisions can be written as $\mathbf{D}(p) = \mathbf{a} - B\mathbf{p}$, i.e. the quantity of each product is a linear function of the corresponding supplier's wholesale price and of the competitor's wholesale price. In particular, we can rewrite it as:

$$d^A(\mathbf{p}) = \alpha - \beta p^A + \beta' p^B, \qquad d^B(\mathbf{p}) = \alpha - \beta p^B + \beta' p^A.$$

Notice, β denotes the demand sensitivity of a supplier with respect to her own price and β' denotes the demand sensitivity of a supplier with respect to her competitor's price.

Corollary 1. *It follows from Assumption 1 that 1. $\beta > 0$: a supplier's demand is a strictly decreasing function of her own wholesale price.*
2. $\beta' \geq 0$: a supplier's demand is an increasing function of her competitor's wholesale price (consistent with the assumption that the products that the two suppliers offer are gross substitutes).
3. $\beta' < \beta$: the demand observed by a given supplier is more sensitive to that supplier's wholesale price rather than to her competitor's wholesale price of the product.

This result can be rephrased in the following way: Supplier A's demand (order quantity he/she receives) increases when her own price decreases, or when supplier B's price increases. Furthermore, if we consider a price change of one unit for each supplier, supplier A's demand (order quantity he/she receives) increases more when her own price has changed rather than when supplier B's price has. In other words, if both prices increase by the same amount simultaneously, both demands decrease. Moreover, if the price applied by supplier B increases, then the demand observed by

supplier B decreases more than the demand observed by supplier A increases. Therefore, the price chosen by a given supplier affects her own demand more than it affects her competitor's demand, which is consistent with standard economics models (see Vives [46]).

Note that properties 1, 2 and 3 of Corollary 1 imply Assumption 1.

For ease of notation in the remainder of the paper, we use α, β, β', a and B rather than $\bar{\alpha}, \bar{\beta}, \bar{\beta}', \bar{a}$ and $\bar{B}$ to be more succinct and we define the following quantities: $b \equiv 2\beta(1+\gamma\beta) - \beta'(1+2\gamma\beta)$, $d \equiv \sqrt{2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta)}$. Notice that

$$b(\beta + \beta') = d^2 + \beta\beta'. \tag{1}$$

2.2 Decentralized Suppliers

In the decentralized supplier setting, the two suppliers compete on prices, anticipating the reaction of the retailer to their decisions, and are subject to production capacity. Such a setting arises for example, in the car industry when different manufacturers (say Toyota and Honda) produce cars targeting the same market segment (say Camry and Accord) and face capacity constraints (see [6]).

In the supplier decentralized setting, supplier $k = A, B$ determines her wholesale price, given a wholesale pricing policy $\tilde{p}^{-k}$ for supplier $-k = B, A$, by solving the following best response, optimization problem:

$$\begin{aligned} J^k = \max_{u^k, p^k} \quad & p^k d^k(p^k, \tilde{p}^{-k}) - \gamma(u^k)^2 \\ \text{such that} \quad & u^k = d^k(p^k, \tilde{p}^{-k}) \\ & 0 \le u^k \le K^k \\ & p^k \ \ge 0. \end{aligned}$$

The following result establishes the existence and uniqueness of an equilibrium in this setting. All the proofs of the results in this paper can be found in Appendix G. (The proof of Theorem 1 follows directly from Proposition 6 in Appendix C and is thus omitted.)

Theorem 1. *Under Assumption 1, there exists a unique Nash equilibrium solution.*

The equilibrium production quantities, prices and profits are derived in Appendix C.

Our goal is to understand the effect of production capacity. Therefore, in order to isolate that effect, we will focus on letting capacity level of supplier B (denoted K^B) vary between zero and K^A while keeping other input parameters constant. This leads us to the following observation. Three regimes are possible at equilibrium, which can be viewed as follows[1]:

[1]Note that it is possible to have $\frac{\alpha\beta(\beta+\beta') - d^2K^A}{\beta\beta'} > \frac{\alpha\beta}{b}$, but, as shown in the Appendix, the case $\frac{\alpha\beta(\beta+\beta') - d^2K^A}{\beta\beta'} > K^B > \frac{\alpha\beta}{b}$ may not occur, and so the results remain correct as stated.

- (Regime a) If $K^B > \frac{\alpha\beta}{b}$, both suppliers produce at an intermediate level (i.e. *not* at full capacity). Because the capacity is not tight, the suppliers are in effect totally symmetric and therefore the production and price decisions at the equilibrium are equal among suppliers and independent of the production capacities.

- (Regime b) If $\frac{\alpha\beta(\beta+\beta')-d^2K^A}{\beta\beta'} < K^B < \frac{\alpha\beta}{b}$, supplier B produces at full capacity and supplier A at an intermediate level (between zero and K^A), while still producing at least as much as supplier B. Since supplier B has a lower production capacity, it is natural to obtain that there is a situation where supplier B produces at full capacity but not supplier A. Note that the lower bound on K^B depends on K^A. Indeed, supplier A needs to produce more once supplier B has reached full capacity, so supplier B's capacity cannot be too low for supplier A's production level to be intermediate.

- (Regime c) If $K^B < \frac{\alpha\beta(\beta+\beta')-d^2K^A}{\beta\beta'}$, both suppliers produce at full capacity.

Let us consider, for comparison purposes, a monopoly setting. The derivation of the monopoly solution can be found in Appendix B. Production level $K_1 = \frac{\alpha}{2(1+\gamma\beta)}$ maximizes the profit in the absence of capacity constraint. Therefore, two cases are possible: if $K > K_1$, the optimal production level is K_1 and is thus intermediate (between 0 and K); if $K \leq K_1$, then it is optimal to produce at full capacity K. Our results in the duopoly setting extend this result to three possible regimes.

To reduce notation we define $l_a \equiv \frac{\alpha\beta}{b}$, $l_b(K^A) \equiv \frac{\alpha\beta(\beta+\beta')-d^2K^A}{\beta\beta'}$. In the decentralized duopoly setting, regime a corresponds to $l_a < K^B < K^A$, regime b to $l_b(K^A) < K^B < l_a$, and regime c to $0 < K^B < l_b(K^A)$. We observe that prices are either constant or decrease linearly with K^B (the latter in regimes b and c, where supplier B is at full capacity). In other words, if capacity is tight for supplier B, customers are charged lower prices when capacity K^B increases, until regime a is reached.

2.3 Supplier Cooperation

In this subsection, we study horizontal integration: we assume that the two suppliers make decisions jointly in order to maximize their total profit. Horizontal competition among the two suppliers is thus eliminated, but the effect of vertical competition with the retailer remains. This setting can be considered simply as a benchmark to understand the value gained/lost due to competition among suppliers. This setting could however arise in practice if for example, the suppliers are two branches of the same company. Consider for instance a corporation like Procter and Gamble, owning the brands of detergent Gain and Tide which are substitute products. While each brand is managed by its own brand manager, upper management could make the two brands cooperate to achieve optimal joint performance.

In what follows we formulate the optimization problem under supplier cooperation and analyze the optimal policy.

$$\begin{aligned} J = J^A + J^B = \max_{u^k, p^k,\ k=A,B} \quad & \sum_{k=A,B} p^k d^k(\mathbf{p}) - \gamma (u^k)^2 \\ \text{such that} \quad & u^k = d^k(\mathbf{p}),\ k = A, B \\ & 0 \le u^k \le K^k,\ k = A, B \\ & p^k \ \ge 0,\ k = A, B. \end{aligned}$$

This problem has a unique optimal solution that is derived in closed form in Appendix D.

We note in particular that unless none of the suppliers produce at full capacity, the prices are different for the two suppliers, even though the decisions are made jointly and only the overall profit is maximized. Indeed, when both suppliers produce at an intermediate level, the actual value of the capacity becomes irrelevant and as a result the suppliers are fully symmetric and thus prices and quantities are symmetric as well. We observe that if A and B are at full capacity, the solution is identical to the decentralized setting equilibrium where both suppliers are also at full capacity – but the condition for this case to hold is different. This follows from the fact that, for both settings, when it is in the suppliers' best interest to be at full capacity, then $K^k = \alpha - \beta p^k + \beta' p^{-k},\ \ k = A, B$. This yields teh same system of two linearly independent equations on $p^A,\ p^B$ in both the coordinated and decentralized settings

$$\begin{cases} \beta p^A - \beta' p^B = \alpha - K^A \\ \beta p^B - \beta' p^A = \alpha - K^B \end{cases}$$

Therefore, at full capacity, the prices are the same in the cooperative and the decentralized setting. As a result, we observe that for low enough capacities, supplier collusion does not make any difference in the outcome.

Three regimes are possible at the optimum, which we can view as follows:

- (Regime a') If $\frac{\frac{\alpha}{2}}{1+\gamma(\beta-\beta')} < K^B < K^A$, then both suppliers produce at an intermediate level (i.e. *not* at full capacity) and the production and price decisions at the optimal level are thus equal for the two suppliers and independent of the production capacity.

- (Regime b') If $\frac{1}{\beta'}\Big((\beta+\beta')\frac{\alpha}{2} - K^A(\beta + \gamma(\beta^2 - \beta'^2))\Big) < K^B < \frac{\frac{\alpha}{2}}{1+\gamma(\beta-\beta')}$, then supplier B produces at full capacity, but supplier A produces at an intermediate level.

- (Regime c') If $K^B < \frac{1}{\beta'}\Big((\beta+\beta')\frac{\alpha}{2} - K^A(\beta + \gamma(\beta^2 - \beta'^2))\Big)$, then both suppliers produce at full capacity, and price (and produce) as they would in a decentralized setting.

These three regimes have the same interpretation as in the decentralized setting, but the conditions are different. We show below some examples of the cooperative optimal solution and compare it with the decentralized equilibrium solution. Figure 2 illustrates the three regimes described above. Figures 8(a) and (b) in Appendix F also show the solutions in other examples. We naturally notice that the overall profits are higher under supplier cooperation than in the decentralized system.

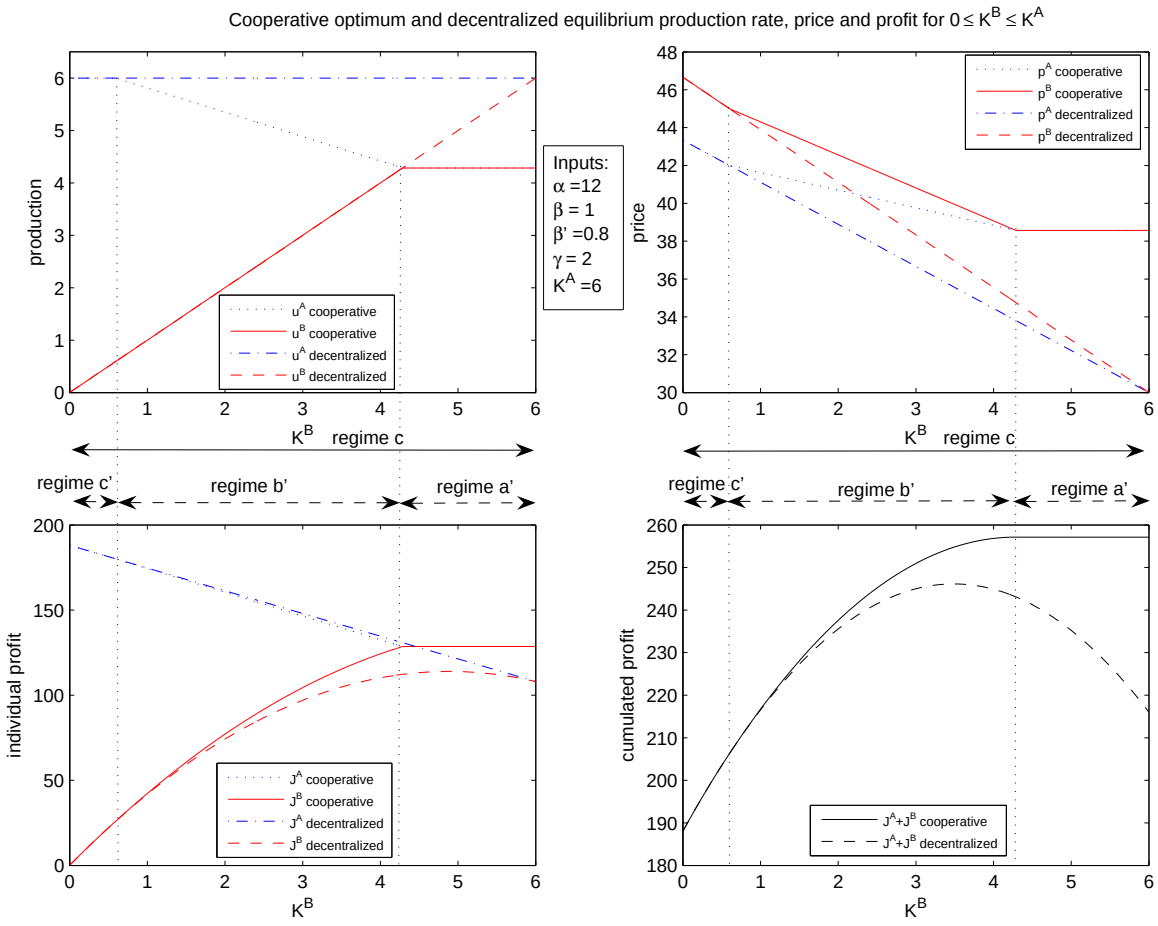


Figure 2: Example: cooperative optimal solution and decentralized equilibrium when B's capacity varies from zero to K^A. The system is in one of 3 possible regimes, depending on whether zero, one or two suppliers produce at full capacity.

Let us now compare the cooperative optimal solution and the decentralized equilibrium solution. Let $l'_a \equiv \frac{\frac{\alpha}{2}}{1+\gamma(\beta-\beta')}$, $l'_b(K^A) \equiv \frac{1}{\beta'}\Big((\beta+\beta')\frac{\alpha}{2} - K^A(\beta+\gamma(\beta^2-\beta'^2))\Big)$ the threshold capacity values between regimes so that regime a' corresponds to $l'_a < K^B < K^A$, regime b' to $l'_b(K^A) < K^B < l'_a$, and regime c' to $0 < K^B < l'_b(K^A)$. It is easy to show that $l_a > l'_a$, i.e. the threshold value

for K^B when supplier B's production quantity reaches full capacity is higher in the decentralized setting than in the cooperative setting. In other words, if supplier B is not at full capacity in the decentralized setting, then she is not at full capacity in the cooperative setting either (i.e. if regime a holds, then regime a' holds). This means that the presence of competition triggers supplier B to produce at full capacity when she might not, should the decisions be taken jointly. In addition, decentralized quantities are higher than centralized quantities.

Lemma 1. *(i) If the inputs are such that regime c' holds under supplier cooperation, then regime c holds at the decentralized equilibrium. (ii) Production quantities under competition are identical or higher than under cooperation for each supplier.*

This result implies that the previous observation for supplier B is valid for supplier A: under competition, for decreasing values of K^B, both suppliers tend to switch "too quickly" (i.e. for too high value of K^B) from intermediate to full production capacity when comparing with a cooperative setting.

2.4 The Centrally Coordinated Supply Chain

In this subsection, we take the point of view of a supply chain central planner who optimizes the overall retailer and suppliers' profit. This allows us to compare the decentralized and cooperative performance with the performance in a fully integrated supply chain where the suppliers and the retailer make joint decisions to optimize the entire system. This comparison also enables us to determine the value of coordinating contracts between the different parties of the supply chain.

The supply chain total profit Π is equal to the sum of the retailer's and the suppliers' profits: $\Pi = J^R + J^A + J^B$. Notice that $\bar{\mathbf{a}} = 2\bar{B}\mathbf{a}$ and $\bar{B} = \frac{1}{2}B^{-1}$. Therefore, we obtain

$$\Pi \;=\; \bar{\mathbf{p}}(\mathbf{D})^T\mathbf{D} - \gamma\mathbf{D}^T\mathbf{D} = (B^{-1}\mathbf{a})^T\mathbf{D} - \frac{1}{2}\mathbf{D}^T B^{-1}\mathbf{D} - \gamma\mathbf{D}^T\mathbf{D}.$$

To view this problem from a pricing perspective, we use $\mathbf{D}(\mathbf{p}) = \mathbf{a} - B\mathbf{p}$ as found in Section 2.1 and we obtain after simplification

$$\begin{aligned} J^R &= \frac{1}{2}\mathbf{p}^T B\mathbf{p} - \mathbf{p}^T\mathbf{a} + \frac{1}{2}\mathbf{a}^T B^{-1}\mathbf{a} \\ \Pi &= -\frac{1}{2}\mathbf{p}^T(B + 2\gamma B^2)\mathbf{p} + 2\gamma\mathbf{a}^T B\mathbf{p} + \frac{1}{2}\mathbf{a}^T B^{-1}\mathbf{a} - \gamma\mathbf{a}^T\mathbf{a}. \end{aligned}$$

The central planner's goal is to maximize the total supply chain profits Π. The central planner's

optimization problem can thus be formulated as a linearly-constrained quadratic problem:

$$\begin{aligned} \max_{\mathbf{p}} \quad & -\frac{1}{2}\mathbf{p}^T(B+2\gamma B^2)\mathbf{p} + 2\gamma \mathbf{a}^T B\mathbf{p} + \frac{1}{2}\mathbf{a}^T B^{-1}\mathbf{a} - \gamma \mathbf{a}^T\mathbf{a} \\ \text{s.t.} \quad & 0 \leq \mathbf{a} - B\mathbf{p} \leq \bar{\mathbf{K}} \\ & \mathbf{p} \geq 0 \end{aligned}$$

where $\bar{\mathbf{K}} = (K^A, K^B)$.

The centrally coordinated optimal solution to the central planner's optimization problem is provided in Appendix E.

We observe that similarly to the decentralized equilibrium and to the cooperative supplier solution, the centrally coordinated optimum solution can be in one of three possible regimes. In the first regime (K^B large), neither of the two suppliers use all of their available capacity, hence the price and production amounts are the same for the two suppliers. In the second regime (K^B intermediate), B is at full capacity, and A is not. In the third regime, both suppliers are at full capacity. The thresholds between these regimes are different than in the decentralized and cooperative settings studied in previous sections. We note that the threshold on K^B between the first and second regimes is larger than for the decentralized setting, and the threshold between the second and third regimes is larger than for the cooperative setting.

