



Scarcity and panic buying: the effect of regulation by subsidizing the supply and customer purchases during a crisis

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Abstract

During the first wave of the COVID-19 pandemic, in France, people cleared the shelves of butter; in Italy, it was pasta; in Great Britain, it was chicken. While there may be cultural disagreement on what is essential, clearly, in times of crisis, consumers stockpile the ‘essentials’. We address the problem of “panic buying”, which is characterized by increasing demand in the face of diminishing inventory. In such cases, prices may hike and firms (retailers) selling the high-demand product are quantity takers, in terms of supply, and price setters. We consider a manufacturer who sells a scarce product to a single retailer. The retailer seeks to maximize her profit, while in contrast, the manufacturer pursues a social objective of regulating and lowering the amount that the end customer (consumer) pays (including the cost of traveling to obtain the scarce product). By analyzing the competition between the two parties, retailer and manufacturer, we find that even when the regulator (manufacturer) makes a significant social commitment, neither subsidizing the retailer nor subsidizing the consumers necessarily curbs price hikes. Furthermore, there is a threshold ratio (i.e., proportion of the end price subsidized by the regulator) that determines the minimal budget that the regulator would need to allocate in order for subsidization to make a difference to consumers.

Keywords Panic buying · Pricing policy · Scarce product · Non cooperative game · Subsidization

1 Introduction

1.1 Abrupt scarcity and panic buying

Oruc (2015) defines *product scarcity* as: “a condition or message that communicates a certain or potential unavailability of a product in the future along with the availability of a product in the present, all of which are directed at all possible recipients of a product.” Products with high consumption and those produced in limited quantities frequently become scarce at the onset

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of unpredicted events such as climate calamities, pandemics and natural disasters. Typical examples include: water allocation to the agricultural sector in dry seasons; vaccines during sudden influenza outbreaks; and daily supplies of bread, eggs, milk and fresh chicken during labor strikes. The uncertainty that arises in such situations creates insecurity (especially food insecurity), which frequently leads to panic buying. For example, after the Japanese nuclear disaster in 2011, many Chinese citizens, concerned about an interruption in the supply of sea salt, were panic buying iodine salt (Post, 2011). As the coronavirus crisis unfolded, disruptions in domestic food supply chains created strong tensions and food insecurity in many countries (The World Bank, 2020). Eggs were among the first food products to see a sharp price increase; the wholesale benchmark pricing for a dozen eggs in California was \$1.73 on March 13, 2020. By April 10, that price had nearly doubled to \$3.47, according to data from the United States Department of Agriculture. Overall, consumers paid 2.6% more for groceries in April 2020 (compared to March), according to the Bureau of Labor Statistics.

Behavioral psychologists explain the excessive demand observed during abrupt adverse events as a need to exert control in uncertain times. The herd instinct plays a role as well. For example, pictures of empty shelves during the first wave of the COVID-19 pandemic increased the public perception of scarcity and motivated people to buy more.

There are rational and irrational elements to panic buying. An indication of this can be seen in the effectiveness of advertising based on limited quantities or limited-time offers—advertising designed to create a feeling of urgency or scarcity. By letting shoppers see how many products are left in stock, sellers (e.g., Amazon, eBay) increase the urgency to purchase immediately.

Commodity scarcity drives up demand. According to Oruc (2015), limited availability increases the value of a product to a greater extent in cases where the scarcity is caused by increased demand (see also Worchel et al., 1975; Verhallen, 1982; Verhallen & Robben, 1994).

1.2 Price regulation

Confidence in the virtues of market-generated outcomes, as well as a distrust in the government's ability to regulate, have led to widespread advocacy for forms of private regulation in lieu of public regulation. As a consequence, most guidance documents on ex ante policy appraisal specify that governments should prioritize self- and co-regulatory solutions before considering more intrusive policy approaches (Saurwein, 2011; Senden, 2005). Co-regulation was described by Van Schooten and Verschuuren (2008) as an element of “non-state law” backed by “some government involvement” that encourages corporate social responsibility. Furthermore, consumers and employees are now looking for more than corporate social responsibility—they are pursuing what is called corporate social justice. Liu et al. (2017) analyze the coordination of a supply chain with a dominant retailer under a government price regulation policy by means of a revenue-sharing contract after demand disruption. Corporate social justice is a new paradigm that imagines a healthier and more mutually beneficial relationship between companies and the communities they interact with (Zheng 2020). Consequently, in this study, we aim to evaluate the efficiency of price regulation by a corporation (manufacturer) that wishes to achieve social justice, i.e., their efficiency at ensuring broader access to necessary products, as well as lower prices, especially at difficult times such as climate calamities and pandemics, where price hikes are common.

An important approach to regulating prices is subsidization. We focus here on two major forms—subsidizing the supply cost of the retailer (see, for example, Li et al., 2020) and subsidizing customer purchases (see, for example, Coady et al., 2004; Le Blanc, 2008; Davis, 2014; Schweiger & Stepanov, 2018; Gu et al., 2019). The former is accomplished by the manufacturer through discounted wholesale pricing. An example of this practice occurred during the COVID-19 pandemic, when travel restrictions prevented Turkmen from traveling abroad for work. Meanwhile, market prices skyrocketed. For example, a one-liter bottle of vegetable oil at the bazaar cost twice as much as the state-subsidized price, and a kilogram of flour cost nearly seven times as much. According to Human Rights Watch and the Turkmenistan Initiative for Human Rights, the government created a commission in late March 2020 to support local producers and keep prices stable. The second type of subsidization occurs when the manufacturer offers a discount coupon or rebate to consumers purchasing his product. For example, the customer completes a form provided by the manufacturer and mails it—along with other items the manufacturer may require.

1.3 Related research

We consider a supply chain that includes one manufacturer (referred to herein as “he”) who supplies a single scarce product-type at a wholesale price to a single retailer (referred to as “she”). The retailer stocks products that she sells at a retail price in response to customer (consumer) demand, where demand is characterized by panic buying; the lower the stock level, the greater the demand. Accordingly, the core factors influencing consumer purchasing decisions are the price offered by the retailer, the stock level and possible shortage. Other dimensions of the problem that we consider are the allocation of scarce resources to the manufacturer and the traveling costs incurred by consumers, which could affect their decisions and the extent of their panic buying. Fox et al. (2004) used the travel time from the consumer’s home to the store to predict consumer patronage and spending at stores with different retail formats. Shopping and spending at grocery, drug and discount retailers were found to be highly sensitive to travel time. Here, we assume that the “price” incorporates the cost of travel from consumer locations to the store (or firm).

There is limited published research related to supply-chain management during panic buying. Despite the significant dynamics that affect supply chains during a crisis and the ensuing panic, this research is mainly based on two-period/stage models. Specifically, Yoon et al. (2018) studied the retailer’s (single and dual) sourcing strategies by considering consumer stockpiling behaviors under supply disruption risk. Their problem setting consisted of a retailer positioned between a supplier and consumers in a three-tier supply chain selling a product to consumers over two periods. They characterized consumer stockpiling behaviors in the presence of supply disruptions and showed that stockpiling increases if consumers have experienced similar problems before and weakens as more inventory is hoarded. Tsao et al. (2018) examined the impact of panic buying on the retailer’s optimal ordering quantities when the retailer sells different substitutable products. This study considered a wholesaler selling two brands of a product with multiple weights and produced by different manufacturers to multiple retailers over two periods. In the first period (the panic situation), the wholesaler attempts to retain the inventory to satisfy the retailers. In the second period (the supply disruption), retailers are willing to accept the substitute products. The wholesaler then segregates retailers into high and low indexed, where high indexed retailers (who provide higher profit) order greater quantities than low indexed retailers.

