Decomposition Method for Oligopolistic Competitive Models with Common Pollution Regulation

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Abstract

We consider the general problem of industrial production in a set of countries subject to a common environmental regulation that limits the emissions of specific sectors. Due to these restrictions, the problem is treated as a generalized non-cooperative game where players (countries) have joint (environmental) constraints caused by the necessity of a common and compulsory emission regulation. The problem is to find a natural mechanism for attaining the corresponding generalized equilibrium state. We suggest a share allocation method, which yields a suitable decomposition type procedure and replaces the initial problem with a sequence of non-cooperative games on Cartesian product sets. We also show that its implementation can be simplified essentially after application of a regularized penalty method. In the case study, we take inspiration from the European Union Emission Trading System (EU-ETS) and we introduce an environmental regulation that restricts the carbon emissions of energy, cement, and steel sectors in some European countries. Our results confirm the important role played by energy sector in reducing carbon emissions.

Keywords: Oligopolistic competition; environmental regulation; non-cooperative games; share allocation method; decomposable penalty method; regularized penalty method.

1 Introduction

Many complex problems in economic systems are formulated as non-cooperative games (see Ichiisi 1983; Okuguchi and Szidarovszky 1990; Owen 1995) which represent systems involving active economic agents (players), so that each payoff function depends on decisions of all of them, but they can take actions independently and simultaneously. The well-known Nash equilibrium concept is the most popular one among the definitions of a solution.

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We here recall the definition of the Nash Equilibrium Problem, NEP for short. Let L be the number of player. Each player $l \in \{1, ..., L\}$ controls the variables $x_l \in \Re^n$. Let $x = (x_1, \ldots, x_L)^\top \in \Re^{nL}$ be the vector formed by all these decision variables. To emphasize the l-th player variables within the vector x we write $x = (x_{-l}, x_l)$ where x_{-l} indicates all the other players' variables. Let $f_l : \Re^{nL} \to \Re$ be the l-th player utility function. In the standard NEP, the variable x_l belongs to a nonempty, closed, and convex set $X_l \subseteq \Re^n$, with $l = 1, ..., L$. Let

$$X = X_1 \times X_2 \times ... \times X_L$$

be the Cartesian product of the strategy set of each player. Then a point $x^* \in X$ is a solution of a NEP if the block component x_l^* satisfies

$$f_l(x_{-l}^*, x_l) \leq f_l(x_{-l}^*, x_l^*) \quad \forall x_l \in X_l \tag{1}$$

for all $l = 1, \ldots, L$.

However, many real economic systems may include share constraints that reflect certain technological, social, and environmental restrictions. This may lead to a Generalized Nash Equilibrium Problem, GNEP for short, in which the strategy set of player l depends on the rival players' strategies.

A shared constraint imposes that x belongs to a set $D \subseteq \Re^{nL}$. A point $x^* = (x_1^*, \ldots, x_L^*)^\top \in D$ is said to be a solution of a GNEP, if

$$f_l(x_{-l}^*, x_l) \leq f_l(x_{-l}^*, x_l^*) \quad \forall x_l \in X_l(x_{-l}^*),\ l = 1, \ldots, L; \tag{2}$$

where $X_l(x_{-l}) = \{x_l \in \Re^n | (x_{-l}, x_l) \in D\}$.

In this paper, we consider one of such problems where competitive behaviour of players are restricted by joint emission bounds. This problem can be treated as a non-cooperative game with joint constraints. Starting from Konnov (2014b, 2016), we propose a two-level mechanism for the allocation of the emission share among market players. In particular, these two-level mechanism allows both for defining the emission share assigned to each player and for determining the emission cap imposed at market level. This mechanism further enables us to replace the initial problem with a sequence of Nash equilibrium problems.

Being based on this approach, we introduce a computational method for finding these generalized game equilibrium points which yield solutions of the production problem with a common environmental regulation that imposes limits on greenhouse gas emissions. More precisely, for modeling this common environmental regulation we take as reference the European Union Emission Trading System (EU-ETS) that imposes a joint constraint on carbon emissions generated by specific installations located in Europe. The EU-ETS regulates European carbon emissions since 2005 thanks to Directive 2003/87/EC. Up to now, the EU-ETS has been subdivided into three trading periods or phases. The first trading period (2005-2007) was a "preparatory" phase for the second trading period that began on the 1st of January 2008 and ran for five years until the end of 2012. This second phase coincided with the first commitment period of the Kyoto Protocol. The third phase, regulated by Directive 2009/29/EC, started in 2013 and will end in 2020. The European

Commission is currently revising the EU emissions trading system for introducing a fourth phase after 2020.[5]

The EU-ETS is a "cap and trade" system that imposes a cap on the level of emission allowed and gives the possibility to the participants to buy and sell allowances as they need within the imposed limit. One allowance gives the holder the right to emit one tonne of CO_2. Power sector and energy-intensive industries are involved in this regulation. In the first two phases, Member States were in charge to define the amount of allowances to allocate in total and to each EU-ETS installation on their territory. This was done through the so-called National Allocation Plans (NAPs) that were based on the historical carbon emissions of the involved installations. On the other side, the role of the European Commission was to check and approve, possibly after some modifications, the NAPs proposed by each Member States. The carbon cap imposed at European level was the result of this from double level procedure and was given by the sum of the national NAPs.

This two level organization of the EU-ETS during the first two phases fits very well with our model proposed in the following sections. For this reason, we take inspiration from the organization of the EU-ETS in the first two phases for our case study. More precisely, we assume that the L players are European countries that are currently involved in the EU-ETS and we model an environmental regulation that limits carbon emission both at country and at whole market levels. More precisely, our mechanism allows for defining both the emission share assigned to each country and the cap imposed on the entire market. We further assume that these L countries produce several commodities, represented by electricity, cement, and steel. We consider electricity production since energy sector is the first responsible of CO_2 emissions at European level, followed by cement and steel industrial sectors that are carbon intensive. Notice that, in the first two EU-ETS phases, a large part of allowances have been assigned for free, namely the 95% and 90% of the NAPs respectively in the first and in the second phases. However, this free allowance allocation created several economic distortions, especially for the power sector (see, e.g., Sijm et al., 2006) and thus Directive 2009/29/EC revised the EU-ETS organization for the third phase. More precisely, Directive 2009/29/EC has imposed a unique carbon cap at EU level and has enlarged both the number of sectors subject to emission limitation and the type of greenhouse gas regulated. In addition, in the third phase, power sector is subject to a full allowance auctioning, while industrial sectors that are deemed to be exposed to carbon leakage still receive free allowances.[6] Cement and steel sectors are recognized to be among those sectors exposed to carbon leakage. However, since the discussion on allowance allocation mechanisms and carbon leakage issues are not the objectives of this work, we assume a full auctioning of carbon permits for all involved sectors. In this way, all sectors are considered in a uniform way.

The results of our analysis confirm the important role played by energy sector in reducing carbon emissions.

The rest of the paper is organized in the following way. Section 2 describes the general formulation of the problem; Section 3 presents the share allocation method and its regularization. Section 4 is devoted to the case study, while Section 5 describes the im-

[5]See http://ec.europa.eu/clima/policies/ets/revision/index_en.htm

[6]See http://ec.europa.eu/clima/policies/ets/cap/leakage/index_en.htm

plementation and the results of our analysis. Finally, conclusions are reported in Section 6.

2 General problem formulation

We now describe the basic problem, which can be regarded as some modification and extension of those in Okuguchi and Szidarovszky (1990), Sections 2.1-2.3.

Consider a system of L players involved in a common environmental regulation system, which independently produce n commodities generating m polluted substances.

Let $x_l = (x_{l1}, \dots, x_{ln})^\top$ be the output vector of the l-th player for a fixed time period.

Assume that the l-th player output vector is limited by lower and upper bounds such that its production set is defined by:

$$X_l = \left\{ x_l \in \Re^n \,\middle|\, \underline{b}_l \le x_l \le \bar{b}_l \right\}, \tag{3}$$

where $\underline{b}_l$, $\bar{b}_l \in \Re^n_+$ and $\Re^n_+$ denotes the non-negative orthant in $\Re^n$.