3 Effect of Capacity on the Equilibrium Profits

3.1 Effect on Individual Profits

We first establish that in the decentralized setting, supplier A is better off when her competitor's production capacity decreases.

Proposition 1. *Supplier A's profit at the Nash equilibrium is non increasing with K^B.*

We next show that supplier B's profit at the equilibrium as a function of her capacity is not monotonic, and may in fact have up to two local maxima (see Figure 4). This implies that under some conditions there exists a critical capacity level beyond which supplier B has no incentive to increase her capacity (at no cost[2]), as an increase would lead her equilibrium profit to decrease (this is illustrated in Figure 3).

[2]For example, increasing capacity would not incur costs when supplier B has the possibility to reallocate existing resources. Note that there would be even less of an incentive to increase capacity if increasing capacity incurred extra costs.

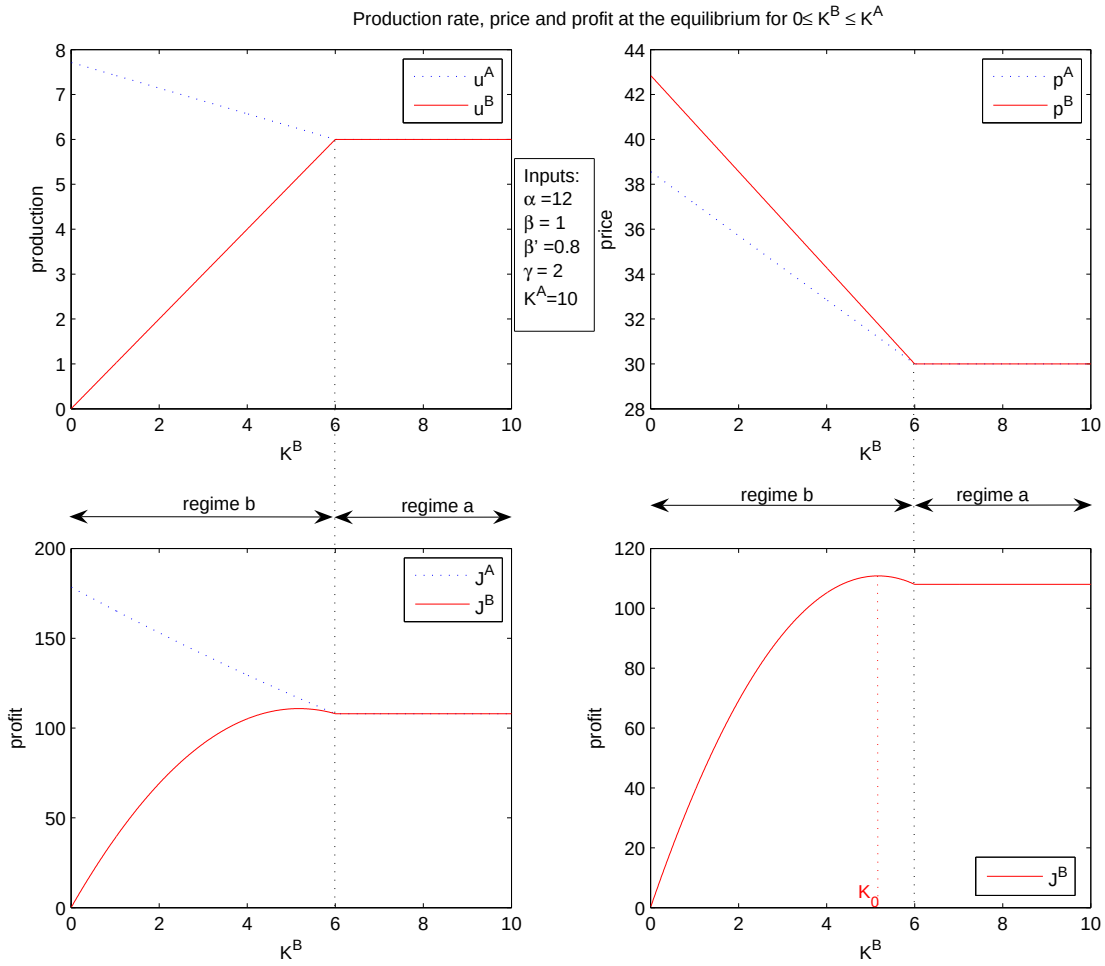


Figure 3: Example: equilibrium solution when supplier B's capacity varies from zero to K^A with inputs as in Figure 8(a). The profit J_d^B reaches a maximum at $K_0 \in (0, K^A)$ in regime b, which implies that there exists a capacity level for supplier B beyond which B's profit at the equilibrium decreases as the capacity increases.

Proposition 2. *Due to Corollary 1, if* $\alpha\beta(\beta+\beta')-K_0\beta\beta'-d^2K^A < 0$, *where* $K_0 \equiv \frac{\alpha(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta))}{2(2\beta(1+\gamma\beta)^2-\gamma\beta'^2(1+2\gamma\beta))}$, *then* $K^B = K_0 \in [0, K^A]$[3] *is a local maximum. Moreover, if*

$$\frac{\alpha(\beta+\beta')}{2\beta+\beta'+2\gamma(\beta^2-\beta'^2)} \leq K^A \leq \frac{\alpha(\beta+\beta')}{\beta'} \tag{2}$$

$$\alpha\beta(\beta+\beta') - \bar{K}_0\beta\beta' - d^2K^A > 0, \tag{3}$$

[3]Since $1+\gamma\beta > \gamma\beta'$, and $\frac{2(1+\gamma\beta)}{1+2\gamma\beta} > 1$, therefore $\beta(1+\gamma\beta)\frac{2(1+\gamma\beta)}{1+2\gamma\beta} > \gamma\beta'^2$ and the denominator is positive.

where $\bar{K}_0 \equiv \frac{\alpha(\beta+\beta')-\beta' K^A}{2\beta+2\gamma(\beta^2-\beta'^2)}$, *then* $\bar{K}_0 \in [0, K^A]$ *is a local maximum. There are no other possible local maxima.*

In particular, this proposition proves that profit J^B viewed as a function of K^B is not monotonic and may have at most two local maxima. We observe that if $\beta' = 0$ (i.e. if there is no competition as the two products offered are fully differentiated), then K_0 equals K_1 (that is, equals the capacity level that maximizes the profit in a monopoly setting, see Appendix B).

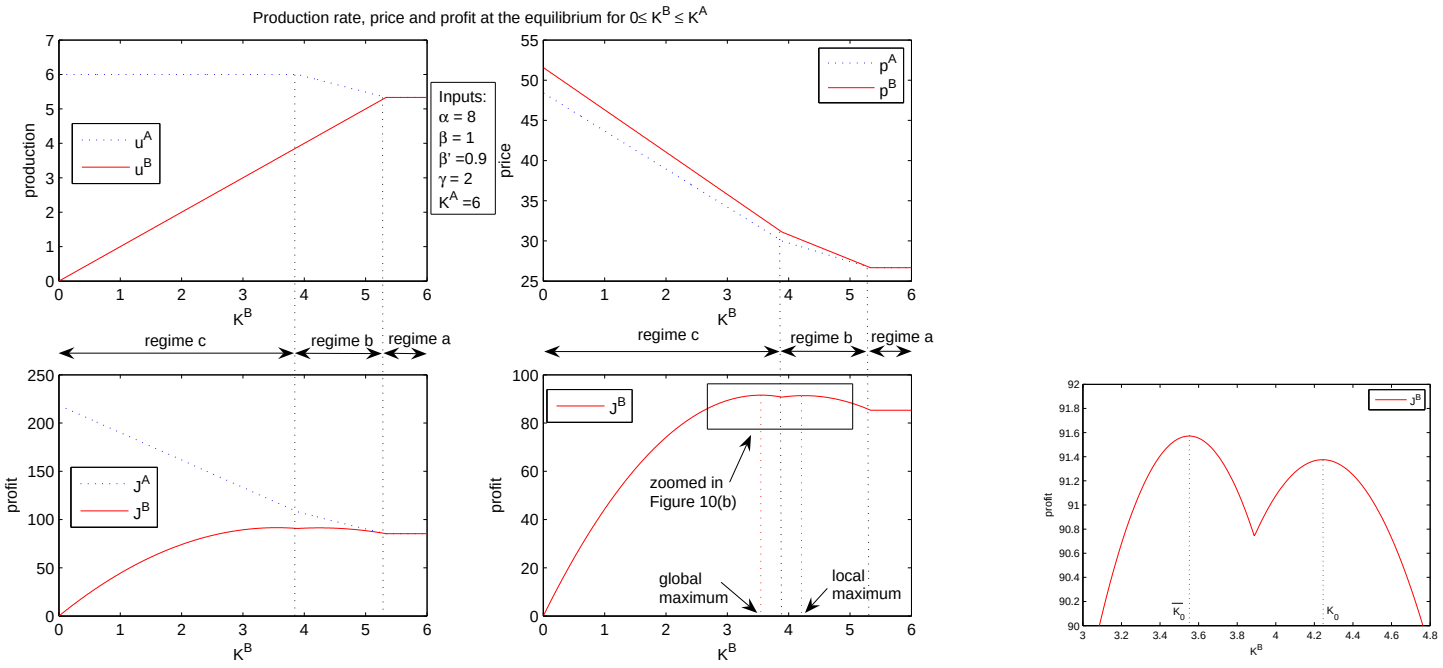


Figure 4: Example with inputs as in Figure 8(b): (a) equilibrium solution and corresponding profits when B's capacity varies from zero to K^A. The profit J^B reaches one local maximum and one global maximum. (b) Zoom of the profits plot.

We have shown what might at first seem as a "counter intuitive" result that, in a supplier duopoly setting, under some assumptions on the inputs, an increase of capacity for supplier B may *decrease* her profit at the equilibrium. In contrast, in a monopoly setting, an increase of capacity of a supplier who is operating at full capacity in the optimal strategy would increase her profit (or leave it unchanged, if the added capacity is unused). In a duopoly setting however, the same intuition is not valid at the equilibrium because competition affects the optimal strategy. If supplier B's capacity level is at a local maximum of profits J^B, then an increase of the capacity level results in *decreased* profits. In order to understand this effect, it is key to keep in mind that the decision making is decentralized and that this solution is an *equilibrium*. The local maxima occur in the interior of regimes b and c, i.e. when supplier B's production capacity constraint is

tight at the equilibrium. If supplier B's capacity increases slightly, we remain in the same regime, thus at the new equilibrium supplier B produces at full capacity (that is his best response to supplier A's strategy), hence he increases production, and therefore the price decreases (to increase demand). Price p^B is input in supplier A's optimization problem through the demand, so a decrease in supplier B's price incurs a decreased demand for supplier A. A new equilibrium is thus reached where supplier A decreases production and price. At the new equilibrium, both suppliers have lower profits. The customers thus gain from the added available capacity as it drives down the prices at equilibrium. Notice that supplier B cannot simply "ignore" the newly available capacity and not increase production to avoid lowering her profits, because this would not be a Nash equilibrium, in the sense that, assuming that supplier A keeps all decisions constant, supplier B is not at his best response and thus has an incentive to deviate. This paradox is analogous to the prisoner's dilemma paradox, where the Nash equilibrium is not Pareto optimum.

3.2 Effect on the Total Supplier Profit

The following result shows that, while supplier A always prefers that supplier B chooses the lowest possible capacity K^B, and supplier B prefers a capacity level that may vary within $[0, K^A]$, the overall system prefers a capacity level K^B that is lower than the one that supplier B prefers when acting selfishly; in other words, the overall system prefers a compromise between the two suppliers' interests: lower capacity than supplier B would want, but possibly higher than what supplier A would want.

Proposition 3. *Let $K^* \in [0, K^A]$ the value of K^B that maximizes the system profit $(J^A + J^B)$. Then K^* is less than or equal to the value of K^B that maximizes J^B.*

This proposition can lead to a potential useful agreement between the two suppliers. Such an agreement is practically relevant for example if the two suppliers represent two different branches of a same firm, that make decisions independently but that could have some level of cooperation to benefit the firm. Consider for instance a firm like Procter and Gamble, owning the brands of detergent Gain and Tide which are substitute products. While each brand is managed by its own brand manager whose goal is to maximize the performance of their product line, upper management could encourage a strategy of cooperation in order to achieve optimal overall corporate performance.

When supplier B decides her production capacity level "selfishly", she will do so in order to maximize her own profits, knowing that price and production for both suppliers will then be determined by the Nash equilibrium solution. As a result, B will select the capacity level K^B that maximizes her equilibrium profits J^B. However, supplier A may find it profitable to provide financial incentives so supplier B selects the capacity level that maximizes the total system profits.

Theorem 2. *An agreement such that supplier A pays supplier B a positive fee and supplier B must select a certain capacity level would benefit both suppliers.*

4 Loss of Supply Chain Profit

While the total supply chain profit under central coordination is by definition optimized, in practice a central planner in many situations cannot impose his/her decisions. Nevertheless, the centrally coordinated chain profit can be viewed as a benchmark to compare the actual decisions with those that would lead to the maximum total profits.

By definition, the total supply chain profit at the centrally coordinated optimum is at least as large as the total supply chain profits at the suppliers' cooperative solution and at least as large as the total profits at the Nash equilibrium. This suggests that both price competition within the Nash game and price collusion under supplier cooperation lead to a supply chain total profit that is lower than the supply chain total profit at the centrally coordinated optimum. A natural question is to measure this gap and determine in particular the worst case total profit at the Nash equilibrium, since the equilibrium is expected to occur when firms compete. Measuring this gap can help quantify the value of efforts to coordinate the supply chain (for example via the implementation of certain types of contracts), by calculating the amount of profits that could potentially be gained.

In what follows we denote by Π_{cc} the total supply chain profit when the supply chain is centrally coordinated, we denote by Π_d (resp. J_d^R, J_d^k) the total supply chain profit (resp. the retailer's profit, supplier k's profit) when the supply chain is decentralized (i.e. when there is horizontal and vertical competition) and finally, we denote by Π_c (resp. J_c^R, J_c^k) the total supply chain profit (resp. the retailer's profit, supplier k's profit) when the two suppliers are coordinated but the two tiers of the supply chain are not coordinated. We therefore define the supply chain loss of total profit as $1 - \frac{\Pi_d}{\Pi_{cc}} \in [0, 1]$. When the loss of total profit is high (close to 1), the Nash equilibrium leads to a low total profit, as compared with the centrally coordinated optimum. When the loss of total profit is low (close to 0), the Nash equilibrium is efficient as the total profit at the equilibrium is close to the maximum possible total profit.

4.1 Symmetric Suppliers

We now consider the symmetric supplier case and compare the retailer's profit as well as the total profit in the two-tier supply chain when the supply chain operates (a) under Nash equilibrium, (b) under supplier cooperation, and finally, (c) when the supply chain is at the centralized optimum. Note that the effect of a lack of coordination on the suppliers' profit is investigated in Section 5.

Theorem 3. *If $K^A = K^B \equiv K$, then*

1. $J_d^R \geq J_c^R$,
2. $\Pi_d \geq \Pi_c$, *and*
3. $$\Pi_{cc} \geq \Pi_d \geq \Pi_c \geq \Pi_{cc}\Big(1 - \frac{1}{4(1+\gamma(\beta-\beta'))^2}\Big) \geq \frac{3}{4}\Pi_{cc}. \tag{4}$$

The first result confirms that the retailer, as a follower, is better off when the symmetric suppliers compete horizontally than when the decisions for the two suppliers are made jointly. The second result determines that the entire supply chain is better off when the two symmetric suppliers compete, rather than when they make joint decisions. This second result may be viewed as non trivial since, while it is intuitive that the *retailer* should benefit from competition among the leaders, the suppliers' profit is by definition higher in the cooperative setting, so it is not obvious whether the system as a whole would benefit or be hurt by horizontal competition among the suppliers. This result can be interpreted as follows. Clearly, the retailer's profit increases and the suppliers' profits decrease when suppliers compete instead of cooperating. Therefore the supply chain profit is affected in two opposite directions by supplier competition. Our result means that the effect on the retailer's profit is stronger than the combined effects on the suppliers' profits, i.e. vertical competition dominates, in a sense, horizontal competition. The third result provides bounds on the worst total profit in both the cooperative and decentralized setting. The bound $1 - \frac{1}{4(1+\gamma(\beta-\beta'))^2}$ is depicted in Figure 5. In particular, it can be seen from the proof in Appendix G.2 that the bounds are tight: $\frac{\Pi_c}{\Pi_{cc}} = 1 - \frac{1}{4(1+\gamma(\beta-\beta'))^2}$ for K large enough, and $\frac{\Pi_c}{\Pi_{cc}} = \frac{3}{4}$ if K is large enough and $\gamma(\beta - \beta') = 0$ (no production cost or non-differentiated suppliers – $\gamma = 0$ or $r = 1$).

We have thus obtained that in the case of symmetric suppliers, the loss of total profit satisfies:

$$0 \leq 1 - \frac{\Pi_d}{\Pi_{cc}} \leq \frac{1}{4(1+\gamma(\beta-\beta'))^2} \leq \frac{1}{4},$$

in other words the supply chain loss of total profit cannot exceed 25%, and may be much lower especially as the production cost rises or the products offered by the firms become more differentiated. The value of this bound has implications to supply chain managers who need to determine the gains that could be obtained from designing and implementing contracts to coordinate the supply chain. Clearly, putting these contracts into practice incurs costs, and these costs could potentially outweigh the benefits that they provide. Hence, it is important to know the potential gains before deciding if implementing such coordinating mechanisms will be worth it.