Zheng et al. (2020) conducted an analytical study on the role of social leaning in the panic-buying decisions of consumers. The authors assumed that some consumers will stockpile products to mitigate the risk of shortages caused by anticipated supply disruption. They considered different distributions of consumer beliefs about the future shortage rate. Their problem setting consisted of a monopolist retailer that sells a staple product to a mass of consumers over two periods. The retailer seeks to optimize her ordering policy during each period while considering potential disruption risk on the supply side as well as consumer reactions on the demand side.

Similar to the above research, we assume that the retailer maximizes her profit and that panic buying is characterized by a finite time horizon. Diverging from the aforementioned work, we use a game-theoretic framework to model the interaction between the supplier and the retailer. Moreover, "panic" according to our model is characterized by increasing demand with diminishing inventory. We further assume that the retailer is a quantity taker due to strained supplies during panic. Unlike previous work, we study panic dynamics over multiple time points in a continuous-time setting rather than just two discrete-time periods. In addition, in contrast to previous research that aimed to determine optimal ordering policies, we focus on optimal pricing policies. Importantly, our work considers efforts to regulate the average cost that supply-chain customers incur during panic through different types of subsidization. That is, we consider a manufacturer whose main motivation is affordability for the customer as a social good during a crisis.

1.4 Contribution

The model that we have developed contributes to the limited number of published studies that analyze the distribution and pricing of scarce products. In particular, we contribute to:

- (a) Modeling a demand rate that increases when approaching shortage.
- (b) Deriving analytical properties of equilibria solutions and a numerical algorithm based on those properties.
- (c) Deriving a closed-form expression for the optimal response retail price when the demand rate weakly depends on the inventory level of the firm.
- (d) Understanding how two different approaches—subsidizing the retailer and offering a discount coupon (or rebate) to customers—can regulate retailer decisions under panic buying.

In Sect. 2 we introduce our assumptions and method of modeling the demand rate. Section 3 focuses on the retailer's inventory dynamics and the objectives of the two parties of our supply chain. Section 4 presents a mathematical analysis of the retailer's optimal strategy. Section 5 illustrates the results with a numerical example and provides a sensitivity analysis in terms of the key parameters of the problem. Section 6 summarizes the results, provides managerial insights and discusses future research directions.

2 Notations, assumptions and the demand model

Consider a retailer (firm) that is located at the left-hand point ($x = 0$) of a straight line $[0,1]$. The firm purchases a scarce product from a manufacturer that can play the role of a regulator. The cost of traveling from the left end to the right end of the straight line is g . Accordingly, a consumer located at point x who travels to the retailer incurs a proportional cost of xg . The manufacturer (regulator) controls the maximal quantities of the scarce product to be

manufactured in each cycle. Scarcity is assumed to mean that there is a limited quantity of the item over a given time period. A total quantity of N is supplied by the manufacturer to the retailer in the given selling period. The wholesale price per unit paid by the retailer is w , while the manufacturer's production cost per unit is c . The retailer replenishes the product inventory by ordering quantity $Q = N$ for the selling (scarcity) period T , which is determined exogenously by the manufacturer. Ordering and transportation costs from the manufacturer to the retailer are denoted by K and are assumed to be the responsibility of the manufacturer. That is, the retailer is a quantity taker due to panic conditions. Such an arrangement is also used in normal conditions (see, for example, Yoon et al., 2018; Tsao et al., 2018; Chernonog, 2020), especially in vendor managed inventory (VMI) contracts, under which the vendor specifies quantities to deliver based on data obtained from Electronic Data Interchange (EDI). The regulating manufacturer may subsidize the product by applying a direct discount s , $0 < s < 1$, to the retail price p (e.g., coupons to discount the price by a given percentage). With discounting, the consumer pays ps for the product, while the regulator incurs an extra cost per unit of $p(1 - s)$. We refer to ps as the *subsidized price*. The retailer's cost of holding a single item in storage for a unit of time is h . The remaining notation list is summarized in the following subsection.

2.1 Main notations

2.1.1 Decision variables

- s regulator's subsidy ratio $0 \leq s \leq 1$ associated with the percentage discount applied to the retail price (a higher ratio implies a lower discount);
- S_1 total subsidy provided by the regulator to customers during the selling period, $S_1 \leq S_1^{\max}$, where $S_1^{\max}$ is the maximal subsidy budget;
- p retail (reference) price of an item without discounting;
- w manufacturer's wholesale price per unit.

2.1.2 Variables

- $\Pi_R(p, s, w)$ retailer's total profit per replenishment cycle T for a given price p , subsidy ratio s and wholesale price w ;
- $I(t)$ inventory level at the retailer's site at time t ;
- $f_p(x)$ probability density function of consumer locations x along the interval $[0, 1]$;
- $\Pi_M(p, s, w, f_p(x), S_1)$ manufacturer's objective over the selling period;
- $\lambda(p, s, x, I(t))$ density function of the demand rate at time t for consumers located at x , inventory level $I(t)$, market price p and subsidy ratio s ;
- $T_A(p, s)$ period of time until the retailer's stock is fully depleted.

2.2 Assumptions

A1 The information regarding the distribution of consumer locations is available to both the retailer and the manufacturer.

A2 Information about prices and total inventory is shared between the supply-chain parties.

A3 The retailer acts rationally, i.e., aims to optimize her own profit.

A4 The inventory level never exceeds the reservation level β^{-1} (a higher level would mean that the demand does not depend on the inventory level, and thus the product is not scarce); i.e., $N \leq \beta^{-1}$.

A5 Shortages are not backlogged, i.e., they are considered lost sales.

A6 The demand rate at time t increases as the total inventory at time t decreases (i.e., scarce inventory).

A7 Leftovers are discarded without additional cost or benefit at the end of the scarcity period, i.e., at $t = T$.

A8 The firm (retailer) buys the entire quantity offered by the manufacturer.

2.3 Modeling demand rate

To account for consumer locations, we use the probability density function $f_p(x)$ along the interval $[0, 1]$. The index p in the probability density function (pdf) implies that in general, the geographic position of the retailer is chosen to serve a community that is characterized by a certain response to price p . In particular, the response is described by the common negative effect of price on demand rate, modeled as $1 - \alpha sp$, where the reservation price, denoted by $(\alpha s)^{-1}$, is the price at which customers no longer purchase the product. A demand that increases with a decrease in the total inventory at time t , $I(t)$, characterizes *panic buying* and is also modeled with an affine function, $1 - \beta I(t)$, where β^{-1} is the *reservation inventory* level, above which the demand is no longer affected by the surplus (i.e., the market does not consider the product as scarce). Sensitivity parameters α and β are case-based (i.e., market and product dependent) and are estimated by historical selling records for a specific market. We construct the total density function of the demand rate $\lambda(p, s, x, I(t), t)$ at time t , for consumers located at x , as follows:

$$\lambda(p, s, x, I(t)) = \begin{cases} f_p(x) (1 - \alpha ps) \left(\lambda_0^{\min} + (\lambda_0 - \lambda_0^{\min}) (1 - \beta I(t)) \right) & 0 \leq x \leq 1, c \leq p \leq (\alpha s)^{-1}, 0 \leq I(t) \leq \beta^{-1} \\ f_p(x) (1 - \alpha sp) \lambda_0^{\min} & 0 \leq x \leq 1, c \leq p \leq (\alpha s)^{-1}, \beta^{-1} < I(t) \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where λ_0 is the density demand rate for an item when the subsidized price sp is set to zero and the retailer is out of stock, and $\lambda_0^{\min}$ is the density demand rate (minimal) for an item when the subsidized price sp is set to zero and the inventory surplus is greater than β^{-1} . Since the retailer cannot predict the subsidy ratio, it is reasonable for her to keep the maximal price at the non-subsidized reservation price α^{-1} rather than to set the price so high (i.e., at the reservation level) that consumers would simply stop buying the product. Consequently, we assume that the retailer does not set the price above α^{-1} . The function $\lambda(p, s, x, I(t))$ accounts for price, subsidy ratio, consumer locations, remaining inventory at time t , sensitivity to the subsidized price and sensitivity to the surplus. This function, however, does not account for the traveling cost, as panic buying implies that consumers are likely to travel to the store for essential goods.