The production output x_l yields the emission vector $y_l = (y_{l1}, \dots, y_{lm})^\top = A_l x_l \in \Re^m$, where A_l is an $m \times n$ matrix and the total pollution volumes within the considered period must be bounded above by the fixed vector $d \in \Re^m$ such that:

$$V = \left\{ x = (x_l)_{l=1,\dots,L} \in \Re^{nL} \,\middle|\, \sum_{i=1}^{L} A_l x_l \le d \right\}. \tag{4}$$

Then, we can define the common feasible set

$$D = X \bigcap V, \tag{5}$$

where

$$X = X_1 \times \cdots \times X_L. \tag{6}$$

Given an output x_l, the l-th player receives the revenue $\mu_l(x)$ and faces the production cost $\eta_l(x)$, where μ_l and η_l are some continuous functions. In addition, the application of the environmental regulation leads to some costs that are represented by the unit vector $p = (p_1, \dots, p_m)^\top \in \Re^m$ that, in its turn, depends on the vector of total pollution:

$$y = \sum_{l=1}^{L} y_l,$$

i.e. $p = p(y)$.

The profit function of the l-th player is then defined as follows:

$$f_l(x) = \mu_l(x) - \eta_l(x) - \sum_{j=1}^{m} p_j(y) y_{lj}, \tag{7}$$

for $l = 1, \dots, L$. The problem consists in determining the unknown output vectors, which also give the corresponding pollution volume. We now place this problem in a somewhat more general setting.

Due to the interdependence of players' pollution charges, the analyzed problem belongs to the class of the oligopolistic competitive models and it reduces to a constrained (generalized) L-person non-cooperative game with the total outcome set D and profit functions f_l for $l = 1, \ldots, L$ and requires the proper extension of the Nash equilibrium point concept. In Okuguchi and Szidarovszky (1990), such problems are called *pseudogames*. A point $x^* = (x_1^*, \ldots, x_L^*)^\top \in D$ is said to be a solution of a *generalized Nash equilibrium problem* (GNEP), if

$$f_l(x_{-l}^*, x_l) \le f_l(x^*) \quad \forall (x_{-l}^*, x_l) \in D, \ l = 1, \ldots, L; \tag{8}$$

where we set $(x_{-l}, z_l) = (x_1, \ldots, x_{l-1}, z_l, x_{l+1}, \ldots, x_L)$. In case $V = X$ we obtain the usual Nash equilibrium problem (NEP).

We now fix our basic assumptions on problem (4)–(8). In what follows, we suppose that *the set D is nonempty, each utility function f_l is continuous, and also concave and differentiable in its l-th variable x_l for $l = 1, \ldots, L$.*

Following Nikaido and Isoda (1955) and Rosen (1965), we can consider the general *equilibrium problem* (EP): Find a point $x^* \in D$ such that

$$\Phi(x^*, x) \ge 0 \quad \forall x \in D, \tag{9}$$

where

$$\Phi(x', x'') = \Psi(x', x'') - \Psi(x', x') \text{ and } \Psi(x', x'') = -\sum_{l=1}^{L} f_l(x'_{-l}, x''_l), \tag{10}$$

$x' = (x'_1, \ldots, x'_L)^\top$ and $x'' = (x''_1, \ldots, x''_L)^\top$. Its solutions are also called *normalized equilibrium points*. From the above assumptions it follows that Φ in (9) is an equilibrium bi-function, i.e., $\Phi(x, x) = 0$ for every $x \in X$, besides, $\Phi(x, \cdot)$ is convex and differentiable for each $x \in X$. It is easy to see that each normalized equilibrium point is a generalized Nash equilibrium point, but the reverse assertion is not true in general.

Besides (see Rosen 1965), under the above assumptions, EP (9)–(10) is equivalent to VI: Find a point $x^* \in D$ such that

$$\sum_{l=1}^{L} \langle G_l(x^*), x_l - x_l^* \rangle \ge 0, \quad \forall x \in D, \tag{11}$$

where

$$G_l(x) = \Phi'_{v_l}(x, v)\big|_{v=x} = -\frac{\partial f_l(x)}{\partial x_l},$$

that is,

$$G_l(x) = \eta'_l(x_l) - \mu'_l(x_l) + A_l^\top p' \left(\sum_{r=1}^{L} A_r x_r \right) A_l x_l + A_l^\top p \left(\sum_{r=1}^{L} A_r x_r \right). \tag{12}$$

Due to the presence of the joint binding constraints the players can not make their choices simultaneously and independently. It seems not so easy to find a suitable regulation mechanism that allows the players to attain an equilibrium state defined in (8) and (9) within the classic non-cooperative game framework.

In fact, there exist a number of iterative solution methods for computation of a solution of EP (9)–(10) or VI (11); see Belen'kii and Volkonskii (1974), Facchinei and Kanzow (2007), Konnov (2007, 2008), Krawczuk and Uryasev (2000), Zukhovitskii et al. (1973) and the references therein. Hence they can be in principle adjusted for finding constrained game equilibria of form (4)–(8). However, they treat the joint constraint set V explicitly, i.e. assume existence of some concordance among players on these constraints and thus contradicting the independence principle.

Next, the streamlined application of the penalty method will consists in inserting a penalty function for the set V, e.g.

$$\tilde{P}(x) = \left\| \left(\sum_{l=1}^{L} A_l x_l - d \right)_+ \right\|^{\sigma}, \ \sigma \geq 1,$$

here $(a)_+$ means the projection of a onto the non-negative orthant $\Re^m_+$. Then, given a number $\tau > 0$, we consider the problem of finding a point $x(\tau) \in X$ such that

$$\Phi(x(\tau), x) + \tau[\tilde{P}(x) - \tilde{P}(x(\tau))] \geq 0 \quad \forall x \in X.$$

However, this EP can not be reduced to a NEP due to the non-separability of the penalty function. If we modify each utility function as

$$\bar{f}_l(x) = f_l(x) - \tau \tilde{P}(x)$$

and remove the set V in (8), we obtain the NEP (see Krawczuk and Uryasev 2000), but then each player will have additional charges after any common violation of the total constraints regardless of individual contributions, which does not seem fair. Similarly, the well-known Lagrangian method (see e.g. Konnov 2014b, Muu and Oettli 1989) also leads to double pollution charges for the players.

Therefore, we need some other flexible control procedures corresponding to the noncooperative game framework.

3 Share allocation method and its regularization

Define the set of partitions of the emission bound vector d:

$$U = \left\{ u \in \Re^{mL} \;\middle|\; \sum_{l=1}^{L} u_l = d \right\}. \tag{13}$$

where $u = (u_1, \dots, u_L)^\top$, $u_l \in \Re^m$ determines the explicit emission share of the l-th player. Given a partition $u \in U$, we can consider the parametric NEP: Find a point $x(u) = (x_1(u), \dots, x_L(u))^\top \in D(u)$ such that

$$f_l(x_{-l}(u), \tilde{x}_l) \leq f_l(x(u)) \quad \forall \tilde{x}_l \in D_l(u_l), l = 1, \dots, L; \tag{14}$$

where

$$D(u) = D_1(u_1) \times \cdots \times D_L(u_L), \tag{15}$$

$$D_l(u_l) = \{x_l \in X_L \mid A_l x_l \leq u_l\},\ l = 1, \dots, L. \tag{16}$$

Hence, an additional upper control level for these shares assignment leads to the desired procedures conforming to the basic information scheme of non-cooperative games. We observe this approach was first applied for separable optimization problems and is known as right-hand side decomposition method; see Kornai and Liptak (1965). Its further extensions for variational inequalities and non-cooperative games were suggested in Konnov (2014a, 2014b, 2016).