4.2 Asymmetric Suppliers

We now consider asymmetric suppliers, i.e. $K^B < K^A$. Clearly, decreasing K^B from the value of K^A can only decrease the optimum centrally coordinated total profit as it allows less flexibility with the capacity constraint; however it is not clear how decreasing K^B may affect the total profit at the equilibrium and thus the loss of total profit. In fact, it is interesting to observe that the supply chain loss of total profit is not in general a monotonic function of K^B (see Figure 9 in Appendix F for some examples). This implies that *decreasing* the capacity available at supplier B may help in some cases *lower the loss of total profit* and thus make the total profit at the equilibrium become closer to the centrally coordinated optimum, and thus in a way make the system more efficient. This

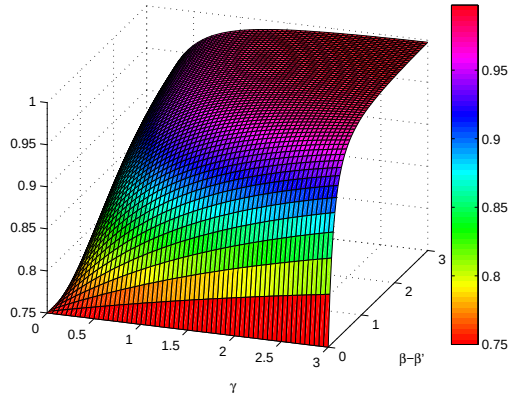


Figure 5: Illustration of the bound $1 - \frac{1}{4(1+\gamma(\beta-\beta'))^2}$ as a function of γ and $\beta - \beta'$.

insight is somewhat counter-intuitive as one may have conjectured that decreasing the capacity of one supplier, hence reducing her feasible space and flexibility, could only increase any measure of inefficiency; however this is not the case in terms of the loss of total profit. We further investigate in the following sections of the paper how changing K^B affects another measure of inefficiency (the loss of supplier profit of the Nash equilibrium compared with the cooperative solution) and the profits at equilibrium.

Numerical experiments indicate that the inequalities (4) and part 1 of Theorem 3, that were proved for symmetric suppliers, also hold in the non symmetric case (see Figure 10 in Appendix F for some examples).

5 Loss of Supplier Profit

The cooperative setting corresponds to a situation without horizontal competition, where a single decision-maker optimizes the sum of the two suppliers' profits. This situation may arise for example, when the two suppliers represent two branches of a common company, and the company management has the option to impose decisions to the two branches instead of letting them operate individually (in a competitive, decentralized way). Hence it is relevant to determine how much overall profit is lost due to the competition of the decentralized setting, as compared with the cooperative decisions, to quantify what would be gained from centralizing the decision making of the two suppliers.

Clearly, the suppliers' total profit in the decentralized setting cannot exceed the suppliers' total profit in the cooperative setting. However, it is possible that one supplier's profit be higher in the decentralized setting than in the cooperative setting, as long as the other supplier suffers more from decentralization that the first gains from it. For example, in Figure 2, in regime b', supplier A's profit is higher in the decentralized setting, and supplier B's profit lower; the sum of the profits is

lower because by having the suppliers make joint decisions, supplier A's profit decreases less than supplier B's profit increases. The following result shows that supplier B benefits from cooperation (or is indifferent). As supplier B is the supplier who is more constrained in terms of production capacity, cooperation enables supplier B to use some of supplier A's resources when necessary and thus to compensate for her handicap. Note that supplier A may earn more or less in the cooperative setting compared to the equilibrium solution depending on the input parameters.

Proposition 4. *Supplier B's profits under competition are no greater than those incurred under cooperation.*

Figure 11 in Appendix F depicts the profit ratios $\rho \equiv \frac{J_d^A+J_d^B}{J_c^A+J_c^B}$, $\rho^A \equiv \frac{J_d^A}{J_c^A}$, and $\rho^B \equiv \frac{J_d^B}{J_c^B}$ of the profit in a decentralized setting over the profit in a cooperative setting, for each supplier and for both, using data from previous examples. Analogously to the system loss of total profit, we call $1-\rho \in [0,1]$ the *loss of supplier profit*, which quantifies the decrease in supplier profit due to the presence of competition among them. When the loss of supplier profit is high (close to 1), the supplier profit at the Nash equilibrium is low compared with the supplier profit in the cooperative system. When the loss of supplier profit is low (close to 0), the Nash equilibrium is efficient in the sense that the supplier profit at the Nash equilibrium is close to its maximum possible value. As noted above, ρ and ρ^B can be no greater than 1, but this is not necessarily true for ρ^A. The term *price of anarchy* has been traditionally used in the recent literature (see [13, 36, 37, 40, 41]) and corresponds to the inverse of the profit ratio.

As illustrated in Figure 11(d), the loss of supplier profit may be quite large (i.e. the profit ratio can be quite low): up to 65% in this example, which raises the question of whether the loss of supplier profit can be bounded. We first start by analyzing the case of symmetric suppliers, and then we generalize to asymmetric suppliers.

5.1 Symmetric Suppliers

Consider the symmetric capacity case: $K^A = K^B \equiv K$. We denote $r = \frac{\beta'}{\beta} \in [0,1]$ the price sensitivity ratio. When r is close to 1, β' is close to β and the products offered by the two suppliers are less differentiated. In other words, if a supplier increases her price by a small amount, the total demand in the market decreases "very little". Conversely, when r is close to 0, β' is very small compared with β and the products offered by the two suppliers are more differentiated; in other words, if a supplier increases her price by a small amount, the total demand in the market decreases "a lot". Coefficient r can thus be viewed as a measure of the substitutability of the products offered by the two suppliers, and hence of the degree of competitiveness of the market. Farahat and Perakis [16], refer to this ratio as *market power*, that is, by how much the total demand in the market is affected when one supplier increases her price. The following proposition provides closed-form solutions for the suppliers profit ratio.

Proposition 5. *In the symmetric supplier setting, the profit ratio is given by*

- $\rho = 1 \quad if \quad K \leq \frac{\alpha}{2(1+\gamma(\beta-\beta'))} \quad$ *(regimes* (c, c')*),*
- $\rho = 1 - \left(2\frac{K}{\alpha}(1+\gamma(\beta-\beta')) - 1\right)^2$ *if* $\quad \frac{\alpha}{2-r+2\gamma(\beta-\beta')} \geq K \geq \frac{\alpha}{2(1+\gamma(\beta-\beta'))} \quad$ *(regimes* (c, a')*),*
- $\rho = 1 - \frac{r^2\beta^2}{b^2} \quad if \quad K \geq \frac{\alpha}{2-r+2\gamma(\beta-\beta')} \quad$ *(regimes* (a, a')*).*

Proposition 5 implies that the loss of supplier profit $1-\rho$ is: (i) zero when the common capacity level is sufficiently low (so that suppliers are at full capacity in both the decentralized and cooperative settings), (ii) is independent of the capacity level K when the common capacity is sufficiently large (so that suppliers are *not* at full capacity in both the decentralized and cooperative settings), and (iii) increases quadratically with the capacity level K for intermediate values (so that suppliers are at full capacity in the decentralized setting but not in the cooperative setting).

It follows from Proposition 5 that in the special case when $\gamma = 0$ (no production cost) or $r = 1$ ($\beta = \beta'$, i.e. products are perfect substitutes), the profit ratio simplifies to:

$$\rho = \begin{cases} 1 & \text{if } \frac{\alpha}{K} \geq 2 \\ 4\frac{K}{\alpha}\left(1 - \frac{K}{\alpha}\right) & \text{if } 1 \leq \frac{\alpha}{K} \leq 2 \\ 0 & \text{if } \frac{\alpha}{K} \leq 1 \end{cases}$$

implying that the loss of supplier profit is worst when the capacity is sufficiently large (greater than or equal to α) and either there is no production cost or the products are perfect substitutes. Note that $\rho = 0$ in the last case above does not mean that the suppliers combined profit at equilibrium is zero. It means that it is infinitely smaller than at the coordinated optimal solution because the optimal coordinated combined profit (for the symmetric case) approaches infinity in this case, while at equilibrium the combined profit remains finite.

The reason why an increase in capacity leads to a decrease in efficiency is related to the source of inefficiency in this problem. Inefficiency comes from the misalignment of decisions at the decentralized equilibrium and cooperative optimal solution, and more specifically, from the difference in quantity decisions: say, one a supplier produces a full capacity at equilibrium but at an intermediate level at the cooperative optimal. Therefore, the higher the capacity level, the bigger the room for a gaps between the quantity decisions of the two suppliers. Indeed, if capacity is very low, then in most cases all available capacity will be used both at equilibrium and at the cooperative optimal. As capacity increases, it becomes more and more likely that the decisions in the two settings differ.

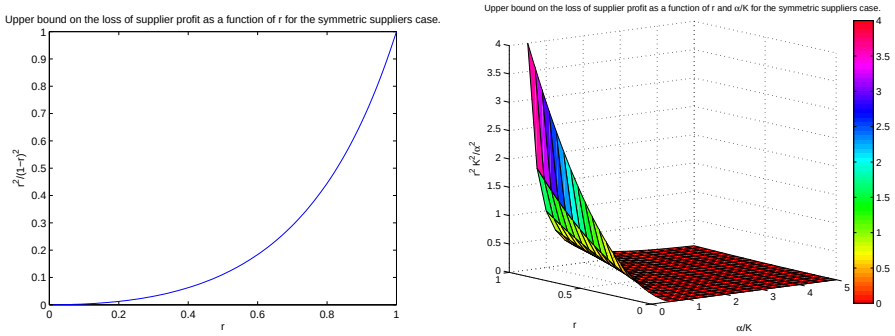
Next, we provide an upper bound on the profit loss $1-\rho$.

Theorem 4. *If* $K^A = K^B = K$*, then the loss of supplier profit* $1-\rho$ *cannot exceed* $\min\{\frac{r^2}{(2-r)^2}, r^2\frac{K^2}{\alpha^2}\}$*, where* $r = \frac{\beta'}{\beta}$*.*

Clearly, the best bound is $\frac{r^2}{(2-r)^2}$ if $\alpha/K < 2-r$, and $r^2\frac{K^2}{\alpha^2}$ otherwise. In other words, when the capacity is large with respect to α (maximum quantity ordered at each supplier), the upper

bound is independent of the capacity, but when the capacity is below a certain threshold, then the upper bound increases quadratically in K. We plot this bound in Figure 6. We observe that the loss of supplier profit bound $\frac{r^2}{(2-r)^2}$ is small as long as the suppliers have some non-negligible "market power"[4] (for example when $r < 0.6$, the loss of profit cannot exceed 20%; see Figure 6(a)). Indeed, it is easy to observe that the upper bound on the loss of supplier profit increases with r. Straightforward calculations lead to noticing that in the extreme case $\gamma = 0$, the bound is tight in regimes (a, a'), and in regimes (c, a') for $\frac{\alpha}{K}$ at the extreme $2 - r$ of its valid range.

Clearly, the bound on the loss of supplier profit $r^2\frac{K^2}{\alpha^2}$ is low for larger values of $\frac{\alpha}{K}$ and lower values of r. Note that by definition the loss of supplier profit is at most one, so this bound is relevant as long as it does not exceed one, i.e. for $\frac{\alpha}{K} > r$.



(a) (b)

Figure 6: Symmetric suppliers case. Upper bound on the loss of supplier profit (a) $\frac{r^2}{(2-r)^2}$ as a function of r; (b) $r^2\frac{K^2}{\alpha^2}$ as a function of r and $\frac{\alpha}{K}$.

To illustrate the tightness of the bound $r^2\frac{K^2}{\alpha^2}$, we compute the gap between the actual loss of supplier profit and the bound for a range of values of r and $\frac{\alpha}{K}$ in Table 1. To obtain the results in this table, we used as inputs $\gamma = 1, \beta = 1$. We would like to note that changing these inputs leads to similar observations. The bound appears to be very close to the actual loss of supplier profit for $\alpha/K \geq 2$ (or 3 if $r > 0.7$). This range of inputs corresponds to a value of the profit ratio close to 1 (i.e. loss of supplier profit close to zero) (see Table 2). The bound is much less tight for smaller values of α/K, (and may even be irrelevant when it takes values higher than 1 for $\frac{\alpha}{K} < r$), when the loss of supplier profit becomes higher. This is due to the fact that smaller values of α/K correspond to values of α/K in regimes (a, a') and far below the threshold value with regimes (c, a'). It is clear from the proof of Theorem 4 that the bound is tight when α/K is at the threshold value. When α/K takes values below this threshold, the actual loss of supplier profit is constant (equal to the

[4]Note that, as we already mentioned above, high market power (i.e. r close to zero) means that an individual supplier can influence significantly the total demand in the market when she increases her price while the other supplier keeps the same price.

		α/K									
		0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
	0.1	0.0393	0.0093	0.0037	0.0018	0.0009	0.0004	0.0001	0.0006	0.0005	0.0004
	0.2	0.1565	0.0365	0.0143	0.0065	0.0029	0.0010	0.0024	0.0025	0.0020	0.0016
	0.3	0.3506	0.0806	0.0306	0.0131	0.0050	0.0006	0.0073	0.0056	0.0044	0.0036
	0.4	0.6196	0.1396	0.0507	0.0196	0.0052	0.0133	0.0131	0.0100	0.0079	0.0064
r	0.5	0.9600	0.2100	0.0711	0.0225	0.0000	0.0278	0.0204	0.0156	0.0123	0.0100
	0.6	1.3656	0.2856	0.0856	0.0156	0.0432	0.0400	0.0294	0.0225	0.0178	0.0144
	0.7	1.8243	0.3543	0.0820	0.0325	0.0768	0.0544	0.0400	0.0306	0.0242	0.0196
	0.8	2.3100	0.3900	0.0344	0.1200	0.1024	0.0711	0.0522	0.0400	0.0316	0.0256
	0.9	2.7607	0.3307	0.1422	0.1925	0.1296	0.0900	0.0661	0.0506	0.0400	0.0324
	0.95	2.9276	0.2201	0.2411	0.2231	0.1444	0.1003	0.0737	0.0564	0.0446	0.0361

Table 1: Difference between the loss of supplier profit and the upper bound $r^2\frac{K^2}{\alpha^2}$ for $\gamma = 1, \beta = 1$

value it took at the threshold). This can be seen in Table 2, where on any given row, the values of the profit ratio are equal in the leftmost columns. However, as α/K decreases, the upper bound increases sharply (quadratic increase; see Figure 7 and Table 3). Therefore, in regimes (a, a'), (i.e. for small α/K), the upper bound $\frac{r^2}{(2-r)^2}$ is tighter than the upper bound $r^2\frac{K^2}{\alpha^2}$.

		α/K									
		0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
	0.1	0.9993	0.9993	0.9993	0.9993	0.9993	0.9993	0.9993	1.0000	1.0000	1.0000
	0.2	0.9965	0.9965	0.9965	0.9965	0.9965	0.9965	0.9992	1.0000	1.0000	1.0000
	0.3	0.9906	0.9906	0.9906	0.9906	0.9906	0.9906	1.0000	1.0000	1.0000	1.0000
	0.4	0.9796	0.9796	0.9796	0.9796	0.9796	0.9956	1.0000	1.0000	1.0000	1.0000
r	0.5	0.9600	0.9600	0.9600	0.9600	0.9600	1.0000	1.0000	1.0000	1.0000	1.0000
	0.6	0.9256	0.9256	0.9256	0.9256	0.9856	1.0000	1.0000	1.0000	1.0000	1.0000
	0.7	0.8643	0.8643	0.8643	0.9100	0.9984	1.0000	1.0000	1.0000	1.0000	1.0000
	0.8	0.7500	0.7500	0.7500	0.9600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.9	0.5207	0.5207	0.7822	0.9900	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.95	0.3176	0.3176	0.8400	0.9975	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 2: Profit ratio ρ for $\gamma = 1, \beta = 1$

5.2 Asymmetric Suppliers

We now focus on the loss of supplier profit in the more general case of asymmetric suppliers.

Theorem 5. *The loss of supplier profit* $1 - \rho$ *is a non decreasing function of* K^B.

This result implies in particular that the upper bound on the loss of supplier profit we obtained for symmetric suppliers (highest possible value of K^B) also holds as an upper bound when suppliers are asymmetric in terms of their capacities. This is stated in the following corollary.

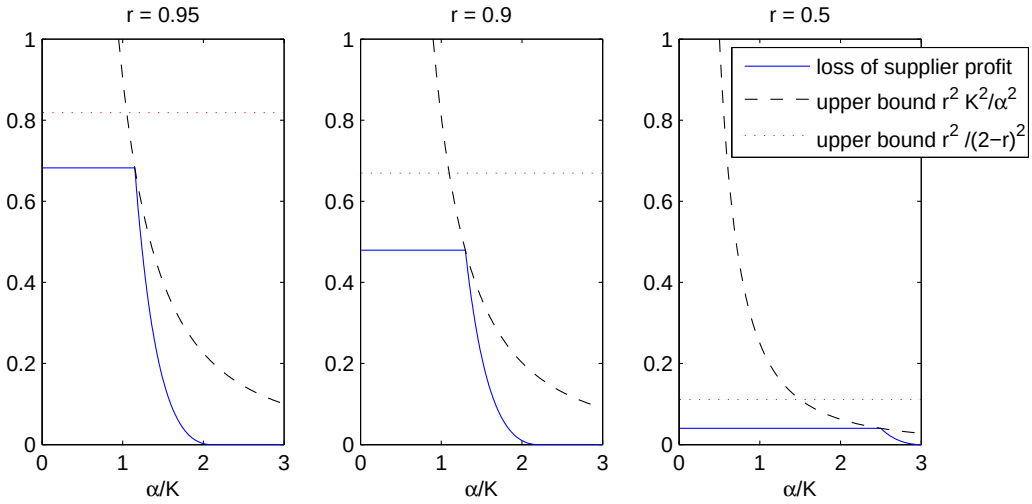


Figure 7: Loss of supplier profit and its upper bounds as a function $\frac{\alpha}{K}$ for the symmetric suppliers case for $\gamma = 1, \beta = 1$.

r	$2 - r + 2\gamma(\beta - \beta')$	$2 + 2\gamma(\beta - \beta')$
0.95	1.15	2.1
0.9	1.3	2.2
0.5	2.5	3

Table 3: Threshold values of $\frac{\alpha}{K}$ between regimes (a, a') and (c, a'), and regimes (c, a') and (c, c'), for $\gamma = 1, \beta = 1$

Corollary 2. *The loss of supplier profit* $1-\rho$ *cannot exceed* $\min\{r^2\frac{(K^A)^2}{\alpha^2}, \frac{r^2}{(2-r)^2}\}$ *for any* $K^B \leq K^A$, *where* $r = \frac{\beta'}{\beta}$.