Based on assumption A4 related to scarcity, the inventory level never exceeds the reservation level β^{-1} . The consumer demand rate $\lambda(p, s, I(t))$ at time t , whenever the retailer's inventory level at t is positive, is,

$$\begin{aligned}\lambda(p, s, I(t)) &= \int_0^1 \lambda(p, s, x, I(t)) dx = (1 - \alpha sp) \left(\lambda_0^{\min} + (\lambda_0 - \lambda_0^{\min})(1 - \beta I(t)) \right) \int_0^1 f_p(x) dx \\ &= (1 - \alpha sp) \left(\lambda_0^{\min} + (\lambda_0 - \lambda_0^{\min})(1 - \beta I(t)) \right),\end{aligned}\quad (2)$$

When the retailer is out of stock (see (1)), $I(t) = 0$, she does not sell the goods, as shortages cannot be backlogged and there is no extra source of supply. From (2) we conclude that the overall demand rate at time t is not affected by the distribution of consumer locations. Yet, it is expected that the overall average unit price that end consumers pay does depend on their locations.

3 Problem formulation

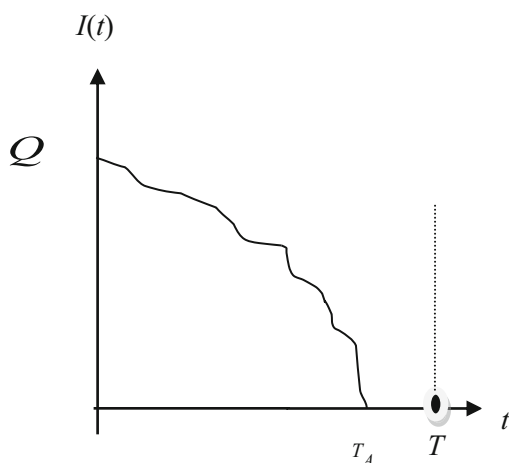
3.1 Inventory dynamics

A change in the inventory level $I(t)$ (which is unknown at this stage) at the retailer's site is due to depletion of the supplied amount $I(0) = N$ by the consumer demand rate (2); see the schematic illustration in Fig. 1.

$$dI(t)/dt = -(1 - \alpha ps)\lambda_0 + \beta(1 - \alpha sp)(\lambda_0 - \lambda_0^{\min})I(t), \quad I(0) = N, \quad (3)$$

We now replace the simplified notation $I(t)$ with the more explicit one, $I(p, s, t)$. The solution of (3) is

Fig. 1 Inventory dynamics over time at the retailer during panic buying



$I(p, s, t) = \frac{\lambda_0}{\beta(\lambda_0 - \lambda_0^{\min})} + k_1 e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})t}$, where k_1 is a constant found from $I(p, s, 0) = N$, i.e., $k_1 = N - \frac{\lambda_0}{\beta(\lambda_0 - \lambda_0^{\min})}$ and thus,

$$I(p, s, t) = \frac{\lambda_0}{\beta(\lambda_0 - \lambda_0^{\min})} + \left[N - \frac{\lambda_0}{\beta(\lambda_0 - \lambda_0^{\min})} \right] e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})t}. \quad (3.1)$$

We next introduce the conflicting objectives of the two supply-chain parties in order to model their interaction as a non-cooperative game.

3.2 The objectives of the two supply-chain parties

3.2.1 Retailer's objective function

The retailer's total profit per replenishment cycle for a given price p , subsidy ratio s and wholesale price w is

$$\Pi_R(p, s, w) = p(N - I(p, s, T)) - wN - h \int_0^T I(p, s, \tau) d\tau. \quad (4)$$

The first term in (4) represents the retailer's revenue based on her quantity sold, $N - I(p, s, T)$, while the other two terms account for purchase and inventory costs respectively.

3.2.2 The manufacturer's (regulator's) objective function

The regulator does not intend to maximize his profit during panic buying. Instead, he balances the wholesale revenue against the subsidy, as a means of recovering his production and transportation costs. The amount invested by the regulator in directly assisting consumers to pay the retail price is given by:

$$S_1 = (1 - s)pN, \quad (5)$$

where $S_1 \geq 0$ and s is the subsidy ratio. In the case where $s = 1$, the retail price is fully paid by consumers. In the case where $s < 1$, the regulator's share of the product price is $(1 - s)p$. The amount spent by the regulator on subsidizing the retailer is given by $cN + K - wN \geq 0$. This term controls the extent to which the regulator's costs associated with production and transportation, K , are recovered. When $cN + K - wN > 0$, the regulator subsidizes part of the costs, while $cN + K - wN = 0$ implies that the costs of production and transportation are fully paid by the retailer via the wholesale price offered by the manufacturer.

We assume that the objective of the regulator consists of achieving a tradeoff between his social agenda of controlling the cost incurred by the end customers (thereby improving consumer welfare) and the level of subsidization needed to achieve such control. Accordingly, the manufacturer's goal is to minimize the following objective:

$$\Pi_M(p, s, w, f_p(x), S_1) = sp + gd(p, s) + \eta_1 S_1/N + \eta_2 (cN + K - wN)/N, \quad (6)$$

where $cN + K - wN$ defines the total subsidy paid by the regulator to the retailer over the selling period, and d is the expected traveling distance to purchase the product, given by:

$$d(p, s) = \int_0^1 x f_p(x) dx = E(x, p) \quad (7)$$

Thus, $sp + gd(p, s)$ is the average unit price, including the traveling cost and taking account of the subsidization of the retail price, that end consumers pay (we remind the reader that g is the traveling cost from the left end to the right end of the straight line). Here, we simplify the analysis by considering a special case in which $f_p(x)$ does not depend on the market price; that is, $f_p(x) = f(x)$ and accordingly, $E(x, p) = E(x)$. In Sect. 5 we consider a more general case under which the firm (retailer) may be located at any point x_0 along the segment mentioned above. The manufacturer regulates his policy with sensitivity parameters $\eta_1, \eta_2 \geq 0$, which quantify his social attitude—the relative importance of each of the goals in objective (6). The smaller the value of η_1, η_2 , the higher the priority of the regulator to reduce the price for the corresponding party (either the retailer or the consumer), possibly resulting in higher subsidies S_1 and S_2 , where $S_2 \equiv cN + K - wN$. Furthermore, since $S_1 = (1 - s)pN$, by simple manipulation of the regulator's objective function we obtain, for $\eta_1 = 1$,

$\Pi_M(p, s, w, f(x), S_1) = gE(x) + p+$. This is an interesting special case to address. The case arises when the manufacturer's priority is to enable the retailer to reduce her reference (selling) price rather than to target the price that consumers pay, i.e., by means of a discounted price.