Following this approach, we first take the parametric VI: Find a point $x(u) \in D(u)$ such that

$$\sum_{l=1}^{L} \langle G_l(x(u)), \tilde{x}_l - x_l(u) \rangle \geq 0, \quad \forall \tilde{x} \in D(u), \tag{17}$$

which is equivalent to (14) under the assumptions made. If this VI is solvable, by duality (see e.g. Konnov 2007), it becomes equivalent to the primal-dual problem of finding a pair $(x(u), v(u)) \in X \times \Re^{mL}_{+}$ such that

$$\langle G_l(x(u)), \tilde{x}_l - x_l(u) \rangle + \langle v_l(u), A_l(\tilde{x}_l - x_l(u)) \rangle \geq 0 \quad \forall \tilde{x}_l \in X_l, \tag{18}$$

$$\langle u_l - A_l x_l(u), v_l - v_l(u) \rangle \geq 0, \quad \forall v_l \in \Re^{m}_{+},\ l = 1, \dots, L; \tag{19}$$

where $v(u) = (v_1(u), \dots, v_L(u))^{\top}$.

We denote by $T(u)$ the set of all the solution points $-v(u)$ (with the negative sign). Treating $T(u)$ as values of the set-valued mapping T, we can define the master VI: Find a point $u^* \in U$ such that

$$\exists t^* \in T(u^*),\ \langle t^*, u - u^* \rangle \geq 0, \quad \forall u \in U. \tag{20}$$

Solutions of VI (20) yield the optimal pollution shares in the sense of GNEP (8) or EP (9)–(10).

Proposition 3.1 *(see [12], Theorem 4.1) If a point u^* solves VI (20), the corresponding solution $x(u^*)$ in (18)–(19) is a solution of VI (11),(12).*

The Lagrange multipliers $v_l(u)$ in (18)–(19) can be thus treated as validity estimates of the particular constraints $A_l x_l \leq u_l$, $l = 1, \dots, L$. In accordance with this approach, a system regulator assigns share allocation values u_l for players, they determine the corresponding Nash equilibrium point together with their validity share constraint estimates. It follows from Proposition 3.1 that this assignment yields the constrained equilibrium solution if all the estimates coincide. In order to calculate a solution of VI (20), we can apply a suitable iterative process (see Konnov 2014b, 2016). However, the mapping T above is set-valued, may have empty values, and does not possess strengthened monotonicity properties in general. This creates serious computational drawbacks in dealing with VI

(20) directly. For this reason, we replace VI (20) with its approximation having better properties, as suggested in Konnov (2014b, 2016).

We recall that a mapping $Q : W \to E$ is said to be

(a) *monotone*, iff, for all $w', w'' \in W$, we have

$$\langle Q(w') - Q(w''), w' - w'' \rangle \geq 0;$$

(b) *strongly monotone* with constant τ, iff, for all $w', w'' \in W$, we have

$$\langle Q(w') - Q(w''), w' - w'' \rangle \geq \tau \|w' - w''\|^2;$$

(c) *co-coercive* with constant τ, iff, for all $v, w \in W$, we have

$$\langle Q(w') - Q(w''), w' - w'' \rangle \geq \tau \|Q(w') - Q(w'')\|^2.$$

From now on, we shall also suppose that $G : X \to \Re^n$ *in (12) is a continuous monotone mapping.*

Choose a number $\varepsilon \geq 0$. Then, for each $u \in \Re^{mL}$, there exists a unique pair $z^\varepsilon(u) = (x^\varepsilon(u), v^\varepsilon(u)) \in X \times \Re^{mL}_+$ such that

$$\begin{aligned} &\langle G(x^\varepsilon(u)), \tilde{x} - x^\varepsilon(u) \rangle + \varepsilon \langle x^\varepsilon(u), \tilde{x} - x^\varepsilon(u) \rangle \\ &+ \sum_{l=1}^{L} \langle v_l^\varepsilon(u), A_l(\tilde{x}_l - x_l^\varepsilon(u)) \rangle \geq 0 \quad \forall \tilde{x} \in X, \end{aligned} \tag{21}$$

$$\langle u_l - A_l x_l^\varepsilon(u) + \varepsilon v_l^\varepsilon(u), v_l - v_l^\varepsilon(u) \rangle \geq 0, \ \forall v_l \in \Re^m_+, \ l = 1, \dots, L. \tag{22}$$

In fact, this problem represents a regularization of system (18)–(19), which corresponds to $\varepsilon = 0$. Adding the regularization terms yields the strong monotonicity for (21)–(22), which gives the existence and uniqueness of its solution in a standard way.

We can set $F^\varepsilon(u) = -v^\varepsilon(u)$ for $\varepsilon > 0$ and consider it as an approximation of $T(u)$ when $\varepsilon \approx 0$. The mapping F^ε is hence single-valued and defined throughout $\Re^{mL}$.

System (21)–(22) admits a suitable re-formulation. Indeed, (22) is a complementarity problem, and we can write its solution explicitly as

$$v_l^\varepsilon(u) = (1/\varepsilon)(A_l x_l^\varepsilon(u) - u_l)_+, \quad \forall l = 1, \dots, L. \tag{23}$$

The corresponding substitution in (21) leads to the problem of finding $x^\varepsilon(u) \in X$ such that

$$\begin{aligned} &\langle G(x^\varepsilon(u)), \tilde{x} - x^\varepsilon(u) \rangle + \varepsilon \langle x^\varepsilon, \tilde{x} - x^\varepsilon(u) \rangle \\ &+ (1/\varepsilon) \sum_{l=1}^{L} \langle (A_l x_l^\varepsilon(u) - u_l)_+, A_l(\tilde{x}_l - x_l^\varepsilon(u)) \rangle \geq 0 \ \forall \tilde{x} \in X. \end{aligned} \tag{24}$$

However, this is nothing but the auxiliary problem of the decomposable regularized penalty method applied to VI (17). Next, under the above assumptions VI (24) is equivalent to the following EP: Find $x^\varepsilon(u) \in X$ such that

$$\begin{aligned} &\Phi(x^\varepsilon(u), \tilde{x}) + (\varepsilon/2) \sum_{l=1}^{L} \left(\|\tilde{x}_l\|^2 - \|x_l^\varepsilon(u)\|^2 \right) \\ &+ (1/(2\varepsilon)) \sum_{l=1}^{L} \left(\|(A_l \tilde{x}_l - u_l)_+\|^2 - \|(A_l x_l^\varepsilon(u) - u_l)_+\|^2 \right) \geq 0 \ \forall \tilde{x} \in X. \end{aligned} \tag{25}$$

Since the outcome set X is a Cartesian product, EP (25) is clearly equivalent to the NEP: Find $x^{\varepsilon}(u) \in X$ such that

$$f_l^{\varepsilon}(x_{-l}^{\varepsilon}(u), \tilde{x}_l) \leq f_l^{\varepsilon}(x^{\varepsilon}(u)) \quad \forall \tilde{x}_l \in X_l,\ l = 1, \dots, L; \tag{26}$$

where the the l-th player has the utility function

$$f_l^{\varepsilon}(x) = f_l(x) - (\varepsilon/2)\|x_l\|^2 - (1/(2\varepsilon))\|(A_l x_l - u_l)_+\|^2. \tag{27}$$

Therefore, we have obtained the basic equivalence result.

Theorem 3.1 *System (21)–(22) is equivalent to VI (24) or to NEP (26)–(27), where $v^{\varepsilon}(u)$ can be then found from (23).*

In addition, we have the basic approximation property.

Proposition 3.2 *(see Konnov 2016, Theorem 6.1) Suppose that the set $T(u)$ is non-empty at some point $u \in \Re^{ml}$, and that we take any sequence $\{\varepsilon_k\} \searrow 0$. Then, for $z^k = (x^k, v^k)$, $x^k = x^{\varepsilon_k}(u)$, $v^k = v^{\varepsilon_k}(u)$ it holds that*

$$\lim_{k \to \infty} z^k = z_n^*, \tag{28}$$

where z_n^* is the minimal norm solution of system (18)–(19).

In order to indicate a suitable method for finding the optimal pollution shares vector $u^* \in U$, we need an additional property of the mapping $F^{\varepsilon}(u)$.