Moreover, the more symmetric the setting is (in terms of capacity), the least "efficient" the system becomes. That is, the loss of supplier profit for *symmetric* suppliers is the worst case (an upper bound) on the loss of supplier profit for *asymmetric* suppliers. This means that collusion is most useful when the two competitors have the same capacity level. As K^B approaches K^A and the setting becomes more and more symmetric, supplier B becomes more and more competitive with respect to supplier A. Furthermore, supplier A has less and less of an advantage over supplier B, and as a result the equilibrium becomes less efficient compared to the cooperative solution.

Conclusions

This paper studies the role of capacity on the profits and efficiency of a two-tier supply chain with one retailer and two suppliers that compete through prices of differentiated substitutable products. As is generally the case in a variety of supply chain settings, the decentralization of decisions leads to inefficiencies in terms of profits: The profits of agents that unilaterally make decisions in their own interest, is lower than if decisions were centralized and optimized for the entire system. This is true both for the two suppliers, where inefficiencies result from horizontal price competition, as well as for the overall supply chain, where inefficiencies are caused by vertical competition. Our results provide a measure of these inefficiencies, by providing a bound on the maximum supply chain profit loss. This bound could provide some guidance to supply chain managers as they consider the possibility of implementing sophisticated, and thus more costly, coordination contracts. These contracts are worthwhile as long as implementation costs do not exceed potential gains. We also analyze the effect of the degree of capacity asymmetry between the suppliers on the profit loss, which may shed light on how to best make long term capacity investment decisions. Finally, our analysis of the effect of capacities on the suppliers profit at equilibrium and the existence of non monotonicities provide an interesting insight on the potential value of reducing capacity for the supplier with lower capacity.

Our results extend to a setting with a non zero initial inventory level. Such a setting would occur if a firm sold a seasonal product and might have inventory remaining from the previous season for example, or if this approach was used only in the last stage of the selling horizon. The major difference that arises with the presence of an initial inventory is that it is possible for a firm to idle and consume existing inventory to satisfy part of the demand. Moreover, our insights also hold in the case of linear production costs.

This paper illustrates the important role that capacity limits and capacity asymmetry play in supply chain management. Future direction of research include further study of the effect of capacity in a variety of supply chain structures, such as multi-tier supply chains, supply chain with multiple retailers and/or suppliers, or retailer-led supply chains. Other interesting related questions to study could address the use of contracts for coordinating the supply chain when capacity is limited.

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A Notation

- wholesale price of supplier k: p^k (subscript c for cooperative optimum, d for Nash equilibrium, cc for centrally coordinated)
- price of retailer for product of supplier k: $\bar{p}^k$
- production level of supplier k: u^k (same subscripts)
- profit of supplier k: J^k (same subscripts)
- profit of retailer: J^R (same subscripts)
- overall supplier profit: $J = J^A + J^B$ (same subscripts)
- overall supply chain profit: $\Pi = J^A + J^B + J^R$ (same subscripts)
- demand level of supplier k: $d^k(\mathbf{p}) = \alpha - \beta p^k + \beta' p^{-k}$
- retail price for product of supplier k: $\bar{p}^k(\mathbf{D}) = \bar{\alpha} - \bar{\beta} d^k - \bar{\beta}' d^{-k}$
- term of the supplier's demand that is independent of prices: α
- price elasticity of the demand of a supplier with respect to her own price: β
- price elasticity of the demand of a supplier with respect to her competitor's price: β'
- $r = \frac{\beta'}{\beta}$
- production capacity of supplier k: K^k
- coefficient of quadratic supplier production cost: γ
- $t = \gamma\beta$
- $b = 2\beta(1 + \gamma\beta) - \beta'(1 + 2\gamma\beta)$
- $d = \sqrt{2\beta^2(1 + \gamma\beta) - \beta'^2(1 + 2\gamma\beta)}$
- capacity threshold in a monopoly setting: $K_1 = \frac{\alpha}{2(1+\gamma\beta)}$
- capacity threshold between regimes a and b (decentralized case): $l_a = \frac{\alpha\beta}{b}$
- capacity threshold between regimes b and c (decentralized case): $l_b(K^A) = \frac{\alpha\beta(\beta+\beta') - d^2 K^A}{\beta\beta'}$
- local maximum of J^B as a function of K^B in regime b: $K_0 = \frac{\alpha(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta))}{2(2\beta(1+\gamma\beta)^2 - \gamma\beta'^2(1+2\gamma\beta))}$
- local maximum of J^B as a function of K^B in regime c: $\bar{K}_0 = \frac{\alpha(\beta+\beta') - \beta K^A}{(\beta+\beta')(1+2\gamma(\beta-\beta'))}$

- $K^* = \text{Arg}\max_{K^B}(J_d^A + J_d^B)(K^B)$
- $\bar{K}_1 = \text{Arg}\max_{K^B} J_d^B(K^B)$
- $\bar{K} = \frac{\beta K^B + \beta' K^A}{\beta + \beta'}$
- capacity threshold between regimes a' and b' (cooperative case): $l'_a = \frac{\frac{\alpha}{2}}{1+\gamma(\beta-\beta')}$
- capacity threshold between regimes b' and c' (cooperative case): $l'_b(K^A) = \frac{(\beta+\beta')\frac{\alpha}{2}) - K^A(\beta+\gamma(\beta^2-\beta'^2))}{\beta'}$
- profit ratio: $\rho = \frac{J_d^A + J_d^B}{J_c^A + J_c^B}$
- profit ratio for supplier k: $\rho^k = \frac{J_d^k}{J_c^k}$
- $d_1 = 4\beta(1+\gamma\beta)^2 - 2\gamma\beta'^2(1+2\gamma\beta)$
- $A = \beta(\beta+\beta')d_1 - \beta\beta'(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta))$
- $B = d_1(2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta))$

B Optimal solution in a monopoly setting

$$\begin{aligned} J(K) = \max_{u,p} \quad & p(\alpha - \beta p) - \gamma u^2 \\ \text{such that} \quad & u = \alpha - \beta p \\ & 0 \le p \le \frac{\alpha}{\beta} \\ & 0 \le u \le K \end{aligned}$$

We obtain the optimal solution:

- if $\frac{\alpha}{2} - K(1+\gamma\beta) \le 0$, then $u = \frac{\alpha}{2(1+\gamma\beta)},\ p = \frac{\alpha}{2\beta} + \frac{\alpha}{2(\beta+\frac{1}{\gamma})} = \frac{1}{1+\gamma\beta}\Big(\frac{\alpha}{2\beta}(1+2\gamma\beta)\Big)$.
- if $0 < \frac{\alpha}{2} - K(1+\gamma\beta)$, then $u = K,\ p = \frac{\alpha - K}{\beta}$.

$$J(K) = \begin{cases} \frac{\alpha^2}{4\beta} - \frac{\gamma\alpha^2}{4(1+\gamma\beta)} = \frac{\alpha^2}{4\beta(1+\gamma\beta)}, & \frac{\alpha}{2} - K(1+\gamma\beta) \le 0 \\ \frac{K}{\beta}(\alpha - K) - \gamma K^2, & 0 < \frac{\alpha}{2} - K(1+\gamma\beta) \end{cases}$$

In particular, $J(K)$ depends on K only when the demand is high enough so that the production is at full capacity, i.e. when $0 < \frac{\alpha}{2} - K(1+\gamma\beta)$, i.e. $K < K_1$, where $K_1 \equiv \frac{\alpha}{2(1+\gamma\beta)}$.
We observe that on the domain $0 < K < K_1$, $J(K)$ is quadratic and increasing. It reaches a maximum at $K = K_1$, while for $K > K_1$, $J(K)$ is independent of K, i.e. remains at the maximum value $J(K_1)$. In other words, beyond K_1, the supplier does not produce at full capacity and

therefore any change of capacity above that value has no impact on the optimal strategy and thus the profits.

C Equilibrium solution derivation

Proposition 6. *Due to Corollary 1, the equilibrium point and equilibrium profits are as follows:*

- *if* $\alpha\beta - bK^B \leq 0$,

$$u^A = u^B = \frac{\alpha\beta}{b}, \quad p^A = p^B = \frac{\alpha(1+2\gamma\beta)}{b}$$

$$J^A(K^A, K^B) = J^B(K^A, K^B) = \frac{\beta(1+\gamma\beta)\alpha^2}{b^2}$$

- *if* $\alpha\beta - bK^B > 0$ *and* $\alpha\beta(\beta+\beta') - K^B\beta\beta' - d^2K^A < 0$,

$$p^A = (1+2\gamma\beta)\frac{\alpha(\beta+\beta') - K^B\beta'}{d^2},$$

$$p^B = \frac{\alpha(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) - 2K^B\beta(1+\gamma\beta)}{d^2}$$

$$u^A = \frac{\alpha\beta(\beta+\beta') - \beta\beta'K^B}{d^2}, \quad u^B = K^B$$

$$J^A(K^A, K^B) = \beta(1+\gamma\beta)\Big(\frac{\alpha(\beta+\beta') - \beta'K^B}{d^2}\Big)^2$$

$$J^B(K^A, K^B) = K^B\Big(\frac{\alpha(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) - 2K^B\beta(1+\gamma\beta)}{2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta)}\Big) - \gamma(K^B)^2$$

- *if* $\alpha\beta(\beta+\beta') - K^B\beta\beta' - d^2K^A \geq 0$ *(which implies* $\alpha\beta - bK^B > 0$*)*,

$$u^A = K^A, \quad u^B = K^B, \quad p^A = \frac{\alpha - \frac{\beta K^A + \beta' K^B}{\beta+\beta'}}{\beta - \beta'}, \quad p^B = \frac{\alpha - \frac{\beta K^B + \beta' K^A}{\beta+\beta'}}{\beta - \beta'}$$

$$J^A(K^A, K^B) = \frac{K^A}{\beta - \beta'}\Big(\alpha - \frac{\beta K^A + \beta' K^B}{\beta+\beta'}\Big) - \gamma(K^A)^2$$

$$J^B(K^A, K^B) = \frac{K^B}{\beta - \beta'}\Big(\alpha - \frac{\beta K^B + \beta' K^A}{\beta+\beta'}\Big) - \gamma(K^B)^2$$

Proof. The best response solution can be derived using Lagrangian duality or using results from the monopoly case (Appendix B):

- case 1

$$\frac{\alpha + \beta' p^{-k}}{2} - K^k(1+\gamma\beta) \leq 0 \;\Rightarrow$$

$$u^k = \frac{\alpha + \beta' p^{-k}}{2(1+\gamma\beta)}, \quad p^k = \frac{\alpha + \beta' p^{-k}}{2\beta} + \frac{\gamma(\alpha + \beta' p^{-k})}{2(1+\gamma\beta)} = \frac{(\alpha + \beta' p^{-k})(1+2\gamma\beta)}{2\beta(1+\gamma\beta)}.$$

- case 2

$$0 < \frac{\alpha + \beta' p^{-k}}{2} - K^k(1+\gamma\beta) \;\Rightarrow\; u^k = K^k, \quad p^k = \frac{\alpha + \beta' p^{-k} - K^k}{\beta}.$$

To determine the equilibrium, we must consider 3 possibilities.

A and B in case 1:

We observe that, only the production capacity differentiates the suppliers, and therefore the problem is symmetric if none of them produces at full capacity. By symmetry, $u^k = u^{-k} = u$, $p^k = p^{-k} = p$ and $u = \frac{\alpha+\beta' p}{2(1+\gamma\beta)}$, $p = \frac{(\alpha+\beta' p)(1+2\gamma\beta)}{2\beta(1+\gamma\beta)}$.

$$\begin{aligned} p &= (\alpha + \beta' p)\frac{1+2\gamma\beta}{2\beta(1+\gamma\beta)} \\ p(1 - \beta'\frac{1+2\gamma\beta}{2\beta(1+\gamma\beta)}) &= \alpha\frac{1+2\gamma\beta}{2\beta(1+\gamma\beta)} \\ p &= \frac{\alpha + 2\gamma\beta\alpha}{2\beta(1+\gamma\beta) - \beta'(1+2\gamma\beta)} \end{aligned}$$

and thus

$$u = \frac{\alpha\beta}{2\beta(1+\gamma\beta) - \beta'(1+2\gamma\beta)}.$$

For this case to hold, we must verify

$$\begin{aligned} & 0 \le \alpha + \beta' p \le 2K^k(1+\gamma\beta) \\ \Leftrightarrow \quad & 0 \le \alpha + \beta'\frac{\alpha(1+2\gamma\beta)}{2\beta(1+\gamma\beta) - \beta'(1+2\gamma\beta)} \le 2K^k(1+\gamma\beta) \\ \Leftrightarrow \quad & (\alpha - 2K^k(1+\gamma\beta))(2\beta(1+\gamma\beta) - \beta'(1+2\gamma\beta)) + \beta'\alpha(1+2\gamma\beta) \le 0 \\ \Leftrightarrow \quad & \alpha\beta - K^k(2\beta(1+\gamma\beta) - \beta'(1+2\gamma\beta)) \le 0, \quad k = A,\ B \\ \Leftrightarrow \quad & \alpha\beta - bK^k \le 0, \quad k = A,\ B \end{aligned}$$

Since $K^B \le K^A$, this case holds if

$$\alpha\beta - bK^B \le 0.$$

A and B in case 2:

$u^k = K^k$, $p^A = \frac{\alpha+\beta' p^B - K^A}{\beta}$, $p^B = \frac{\alpha+\beta' p^A - K^B}{\beta}$, therefore

$$\begin{aligned} p^A &= \frac{\alpha - K^A}{\beta} + \frac{\beta'}{\beta}\frac{\alpha+\beta' p^A - K^B}{\beta} \\ p^A(1-\frac{\beta'^2}{\beta^2}) &= \frac{\beta\alpha - \beta K^A + \beta'(\alpha - K^B)}{\beta^2} \\ p^A &= \frac{\alpha - \frac{\beta K^A + \beta' K^B}{\beta+\beta'}}{\beta - \beta'} \end{aligned}$$

and by symmetry

$$p^B = \frac{\alpha - \frac{\beta K^B + \beta' K^A}{\beta+\beta'}}{\beta - \beta'}.$$

For this case to hold, we must verify

$$0 < \frac{\alpha}{2} - K^k(1+\gamma\beta) + \frac{\beta'}{2}\frac{\alpha - \frac{\beta K^{-k} + \beta' K^k}{\beta+\beta'}}{\beta - \beta'}, \quad k = A,\ B$$

$$\begin{aligned} 0 &< \alpha\beta - 2K^k(\beta-\beta')(1+\gamma\beta) - \beta'\frac{\beta K^{-k} + \beta' K^k}{\beta+\beta'}, \quad k = A,\ B \\ \Leftrightarrow\ 0 &< \alpha\beta(\beta+\beta') - \beta\beta' K^{-k} - K^k(2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta)), \quad k = A,\ B \\ \Leftrightarrow\ 0 &< \alpha\beta(\beta+\beta') - \beta\beta' K^{-k} - d^2 K^k, \quad k = A,\ B \end{aligned}$$

Lemma 2. $0 < \alpha\beta(\beta+\beta') - \beta\beta' K^B - d^2 K^A$ *implies* $0 < \alpha\beta(\beta+\beta') - \beta\beta' K^A - d^2 K^B$

Proof. To prove the lemma, it is sufficient to show that $\beta\beta' K^B + d^2 K^A \geq \beta\beta' K^A + d^2 K^B$.

$$\begin{aligned} \beta\beta' K^B + d^2 K^A - \beta\beta' K^A - d^2 K^B &= (K^A - K^B)(d^2 - \beta\beta') \\ &= (K^A - K^B)(-\beta\beta' + 2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta) \\ &= (K^A - K^B)((2\beta+\beta')(\beta-\beta') + 2\gamma\beta(\beta^2 - \beta'^2)) \\ &= (K^A - K^B)(\beta-\beta')(2\beta+\beta'+2\gamma\beta(\beta+\beta')) \geq 0 \end{aligned}$$

□

Therefore, this case holds for

$$0 < \alpha\beta(\beta+\beta') - \beta\beta' K^B - d^2 K^A$$

Remark: The inequality above implies $\alpha\beta - bK^B > 0$. Indeed, since $K^A \geq K^B$, this inequality

implies

$$0 < \alpha\beta(\beta+\beta') - \beta\beta' K^B - d^2 K^A \leq \alpha\beta(\beta+\beta') - (\beta\beta'+d^2)K^B = (\alpha\beta - bK^B)(\beta+\beta').$$

A in case 1, B in case 2:

$$u^A = \frac{\alpha+\beta' p^B}{2(1+\gamma\beta)},\ u^B = K^B,\ \ p^A = \frac{(\alpha+\beta' p^B)(1+2\gamma\beta)}{2\beta(1+\gamma\beta)},\ \ p^B = \frac{\alpha+\beta' p^A - K^B}{\beta}$$

thus

$$\begin{aligned}
p^A &= \frac{\alpha(1+2\gamma\beta)}{2\beta(1+\gamma\beta)} + \frac{\beta'(1+2\gamma\beta)}{2\beta(1+\gamma\beta)}\frac{\alpha+\beta' p^A - K^B}{\beta} \\
p^A &= \frac{\alpha(1+2\gamma\beta)(\beta+\beta') - K^B\beta'(1+2\gamma\beta))}{2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta)} \\
p^B &= \frac{\alpha - K^B}{\beta} + \frac{\beta'}{\beta}\left(\frac{\alpha(1+2\gamma\beta)(\beta+\beta') - K^B\beta'(1+2\gamma\beta)}{2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta)}\right) \\
&= \frac{\alpha\beta(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) - 2K^B\beta^2(1+\gamma\beta)}{\beta(2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta))}
\end{aligned}$$