The decision each party makes affects the other. Therefore, we use the game-theoretic framework to model the resultant vertical competition. Under the assumption of a non-cooperative game, the two players compete by simultaneously announcing their decisions. As these decisions are monetary or quantity oriented, using current technology-based communication protocols (e.g., EDI), they can be updated frequently in real time—efficiently and with negligible cost. This reality allows all players to adjust their decisions before signing a supply contract, waiting until equilibrium is reached. Given an available supply quantity N , the regulator determines the best wholesale price w , subsidy ratio s and subsidy payments given to consumers S_1 and to the retailer S_2 , while the retailer decides on the reference (selling) price p , which means that the discounted price is sp . Consequently, the competition is characterized by Nash equilibria and the optimal pricing policy (retailer's best response function) for given (w, s) is obtained by solving

$$\begin{aligned} \max_p \quad & \Pi_R(p, s, w) \\ \text{s.t.} \quad & \\ & w \leq p \leq (\alpha s)^{-1}, \forall w, s \end{aligned} \quad (8)$$

The constraint in problem (8) prevents the retailer from selecting prices that exceed the reservation price $(\alpha s)^{-1}$ or would cause a loss. The manufacturer's best response function to any given price p is obtained by solving

$$\begin{aligned} \min_{s, S_1, w} \quad & \Pi_M(p, s, w, f(x), S_1) \\ \text{s.t.} \quad & \\ (1) \quad & S_1 = (1 - s)pN \\ (2) \quad & S_1 \geq 0 \\ (3) \quad & cN + K - wN \geq 0 \\ (4) \quad & 0 \leq s \leq 1 \\ (5) \quad & S_1 \leq S_1^{\max} \end{aligned} \quad (9)$$

Simultaneously solving problems (8) and (9) enables identification of the equilibrium prices and regulatory decisions. Constraint (3) in problem (9) avoids the scenario under which the regulator seeks to make a profit. Note, from constraints (1) and (5) in problem (9),

$(1-s)pN \leq S_1^{\max}$, or

$$1 - \frac{S_1^{\max}}{pN} \leq s. \quad (9.1)$$

Constraint (9.1) implies that the regulator may encounter a budget limitation when attempting to apply too high a discount to the products sold by the retailer.

4 Mathematical analysis

By setting the inventory level in (3.1) to zero, we find the point in time T_A (which could be beyond T) where the retailer's stock empties, $0 = \theta + (N - \theta)e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})T_A}$, i.e.,

$$T_A = \frac{\ln\left(\frac{-\theta}{N-\theta}\right)}{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})}, \quad (10)$$

where we define $\theta \equiv \frac{\lambda_0}{\beta(\lambda_0 - \lambda_0^{\min})}$. Accordingly, the inventory level at time t is

$$I(t) = \theta + (N - \theta)e^{\beta(1-\alpha ps)(\lambda_0 - \lambda_0^{\min})t}, \quad t \in [0, T'], \text{ and } I(t) = 0, \quad t \in [T', T], \quad (11)$$

where $T' = \min(T_A, T)$. Following (11), $N < \theta$ must hold; otherwise the inventory level (11) increases with time, and such a scenario is infeasible for our setting of panic buying and scarcity. Recalling that $I(t) = I(p, s, t)$, the retailer's total profit in a selling period is

$$\Pi_R(p, s, w) = p(N - I(p, s, T)) - wN - h \int_0^{T'} \left(\theta + (N - \theta)e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})\tau} \right) d\tau.$$

That is,

$$\begin{aligned} \Pi_R(p, s, w) = & p(N - I(p, s, T)) - wN - h\theta T' \\ & - \frac{h(N - \theta)}{\beta(1 - \alpha sp)(\lambda_0 - \lambda_0^{\min})} \left(e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})T'} - 1 \right). \end{aligned} \quad (12)$$

From (11) and (12) we observe that, as long as the retailer does not incur losses, thereby shutting down her business, the wholesale price does not affect the price she chooses. The same conclusion is obtained if we add to the retailer's objective $S_2 = cN + K - wN$. The following straightforward conclusion has practical implications for facilitating the search for an equilibrium solution as well as for achieving a more profound understanding of the relative leverage of the retailer over the regulator.

Lemma 1 *The optimal response function $p^*(w, s, S_1)$ is independent of both the wholesale price w (as long as the retailer's business is sustainable) and the amount the regulator offers the retailer as a subsidy, $cN + K - wN$.*

Lemma 1 shows that a manufacturer with a non-profit intent (or a social good priority) is left with only a single tool to regulate the retail price: the subsidy ratio s . This is because neither the wholesale price nor direct subsidization of the retailer affect the retailer's decision on price p , while on the other hand, the total subsidy given to consumers, S_1 , which is determined by the subsidy ratio, s , indirectly affects the retailer's policy. For example, in the case where $S_1^{\max}$ is binding in terms of constraint (9.1), this bound would also limit the possible values

of s and, accordingly, would affect the retailer's price. To reflect the above result, we replace $p^*(w, s, S_1)$ with $p^*(s)$.

The outcome of Lemma 1 is due to the interaction of two major factors: (1) the standard wholesale practice of selling at price w per unit, for a batch of N units, resulting in a payment of Nw ; and (2) panic buying, which implies that all N available units will be purchased by the retailer. Consequently, as long as the wholesale price offered by the manufacturer does not cause the retailer to shut down her business, the specific level of w will not affect the retailer's response.

To find the retailer's optimal response, we employ the first-order optimality condition (FOC) for the retailer's objective function, which, when denoting by p^T the minimal price under which $T' = T$, results in the solution $\frac{\ln\left(\frac{-\theta}{N-\theta}\right)}{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})} = T$ and

$$p^T(s) = (\alpha s)^{-1} \left[1 - \frac{\ln\left(\frac{-\theta}{N-\theta}\right)}{\beta(\lambda_0 - \lambda_0^{\min})T} \right] \quad (13)$$

We first assume the case where $T' = T_A$, which is equivalent to $p < p^T$. Then

$$\Pi_R(p, s, w) = pN - wN - h\theta T_A - \frac{h(N-\theta)}{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})} \left(e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})T_A} - 1 \right),$$

or equivalently,

$$\Pi_R(p, s, w) = pN - wN + \frac{hN - h\theta \ln\left(\frac{-\theta}{N-\theta}\right)}{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})}. \quad (14)$$

In what follows we show that the objective function (14) is strictly concave in p . We introduce the function

$$R(\theta) = \frac{hN - h\theta \ln\left(\frac{-\theta}{N-\theta}\right)}{\beta(\lambda_0 - \lambda_0^{\min})}, \quad (15)$$

which is non-positive, tends to minus infinity when $\theta = N$ and $\lim_{\theta \rightarrow \infty} R(\theta) = 0$. Then, $\Pi_R(p, s, w) = pN - wN + \frac{R(\theta)}{(1-\alpha sp)}$ and $\partial \Pi_R(p, s, w) / \partial p = N + \frac{\alpha s R(\theta)}{(1-\alpha sp)^2}$, which, when solved for p^0 , results in

$$p^0(s) = (\alpha s)^{-1} \left[1 - \sqrt{\frac{-\alpha s R(\theta)}{N}} \right]. \quad (16)$$

Noting that $\partial^2 \Pi_R(p, s, w) / \partial p^2 = \frac{2(\alpha s)^2 R(\theta)}{(1-\alpha sp)^3}$, together with the fact that $R(\theta)$ remains negative while increasing with θ , we summarize:

Corollary 1 For the case of $T' = T_A$ and $p < (\alpha s)^{-1}$,

$\Pi_R(p, s, w)$ is a strictly concave function of price p for all (s, w) .

We now address the case where $T' = T$, which is equivalent to $p \geq p^T$.

$$\begin{aligned} \Pi_R(p, s, w) = & p(N - I(p, s, T)) - wN - h\theta T \\ & - \frac{h(N-\theta)}{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})} \left(e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})T} - 1 \right) \end{aligned} \quad (17)$$

From the FOC, $\partial \Pi_R(p, s, w)/\partial p = 0$, and after dividing by $N - \theta$, we have:

$$\begin{aligned} & 1 - e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})T} + p\beta\alpha s(\lambda_0 - \lambda_0^{\min})T e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})T} \\ & + \frac{h}{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})} \left(\beta\alpha s(\lambda_0 - \lambda_0^{\min})T e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})T} \right) \\ & - \frac{\alpha sh}{\beta(1-\alpha sp)^2(\lambda_0 - \lambda_0^{\min})} \left(e^{\beta(1-\alpha sp)(\lambda_0 - \lambda_0^{\min})T} - 1 \right) = 0 \end{aligned} \quad (18)$$

To formalize our next result, we introduce the following technical conditions:

$$w \leq w^1, \text{ where } w^1 \equiv \alpha^{-1} \left[1 - \frac{\ln\left(\frac{-\theta}{N-\theta}\right)}{T\beta(\lambda_0 - \lambda_0^{\min})} \right] \quad (19.1)$$

and

$$w \leq \alpha^{-1} \left[1 - \sqrt{\frac{-\alpha R(\theta)}{N}} \right] \quad (19.2)$$

The technical conditions (19.1) and (19.2) ensure that the wholesale price does not cause the retailer to incur losses (and therefore to shut down), i.e., $w \leq p_1^*$. Note that these two inequalities represent tighter constraints (i.e., with $s = 1$) than the original constraint in (8).