We suggest to find an approximate solution of set-valued VI (20) via the single-valued VI: Find $u^* \in U$ such that:

$$\langle F^{\varepsilon}(u^*), u - u^* \rangle \geq 0, \quad \forall u \in U, \tag{29}$$

where $\varepsilon > 0$ is small enough. However, it seems better to take the equivalent single-valued equation:

$$u^* \in U,\ \bar{F}^{\varepsilon}(u^*) = \mathbf{0}, \tag{30}$$

where $\pi[g]$ denotes the projection of a point g onto the set

$$U_0 = \left\{ u \in \Re^{mL} \;\middle|\; \sum_{l=1}^{L} u_l = \mathbf{0} \right\},$$

which is given by the explicit formula

$$[\pi(g)]_l = g_l - (1/L) \sum_{r=1}^{L} g_r \quad \forall l = 1, \dots, L.$$

In addition, we give a strengthened monotonicity property of F^{ε}.

Proposition 3.3 *For any fixed $\varepsilon > 0$, F^{ε} is co-coercive with constant ε.*

Proof. Take arbitrary points $u', u'' \in \Re^{mL}$. Then there exist the corresponding unique solutions (x', u') and (x'', u'') of (21)–(22). It follows from (21) that

$$\langle G(x'), x'' - x' \rangle + \varepsilon \langle x', x'' - x' \rangle + \sum_{l=1}^{L} \langle v'_l, A_l(x''_l - x'_l) \rangle \geq 0,$$

$$\langle G(x''), x' - x'' \rangle + \varepsilon \langle x'', x' - x'' \rangle + \sum_{i=1}^{L} \langle v''_l, A_l(x'_l - x''_l) \rangle \geq 0;$$

hence

$$\begin{aligned} &\sum_{l=1}^{L} \langle v''_l - v'_l, A_l(x'_l - x''_l) \rangle \\ &\geq \langle G(x') - G(x''), x' - x'' \rangle + \varepsilon \|x'' - x'\|^2 \geq \varepsilon \|x'' - x'\|^2 \end{aligned}$$

since G is monotone. At the same time, (22) gives

$$\langle A_l x'_l - u'_l - \varepsilon v'_l, v'_l - v''_l \rangle \geq 0 \ \text{ and } \langle A_l x''_l - u''_l - \varepsilon v''_l, v''_l - v'_l \rangle \geq 0;$$

hence,

$$\langle A_l(x'_l - x''_l), v'_l - v''_l \rangle \geq \langle u'_l - u''_l, v'_l - v''_l \rangle + \varepsilon \|v''_l - v'_l\|^2$$

for each $l = 1, \dots, L$. Adding these inequalities and combining with the above, we obtain

$$\sum_{l=1}^{L} \langle v'_l - v''_l, u''_l - u'_l \rangle \geq \varepsilon (\|x'' - x'\|^2 + \|v'' - v'\|^2)$$

therefore,

$$\langle u'' - u', F^{\varepsilon}(u'') - F^{\varepsilon}(u') \rangle \geq \varepsilon \|v'' - v'\|^2 = \varepsilon \|F^{\varepsilon}(u'') - F^{\varepsilon}(u')\|^2. \quad \spadesuit$$

It differs from the similar results in Konnov (2014b, 2016) because we deduce co-coercivity of F^{ε} from monotonicity of G in (12), rather than from that bi-function Φ in (9). Indeed the monotonicity of the bi-functions is more restrictive than that of the corresponding pseudo-gradient mappings and may not hold for some oligopolistic game problems.

We can now apply a number of iterative methods for problem (30), see e.g. Konnov (2007). For instance, the simplest projection method, starting from a point $u^0 \in U$, generates a sequence

$$u^{k+1} = u^k - \alpha_k \pi[F^{\varepsilon}(u^k)] \tag{31}$$

with $\alpha_k > 0$. We can tune the value ε during the computation process.

This solution process has a rather simple interpretation. The system regulator assigns sequentially share allocation values. After calculation of the current iterate u^k, the regulator reports the particular share allocation values to the players. They solve the usual game problem (26)–(27) and determine the corresponding validity share constraint estimates $v^{\varepsilon}(u^k)$ in (23). The system regulator receives these values and changes the share assignment with (31), and so on. This procedure seems rather natural and corresponds to the basic noncooperative game framework. Hence, it can be applied to solve the competitive industrial production problem with upper bounds on pollution volumes.

In order to illustrate the applicability of the method, we consider the affine case, i.e. suppose that all the functions μ_l and η_l and the charge mapping p are affine. Then, $\mu_l(x_l) = \langle a'_l, x_l\rangle + \alpha'_l$, $\eta_l(x_l) = \langle a''_l, x_l\rangle + \alpha''_l$ for $l = 1, \dots, L$, and $p(y) = Py + q$, where P is an $m \times m$ matrix and q is a fixed vector in $\Re^m$. It follows from (12) that

$$\begin{aligned} G_l(x) &= a''_l - a'_l + A_l^\top P^\top A_l x_l + A_l^\top \left(\sum_{r=1}^{L} P A_r x_r + q \right) \\ &= a_l + A_l^\top P^\top A_l x_l + \sum_{r=1}^{L} A_l^\top P A_r x_r. \end{aligned}$$

where $a_l = a''_l - a'_l + A_l^\top q$. It is natural to suppose that the matrix P is positive semi-definite, but non symmetric in general.

Take arbitrary points $x', x'' \in \Re^n$. Then

$$\begin{aligned} &\langle G(x') - G(x''), x' - x''\rangle = \sum_{l=1}^{L} \langle G_l(x') - G_l(x''), x'_l - x''_l\rangle \\ &= \sum_{l=1}^{L} \langle A_l^\top P^\top A_l(x'_l - x''_l), x'_l - x''_l\rangle + \sum_{l=1}^{L} \left\langle \sum_{r=1}^{L} A_l^\top P A_r (x'_r - x''_r), x'_l - x''_l \right\rangle \\ &= \sum_{l=1}^{L} \langle P^\top h_l, h_l\rangle + \sum_{l=1}^{L}\sum_{r=1}^{L} \langle P h_r, h_l\rangle = \langle Qh, h\rangle; \end{aligned}$$

where we set $h_l = A_l(x'_l - x''_l)$, $l = 1, \dots, L$, $h = (h_1, \dots, h_L)^\top$ and

$$Q = \begin{pmatrix} P + P^\top & P & \dots & P \\ P & P + P^\top & \dots & P \\ \dots & \dots & \dots & \dots \\ P & P & \dots & P + P^\top \end{pmatrix}.$$

Since

$$(Q + Q^\top) = \begin{pmatrix} 2 & 1 & \dots & 1 \\ 1 & 2 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 1 & 1 & \dots & 2 \end{pmatrix} \otimes (P + P^\top),$$

where $\otimes$ denotes the Kronecker product of matrices, the eigenvalues of $(Q + Q^\top)$ coincide with all the products of the eigenvalues of both the factors; see Lemma 4.1.1 in Okuguchi and Szidarovszky (1990). Hence the matrix Q is positive semi-definite and G is monotone, as desired. We conclude that all the results obtained remain true for this affine case.

4 Case Study

4.1 Problem formulation and environmental regulation

The two level organization of the EU-ETS during the first two phases fits very well with our model proposed in the previous sections. For this reason, we take inspiration from the organization of the EU-ETS in the first two phases for our case study and we assume that the L players are European countries that are currently involved in the EU-ETS. More precisely, we model an environmental regulation that limits carbon emission both at country and at whole market levels. These L countries produce electricity, cement, and steel. We consider electricity production since energy sector is the first responsible of CO_2 emissions at European level, followed by cement and steel industrial sectors that are carbon intensive. Moreover, for the reasons explained in the Introduction, we assume full auctioning of allowances.

For many years, electricity production has been based on fossil fuels and nuclear power plants where available. The application of environmental policies, as the 20-20-20 targets and the Energy Roadmap 2050 (see European Commission 2009, 2011), aiming at decarbonizing the European electricity sector, has led to an increase of the utilization of clean Renewable Energy Sources (RES), like wind and solar energy, and to a consequent reduction of dirty fossil fuels for the production of electricity. In order to account for this renewed mix of energy sources used for the generation of electrical energy, in our case study we consider the electricity produced by RES, hydroelectric, nuclear, and fossil fuel-based power plants. This classification of electricity production per source is important because it allows us to attribute the specific emission factor and variable production costs associated to each of these sources that are quite different among each other.