For this case to hold, we must verify

$$\begin{aligned}
0 &\geq \frac{\alpha+\beta' p^B}{2} - K^A(1+\gamma\beta) \\
&= \beta\frac{\alpha(\beta+\beta')(1+\gamma\beta) - K^B\beta'(1+\gamma\beta)}{2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta)} - K^A(1+\gamma\beta) \\
\Leftrightarrow 0 &\geq \alpha\beta(\beta+\beta') - K^B\beta\beta' - K^A(2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta)) \\
\Leftrightarrow 0 &\geq \alpha\beta(\beta+\beta') - K^B\beta\beta' - d^2K^A
\end{aligned}$$

and

$$\begin{aligned}
0 &\leq \frac{\alpha+\beta' p^A}{2} - K^B(1+\gamma\beta) \\
&= \frac{\alpha\beta(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) - K^B\beta'^2(1+2\gamma\beta)}{4\beta^2(1+\gamma\beta) - 2\beta'^2(1+2\gamma\beta)} - K^B(1+\gamma\beta) \\
&= \frac{\alpha\beta(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) - K^B(4\beta^2(1+\gamma\beta)^2 - \beta'^2(1+2\gamma\beta)^2)}{4\beta^2(1+\gamma\beta) - 2\beta'^2(1+2\gamma\beta)} \\
\Leftrightarrow 0 &\leq \alpha\beta(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) - K^B(4\beta^2(1+\gamma\beta)^2 - \beta'^2(1+2\gamma\beta)^2) \\
&= \alpha\beta - K^B(2\beta(1+\gamma\beta) - \beta'(1+2\gamma\beta)) \\
&= \alpha\beta - bK^B
\end{aligned}$$

Remark: it is impossible to have A in case 2 and B in case 1. Indeed, for this case to hold, we

would need $0 \geq \alpha\beta(\beta+\beta') - K^A\beta\beta' - d^2K^B$ and $0 < \alpha\beta - bK^A$, but $0 \geq \alpha\beta(\beta+\beta') - K^A\beta\beta' - d^2K^B$ implies

$$0 \geq \alpha\beta(\beta+\beta') - K^A\beta\beta' - d^2K^A = (\beta+\beta')(\alpha\beta - K^A b).$$

□

D Optimal solution under supplier cooperation

While the capacity constraint is not tight for either supplier, by symmetry, the solutions are identical for each supplier, and they are a solution to the monopoly problem with price sensitivity $\beta - \beta'$, i.e.:

- if $\frac{\alpha}{2} - K^B(1+\gamma(\beta-\beta')) \leq 0$, then

$$u^A = u^B = \frac{\alpha}{2(1+\gamma(\beta-\beta'))}, \quad p^A = p^B = \frac{\alpha(1+2\gamma(\beta-\beta'))}{2(\beta-\beta')(1+\gamma(\beta-\beta'))}.$$

$$J^A = J^B = \frac{\alpha^2}{4(\beta-\beta')(1+\gamma(\beta-\beta'))}$$

Note that we can show

$$\frac{\alpha}{2} - K^A(1+\gamma(\beta-\beta')) + \frac{\beta'}{\beta+\beta'}(K^A - K^B) = \frac{\alpha}{2} - \gamma K^A(\beta-\beta') - \frac{\beta K^A + \beta' K^B}{\beta+\beta'} < \frac{\alpha}{2} - K^B(1+\gamma(\beta-\beta')).$$

Some calculations involving minimizing the Lagrangian similarly to Appendix C lead to the additional cases:

- if $\frac{\alpha}{2} - K^A(1+\gamma(\beta-\beta')) + \frac{\beta'(K^A-K^B)}{\beta+\beta'} < 0 < \frac{\alpha}{2} - K^B(1+\gamma(\beta-\beta'))$, then

$$u^A = \frac{\frac{\alpha}{2}(\beta+\beta') - \beta' K^B}{\beta + \gamma(\beta^2 - \beta'^2)}, \quad u^B = K^B,$$

$$p^A = \frac{\alpha}{2(\beta-\beta')} + \frac{\frac{\alpha}{2}\gamma(\beta+\beta') - \gamma\beta' K^B}{\beta + \gamma(\beta^2-\beta'^2)},$$

$$p^B = \frac{\alpha}{2(\beta-\beta')} + \frac{\frac{\alpha}{2}(1+\gamma(\beta+\beta')) - K^B(1+\gamma\beta)}{\beta + \gamma(\beta^2-\beta'^2)}$$

$$J^A = \frac{\alpha\big(\frac{\alpha}{2}(\beta+\beta') - \beta' K^B\big)}{2(\beta-\beta')(\beta+\gamma(\beta^2-\beta'^2))},$$

$$J^B = -\gamma(K^B)^2 + K^B\Big(\frac{\alpha}{2(\beta-\beta')} + \frac{\frac{\alpha}{2}(1+\gamma(\beta+\beta')) - K^B(1+\gamma\beta)}{\beta+\gamma(\beta^2-\beta'^2)}\Big)$$

- if $0 \leq \frac{\alpha}{2} - K^A(1+\gamma(\beta-\beta')) + \frac{\beta'(K^A-K^B)}{\beta+\beta'}$, then

$$u^B = K^B,\ u^A = K^A,$$

$$p^A = \frac{1}{\beta-\beta'}\Big(\alpha - \frac{\beta K^A + \beta' K^B}{\beta+\beta'}\Big),\ p^B = \frac{1}{\beta-\beta'}\Big(\alpha - \frac{\beta K^B + \beta' K^A}{\beta+\beta'}\Big)$$

$$J^A = -\gamma(K^A)^2 + \frac{K^A}{\beta-\beta'}\Big(\alpha - \frac{\beta K^A + \beta' K^B}{\beta+\beta'}\Big),\ \ J^B = -\gamma(K^B)^2 + \frac{K^B}{\beta-\beta'}\Big(\alpha - \frac{\beta K^B + \beta' K^A}{\beta+\beta'}\Big)$$

E The centrally coordinated optimal solution

Proposition 7. *The centrally coordinated optimal solution to the central planner's optimization problem is:*

- *if* $K^B \geq \frac{\alpha}{1+2\gamma(\beta-\beta')}$, *then*

$$u^A = u^B = \frac{\alpha}{1+2\gamma(\beta-\beta')} \equiv u_0,\ \ p^A = p^B = \frac{2\gamma\alpha}{1+2\gamma(\beta-\beta')} \equiv p_0$$

$$\Pi_{cc} = \frac{\alpha^2}{(\beta-\beta')(1+2\gamma(\beta-\beta'))}$$

- *if* $\frac{\alpha(\beta+\beta')-K^A(\beta+2\gamma(\beta^2-\beta'^2))}{\beta'} \leq K^B < \frac{\alpha}{1+2\gamma(\beta-\beta')}$ *then*

$$u^A = u_0 + \frac{\beta'(u_0-K^B)}{\beta+2\gamma(\beta^2-\beta'^2)} = \frac{\alpha(\beta+\beta')-\beta' K^B}{\beta+2\gamma(\beta^2-\beta'^2)},\ \ u^B = K^B,$$

$$p^A = p_0 + \frac{2\gamma\beta'(u_0-K^B)}{\beta+2\gamma(\beta^2-\beta'^2)},\quad p^B = p_0 + \frac{(1+2\gamma\beta)(u_0-K^B)}{\beta+2\gamma(\beta^2-\beta'^2)}$$

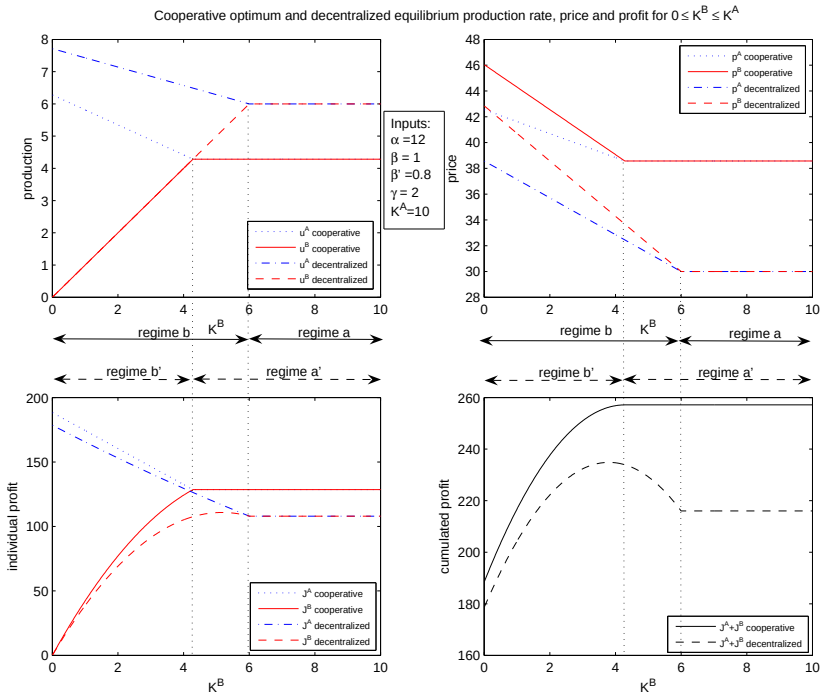
- *if* $K^B < \frac{\alpha(\beta+\beta')-K^A(\beta+2\gamma(\beta^2-\beta'2))}{\beta'}$, *then*

$$u^B = K^B,\ u^A = K^A,$$

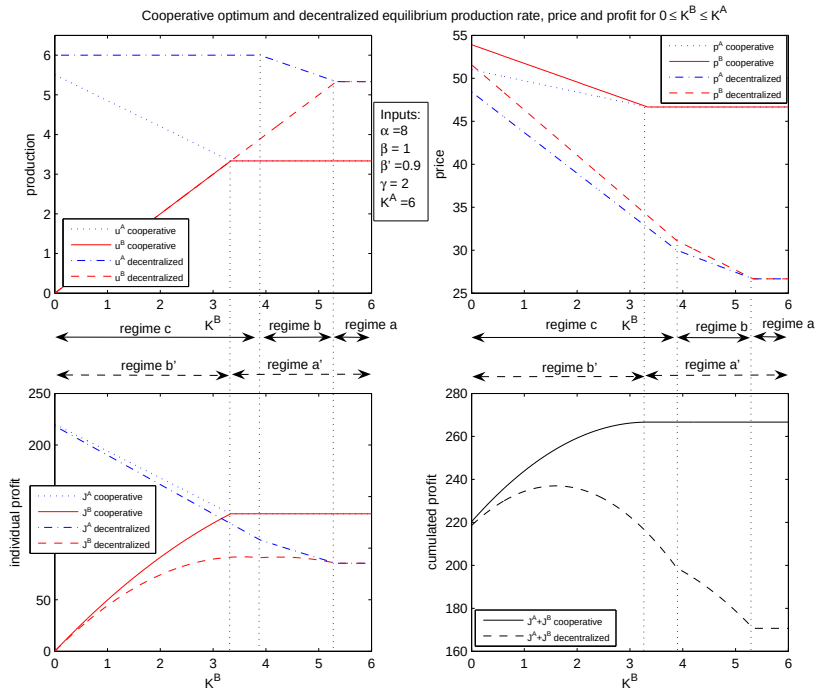
$$p^A = \frac{\alpha(\beta+\beta')-\beta K^A-\beta' K^B}{\beta^2-\beta'^2},\quad p^B = \frac{\alpha(\beta+\beta')-\beta' K^A-\beta K^B}{\beta^2-\beta'^2}$$

The proof uses Lagrangian duality and is straightforward; it is thus omitted.

F Figures

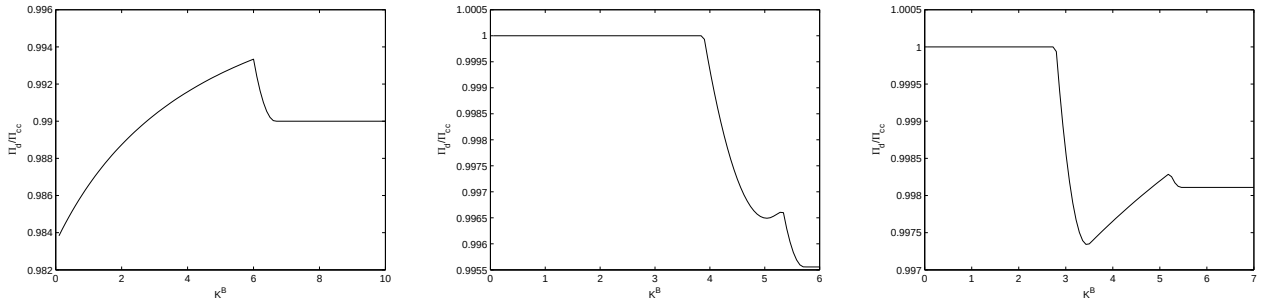


(a)



(b)

Figure 8: Cooperative and decentralized solution on two examples.



(a) (b) (c)

Figure 9: Example: total profit ratio $\frac{\Pi_d}{\Pi_{cc}}$ as a function of K^B varying from 0 to K^A with inputs (a) as in Figure 8(a); (b) as in Figure 8(b); (c) $\beta' = 0.95$, $\beta = 1$, $\alpha = 6$, $K^A = 7$, $\gamma = 1$.

G Proofs

G.1 Proof of Lemma 1

(i) We can show that

$$l_b(K^A) - l'_b(K^A) = \frac{\beta + \beta'}{\beta\beta'}\big(\frac{\alpha}{2}\beta - (\beta - \beta')K^A(1 + \gamma\beta)\big)$$

Regime c' implies $l'_b(K^A) > 0$, that is

$$K^A(\beta(1 + \gamma\beta) - \gamma\beta'^2) < (\beta + \beta')\frac{\alpha}{2}.$$

Moreover, we can show that

$$\frac{\beta + \beta'}{\beta(1 + \gamma\beta) - \gamma\beta'^2} < \frac{1}{1 + \gamma\beta}\frac{\beta}{\beta - \beta'}$$

as follows:

$$\begin{aligned} & \frac{\beta + \beta'}{\beta(1 + \gamma\beta) - \gamma\beta'^2} < \frac{1}{1 + \gamma\beta}\frac{\beta}{\beta - \beta'} \\ \Leftrightarrow \quad & (1 + \gamma\beta)(\beta^2 - \beta'^2) < \beta(\beta(1 + \gamma\beta) - \gamma\beta'^2) \\ \Leftrightarrow \quad & -\beta'^2(1 + \gamma\beta) < -\gamma\beta\beta'^2 \end{aligned}$$

which clearly holds.
Therefore, $l'_b(K^A) > 0$ implies

$$K^A < \frac{(\beta + \beta')\frac{\alpha}{2}}{\beta(1 + \gamma\beta) - \gamma\beta'^2} < \frac{1}{1 + \gamma\beta}\frac{\alpha}{2}\frac{\beta}{\beta - \beta'}$$

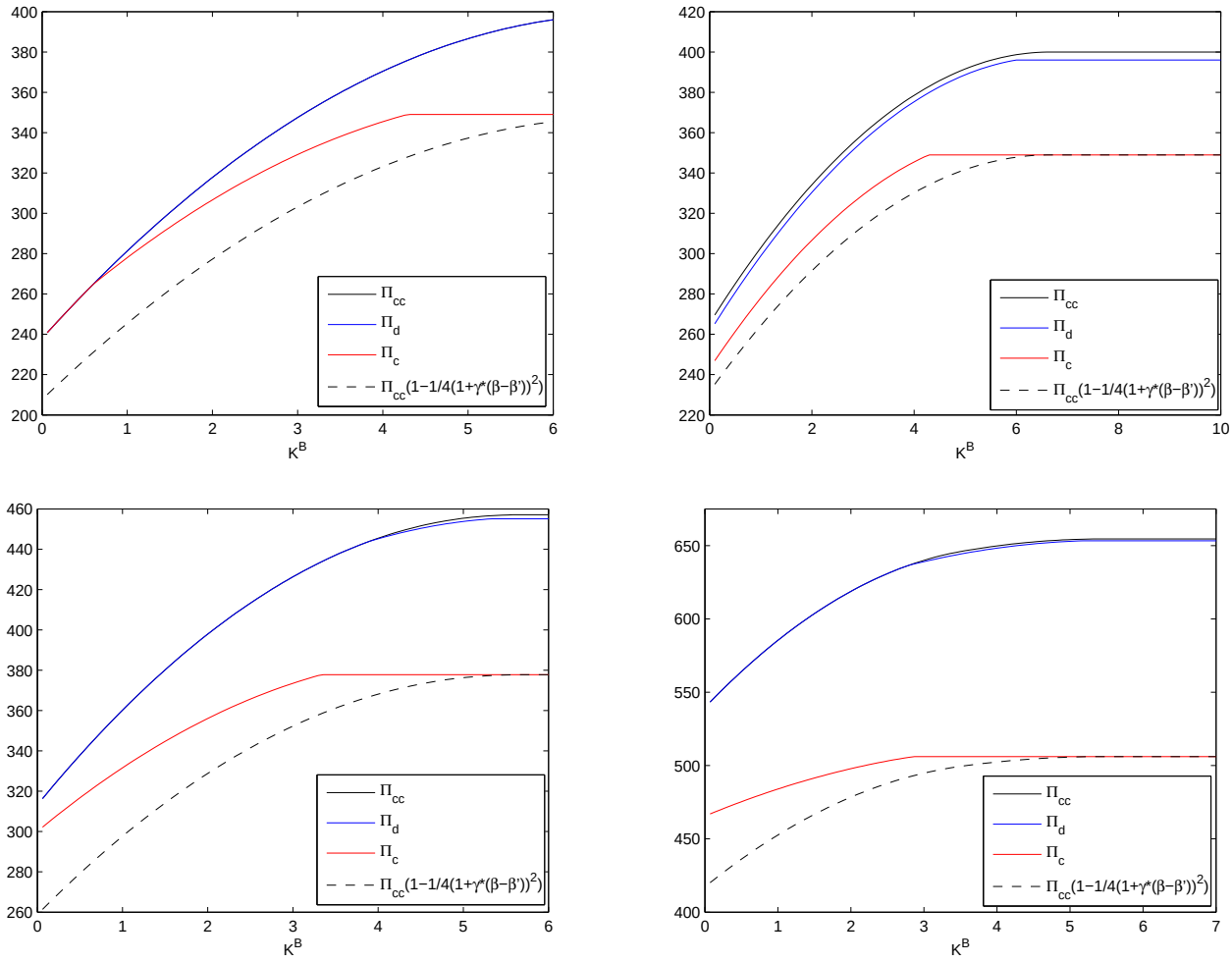


Figure 10: Example for asymmetric suppliers: Π_{cc}, Π_d, Π_c and $\Pi_{cc}\left(1-\frac{1}{4(1+\gamma(\beta-\beta'))^2}\right)$ as a function of K^B varying from 0 to K^A with inputs as in Figure 2, Figure 8(a), Figure 8(b), and Figure 9(c). (In the first plot, Π_{cc} and Π_d overlap.)