Theorem 1 Under technical conditions (19.1) and (19.2), the retailer's optimal response function is obtained by the following solution algorithm:

Step 1. Assume $T_A \leq T$.

$$p_1^*(s) = \min \{ (\alpha s)^{-1}, p^T(s), p^0(s) \}.$$

Step 2. Assume $T_A > T$. If $(\alpha s)^{-1} < p^T(s)$ then there is no feasible solution. Go to Step 3.

Solve FOC (18) numerically to obtain $p_2^0(s)$ and obtain

$$p_2^*(s) = \begin{cases} p^T(s) & p_2^0(s) \leq p^T(s) \leq (\alpha s)^{-1} \\ p_2^0(s) & p_2^0(s) \leq p^T(s) \leq (\alpha s)^{-1} \\ (\alpha s)^{-1} & p^T(s) \leq (\alpha s)^{-1} \leq p_2^0(s) \end{cases}$$

Step 4. $p^*(s) = \arg \max_{p_1^*(s), p_2^*(s)} \{ \Pi_R(p, s, w) \}.$

Theorem 1 implies that under conditions (19), as well as the condition in Step 2 ($(\alpha s)^{-1} < p^T(s)$), shortages are never observed. This is because the retailer exploits the fact that when the inventory accumulates, the demand decreases; i.e., she raises the price to "adjust" to the new level of demand. This flattens the demand over the entire selling horizon. The retailer does not incur losses from this strategy since the entire supply is in her hands. It should be noted that since leftovers are feasible, she may choose to further increase the price (i.e., beyond p^T) and sell a smaller quantity than the originally received supply in order to increase her profits during a panic.

The following result implies that when $S_1 \leq S_1^{\max}$, the "game" between the retailer and the regulator is almost always redundant (except in the case where $\eta_1 \neq 1$).

Theorem 2 Under the technical condition $c + \frac{K}{N} \leq \min(w^1, p^0)$, the optimal response of the manufacturer is given by.

$$w^*(p) = c + \frac{K}{N}; \quad (20.1)$$

$$s^*(p) = \begin{cases} 1 & \eta_1 > 1 \\ s \in \left[\max\left\{1 - \frac{S_1^{\max}}{pN}, 0\right\}, 1 \right] & \eta_1 = 1 \\ \max\left\{1 - \frac{S_1^{\max}}{pN}, 0\right\} & 0 \leq \eta_1 < 1 \end{cases}; \quad (20.2)$$

Proof The derivative of the objective in (6) with respect to w is negative. Considering expressions (5) and (7), the manufacturer's objective (6) is

$$\Pi_M(p, s, w, f(x), S_1) = sp(1 - \eta_1) + \eta_1 p + gE(x) + \eta_2(cN + K - wN)/N, \quad (21)$$

which, when minimizing with respect to s for any p , straightforwardly results in (20.2). Finally, to comply with the technical conditions (19.1) and (19.2) we need

$$c + \frac{K}{N} \leq \min(c^1, p^0(s = 1)). \quad \square \quad (22)$$

Theorem 2 indicates that under various social attitude cases, the regulator has to choose between the following two pure strategies: not to subsidize at all or to fully subsidize the consumer (i.e., the consumer only pays travel costs) assuming that this is possible within budget $S_1^{\max}$ (i.e., assuming the budget is non-binding). Furthermore, the optimal wholesale price does not depend on the retail price (see (20.1)). On the other hand, when the regulator's priority is to enable the retailer to reduce her selling price rather than to subsidize the consumer (i.e., when $\eta_1 = 1$), he may choose a mixed strategy, $s \in [0, 1]$, again subject to the maximum budget. We conclude from Theorem 2 that for $\eta_1 = 1$ and any η_2 , $\Pi_M(p, s, w, f(x), S_1) = gE(x) + p$.

The particular case of $\eta_1 \leq 1$ deserves special attention. We next show, for this case, the important properties that affect the equilibria and hence the potential outcomes of the game.

Theorem 3 Let satisfy $N((1 - s)p^*(s)) \leq S_1^{\max}$ and let $\eta_1 \leq 1$; then.

- (1) There exists a threshold subsidy ratio $0 \leq s^0 < 1$ such that the optimal subsidized price is constant, $sp^*(s) = \text{const}$, for $s^0 < s \leq 1$.
- (2) The threshold subsidy ratio is given by $s^0 = \alpha p^*(s = 1)$.
- (3) The retailer's optimal response price $p^*(s)$ is a non-increasing function of the subsidy ratio s :

$$p^*(s) = \begin{cases} \alpha^{-1} & 0 \leq s \leq s^0 \\ s^{-1} p^*(s = 1) & s^0 < s \leq 1 \end{cases} \quad \text{and} \quad sp^*(s) = \begin{cases} s\alpha^{-1}, & 0 \leq s < s^0 \\ p^*(s = 1), & s^0 < s \leq 1 \end{cases}.$$

Proof See Appendix A.

Theorem 3 implies that, for the case of $\eta_1 \leq 1$, there exists a threshold ratio, s^0 , such that above this value, as the regulator increases the subsidy ratio (i.e., reduces the amount of discount), the retailer decreases the retail price, in such a manner that consumers end up paying the same price regardless of the subsidy ratio—a price that is identical to that paid when there is no discount at all ($s = 1$). Thus, if the regulator wants to ensure that a real discount is applied to the reference price, his investment must result in a subsidy ratio below this threshold (i.e., $s < s^0$).

We summarize our observations and account for the budget constraint below.

Proposition 1

- (a) Assume $\frac{S_1^{\max}}{N} \geq \alpha^{-1}$ and $0 \leq \eta_1 < 1$, and if $T_A(p, s = 0) \leq T$, then the equilibrium price is $p^*(s) = \alpha^{-1}$; otherwise, if $T_A(p, s = 0) > T$, then $p^*(s) = \max\{p^T(s = 0), p_2^0(s = 0)\}$ as determined by Step 2 in Theorem 1. The equilibrium subsidy ratio is $s^* = 0$.
- (b) Assume $\frac{S_1^{\max}}{N} < \alpha^{-1}$ and $0 \leq \eta_1 < 1$. If $0 \leq S_1^{\max} \leq (1 - s^0)\alpha^{-1}N$ then the equilibrium solution is $s^* = 1 - \frac{S_1^{\max}}{p^*(s=1)N}$ and $p^* = \left(\frac{p^*(s=1)N}{p^*(s=1)N - S_1^{\max}}\right)p^*(s = 1)$. Otherwise, if $(1 - s^0)\alpha^{-1}N < S_1^{\max} < \alpha^{-1}N$, then the equilibrium solution is $s^* = 1 - \frac{S_1^{\max}}{N\alpha^{-1}}$ and $p^* = \alpha^{-1}$.
- (c) Assume $\eta_1 > 1$; then $s^* = 1$ and the equilibrium retail price p^* is given by Theorem 1.
- (d) Assume $\eta_1 = 1$; then the game may have multiple equilibria with $s \in [\rho, 1]$, $0 \leq \rho = \max\left\{1 - \frac{S_1^{\max}}{pN}, 0\right\} \leq 1$, $p = p(s)$, as given by Theorem 1 and condition (3) of Theorem 3.