Cement manufacturing process is high carbon intensive. Its production is based on three main stages: first, limestone and other raw materials are extracted and milled; these are then heated in kilns to produce clinker and, finally, clinker is cooled and milled with other additives to obtain cement. Depending on the kiln used to produce clinker, the cement production process takes the name of wet, semi-wet, dry or semi-dry. Whatever the process adopted, clinker manufacturing releases to air nitrogen oxides (NO_x), sulphur dioxide (SO_2) and carbon dioxide (CO_2). CO_2 emissions are the by-products of the chemical conversion process used in the production of clinker that transforms the limestone ($CaCO_3$) into lime (CaO). In addition, the thermal energy that these chemical reactions need is usually produced by burning carbon intensive fuels, like coal and pet-coke, that enhance carbon emissions. While CO_2 emissions coming from fuel burning can be reduced with the utilization of alternative fuels, those generated by the conversion of limestone into lime are unavoidable. For this reason, the cement production is classified as carbon intensive.

Crude steel can be produced through two routes: the first one is the integrated steelmaking route that is based on the Blast Furnace (BF) ironmaking and Basic Oxygen Furnace (BOF) and uses raw material including iron ore, coal and coke, limestone and dolomite. The second one, known as recycling route, produces steel in the Electric Arc Furnace (EAF) using primarily ferrous scrap as main raw material. In the integrated route, the production of crude steel starts from the reduction of iron ore into pig iron with the addition of reducing agents and fluxes in the BF. The main reducing agent used is coke, which is produced from coal, while fluxes involved are mainly limestone and dolomite. Iron oxides, coke, coal and fluxes react with the heated blast hair injected on the bottom of the furnace to form pig iron, carbon monoxide (CO), and slag which are periodically removed from the furnace. Then, the remaining carbon in the pig iron is oxidized in the BOF by injecting a high-purity oxygen to remove carbon as CO and CO_2. On the contrary, the EAF route uses scrap as main input and melts it with electricity. The different production mechanisms make the BOF process much more pollutant than the EAF-based one. These two different production processes are characterized by very different emission factors and for this reason we take them separated.

Taking into account the European Commission decarbonization policies applied to electricity sector and the technological aspect characterizing the cement and steel industrial processes, in our analysis we consider seven commodities ($n = 7$) that are represented by electricity generation, subdivided into RES, hydroelectric, nuclear and fossil fuel power, cement, and steel production that is further partitioned into BOF and EAF. In the following we denote these commodities as "RES", "hydro", "nuclear", "fossil", "cement", "BOF", and "EAF".

Considering the market players that, in our case, are represented by countries, we select Germany, France, Italy, and Spain ($l = 1, ..., 4$) because these are the largest producers of these commodities in Europe. According to Eurostat,[7] in the years 2012-2014 about the 55% of the EU-28 energy production has been covered by these four countries. We have similar trend also for cement and steel productions. On the basis of the analysis conducted by Cembureau,[8], the quota of cement production covered by these countries over the EU-28 was around 55% in the same period. The crude steel production is on the same line. As indicated by the Worldsteel Association (2014), the 58% of the 2012-2013 crude steel production in EU-27 has been covered by these four countries.

We consider CO_2 emissions as the common pollutant of all these production processes and we thus set $m = 1$. In this case study, $A_l x_l$ is the total amount of emission generated by the each country l by producing the commodities $x_l \in R^7$ and u_l indicates the national emission share, namely the maximum quantity of emissions that each country l can generate. Making a parallelism with the EU-ETS, these u_l corresponds to the national NAPs. This implies that each country l takes its own production decision with the aim of maximizing the profits (7) while taking into account the capacity constraints indicated by condition (3) and the national carbon emission limit (16). This means that each country l has to decide the level of production of the seven commodities such that the total emissions $A_l x_l$ generated do not exceed the limit u_l imposed at national level. In addition, as applied in the first two phase of the EU-ETS regulation, the sum of the national emission share u_l is equal to the CO_2 cap d imposed on the entire market that, in our problem formulation, includes Germany, France, Italy, and Spain. The relation between the national allowance shares u_l and the market cap d is defined by condition (13). In the following section, we describe the dataset used for our analysis.

[7]See http://ec.europa.eu/eurostat/web/energy/data/main-tables
[8]See http://www.cembureau.be/activity-reports

Table 1: Values of the upper bounds $\overline{b}_l$ for the year 2012

	Germany	**France**	**Italy**	**Spain**	**Unit of Measure**
RES	93,788	27,055	50,692	67,829	GW
Hydro	24,201	66,720	57,125	50,681	GW
Nuclear	94,986	442,415	-	59,776	GW
Fossil	482,003	204,504	546,792	355,207	GW
Cement	36,865	26,700	49,030	50,000	kton
EAF	19,040	6,196	17,500	15,900	kton
BOF	36,700	14,500	14,500	8,000	kton

4.2 Dataset

In our analysis, we take as reference year 2012 that is the last year of application of the second EU-ETS phase.

For each country, we first set the lower and the upper bounds on the quantity of commodity produced, namely $\underline{b}_l$ and $\overline{b}_l$. Notice that the amount of commodities x_l produced refers to the entire year. Table 1 reports the upper bounds $\overline{b}_l$ on the annual production of the different commodities. These upper bounds correspond to the annual capacities of all installations used to produce commodities x_l in a selected country. For the upper bounds on electricity production, namely RES, hydro, nuclear and fossil, we evaluate the capacity on the basis of the data reported in the ENTSO-E (2012) and taking into account the technology availability over the year. Recall that there are no nuclear power plants in Italy. Annual cement capacity data are taken from the reports of the national cement associations that are Vdz,[9] Infociments,[10] Aitec,[11] and Oficemen[12] respectively for Germany, France, Italy, and Spain. Finally, the annual capacity data for BOF and EAF crude steel production are available in Worldsteel Association (2014). On the other side, the values of parameter $\underline{b}_l$ have been set equal to 10% of the corresponding values of the upper bound $\overline{b}_l$.

Table 2: Emission factors A_l

	Germany	**France**	**Italy**	**Spain**	**Unit of Measure**
RES	0	0	0	0	ton CO_2/MWh
Hydro	0	0	0	0	ton CO_2/MWh
Nuclear	0	0	-	0	ton CO_2/MWh
Fossil	0.766	0.705	0.639	0.586	ton CO_2/MWh
Cement	0.586	0.643	0.634	0.664	ton CO_2/ton
EAF	0.283	0.283	0.283	0.283	ton CO_2/ton
BOF	1.328	1.328	1.328	1.328	ton CO_2/ton

Table 2 reports the CO_2 emission factors A_l associated to the considered commodities. RES and hydroelectric power plants are clean technologies and do not cause carbon emissions. Nuclear power plants do not generate CO_2 emissions as well. On the other side, electricity

[9]See https://www.vdz-online.de/en/
[10]See http://www.infociments.fr/
[11]See http://www.aitecweb.com/
[12]See https://www.oficemen.com/default.asp?id_cat=10

production from conventional plants is very pollutant. The corresponding emission factors reported in Table 2 differ per country because they are estimated on the basis of the conventional fuel mix used to produce electricity at national level (see ENTSO-E, 2012). This fuel mix accounts for coal, natural gas and oil based power plants. The emission factors for cement are taken from the GNR dataset that is available on the World Business Council for Sustainable Development website.[13] The EAF process is less carbon intensive than the BOF as explained in Riccardi et al. (2015).

Concerning the revenue $\mu_l(x)$ and the production cost $\eta_l(x)$ in condition (7), we assume that are linear functions where the selling price and production costs are fixed. These are respectively reported in Table 3 and Table 4.

Electricity selling prices are taken from Eurostat.[14] Notice that these prices differ per country but not per fuel since electricity is a homogenous good and when it is sold it does not matter which source has been used to produce it. Cement prices are taken from Armstrong (2012), while we estimate the crude steel prices starting from the data reported in Riccardi et al. (2015).