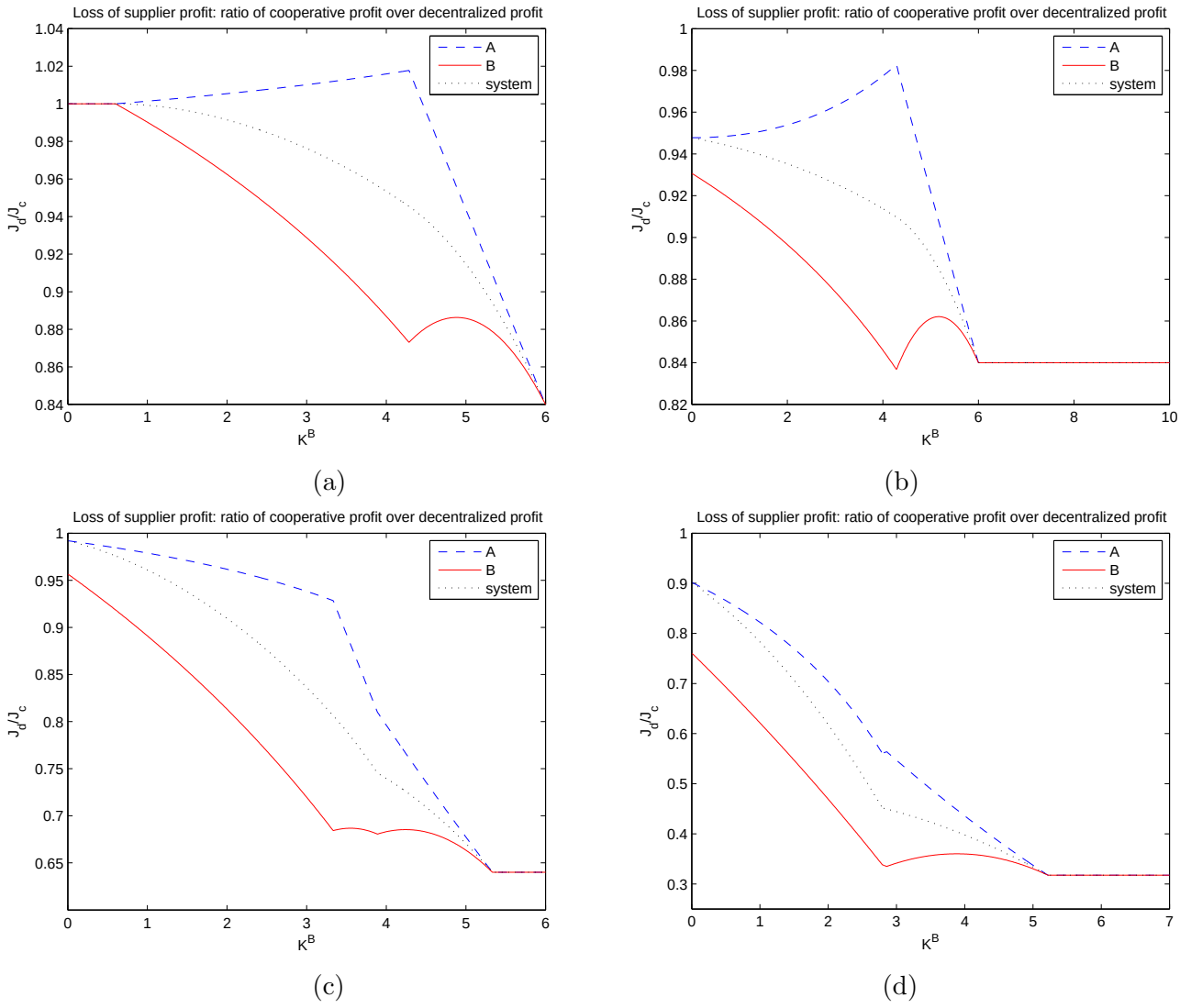


Figure 11: Example: profit ratio as a function of K^B varying from 0 to K^A with inputs as in (a) Figure 2; (b) Figure 8(a); (c) Figure 8(b); (d) Figure 9(c).

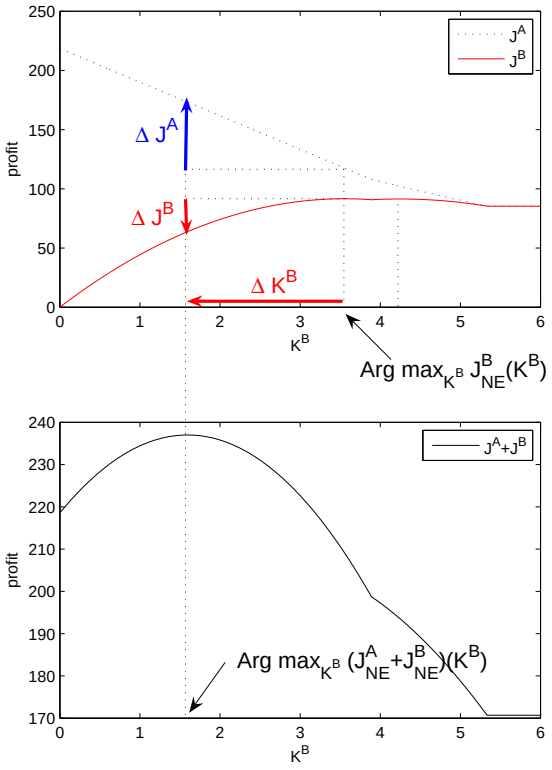


Figure 12: Illustration of an agreement between the suppliers to determine supplier B's capacity.

which in turn implies $l_b(K^A) - l'_b(K^A) > 0$. Finally, if regime c' holds, then $K^B < l'_b(K^A) < l_b(K^A)$ and therefore, in a decentralized setting, regime c holds.

(ii) In this proof, the subscript d (resp. c) refers to the decentralized (resp. cooperative) solution.

As seen in Section 2.3, regime a implies regime a' and regime c' implies regime c. Therefore, the inputs may correspond to one of 7 possible situations: regimes a and a', b and a', b and b', c and a', c and b', c and c', for respectively the decentralized and centralized settings.

- **regimes (a,a')**: Since the optimal and equilibrium (resp.) production quantities are given by $u_c^A = u_c^B = \frac{\alpha}{2(1+\gamma\beta)-2\gamma\beta'}$ and $u_d^A = u_d^B = \frac{\alpha}{2(1+\gamma\beta)-\frac{\beta'}{\beta}-2\gamma\beta'}$, it is clear that $u_c^A = u_c^B < u_d^A = u_d^B$.

- **regimes (b,a')**: The equilibrium production quantity for B is $u_d^B = K^B$, so by feasibility it follows that $u_c^B \leq u_d^B$. Moreover, for supplier A, $u_d^A = \frac{\alpha\beta(\beta+\beta')-\beta\beta' K^B}{2\beta^2(1+\gamma\beta)-\beta'^2(1+2\gamma\beta)}$, $u_c^A = \frac{\alpha}{2(1+\gamma(\beta-\beta'))}$. Some calculations lead to

$$\begin{aligned} u_d^A \geq u_c^A &\Leftrightarrow \Big(\alpha\beta(\beta+\beta') - \beta\beta' K^B\Big)\Big(2(1+\gamma(\beta-\beta'))\Big) < \alpha\Big(2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta)\Big) \\ &\Leftrightarrow \alpha(2\beta+\beta') \geq 2\beta K^B(1+\gamma(\beta-\beta')) \end{aligned}$$

  Under regime b, we have $\alpha\beta \geq K^B(2\beta(1+\gamma\beta) - \beta'(1+2\gamma\beta))$, and therefore

$$\alpha(2\beta+\beta') \geq \frac{2\beta+\beta'}{\beta}\Big(K^B(2\beta(1+\gamma\beta) - \beta'(1+2\gamma\beta))\Big).$$

  As a result, to show $u_d^A \geq u_c^A$, it is sufficient to show that the right hand side in the inequality above is greater than or equal to $2\beta K^B(1+\gamma(\beta-\beta'))$. We observe that

$$\begin{aligned} &\frac{2\beta+\beta'}{\beta}\Big(K^B(2\beta(1+\gamma\beta) - \beta'(1+2\gamma\beta))\Big) \\ &\quad -2\beta K^B(1+\gamma(\beta-\beta')) = K^B(\frac{2\beta^2-\beta'^2}{\beta} + 2\gamma(\beta^2-\beta'^2)) > 0 \end{aligned}$$

  and the result follows.

- **regimes (b,b')**: The optimal and equilibrium (resp.) production quantities are given by $u_d^A = \frac{\alpha(\beta+\beta')-\beta' K^B}{2\beta(1+\gamma\beta)-\frac{\beta'^2}{\beta}-2\gamma\beta'^2}$, $u_c^A = \frac{\alpha(\beta+\beta')-2\beta' K^B}{2\beta(1+\gamma\beta)-2\gamma\beta'^2}$, $u_d^B = u_c^B = K^B$, thus it is clear that $u_d^A > u_c^A$.

- **regimes (c,a'), (c,b') and (c,c')**: The equilibrium production quantities are given by $u_d^A = K^A$, $u_d^B = K^B$ so by feasibility $u_c^A \leq u_d^A$ and $u_c^B \leq u_d^B$ and the result follows.

□

G.2 Proof of Theorem 3

Part 1.

It is straightforward to derive that if $K^A = K^B \equiv K$, then

$$J_d^R = \begin{cases} \frac{\alpha^2\beta^2}{b^2(\beta-\beta')} & \text{if } \alpha\beta - bK \leq 0 \\ \frac{K^2}{\beta-\beta'} & \text{else} \end{cases}$$

$$J_c^R = \begin{cases} \frac{\alpha^2}{4(\beta-\beta')(1+\gamma(\beta-\beta'))^2} & \text{if } \frac{1}{2}\alpha - K(1+\gamma(\beta-\beta')) \leq 0 \\ \frac{K^2}{\beta-\beta'} & \text{else} \end{cases}$$

In particular,

$$\frac{J_d^R}{J_c^R} = \begin{cases} \frac{4\beta^2(1+\gamma(\beta-\beta'))^2}{b^2} & \text{if } K \geq \alpha\beta/b \\ \frac{4K^2(1+\gamma(\beta-\beta'))^2}{\alpha^2} & \text{if } \frac{\alpha}{2(1+\gamma(\beta-\beta'))} \leq K < \alpha\beta/b \\ 1 & \text{else} \end{cases}$$

We have $b^2 = (2\beta - \beta' + 2\gamma\beta(\beta-\beta'))^2 < (2\beta + 2\gamma\beta(\beta-\beta'))^2 = 4\beta^2(1+\gamma(\beta-\beta'))^2$. Moreover, if $\frac{\alpha}{2(1+\gamma(\beta-\beta'))} \leq K$, then it is clear that $4K^2(1+\gamma(\beta-\beta'))^2 \geq \alpha^2$. As a result, it follows that $J_d^R/J_c^R \geq 1$.

Part 2.

It is straightforward to derive that if $K^A = K^B \equiv K$, then

$$\Pi_d = \begin{cases} \frac{\alpha^2\beta(3\beta-2\beta'+2\gamma\beta(\beta-\beta'))}{b^2(\beta-\beta')} & \text{if } \alpha\beta - bK \leq 0 \\ \frac{K(2\alpha-K(1+2\gamma(\beta-\beta')))}{\beta-\beta'} & \text{else} \end{cases}$$

$$\Pi_c = \begin{cases} \frac{\alpha^2(3+2\gamma(\beta-\beta'))}{4(\beta-\beta')(1+\gamma(\beta-\beta'))^2} & \text{if } \frac{1}{2}\alpha - K(1+\gamma(\beta-\beta')) \leq 0 \\ \frac{K(2\alpha-K(1+2\gamma(\beta-\beta')))}{\beta-\beta'} & \text{else} \end{cases}$$

$$\Pi_{cc} = \begin{cases} \frac{\alpha^2}{(\beta-\beta')(1+2\gamma(\beta-\beta'))} & \text{if } K \geq \frac{\alpha}{1+2\gamma(\beta-\beta')} \\ \frac{K(2\alpha-K(1+2\gamma(\beta-\beta')))}{\beta-\beta'} & \text{else} \end{cases}$$

In particular,

$$\frac{\Pi_d}{\Pi_c} = \begin{cases} \frac{4\beta(3\beta-2\beta'+2\gamma\beta(\beta-\beta'))(1+\gamma(\beta-\beta'))^2}{b^2(3+2\gamma(\beta-\beta'))} & \text{if } K \geq \alpha\beta/b \\ \frac{4K(2\alpha-K(1+2\gamma(\beta-\beta')))(1+\gamma(\beta-\beta'))^2}{\alpha^2(3+2\gamma(\beta-\beta'))} & \text{if } \frac{\alpha}{2(1+\gamma(\beta-\beta'))} \leq K < \alpha\beta/b \\ 1 & \text{else} \end{cases}$$

We obtain after simplification:

$$4\beta(3\beta-2\beta'+2\gamma\beta(\beta-\beta'))(1+\gamma(\beta-\beta'))^2 - b^2(3+2\gamma(\beta-\beta')) = \beta'(4\beta-3\beta') + \gamma\beta'(\beta-\beta')(4\beta-2\beta') > 0$$

Moreover, we find that

$$\begin{aligned} & 4K(2\alpha - K(1+2\gamma(\beta-\beta'))(1+\gamma(\beta-\beta'))^2 - \alpha^2(3+2\gamma(\beta-\beta')) \\ = \; & -4(1+2\gamma(\beta-\beta'))(1+\gamma(\beta-\beta'))^2\Big(K - \frac{\alpha}{2(1+\gamma(\beta-\beta'))}\Big)\Big(K - \frac{\alpha(3+2\gamma(\beta-\beta'))}{2(1+2\gamma(\beta-\beta'))(1+\gamma(\beta-\beta'))}\Big) \end{aligned}$$

Clearly, for $\frac{\alpha}{2(1+\gamma(\beta-\beta'))} \leq K$ we have $-4(1+2\gamma(\beta-\beta'))(1+\gamma(\beta-\beta'))^2\Big(K - \frac{\alpha}{2(1+\gamma(\beta-\beta'))}\Big) \leq 0$. In addition, it is easy to obtain that

$$\frac{\beta}{b} \leq \frac{3+2\gamma(\beta-\beta')}{2(1+2\gamma(\beta-\beta'))(1+\gamma(\beta-\beta'))},$$

therefore, for $K < \alpha\beta/b$, $K - \frac{\alpha(3+2\gamma(\beta-\beta'))}{2(1+2\gamma(\beta-\beta'))(1+\gamma(\beta-\beta'))} < 0$. It follows that

$$4K(2\alpha - K(1+2\gamma(\beta-\beta'))(1+\gamma(\beta-\beta'))^2 - \alpha^2(3+2\gamma(\beta-\beta')) < 0$$

for $\frac{\alpha}{2(1+\gamma(\beta-\beta'))} \leq K < \alpha\beta/b$, and hence $\Pi_d \leq \Pi_c$.

Part 3.

Similarly,

$$\frac{\Pi_c}{\Pi_{cc}} = \begin{cases} \frac{(3+2\gamma(\beta-\beta'))(1+2\gamma(\beta-\beta'))}{4(1+\gamma(\beta-\beta'))^2} & \text{if } K \geq \frac{\alpha}{1+2\gamma(\beta-\beta')} \\ \frac{\alpha^2(3+2\gamma(\beta-\beta'))}{4K(2\alpha-K(1+2\gamma(\beta-\beta')))(1+\gamma(\beta-\beta'))^2} & \text{if } \frac{\alpha}{2(1+\gamma(\beta-\beta'))} \leq K < \frac{\alpha}{1+2\gamma(\beta-\beta')} \\ 1 & \text{else} \end{cases}$$

We notice that

$$\frac{(3+2\gamma(\beta-\beta'))(1+2\gamma(\beta-\beta'))}{4(1+\gamma(\beta-\beta'))^2} = 1 - \frac{1}{4(1+\gamma(\beta-\beta'))^2}$$

Moreover, in the second expression, the denominator is clearly a increasing function of K for $K < \frac{\alpha}{1+2\gamma(\beta-\beta')}$ so the ratio is decreasing with K. For the largest possible value of K in the range, the ratio (at its minimum value) equals the ratio of the first case, which we showed above cannot go below $1 - \frac{1}{4(1+\gamma(\beta-\beta'))^2}$. Therefore,

$$\frac{\Pi_c}{\Pi_{cc}} \geq 1 - \frac{1}{4(1+\gamma(\beta-\beta'))^2} \geq \frac{3}{4}$$

where the first inequality is tight for large capacity level and the second inequality is tight in the case of no production cost ($\gamma = 0$) or non differentiated suppliers ($\beta = \beta'$).

□

G.3 Proof of Proposition 4

We consider the 7 possible combinations of regimes in the decentralized and cooperative settings.