Proof See Appendix B.

It is important to stress that although the case of $\eta_1 = 1$ is characterized by multiple equilibria, thereby implying an uncertain outcome, it is likely that the equilibrium that both supply-chain parties select will be unique. The logic behind such an outcome is as follows. Since the subsidy ratio s does not affect the manufacturer's objective function, it is reasonable that the manufacturer will choose to discount as much as possible, $s^* = 1 - \frac{S_1^{\max}}{N\alpha^{-1}}$, if he has a sufficient budget, $S_1^{\max} > (1 - s^0)\alpha^{-1}N$. Accordingly, the retailer will set an equilibrium price, $p^* = \alpha^{-1}$, that matches this expected behavior. That is, the manufacturer is able to regulate the subsidized price, $p^*s^* = \alpha^{-1} - \frac{S_1^{\max}}{N}$. Furthermore, as we show later numerically, it is likely that such an equilibrium will maximize the retailer's profit while having no effect on the regulator's objective; that is, it will be pareto improving or even pareto optimal.

Due to the complexity of the FOC (18), Theorem 1 and Proposition 1 refer to an algorithm for obtaining the retailer's pricing strategy either fully or partially numerically. The following result addresses a special case under which an explicit solution is available:

Proposition 2 When the demand rate weakly depends on the inventory level, i.e., $\beta \rightarrow 0$ (meaning that the panic effect is weak), the first-order condition (FOC) presented in (18) is a quadratic function of p ,

$$-(\alpha s)^2 \beta (\lambda_0 - \lambda_0^{\min}) T p^2 + (2\alpha s + \alpha s \beta (\lambda_0 - \lambda_0^{\min}) T) p + \alpha s h T - 1 = 0 \quad (23)$$

with an equilibrium price given by

$$p_2^0(s) = \frac{-(2\alpha s + \alpha s \beta (\lambda_0 - \lambda_0^{\min}) T) + \sqrt{[(2\alpha s + \alpha s \beta (\lambda_0 - \lambda_0^{\min}) T)^2 + 4(\alpha s)^2 \beta (\lambda_0 - \lambda_0^{\min}) T (\alpha s h T - 1)]}}{-2(\alpha s)^2 \beta (\lambda_0 - \lambda_0^{\min}) T} \quad (24)$$

Proof See Appendix C.

5 Numerical illustrations

The applicability and significance of the proposed model are exemplified in this section. TIHR, together with media outlets Turkmen News and Azadlyk Radiosi, have monitored food

Table 1 The data

Baseline parameters										
λ_0	α^{-1}	g	T	$\lambda_0^{\min}$	β^{-1}	h	N	c	K	η_1
20,000	30	2	6	10,000	100,000	0.5	50,000	5	5000	2

availability in Turkmenistan during the COVID-19 crisis. They report that staple foods, e.g., eggs and frozen chicken, have been periodically unavailable in state stores in various regions of the country. We develop a numerical illustration motivated by these real-life circumstances.

Consider a scarce product (e.g., a carton of eggs in the first week of a producers' strike) sold in a small city in which there is a maximum weekly total demand of 20,000 units (i.e., this level of demand would empty the local stock) and a maximum weekly demand of 10,000 units when the inventory exceeds the reservation inventory level. The product has a reservation price of 30 NIS (New Israeli Shekel). Table 1 below presents the data we use.

5.1 The case of $\eta_1 > 1$: no subsidy

We used the solution method presented in Theorems 1 and 2 and obtained the equilibrium solutions as stated in Proposition 1. This resulted in $w^* = \text{NIS}5.1$, $S_1^* = S_2^* = 0$, $s^* = 1$, $p^* = \text{NIS } 15.616$, and accordingly a profit per unit time of $\pi_R^*(T) = \Pi_R^*/T = \text{NIS } 74533.65$ along with a final stock level of $I^*(T) = 0.415$. This equilibrium reflects a strategy under which the regulator does not invest in any type of subsidy. After substituting the parameter values presented in Table 1 into (19.1) and (19.2), we obtain $w \leq 15.61$ and $w \leq 25.24$, respectively. Since the optimal wholesale price is $w^* = \text{NIS}5.1$, this complies with the technical condition of preventing the retailer from incurring losses.

5.1.1 The effect of the scarcity period length

Figures 2, 3 and 4 present the equilibrium price $p^*(T)$, profit per unit time $\pi^*(T)$, and remaining inventory at the end of the planning horizon $I^*(T)$ for different values of T . Figures 2, 3 and 4 indicate that the retailer clearly exploits her monopolistic power. When the selling period increases, the retailer keeps emptying her entire inventory; however the price per unit increases (and accordingly, the profits—at least for some range of T).

For smaller planning horizons, there are leftovers. Even at $T = 5$, the firm chooses to leave 2115 units unsold (4.23% of the initial supply) in spite of the demand rate being relatively high. The sole cause of this outcome (which is counter to the interests of both consumers and the regulator) is that the price set by the retailer is so high that many consumers give up on attempting to purchase. The results indicate that the length of the time horizon T is an influential factor for the three key performance measures. It actually determines whether or not the retailer gains a monetary benefit; in particular, a selling period of less than approximately 4.5 weeks would make the firm prefer a shutdown (i.e., the retailer would prefer not to sell at all).