Table 3: Selling price of commodities x_l

	Germany	**France**	**Italy**	**Spain**	**Unit of Measure**
RES	130	79	178	120	euro/MWh
Hydro	130	79	178	120	euro/MWh
Nuclear	130	79	-	120	euro/MWh
Fossil	130	79	178	120	euro/MWh
Cement	67	95	64	64	euro/ton
EAF	580	570	545	575	euro/ton
BOF	580	570	545	575	euro/ton

Table 4: Production costs of commodities x_l

	Germany	**France**	**Italy**	**Spain**	**Unit of Measure**
RES	0	0	0	0	euro/MWh
Hydro	0	0	0	0	euro/MWh
Nuclear	5.00	5.00	-	5.00	euro/MWh
Fossil	50.79	68.81	62.57	42.57	euro/MWh
Cement	31.60	31.60	38.80	31.60	euro/ton
EAF	256.80	232.80	224.80	274.80	euro/ton
BOF	435.19	411.19	403.19	453.19	euro/ton

The values reported in Table 4 are variable production costs associated to the different commodities. Fixed costs are not considered. The electricity production costs is mainly affected by fuel costs. Since RES and hydro are freely available natural sources, the associated variable production costs are 0 euro/MWh. The variable costs of the nuclear power production is based on the uranium costs, while those associated to electricity generated by fossil fuels have been estimated taking into account the conventional fuel mix used in the countries. For the price of uranium, coal, natural gas and oil we take as reference the

[13]See http://www.wbcsdcement.org/GNR-2013/index.html

[14]See http://ec.europa.eu/eurostat/statistics-explained/index.php/Energy_price_statistics

report by NREL (2015). The cement and crude steel production costs are respectively taken from Allevi et al. (2016) and Riccardi et al. (2015).

5 Implementation and results

In this section we present the details of the implementation and the results of the numerical experiments.

5.1 Implementation

In order to find the optimal emission shares vector $u^* \in U$, we use the projection method generating a sequence as in (31) with a starting a point $u^0 \in U$ and tuning the value α during the computation process.
To solve the NEP (26)–(27) equivalent to the auxiliary problem EP (25), we used the following Nikaido-Isoda-function:

$$\Psi_\varepsilon(x, \tilde{x}) = \sum_{l=1}^{L} [f_l^\varepsilon(x_{-l}, \tilde{x}_l) - f_l^\varepsilon(x)] \tag{32}$$

where $\tilde{x}_l \in X_l, x \in X$.
In our implementation we find the solution of NEP (26)–(27) solving the problem

$$\min_{x \in X} V_\varepsilon(x) \tag{33}$$

where $V_\varepsilon(x) = \max_{\tilde{x} \in X} \Psi_\varepsilon(x, \tilde{x})$.

All experiments were carried out on an CPU Intel Core i7 2.66 GHz; Memory 8 GB; OS Software Matlab 7.14.0.739 (R2012a). In our implementation we find the solution of problem (33) by using the Matlab procedure to minimize a constrained non differentiable multivariable function with genetic algorithm. The stopping criterion for the main iteration on k is $||(x_k - x_{k-1})|| < 10^{-3}$.
The results are obtained with $\epsilon = (1/2)^k$ in each main iteration k.
In all the experiments the number the maximum number of k in main iterations is 10.

More precisely our experiments aim at evaluating the effects of a progressively stringent environmental regulation on production and emissions. For this reason, we conduct a sensitivity analysis on the national emission share and the market cap levels. For this reason, we consider different starting points for the algorithm where u_l, d, and the carbon price $p(y)$ have been fixed as indicated in Table 5. We first consider a reference case that we denote as *Case 0*, where there is no environmental regulation. This means that carbon price is 0 euro/ton and there are no limits on the CO_2 emissions generated both at national and European levels. *Case 1* considers the environmental regulation that imposes limit both at national and at market levels. The national emission shares u_l of *Case 1* reported in Table 5 (from the third to the sixth column) have been estimated using the NAPs imposed in 2012 by the EU-ETS on the emissions generated by the electricity, cement, and steel sectors in the four considered countries.[15] The value of market cap d in the last column of Table 5 is equal to the sum of the national emission shares. We set the CO_2 price at 10 euro/ton.[16]

The national emission share u_l and consequently the market cap d are proportionally reduced in *Case 2* and *Case 3* compared to *Case 1*. The imposition of more restrictive emission limits leads to an increase of the emission prices. For this reason, we assume that *Case 2* and *Case 3* are respectively equal to 20 and 30 euro/ton CO_2.

Table 5: Considered cases

	p(y) euro/ton CO_2	**u_l Germany ton CO_2**	**u_l France ton CO_2**	**u_l Italy ton CO_2**	**u_l Spain ton CO_2**	**d ton CO_2**
Case 0	0	-	-	-	-	-
Case 1	10	377,633,216	79,265,248	160,845,358	108,175,718	725,919,540
Case 2	20	283,224,912	59,448,936	120,634,019	81,131,789	544,439,655
Case 3	30	188,816,608	39,632,624	80,422,679	54,087,859	362,959,770

5.2 Results

We conduct our analysis on the basis of the *Cases* indicated in Section 4.2. Our two-level mechanism allows for determining the emission share of each country and the whole emission cap imposed on the considered market. Table 6 reports the national emission share and the market cap that are obtained from the implementation of our algorithm in *Cases 1-3*. These results show that, in all *Cases*, the market caps d are as indicated in Table 5, while the national emission shares u_l are different. In other words, the the national emission shares are re-distributed among the four countries in a such a way that the an higher share in a country is compensated by a lower share in an other country.

Table 6: National emission share u_l and market cap d in *Cases 1-3*

	p(y) euro/ton CO_2	**u_l Germany ton CO_2**	**u_l France ton CO_2**	**u_l Italy ton CO_2**	**u_l Spain ton CO_2**	**d ton CO_2**
Case 1	10	413,130,552	59,257,449	156,905,340	96,626,199	725,919,540
Case 2	20	285,863,051	48,329,236	124,748,119	85,499,249	544,439,655
Case 3	30	195,617,714	27,978,805	87,913,351	51,449,899	362,959,770

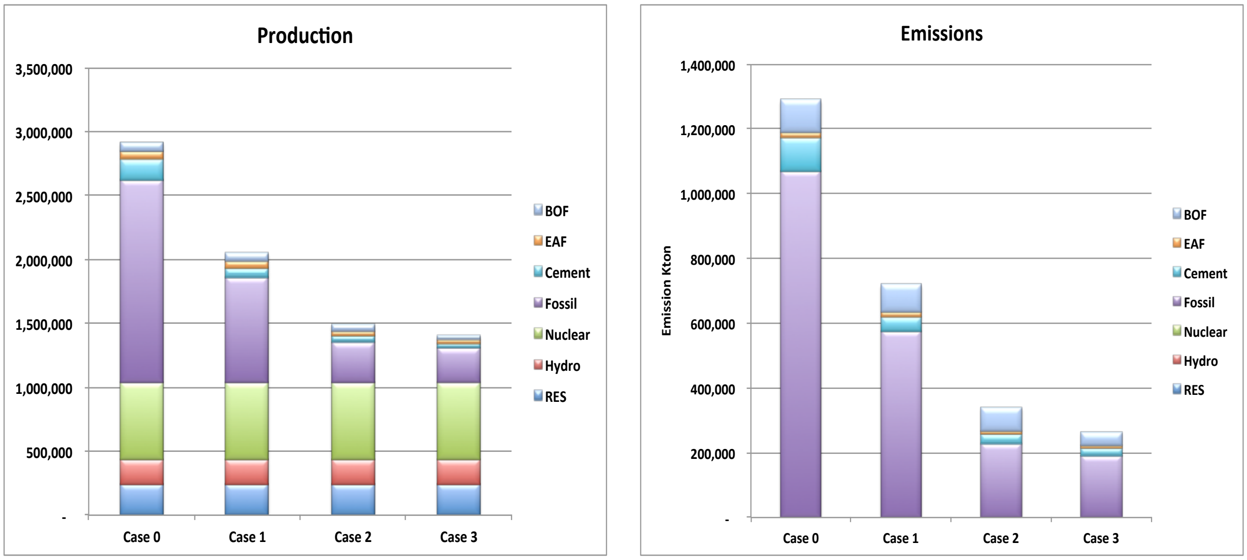
We first evaluate the effect of the application of the carbon regulation on CO_2 emissions, commodity production, and profits.