- **regimes** (a, a'): Let's first note that

$$b^2 - 4\beta(\beta-\beta')(1+\gamma(\beta-\beta'))(1+\gamma\beta) = \beta'^2.$$

Some calculations then lead to

$$J_c^B - J_d^B = \frac{\frac{\alpha^2}{4(\beta-\beta')}}{1+\gamma(\beta-\beta')} - \frac{\beta(1+\gamma\beta)\alpha^2}{b^2} = \frac{\beta'^2\alpha^2}{4b^2(\beta-\beta')(1+\gamma(\beta-\beta'))} \geq 0.$$

- **regimes** (b, a'): Some calculations then lead to

$$\begin{aligned} J_c^B - J_d^B &= \frac{\frac{\alpha^2}{4(\beta-\beta')}}{1+\gamma(\beta-\beta')} + \gamma(K^B)^2 - K^B\Big(\frac{\alpha(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta)) - 2K^B\beta(1+\gamma\beta)}{d^2}\Big) \\ &= \frac{-\alpha K^B(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta)) + (K^B)^2(2\beta(1+\gamma\beta)^2 - \gamma\beta'^2(1+2\gamma\beta))}{d^2} \\ &\qquad + \frac{\alpha^2}{4(\beta-\beta')(1+\gamma(\beta-\beta'))} \end{aligned}$$

We observe that this is a polynomial of degree 2 in α with concavity turned up. Straightforward calculations enable to find that this polynomial reaches a minimum at

$$\frac{2(\beta-\beta')(1+\gamma(\beta-\beta'))K^B(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta))}{d^2}$$

and thus the minimum value of the polynomial is

$$\begin{aligned} & \frac{(K^B)^2}{d^4}\Big(-(\beta-\beta')(1+\gamma(\beta-\beta'))(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta))^2 + d^2(2\beta(1+\gamma\beta)^2 - \gamma\beta'^2(1+2\gamma\beta))\Big) \\ = & \frac{(K^B)^2}{d^4}\Big(\beta'^3(1+2\gamma\beta)^3 + \beta\beta'^2(1+\gamma\beta)(1+2\gamma\beta)^2 - \gamma\beta^2\beta'^2(1+\gamma\beta)^2 - 4\gamma\beta\beta'^3(1+\gamma\beta)(1+2\gamma\beta)\Big) \\ = & \frac{(K^B)^2}{d^4}\Big(\beta'^3(1+2\gamma\beta) + \beta\beta'^2(1+\gamma\beta)(1+3\gamma\beta+3\gamma^2\beta^2)\Big) > 0 \end{aligned}$$

- **regimes** (b, b'):

$$\begin{aligned} J_c^B - J_d^B &= K^B\Big(\frac{\alpha}{2(\beta-\beta')} + \frac{\frac{\alpha}{2}(1+\gamma(\beta+\beta')) - K^B(1+\gamma\beta)}{\beta+\gamma(\beta^2-\beta'^2)} \\ &\qquad - \frac{\alpha(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta)) - 2K^B\beta(1+\gamma\beta)}{d^2}\Big) \end{aligned}$$

thus $J_c^B \geq J_d^B$ iff

$$\frac{\alpha}{2(\beta-\beta')} + \frac{\frac{\alpha}{2}(1+\gamma(\beta+\beta')) - K^B(1+\gamma\beta)}{\beta+\gamma(\beta^2-\beta'^2)} - \frac{\alpha(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta)) - 2K^B\beta(1+\gamma\beta)}{d^2} > 0$$

$$\Leftrightarrow \quad \alpha\Big(\frac{2\beta-\beta'+2\gamma(\beta^2-\beta'^2)}{2(\beta-\beta')(\beta+\gamma(\beta^2-\beta'^2))} - \frac{2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta)}{d^2}\Big) + \frac{K^B\beta'^2(1+\gamma\beta)}{d^2(\beta+\gamma(\beta^2-\beta'^2))} > 0$$

It is sufficient to show that the slope of α above is positive. Since we observe that $(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta))(\beta-\beta') = d^2 - \beta\beta'$, the slope of α is positive iff

$$d^2(2(\beta+\gamma(\beta^2-\beta'^2))-\beta') - 2(\beta+\gamma(\beta^2-\beta'^2))(d^2-\beta\beta') > 0$$

which simplifies to $\beta'^3 > 0$. Therefore, $J_c^B \geq J_d^B$.

- **regimes** (c, a'):

$$\begin{aligned} J_c^B - J_d^B &= \frac{\frac{\alpha^2}{4(\beta-\beta')}}{1+\gamma(\beta-\beta')} - \frac{K^B}{\beta-\beta'}\Big(\alpha-\bar{K}\Big) + \gamma(K^B)^2 \\ &= \frac{\alpha^2}{4(\beta-\beta')(1+\gamma(\beta-\beta'))} - \frac{\alpha K^B}{\beta-\beta'} + \frac{K^B\bar{K}}{\beta-\beta'} + \gamma(K^B)^2 \end{aligned}$$

where $\bar{K} = \frac{\beta K^B + \beta' K^A}{\beta+\beta'} > K^B$. We observe that $J_c^B - J_d^B$ is quadratic in α with the concavity turned up. Therefore, it if necessary and sufficient to check the sign of $J_c^B - J_d^B$ at the point where the polynomial reaches its minimum: $2K^B(1+\gamma(\beta-\beta'))$, where the polynomial takes value

$$\frac{K^B\bar{K}}{\beta-\beta'} + \gamma(K^B)^2 - \frac{(K^B)^2(1+\gamma(\beta-\beta'))}{\beta-\beta'} = \frac{K^B(\bar{K}-K^B}{\beta-\beta'} > 0$$

Therefore, $J_c^B \geq J_d^B$.

- **regimes** (c, b'):

$$\begin{aligned} J_c^B - J_d^B &= K^B\Big(-\frac{\alpha}{2(\beta-\beta')} + \frac{\frac{\alpha}{2}(1+\gamma(\beta+\beta')) - K^B(1+\gamma\beta)}{\beta+\gamma(\beta^2-\beta'^2)} + \frac{\beta K^B+\beta' K^A}{\beta^2-\beta'^2}\Big) \\ &= K^B\Big(\frac{-\alpha\beta'}{2(\beta-\beta')(\beta+\gamma(\beta^2-\beta'^2))} \\ &\qquad + \frac{-K^B(1+\gamma\beta)(\beta^2-\beta'^2) + (\beta K^B+\beta' K^A)(\beta+\gamma(\beta^2-\beta'^2))}{(\beta+\gamma(\beta^2-\beta'^2))(\beta^2-\beta'^2)}\Big) \\ &= K^B\beta'\Big(\frac{-\alpha}{2(\beta-\beta')(\beta+\gamma(\beta^2-\beta'^2))} + \frac{K^B\beta' + K^A(\beta+\gamma(\beta^2-\beta'^2))}{(\beta+\gamma(\beta^2-\beta'^2))(\beta^2-\beta'^2)}\Big) \end{aligned}$$

Since in regime b', $\frac{\alpha}{2} < K^A(1+\gamma(\beta-\beta')) - \frac{\beta'(K^A-K^B)}{\beta+\beta'}$, we have

$$\begin{aligned} J_c^B - J_d^B &> K^B\beta' \frac{-K^A(\beta+\beta'+\gamma(\beta^2-\beta'^2)) + \beta'(K^A-K^B) + K^B\beta' + K^A(\beta+\gamma(\beta^2-\beta'^2))}{(\beta+\gamma(\beta^2-\beta'^2))(\beta^2-\beta'^2)} \\ &= 0 \end{aligned}$$

which proves that on the valid domain $J_c^B \geq J_d^B$.

- **regimes** (c,c'): $J_c^B = J_d^B = \frac{K^B}{\beta-\beta'}\Big(\alpha - \frac{\beta K^B + \beta' K^A}{\beta+\beta'}\Big)$ (the cooperative solution and the decentralized equilibrium coincide).

□

G.4 Proof of Proposition 5 and Theorem 4

Three regimes are possible: (a,a'), (c,a') and (c,c').

- regimes (c,c'): In this regime, the optimal solution under cooperation is identical to the equilibrium, therefore $\rho = 1$.
  The condition to be in this regime is $0 \leq \frac{\alpha}{2} - K(1+\gamma(\beta-\beta'))$.

- regimes (a,a'): This regime holds if $\alpha\beta - bK \leq 0$, i.e. $\alpha - K(2-r+2\gamma(\beta-\beta')) \leq 0$.
  Since $b^2 - 4\beta(\beta-\beta')(1+\gamma(\beta-\beta'))(1+\gamma\beta) = \beta'^2$,

$$\rho = \frac{4\beta(\beta-\beta')(1+\gamma\beta)(1+\gamma(\beta-\beta'))}{b^2} = \frac{b^2 - r^2\beta^2}{b^2} = 1 - \frac{r^2\beta^2}{b^2}$$

  Moreover, $\frac{b}{\beta} = 2 - r + 2\gamma(\beta-\beta') > 2 - r$ so $\frac{r^2\beta^2}{b^2} < \frac{r^2}{(2-r)^2}$. Therefore,

$$\rho > 1 - \frac{r^2}{(2-r)^2}.$$

- regimes (c,a'): This regime holds if $\alpha - K(2-r+2\gamma(\beta-\beta')) \geq 0$ and $0 \geq \frac{\alpha}{2} - K(1+\gamma(\beta-\beta'))$, i.e.

$$\frac{1}{2(1+\gamma(\beta-\beta'))} \leq \frac{K}{\alpha} \leq \frac{1}{2-r+2\gamma(\beta-\beta')}.$$

$$\begin{aligned} \rho &= 4(1+\gamma(\beta-\beta'))\frac{K(\alpha-K) - \gamma(\beta-\beta')K^2}{\alpha^2} \\ &= 4(1+\gamma(\beta-\beta'))\frac{K}{\alpha}\Big(1 - \frac{K}{\alpha}(1+\gamma(\beta-\beta'))\Big) \end{aligned}$$

  Denoting $X = (1+\gamma(\beta-\beta'))\frac{K}{\alpha}$, we have $\rho = 4X(1-X)$, where $\frac{1}{2} \leq X \leq X_0 \equiv \frac{1+\gamma(\beta-\beta')}{2-r+2\gamma(\beta-\beta')}$.

On this domain, ρ reaches its minimum at X_0, i.e.

$$\begin{aligned}\rho &\geq 4X_0(1-X_0)\\ &= \frac{4(1+\gamma(\beta-\beta'))(1+\gamma(\beta-\beta')-r)}{(2+2\gamma(\beta-\beta')-r)^2}\\ &= 1-\frac{r^2}{(2+2\gamma(\beta-\beta')-r)^2}\\ &\geq 1-\frac{r^2}{(2-r)^2}\end{aligned}$$

It follows that $1-\rho \leq \frac{r^2}{(2-r)^2}$. We now show that $1-\rho \leq r^2\frac{K^2}{\alpha^2}$.

In regimes (c,c'), $\rho=1$ so the bound clearly holds.
In regimes (a,a'), $\frac{b}{\beta}=2-r+2\gamma(\beta-\beta')\geq\frac{\alpha}{K}$ and thus $\rho=1-\frac{r^2\beta^2}{b^2}\geq 1-r^2\frac{K^2}{\alpha^2}$.
In regimes (c,a'), for brevity let us denote $u\equiv 1+\gamma(\beta-\beta')$. We have (as seen in Appendix G.4) $\rho=1-(1-2u\frac{K}{\alpha})^2$ and the valid domain in this regime is $2u-r\leq\frac{\alpha}{K}\leq 2u$. Therefore, we have in particular $\frac{K}{\alpha}(2u-r)\leq 1$, which implies $0\leq 2u\frac{K}{\alpha}-1\leq r\frac{K}{\alpha}$. Therefore, $\left(2u\frac{K}{\alpha}-1\right)^2\leq r^2\frac{K^2}{\alpha^2}$, so we have $\rho=1-(2u\frac{K}{\alpha}-1)^2\geq 1-r^2\frac{K^2}{\alpha^2}$. □

G.5 Proof of Theorem 5

The derivative of ρ with respect to K^B is

$$\frac{\partial\rho}{\partial K^B}=\frac{\partial}{\partial K^B}\Big(\frac{J_d^A+J_d^B}{J_c^A+J_c^B}\Big)$$

therefore ρ is non increasing with K^B iff

$$(J_d^A+J_d^B)\frac{\partial(J_c^A+J_c^B)}{\partial K^B}-(J_c^A+J_c^B)\frac{\partial(J_d^A+J_d^B)}{\partial K^B}\geq 0 \tag{5}$$

- **regimes** (c,c'): $\rho=1$ is independent of K^B.
- **regimes** (a,a'): $J_d^A,\ J_d^B,\ J_c^A$ and J_c^B are independent of K^B, so ρ is independent of K^B.
- **regimes** (c,a'): We have $\frac{\partial(J_c^A+J_c^B)}{\partial K^B}=0$ and $J_c^A+J_c^B\geq 0$, therefore (5) holds iff $\frac{\partial(J_d^A+J_d^B)}{\partial K^B}\leq 0$. We have

$$\begin{aligned}\frac{\partial(J_d^A+J_d^B)}{\partial K^B} &= -\frac{\beta'}{\beta+\beta'}\frac{K^A}{\beta-\beta'}-2\gamma K^B+\frac{\alpha}{\beta-\beta'}-\frac{\beta K^B+\beta' K^A}{\beta^2-\beta'^2}-\frac{\beta}{\beta+\beta'}\frac{K^B}{\beta-\beta'}\\ &= \frac{\alpha}{\beta-\beta'}-2\gamma K^B-2\frac{\beta K^B+\beta' K^A}{\beta^2-\beta'^2}\\ &\leq \frac{\alpha-2\gamma K^B(\beta-\beta')-2K^B}{\beta-\beta'}\end{aligned}$$

where the last inequality follows from the observation that $\frac{\beta K^B + \beta' K^A}{\beta+\beta'} \geq K^B$ for $K^A \geq K^B$. In regime a', $\alpha - 2K^B(1+\gamma(\beta-\beta') < 0$, so the result follows.

- **regimes** (b, a'): We have $\frac{\partial(J_c^A + J_c^B)}{\partial K^B} = 0$ and $J_c^A + J_c^B \geq 0$, therefore (5) holds iff

$$\frac{\partial(J_d^A + J_d^B)}{\partial K^B} \leq 0 \tag{6}$$

We observe that $\frac{\partial J_d^A}{\partial K^B}$ and $\frac{\partial J_d^B}{\partial K^B}$ are linear in K^B, with a slope respectively equal to $\frac{2\beta\beta'^2(1+\gamma\beta)}{d^4} > 0$ and $-2\gamma - \frac{4\beta(1+\gamma\beta)}{d^2} < 0$. Therefore, $\frac{\partial(J_d^A+J_d^B)}{\partial K^B}$ is linear in K^B with slope

$$\frac{2}{d^4}\Big(\beta\beta'^2(1+\gamma\beta) - \gamma d^4 - 2\beta(1+\gamma\beta)d^2\Big) < 0$$

since $d^2 \geq 2\beta^2 - \beta'^2$ implies $\beta'^2 - 2d^2 \leq -4\beta^2 + \beta'^2 < 0$. Therefore, (6) holds iff $\frac{\partial(J_d^A+J_d^B)}{\partial K^B}$ is non positive at the lowest value allowed for K^B in regimes (b, a'), i.e.

$$\max\{\frac{\alpha\beta(\beta+\beta') - d^2K^A}{\beta\beta'},\ \frac{\frac{\alpha}{2}}{1+\gamma(\beta-\beta')}\}.$$

We will prove the sufficient condition that $\frac{\partial(J_d^A+J_d^B)}{\partial K^B}$ is non positive for $K^B = \frac{\frac{\alpha}{2}}{1+\gamma(\beta-\beta')}$. $\frac{\partial(J_d^A+J_d^B)}{\partial K^B}$ is linear decreasing in K^B, and some calculations lead to showing that it takes value 0 for

$$K^B = \frac{\alpha}{2}\frac{d^2(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) - 2\beta\beta'(1+\gamma\beta)(\beta+\beta')}{-\beta\beta'^2(1+\gamma\beta) + \gamma d^4 + 2\beta(1+\gamma\beta)d^2}.$$

Therefore $\frac{\partial(J_d^A+J_d^B)}{\partial K^B}$ is non positive for $K^B = \frac{\frac{\alpha}{2}}{1+\gamma(\beta-\beta')}$ iff

$$\frac{d^2(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) - 2\beta\beta'(1+\gamma\beta)(\beta+\beta')}{-\beta\beta'^2(1+\gamma\beta) + \gamma d^4 + 2\beta(1+\gamma\beta)d^2} \leq 1,$$

or equivalently,

$$\gamma d^4 - d^2\beta'(1+2\gamma\beta) + \beta\beta'(1+\gamma\beta)(2\beta+\beta') \geq 0.$$

Denoting $r = \frac{\beta'}{\beta} \in [0,1]$ and $t = \gamma\beta > 0$, we have $\frac{d^2}{\beta^2} = (2-r^2) + 2t(1-r^2)$, so we observe that the inequality above is equivalent to

$$t\big((2-r^2) + 2t(1-r^2)\big)^2 - \big((2-r^2) + 2t(1-r^2)\big)r(1+2t) + r(1+t)(2+r) \geq 0$$

which, as illustrated in Figure 13, holds for any r in $[0,1]$ and $t > 0$. This ends the proof.

Lemma 3. *Consider two polynomials of degree 2:* $P_1(X) = aX^2 + bX + c$ *and* $P_2(X) = a'X^2 + b'X + c'$. *Then* $P_1(X)P_2'(X) - P_1'(X)P_2(X)$ *is a polynomial of degree 2 given by*

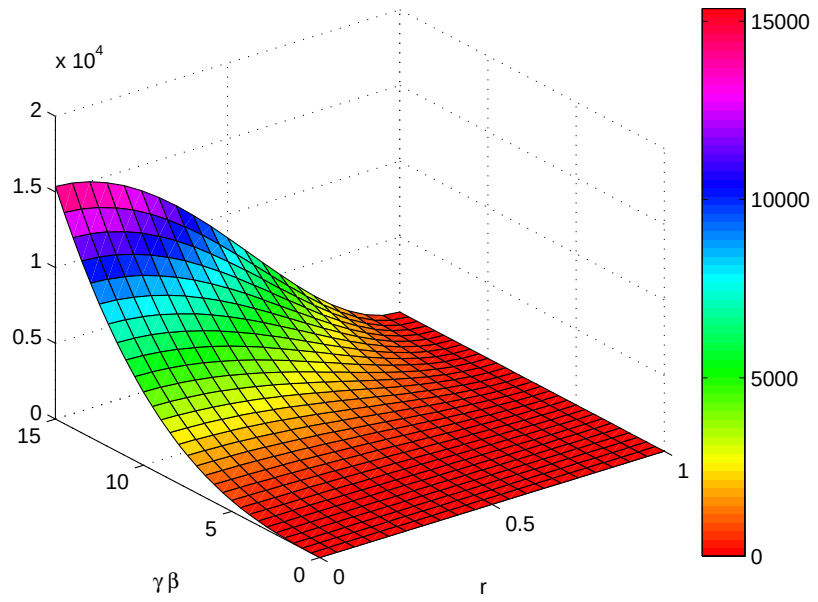


Figure 13: Plot of $t\big((2-r^2)+2t(1-r^2)\big)^2-\big((2-r^2)+2t(1-r^2)\big)r(1+2t)+r(1+t)(2+r)$ as a function of r and t, where $r=\frac{\beta'}{\beta}$ and $t=\gamma\beta$.