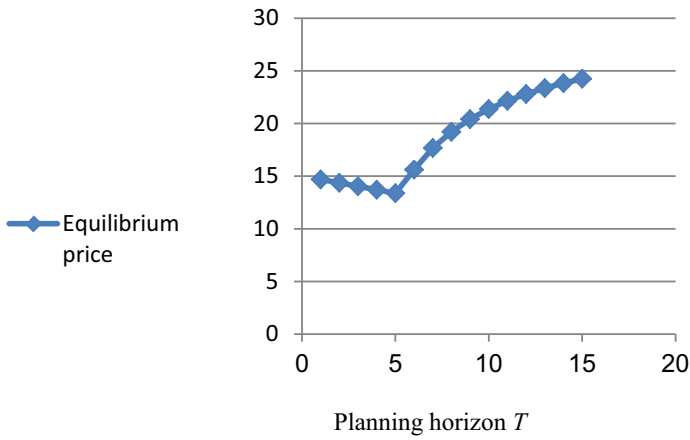


Fig. 2 Equilibrium retail price $p^*(T)$

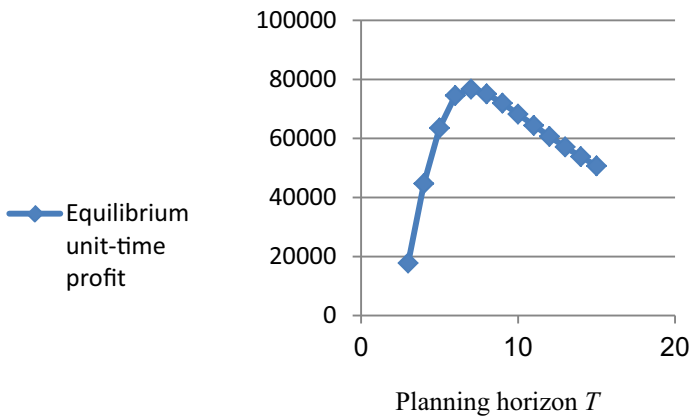


Fig. 3 Retailer's equilibrium profit per unit time $\pi_R^*(T)$

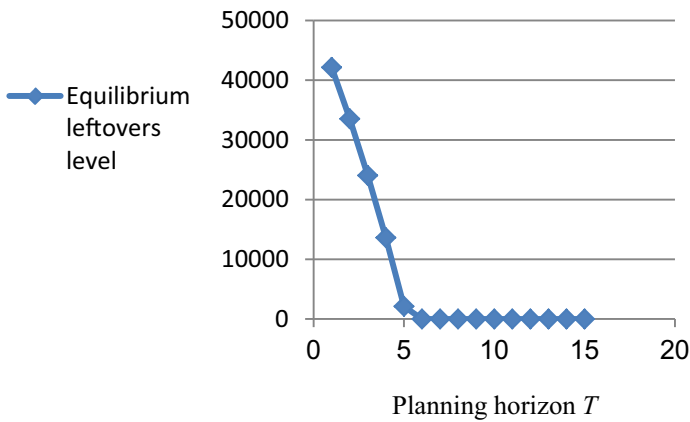


Fig. 4 Equilibrium leftovers level $I^*(T)$

5.1.2 The effect of consumer locations relative to the retailer's location

Traveling costs affect the total price consumers pay for a product. We illustrate this with a *triangular* pdf $f(x)$ of consumer locations on the segment $[0,1]$, which permits flexibility in positioning the peak where most consumers are located. Information on product prices is presented to consumers in store and online. It is of particular interest to examine the situation in which consumer locations are biased towards one of the segment, as this is the more likely scenario in practice. Figures 5 and 6 show, respectively, the effect of consumer locations and the firm's location on the unit price (including traveling cost). Specifically, Fig. 5 shows that changes in the peak location may alter the total unit price by a maximum of approximately 4%. Furthermore, the average traveling distance can double (e.g., from $0.333d$ to $0.666d$) depending on the exact location of the peak.

Note that utilizing an assumption under which the firm is located at the far left end of $[0,1]$, even when there is no competition (with other firms), may be too subjective. In the

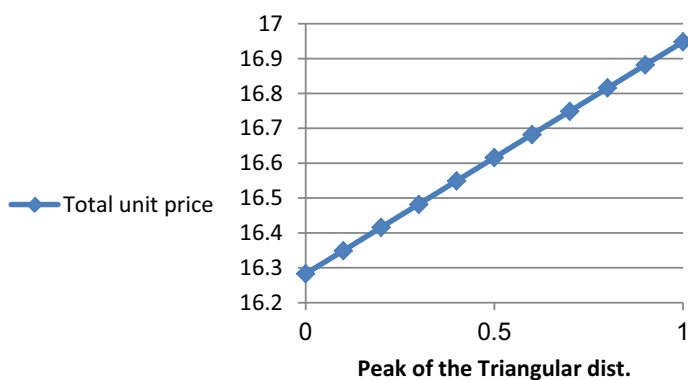


Fig. 5 Total unit price (in NIS) for different peaks of the triangular distribution of consumer locations

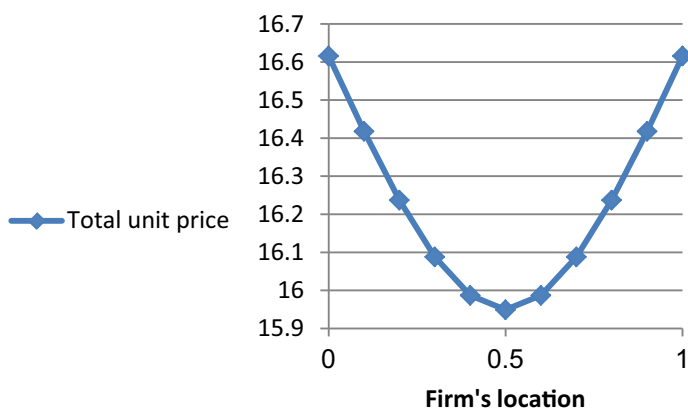


Fig. 6 Total unit price (the equilibrium cost to consumers, in NIS, of purchasing the product) for different firm locations

case where the firm is located at $x = x_0$, the average traveling distance is

$$d(x_0) = \int_0^{x_0} (x_0 - x)f(x)dx + \int_{x_0}^1 (x - x_0)f(x)dx, \quad (25)$$

which leads to a total unit price that varies as a function of firm location. Figure 6 presents the results under the assumption that the retailer may be located at points other than the far left end.

Figure 6 shows that, as expected, the smaller the average traveling distance (i.e., the closer the firm is to the peak of the distribution of consumer locations), the lower the total cost that end customers pay. The price ranges from NIS 15.949 to NIS 16.616, and the average price (across all firm locations) is approximately NIS 0.3 less (which equates to nearly 2% of the unit price) than when the firm is located at the far left end. Furthermore, the average traveling distance, calculated by (25), ranges from $0.166d$ to $0.5d$, which is at least $0.166d$ less than when the retailer is located at the far left end (i.e., a reduction of at least 25%). Of course, the closer the firm is to the peak of the consumer locations, the smaller the total average distance that consumers travel. The symmetric shape of both curves is attributed to the location of the peak at the midpoint (i.e., $x = 0.5$).

5.2 The equilibrium under the remaining cases of η_1

In the previous section it was assumed that $\eta_1 > 1$, meaning the regulator does not invest in any subsidies. According to Proposition 1, there are two additional alternatives, the first being $0 \leq \eta_1 < 1$ and the other, $\eta_1 = 1$. We next illustrate solutions for these alternatives.

5.2.1 The case of $0 \leq \eta_1 < 1$ and a non-binding budget

This range of the regulator's social attitude implies a strong preference for discounting the reference price as much as possible. According to (20) and Proposition 1, the outcome in this case is characterized by $w^*(p) = \text{NIS } 5.1$, $s^*(p) = 0$, and $S_1^*(p) = 50000p$. The meaning of $s^*(p) = 0$ is that the product can be purchased for free if the regulator has sufficient budget. Since the retailer is insensitive to the manufacturer's decisions, she fully determines the value of the regulator's objective. In this case, a condition should be imposed (e.g., by means of a regulatory request) that the non-subsidized reservation price α^{-1} is also valid for the regulator. According to our settings (see Table 1), $\alpha^{-1} = \text{NIS } 30$, so the optimal response function of the retailer is $p^*(s) = \alpha^{-1} = \text{NIS } 30$, $S_1^* = \text{NIS } 1500000$, the optimal profit per unit time is $\pi_R^*(T) = \Pi_R^*/T = \text{NIS } 201194.70$, and $I^*(T) = 0$. That is, the increased demand for cheap products results in a quick stockout within $T_A = 2.877$ weeks (instead of 6 weeks). Since the regulator has a virtually unlimited subsidy budget, the result is extreme. The retailer suffers shortages for more than 50% of the selling period, due to increased panic buying, which enables her to achieve more than 2.5 times the profit obtained in the previous case.

5.2.2 The case of $0 \leq \eta_1 < 1$ and a binding budget

This case is affected by the threshold ratio determined in Theorem 3. Figures 7 and 8 illustrate this effect for various amounts of budget $S_1^{\max}$.