Figure 1 compares the commodity production and the global amount of emission generated in the considered *Cases*.

In *Case 0*, the production of all commodities is equal to the upper bounds $\bar{b}_l$ (see Figure 1a) as defined in Table 1. The application of the carbon regulation in *Cases 1-3* imposes a reduction of the emissions generated by production activities. The imposition

[15]The NAPs of the second EU-ETS phase are available at http://ec.europa.eu/clima/policies/ets/registry/documentation_en.htm

[16]See https://www.eex.com/en/



(a) (b)

Figure 1: Global production of the n commodities and related emission in *Cases 0-3*

of the environmental regulation reduces the CO_2 emissions but, at the same time, implies a reduction of the global amount of commodity produced. The reduction of both emission and total production is higher in *Case 2* and *Case 3* when the regulation becomes more stringent. As highlighted by Figure 1b, in all *Cases*, the main responsible of carbon emissions in the considered market is electricity generation from fossil fuel, followed by cement, BOF and EAF crude steel productions. For this reason, the environmental targets are achieved through significant curtailment of the amount of electricity produced with dirty fossil fuels and progressive cuts of the cement and crude steel productions. More precisely, the reductions of fossil fuel electricity production in *Cases 1, 2, 3* compared to in *Case 0* are respectively equal to -48%, -80%, and -83%. For cement production, decreases are respectively equal to equal to -56%, -71%, and -76%; for EAF crude steel they are equal to 0%, -33%, and -42%; finally for BOF crude steel they are equal to -15%, -27%, and -60%.[17] On the contrary the production of electricity using RES and hydroelectric power plants remains at maximum level because it is cheap and, most of all, does not generate CO_2 emissions. The same holds for electricity production from nuclear plants.

Table 7 is devoted to profit analysis. In this Table, we compare the country and the total profits in the *Cases 0-3*. The highest profit level is achieved in *Case 0* when environmental regulation is not applied. The introduction of the environmental regulation has two effects: the first one is represented by the reduction of the revenues due to the fall of commodity production and the second is the rise of CO_2 costs. The combination of these two effects leads to a reduction of the national and total profits. The more stringent

[17]Notice that EAF crude steel production has the lowest carbon emission factors among the considered pollutant production processes (see Table 2). This justifies the fact that EAF crude steel has the lowest production drops.

Table 7: Profit analysis

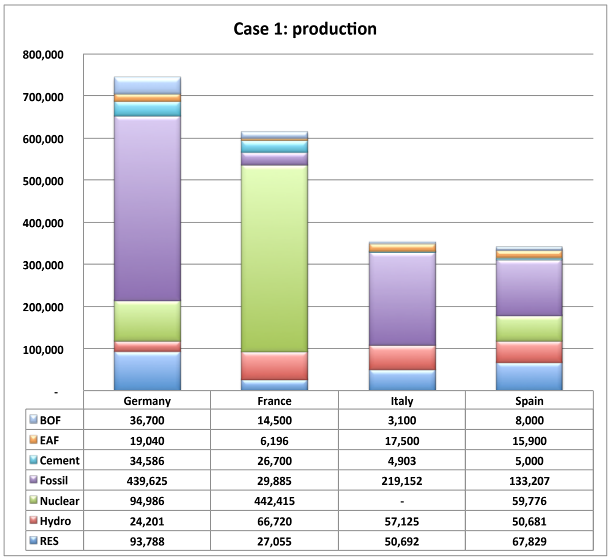
	Keuro	Germany	France	Italy	Spain	Total profits
Case 0	**Revenues**	122,367,545	69,467,636	132,146,896	77,303,340	401,285,417
	Production costs	46,979,935	24,343,492	45,894,339	24,989,741	142,207,506
	CO_2 costs	-	-	-	-	-
	Profits	**75,387,610**	**45,124,144**	**86,252,557**	**52,313,599**	**259,077,911**
Case 1	**Revenues**	119,484,529	59,053,177	69,741,365	51,441,660	299,720,731
	Production costs	44,757,478	12,516,684	19,086,073	14,122,762	90,482,997
	CO_2 costs	4,111,465	592,468	1,522,161	965,030	7,191,123
	Profits	**70,615,586**	**45,944,024**	**49,133,132**	**36,353,868**	**202,046,610**
Case 2	**Revenues**	88,570,162	45,407,914	47,589,389	27,957,045	209,524,509
	Production costs	32,678,478	4,443,974	13,721,200	3,058,057	53,901,709
	CO_2 costs	4,579,164	364,706	1,315,172	533,573	6,792,614
	Profits	**51,312,521**	**40,599,234**	**32,553,017**	**24,365,415**	**148,830,187**
Case 3	**Revenues**	71,930,409	45,407,914	33,627,374	35,579,304	186,545,000
	Production costs	23,545,454	4,443,974	5,758,127	6,700,921	40,448,476
	CO_2 costs	5,005,729	547,059	1,440,579	890,917	7,884,284
	Profits	**43,379,226**	**40,416,881**	**26,428,668**	**27,987,467**	**138,212,241**

is the environmental regulation and the higher is the profit drop.

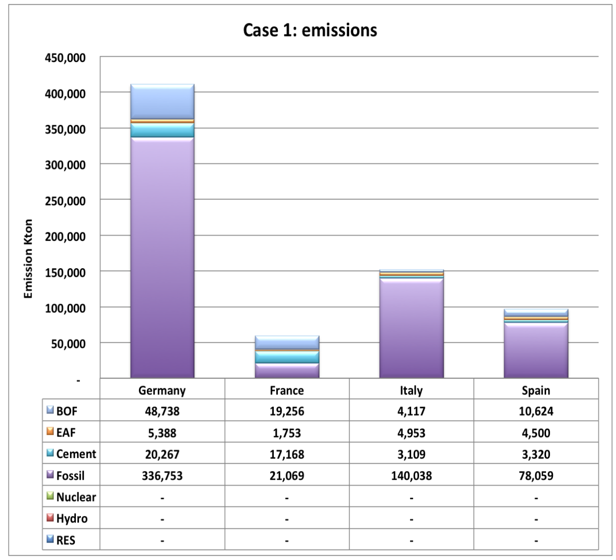
Figure 2 illustrates in details the production and the total emissions in each country in *Case 1*. Similar trends are also registered in *Case 2* and *Case 3*.

In all countries, with the exception of France, carbon emissions are mainly due to electricity production. This is also highlighted by Figures 3a, 3b, 3c, and 3d that indicate how the different commodities contribute to the generation of carbon emissions. In particular, these Figures compare the quota of emissions caused by electricity production with those due to energy-intensive industrial sectors. The emissions generated by industrial sectors are further partitioned among emissions from cement, EAF, and BOF production. In Germany, in Italy, and in Spain the electricity production is respectively responsible of the 82%, 92% and 81% of the national CO_2 emissions. In France, this proportion drops to 36%. This is a direct consequence of the fuel mix used to produce electricity in the four countries. Our results show that the French electricity production is based for the 78% on nuclear power plants that do not emit CO_2 and only the 5% is covered by dirty conventional units that burn fossil fuels (see Figure 2a). The remaining part is produced by clean RES and hydroelectric plants that are run at full capacity since do not emit CO_2. Also in the other three countries, the capacity of RES and hydroelectric power plants is fully employed, but the highest quota of electricity is generated with highly pollutant fossil fuels. Note that in the last years, Germany has strongly boosted the production of electricity using RES-based plants. This trend is also highlighted by our results, which show that the amount of electricity produced by RES in Germany is the highest among the four countries. This incentive in investing in RES-based plants is also registered in the other country, but conventional power plants based on fossil fuels are still needed to cover electricity demand.