$(a'b-ab')X^2+2(a'c-ac')X+(cb'-c'b)$.

- **regimes** (c,b'): $J_c^A+J_c^B$ and $J_d^A+J_d^B$ are quadratic in K^B, so, using the Lemma above, inequality (5) is a quadratic expression in K^B, which we will denote polynomial $P(K^B)=sK^{B^2}+t_0K^B+v$. The coefficient of K^{B^2} is, after calculations, equal to

$$\begin{aligned} s &= \frac{(1+\gamma(\beta+\beta'))\alpha}{\beta^2-\beta'^2}-\frac{\alpha-2K^A\frac{\beta'}{\beta+\beta'}}{\beta-\beta'}\Big(\gamma+\frac{1+\gamma\beta}{\beta+\gamma(\beta^2-\beta'^2)}\Big) \\ &= \frac{\beta'(1+\gamma(\beta+\beta'))}{(\beta^2-\beta'^2)(\beta+\gamma(\beta^2-\beta'^2))}(-\alpha+2K^A(1+\gamma(\beta-\beta'))) \end{aligned}$$

This coefficient is positive iff

$$\alpha<2K^A(1+\gamma(\beta-\beta'))$$

Regime b' implies $(\beta+\beta')\alpha-2K^A(\beta+\gamma(\beta^2-\beta'^2))<2\beta'K^B$. Since $K^B\leq K^A$, we have in this regime

$$(\beta+\beta')\alpha<2K^A(\beta+\beta'+\gamma(\beta^2-\beta'^2)),$$

i.e. $\alpha<2K^A(1+\gamma(\beta-\beta'))$.
This proves that $(J_d^A+J_d^B)\frac{\partial(J_c^A+J_c^B)}{\partial K^B}-(J_c^A+J_c^B)\frac{\partial(J_d^A+J_d^B)}{\partial K^B}$ is quadratic in K^B with a coefficient of $(K^B)^2$ that is positive. To show that polynomial $P(K^B)$ is non negative on

the valid domain for K^B in regimes (c, b'), we will show that at the minimum allowed value in this regime $\max(0, K_1^B)$, $P(K^B)$ and its first derivative are non negative, where $l'_b(K^A) = \frac{(\beta+\beta')\frac{\alpha}{2} - K^A(\beta+\gamma(\beta^2-\beta'^2))}{\beta'}$.
First, it is clear that J_c^A, J_c^B, J_d^A and J_d^B are continuous in K^B. Furthermore, we observe that $\frac{\partial(J_c^A+J_c^B)}{\partial K^B}$ is continuous in K^B at the threshold $l'_b(K^A)$ between regimes b' and c'. Indeed, after calculations, to the left (regime b'), it is equal to

$$\frac{\alpha(1+\gamma(\beta+\beta'))}{\beta+\gamma(\beta^2-\beta'^2)} - 2l'_b(K^A)\frac{(1+\gamma(\beta+\beta'))(1+\gamma(\beta-\beta'))}{\beta+\gamma(\beta^2-\beta'^2)}$$

while to the right (regime c'), it is equal to

$$\frac{\alpha - 2\frac{\beta'}{\beta+\beta'}K^A}{\beta-\beta'} - 2l'_b(K^A)\frac{\beta+\gamma(\beta^2-\beta'^2)}{\beta^2-\beta'^2}$$

Straightforward simplifications show that these two quantities are equal. Since $l'_b(K^A)$ is not a threshold between two regimes for J_d, $\frac{\partial(J_d^A+J_d^B)}{\partial K^B}$ is continuous in K^B at $l'_b(K^A)$. Therefore, the first derivative of ρ with respect to K^B is continuous at $l'_b(K^A)$. To the left of $l'_b(K^A)$, regimes (c, b') become regimes (c, c') where $\rho = 1$ and in particular its first derivative is equal to zero. Therefore, by continuity, polynomial $P(K^B)$ takes value zero at $l'_b(K^A)$.
If $l'_b(K^A)$ is the larger root (i.e. $l'_b(K^A) > -\frac{t_0}{2s}$), then in regime (c, b'), $K^B > \max(0, l'_b(K^A)) \geq l'_b(K^A)$ and thus $P(K^B)$ is non negative and the result is shown. Assume $l'_b(K^A)$ is the smaller root (i.e. $l'_b(K^A) < -\frac{t_0}{2s}$); we need to show that t_0 the derivative of polynomial $P(K^B)$ at zero is non negative. It will then follow that $l'_b(K^A) < 0$ (because we showed $s > 0$), hence the result.

The coefficient of the linear term in $P(K^B)$ is:

$$\begin{aligned} t_0 &= -\frac{2}{\beta-\beta'}(1+\gamma(\beta+\beta'))(1+\gamma(\beta-\beta'))\Big(\frac{K^A\alpha}{\beta+\gamma(\beta^2-\beta'^2)} - \frac{(K^A)^2}{(\beta+\beta')}\Big) + \frac{2}{\beta-\beta'}\Big(\frac{\alpha^2}{4(\beta-\beta')}\Big) \\ &= \frac{2}{\beta-\beta'}\Big(\frac{\alpha^2}{4(\beta-\beta')} - (1+\gamma(\beta+\beta'))(1+\gamma(\beta-\beta'))\Big(\frac{K^A\alpha}{\beta+\gamma(\beta^2-\beta'^2)} - \frac{(K^A)^2}{\beta+\beta'}\Big)\Big) \end{aligned}$$

It is easy to check (using for example the Excel solver) that t_0 above is minimized for $\beta' = 0$ and then equals $\frac{2}{\beta^2}\Big((\frac{\alpha}{2} - K^A)^2 + \gamma(K^A)^2\Big) \geq 0$. Therefore $t_0 \geq 0$, which ends the proof.

- **regimes** (b, b'): $J_c^A + J_c^B$ and $J_d^A + J_d^B$ are quadratic in K^B, so, using the Lemma above, inequality (5) is a quadratic expression in K^B: $P_1(K^B) = s_1(K^B)^2 + t_1K^B + v_1$. First, let's show that the coefficient s_1 of $(K^B)^2$ is positive. This coefficient is equal to

$$s_1 = -\frac{(1+\gamma(\beta+\beta'))(1+\gamma(\beta-\beta'))}{\beta+\gamma(\beta^2-\beta'^2)}\Big(-\frac{2\beta\beta'(1+\gamma\beta)}{d^4}(\alpha(\beta+\beta')) + \frac{\alpha(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta))}{d^2}\Big)$$

$$-\Big(\big(-\gamma+\frac{\beta\beta'^2(1+\gamma\beta)}{d^4}-\frac{2\beta(1+\gamma\beta)}{d^2}\big)\frac{\alpha-(1+\gamma(\beta+\beta'))}{\beta+\gamma(\beta^2-\beta'^2)}\Big)$$

It is easy to check (using for example the Excel solver) that this expression is minimized for $\beta'=0$, in which case it equals

$$\frac{\alpha}{\beta}\frac{1+\gamma\beta}{\beta}+(\gamma+\frac{1}{\beta})\frac{\alpha}{\beta}=0.$$

Therefore s_1 is non negative. If polynomial $P_1(K^B)$ had less than 2 roots, then it would always be non negative and the result follows. Let's assume that it has 2 roots.

Moreover, we have that the coefficient of the linear term in $P_1(K^B)$ is given by

$$t_1=-2\frac{(1+\gamma(\beta+\beta'))(1+\gamma(\beta-\beta'))}{\beta+\gamma(\beta^2-\beta'^2)}\Big(\frac{\beta(1+\gamma\beta)}{d^4}\big(\alpha(\beta+\beta')\big)^2\Big)$$

$$-\Big(-\gamma+\frac{\beta\beta'^2(1+\gamma\beta)}{d^4}-\frac{2\beta(1+\gamma\beta)}{d^2}\Big)\frac{\frac{\alpha^2}{2}(\beta+\beta')}{(\beta-\beta')(\beta+\gamma(\beta^2-\beta'^2))}$$

It is easy to check (using for example the Excel solver) that t_1 above is minimized for $\beta'=0$ and then equals

$$-\frac{2}{\beta}(1+\gamma\beta)\Big(\frac{\alpha^2}{4\beta(1+\gamma\beta)}\Big)+\frac{\alpha^2}{2\beta^2}=\frac{-\frac{\alpha)^2}{2}+\frac{\alpha^2}{2}}{\beta^2}=0$$

Therefore $t_1\geq 0$. In other words, polynomial $P_1(K^B)$ has a derivative at 0 that is non negative. Finally, we have to show that $P_1(0)\geq 0$ to conclude that both roots are negative, and as a result on the valid domain for K^B, we have $P_1(K^B)\geq 0$. We have

$$v_1=\frac{\alpha(1+\gamma(\beta+\beta'))}{\beta+\gamma(\beta^2-\beta'^2)}\Big(\frac{\beta(1+\gamma\beta)}{d^4}\big(\alpha(\beta+\beta')\big)^2\Big)-\frac{\frac{\alpha^2}{2}(\beta+\beta')}{2(\beta-\beta')(\beta+\gamma(\beta^2-\beta'^2))}\Big(-\frac{2\beta\beta'(1+\gamma\beta)}{d^4}(\alpha(\beta+\beta'))$$

$$+\frac{\alpha(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta))}{d^2}\Big)$$

It is easy to check (using for example the Excel solver) that v_1 above is minimized for $\beta'=0$ and then equals

$$\frac{\alpha-2I_0}{\beta}\Big(-\gamma I_0^2+\frac{(\alpha+2\gamma\beta I_0)^2}{4\beta(1+\gamma\beta)}+\frac{I_0(\alpha-I_0)}{\beta}\Big)-\frac{\frac{\alpha^2}{4}+I_0(\alpha-I_0)(1+2\gamma\beta)}{\beta(1+\gamma\beta)}\frac{\alpha-2I_0}{\beta}=0$$

Therefore $v_1\geq 0$.

□

G.6 Proof of Proposition 1

In regime a, $\frac{\partial J^A}{\partial K^B}(K^A, K^B) = 0$.
In regime c,

$$\frac{\partial J^A}{\partial K^B}(K^A, K^B) = -\frac{\beta' K^A}{\beta^2 - \beta'^2} < 0.$$

In regime b,

$$\frac{\partial J^A}{\partial K^B}(K^A, K^B) = -\frac{2\beta\beta'(1+\gamma\beta)}{d^4}(\alpha(\beta+\beta') - \beta' K^B)$$

and in that regime,

$$\beta' K^B < \alpha\frac{\beta\beta'}{b} = \alpha(\beta+\beta') - \alpha\frac{d^2}{b}$$

therefore $\alpha(\beta+\beta') - \beta' K^B > \alpha\frac{\beta\beta'}{b} > 0$ and thus $\frac{\partial J^A}{\partial K^B}(K^A, K^B) < 0$. □

G.7 Proof of Proposition 2

We first show that $\alpha\beta - bK_0 > 0$ and that if $\alpha\beta(\beta+\beta') - K_0\beta\beta' - d^2K^A < 0$, then $K_0 \leq K^A$, and thus $K^B = K_0$ corresponds to regime b.
Let $d_1 \equiv 4\beta(1+\gamma\beta)^2 - 2\gamma\beta'^2(1+2\gamma\beta)$ so that $K_0 = \frac{\alpha(2\beta(1+\gamma\beta)+\beta'(1+2\gamma\beta))}{d_1}$.

$$\begin{aligned} \alpha\beta - bK_0 > 0 \quad &\Leftrightarrow \quad \beta d_1 - b(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) > 0 \\ &\Leftrightarrow \quad \beta'^2(1+2\gamma\beta) > 0 \end{aligned}$$

which holds.

$$\begin{aligned} \alpha\beta(\beta+\beta') - K_0\beta\beta' - d^2K^A < 0 \quad \Rightarrow \quad & \alpha\beta(\beta+\beta')d_1 - \beta\beta'\alpha(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) \\ & - K^A d_1(2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta)) < 0 \end{aligned}$$

Let

$$\begin{aligned} A &= \beta(\beta+\beta')d_1 - \beta\beta'(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) \\ B &= d_1(2\beta^2(1+\gamma\beta) - \beta'^2(1+2\gamma\beta)) > 0 \end{aligned}$$

so that the inequality above is: $A\alpha - BK^A < 0$. We can show that $A > 0$: since

$$d_1 > 2\beta'(2(1+2\gamma\beta+\gamma^2\beta^2) - \gamma\beta'(1+2\gamma\beta) > 2\beta'(2+3\gamma\beta)$$

we have

$$A = \beta^2(d_1 - 2\beta'(1+\gamma\beta)) + \beta'\beta(d_1 - \beta'(1+2\gamma\beta)) > 0$$

$K_0 \leq K^A$ iff

$$\alpha(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) - d_1 K^A \leq 0.$$

In order to show that $K_0 \leq K^A$, it is sufficient to show

$$\frac{B}{A} < \frac{d_1}{2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)}.$$

After calculations,

$$B(2\beta(1+\gamma\beta) + \beta'(1+2\gamma\beta)) - Ad_1 = -d_1\beta'^2(\beta+\beta')(1+2\gamma\beta) < 0.$$

This proves that if $\alpha\beta(\beta+\beta') - K_0\beta\beta' - d^2K^A < 0$, then $K_0 \in [0, K^A]$ and $K^B = K_0$ is in regime b. Therefore, a simple calculation of derivatives (using the expression found in Proposition 6 leads to $\frac{\partial J^B}{\partial K^B}\Big|_{K^B=K_0} = 0$, which means that K_0 is a local maximum of J^B. (It is not a local minimum or a point of inflexion because we proved that the profit was quadratic concave in regime b.)

Similarly, it is easy to show that under conditions (2) and (3), if $\bar{K}_0 \in [0, K^A]$, regime c holds for $K^B = \bar{K}_0$ and $\frac{\partial J^B}{\partial K^B}\Big|_{K^B=\bar{K}_0} = 0$.

□

G.8 Proof of Proposition 3

$J^A + J^B$ is independent of K^B in regime a. Therefore if K^* is in regime a, then $l_a = \frac{\alpha\beta}{b}$ the threshold between regime a and b is also a maximum for $J^A + J^B$, and $K^* \geq l_a$.
Assume that $\bar{K}_1$ the value of K^B that maximizes $J^B(K^B)$ satisfies $\bar{K}_1 < l_a$. Then $\bar{K}_1$ is in regime b or c, and $J^B(\bar{K}_1) \geq J^B(l_a)$. Since J^A is decreasing with K^B in regimes b and c, $J^A(\bar{K}_1) > J^A(l_a)$, and therefore $(J^A+J^B)(\bar{K}_1) > (J^A+J^B)(l_a)$, which contradicts that l_a is a maximum for J^A+J^B.
Now consider the case when K^* corresponds to either regime b or c. Thus we have

$$\frac{\partial(J^A + J^B)(K^A, K_B)}{\partial K^B}\Big|_{K^B=K^*} = 0.$$

As we proved above, J^A is non decreasing with K^B, and in particular $\frac{\partial J^A}{\partial K^B}(K^A, K^B)\Big|_{K^B=K^*} < 0$.
Therefore, $\frac{\partial J^B}{\partial K^B}(K^A, K^B)\Big|_{K^B=K^*} > 0$.
Moreover, in regimes b and c respectively, $\frac{\partial J^B}{\partial K^B}(K^A, K^B)$ is linear decreasing with K^B, possibly crossing the horizontal axis at respectively K_0 and $\bar{K}_0$. First suppose that J^B's global maximum is K_0 (regime b). Then for $K^B \geq K_0$ (regime a and rest of regime b), $\frac{\partial J^B}{\partial K^B}(K^A, K^B) \leq 0$ and thus

$K^* < K_0$. Now suppose the global maximum is $\bar{K}_0$ (regime c). Then for $K^B > \bar{K}_0$ and in regime c, $\frac{\partial J^B}{\partial K^B}(K^A, K^B) < 0$, so if K^* is in regime c, $K^* < \bar{K}_0$. To end the proof, we only need to show that if J^B's global maximum is $\bar{K}_0$, then K^* is in regime c (not b).
Assume that J^B's global maximum is $\bar{K}_0$ and K^* is not in regime c (and as a result, $\bar{K}_0 < K^*$). Then $J^B(\bar{K}_0) > J^B(K^*)$. Moreover, J^A is continuous with K^B and non increasing, therefore $J^A(\bar{K}_0) \geq J^A(K^*)$, which implies $(J^A + J^B)(\bar{K}_0) > (J^A + J^B)(K^*)$ and is a contradiction. □

G.9 Proof of Theorem 2

It is in supplier B's best interests to choose her capacity level at K_0 or $\bar{K}_0$, whichever value maximizes her equilibrium profit (depending on which of the local maxima is a global maximum). Denote $\bar{K}_1$ the value that supplier B would select, i.e., $argmax_{K^B} J^B(K^B)$. From Proposition 3, it follows that $\bar{K}_1 \geq K^*$. At that level $\bar{K}_1$, the system total profits are not maximized. In other words, by decreasing supplier B's capacity level from $\bar{K}_1$ to K^*, supplier B's profits decrease less than supplier A's profits increase (see Figure 12). Supplier A would benefit from paying supplier B a fee of $J^B(\bar{K}_1) - J^B(K^*) + \epsilon$ to change her capacity level from $\bar{K}_1$ to K^*. This would leave supplier B better off, and incurs for supplier A a profit of

$$(J^A + J^B)(K^*) - J^B(\bar{K}_1) - \epsilon \geq (J^A + J^B)(\bar{K}_1) - J^B(\bar{K}_1) - \epsilon = J^A(\bar{K}_1) - \epsilon.$$

Thus, such a fee together with a change from supplier B capacity level from $\bar{K}_1$ to K^* incurs a benefit of ϵ for supplier B and of $-\epsilon + (J^A + J^B)(K^*) - (J^A + J^B)(\bar{K}_1)$ for supplier A (supplier A benefits as long as the premium ϵ is no greater than $(J^A + J^B)(K^*) - (J^A + J^B)(\bar{K}_1)$). Notice that the setting is still decentralized, in the sense that both suppliers make decisions "selfishly" in order to optimize their own profits, not the system profits. □