Considering the industrial sectors, the production of crude steel using the BOF route is highly pollutant compared to the other industrial production processes as highlighted in Table 2. This justifies the fact that the BOF crude steel is responsible of the 12%, 32%, 3%, and 11% of the total carbon emissions respectively in Germany, France, Italy, and

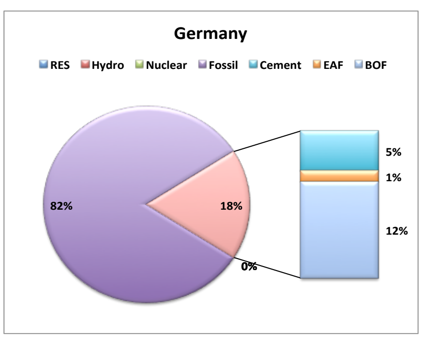


(a)

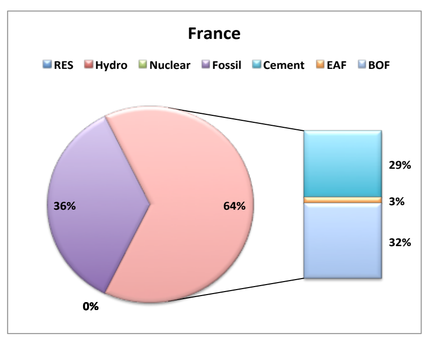


(b)

Figure 2: Production of the n commodities per country l and related emission in *Case 1*



(a)



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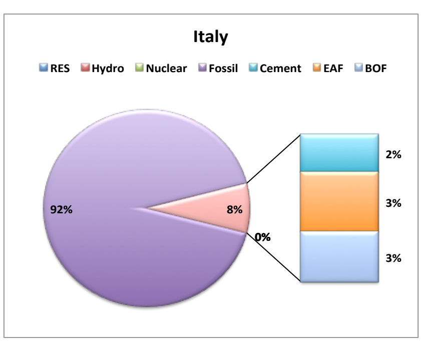

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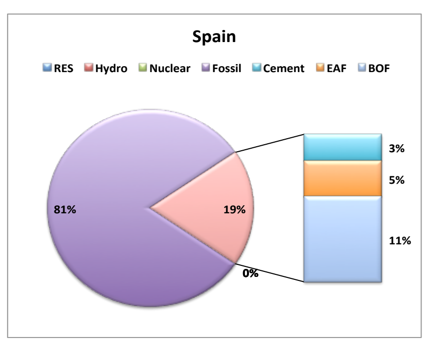

(d)

Figure 3: Carbon emission proportions per country l and commodity n

Spain even though the corresponding production levels are lower than those of cement and crude steel from EAF process. To sum up, our results confirm the important role that energy sector plays in tackling climate change and abating carbon emissions. A full decarbonization of the electricity production through investments in RES based technologies, as required by the European Commission, could significantly reduce carbon emissions.

6 Conclusions

In this paper, we consider a generalized non-cooperative game where players are restricted by joint emission bounds and we propose a two-level mechanism for allocating emission shares. This enables us to replace the initial problem with a sequence of Nash equilibrium problems. Being based on this approach, we now develop a computational method for finding these generalized game equilibrium points which yield solutions of the production problem with common pollution regulation.

More precisely, we introduce an environmental regulation applied to emissions generated by electricity, cement and steel sectors taking inspiration from the EU-ETS scheme that is currently applied in Europe. We select Germany, France, Italy, and Spain as market players because these are the largest producers of these commodities in Europe. Our analysis, conducted by the application of a regularized penalty method, shows the importance played by electricity sector in reducing carbon emissions. Except from France where electricity is mainly produced using nuclear power plants, energy production is the main sources of CO_2 emissions in all considered countries and carbon emission cut is achieved by reducing the use of fossil fuel power plants and enhancing the utilization of RES and hydroelectric technologies. These results are in line with the 20-20-20 targets and the decarbonization policies implemented by the European Commission.

References

[1] Allevi, E., Oggioni, G., Riccardi, R. & Rocco, M. (2016). Evaluating the carbon leakage effect on cement sector under different climate policies. *Journal of Cleaner Production*, DOI: 10.1016/j.jclepro.2015.12.072.

[2] Armstrong T. (2012). The cement industry in figures. An overview of global cement sector trends. Available at http://www.ficem.org/pres/THOMAS-ARMSTRONG-LA-INDUSTRIA-DEL-CEMENTO-EN-CIFRAS.pdf

[3] Belen'kii, V.Z. & Volkonskii, V.A. (eds) (1974). *Iterative Methods in Game Theory and Programming.* Moscow: Nauka, (in Russian).

[4] ENTSO-E. Memo (2012). Available at https://www.entsoe.eu/publications/statistics/statistical-factsheet/Pages/default.aspx

[5] European Commmision (2009). Directive 2009/28/EC of the European Parliament and of the Council of 23 April 2009 on the Promotion of the Use of

Energy from Renewable Sources and Amending and Subsequently Repealing Directives 2001/77/EC and 2003/30/EC. Available at http://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32009L0028& from=EN

[6] European Commmision (2011). Energy Roadmap 2050. Available at http://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:52011DC0885& from=EN.

[7] Facchinei, F. & Kanzow, C. (2007). Generalized Nash Equilibrium Problems. *4OR*, 5(3), 173-210.

[8] Konnov, I.V. (2007). *Equilibrium Models and Variational Inequalities*. Amsterdam: Elsevier.

[9] Konnov, I.V. (2008). Iterative Solution Methods for Mixed Equilibrium Problems and Variational Inequalities with Non-smooth Functions. In: Haugen, I.N. & Nilsen, A.S. (eds) *Game Theory: Strategies, Equilibria, and Theorems*, Ch. 4. Hauppauge: NOVA, 117-160.

[10] Konnov, I.V. (2014a). Right-hand Side Decomposition for Variational Inequalities. *Journal of Optimization Theory and Applications*, 160(1), 221-238.

[11] Konnov, I.V. (2014b). Justification of Constrained Game Equilibrium Models. *Optimization Online*. Available at http://www.optimization-online.org/DB_HTML/2014/07/4439.html

[12] Konnov, I.V. (2016). Share Allocation Methods for Generalized Game Problems with Joint Constraints. *Set Valued and Variational Analysis*. DOI 10.1007/s11228-015-0347-2.

[13] Kornai, J. & Liptak, T. (1965). Two-level Planning. *Econometrica*, 33(1),141-169.

[14] Krawczuk, J.B. & Uryasev, S. (2000). Relaxation Algorithms to Find Nash Equilibria with Economic Applications, *Environmental Modeling and Assessment*, 5(1), 63-73.

[15] Ichiisi, I. (1983). *Game Theory for Economic Analysis*, New York: Academic Press.

[16] Muu, L.D., Oettli, W. (1989). A Lagrangian Penalty Function Method for Monotone Variational Inequalities. *Numerical Functional Analysis and Optimization*, 10(9-10), 1003-1017.

[17] NREL (2015). 2015 Standard Scenarios Annual Report: U.S. Electric Sector Scenario Exploration. Available at http://www.nrel.gov/docs/fy15osti/64072.pdf

[18] Nikaido, H. & Isoda, K. (1955). Note on Noncooperative Convex Games. *Pacific Journal of Mathematics*, 5(1), 807-815.

[19] Okuguchi, K. & Szidarovszky, F. (1990). *The Theory of Oligopoly with Multi-product Firms*, Berlin: Springer-Verlag.

[20] Owen, G. (1995). *Game theory*. 3rd Ed. San Diego: Academic Press.

[21] Riccardi, R., Bonenti F., Allevi, E., Avanzi, C., & Gnudi, A. (2015). The steel industry: A mathematical model under environmental regulations, *European Journal of Operational Research*, **242**, 1017-1027.

[22] Rosen, J.B. (1965). Existence and Uniqueness of Equilibrium Points for Concave N-person Games. *Econometrica*, 33(3), 520-533.

[23] Sijm J., Neuhoff, K. & Chen Y. (2006). CO2 cost pass-through and windfall profits in the power sector, *Climate Policy*, 6(1), 49-72.

[24] Worldsteel Association (2014). Steel Statistical Yearbook 2014. Available at https://www.worldsteel.org/dms/internetDocumentList/statistics-archive/yearbook-archive/Steel-Statistical-Yearbook-2014/document/Steel-Statistical-Yearbook-2014.pdf.

[25] Zukhovitskii, S.I., Polyak, R.A. & Primak, M.E. (1973). Concave Multi-person Games: Numerical Methods. *Matekon*, 7(1), 11-30.