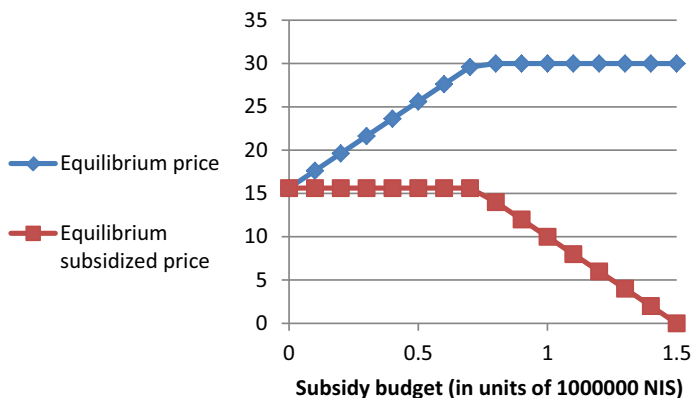


Fig. 7 Equilibrium price p^* and subsidized price $s^* p^*$ for different subsidy budget sizes

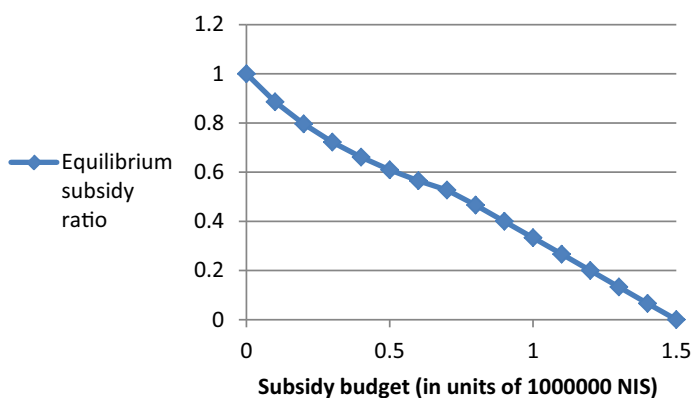


Fig. 8 Equilibrium subsidy ratio s^* for different subsidy budget sizes

Figures 7 and 8 indicate that the size of the budget significantly impacts all supply-chain parties. In particular, for a higher budget, the regulator is able to reduce the subsidy ratio, i.e., offer a greater discount to the consumer; see Fig. 8. However, the price that end consumers pay (i.e., the subsidized price) does not decrease until s falls below the threshold ratio. When there is a tighter limit on the budget, the subsidized price is higher, thereby decreasing the shortage period. In the extreme scenario under which the regulator does not invest in subsidies at all, there is no shortage, similar to the case of $\eta_1 > 1$ discussed above.

5.2.3 The case of $\eta_1 = 1$ and a binding budget

The regulator aims to discount the reference price in this case. According to Proposition 1, the subsidy ratio can be selected randomly, to represent any mixed strategy as well as a pure strategy, because this parameter does not affect the regulator's objective. Thus, we observe multiple equilibria. Note that the retailer is significantly affected by different choices of subsidy ratio s . The optimal pricing response of the retailer is presented in Fig. 9.

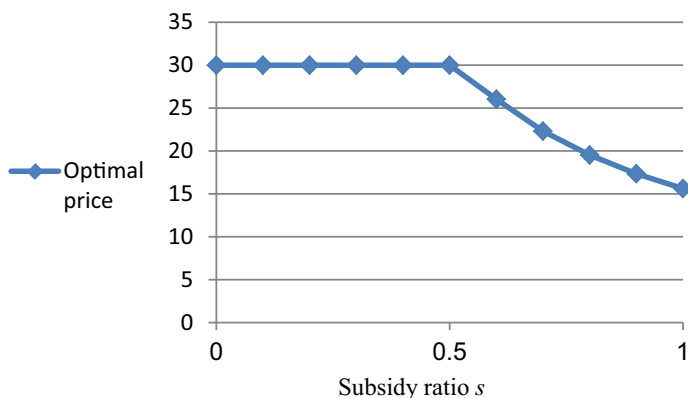


Fig. 9 Retailer's optimal pricing response (in NIS) for different values of the subsidy ratio s

Figure 10 presents the corresponding subsidized price for different values of the subsidy ratio s . The subsidized unit price sp linearly increases (Fig. 10) as long as the reference price is identical to the reservation price (i.e., for $s \leq 0.52$), as shown in Fig. 9. For $s > 0.52$, the subsidized unit price sp remains fixed at the level of the equilibrium price $p^* = \text{NIS } 15.616$.

In other words, in response to the regulator increasing the size of the discount (from 0% up to 48%), the retailer increases the reference price. As a result, the price that consumers pay remains the same. On the other hand, once the reference price is discounted by more than 48%, any further discount results in a lower final (discounted) price paid by the consumer. This can be as low as zero (i.e., the product is free), subject to the available subsidy budget. Consequently, if the subsidy budget $S_1^{\max}$ is sufficient to discount the price by more than 48%, the regulator is likely to choose an equilibrium such that $s = \max\left\{1 - \frac{S_1^{\max}}{\alpha^{-1}N}, 0\right\} = \max\left\{1 - \frac{S_1^{\max}}{1500000}, 0\right\}$. Figure 11 details the regulator's investment $S_1^*(s)$ as a function of the subsidy ratio.

Figure 11 shows a curve that consists of two segments. The first linearly decreases with subsidy ratio for $s \leq 0.52$, and the complementary segment decreases more moderately in

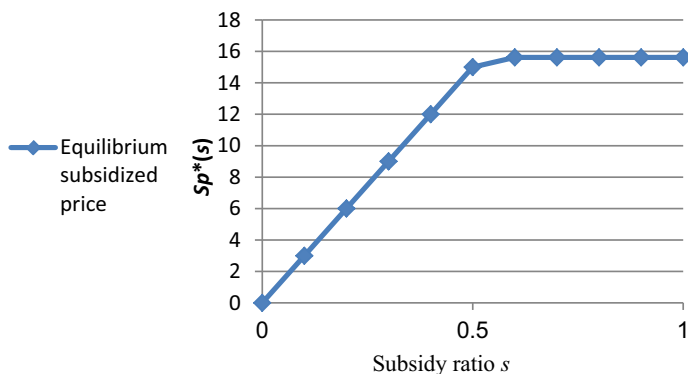


Fig. 10 Equilibrium discounted unit price (in NIS) for different values of the subsidy ratio s

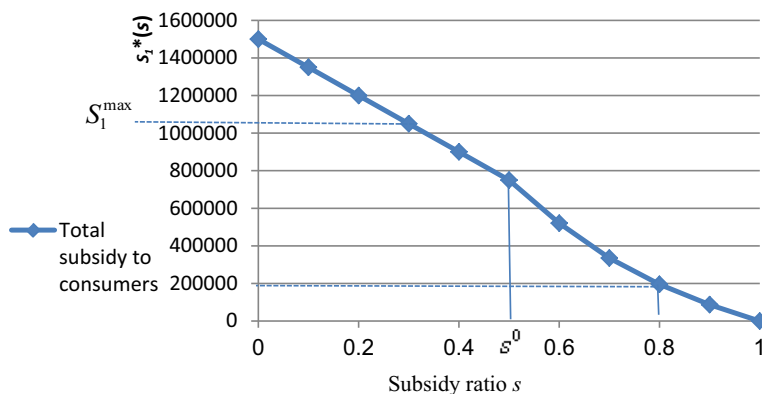


Fig. 11 Total subsidy invested by the regulator (in NIS) for the entire selling period for different values of the subsidy ratio s

a convex manner for $0.52 < s \leq 1$. This result indicates that the regulator has two modes of subsidy: one, which corresponds to the second segment, where the "discount" paid by the regulator does not actually benefit the consumer, and a second mode of subsidy, corresponding to the first segment, which is a "real" subsidy and is more expensive for the regulator. The following two examples better clarify those modes by assuming that the maximal budget the regulator can invest is $S_1^{\max} = \text{NIS } 1050000$.

We select two equilibria among multiple possibilities to illustrate the effect of the relationship between the threshold ratio and the available subsidy budget. Consider a 20% discount on the retailer's reference price, i.e., $s = 0.8$. This discount requires a total investment of NIS 195200 by the regulator, which is feasible under the given limitation of the budget. This amount is equivalent to NIS 3.904 for each item. The reference price is, however, raised by the retailer to NIS 19.520, meaning that consumers end up paying exactly the same unit price, NIS 15.616, as that proposed when the regulator did not support consumers at all (i.e., $p^*(s = 1)$). Here, the "discount" is not real, and the subsidy goes to the retailer rather than the consumers.

For $s = 0.2$, i.e., a discount of 80% on the reference price, the total investment by the regulator is NIS 1199880. This amount exceeds the available budget of NIS 1050000; therefore it is not feasible. For $s = 0.3$ (which is slightly greater than the threshold s^0 ; see Theorem 3), corresponding to a discount of 70% on the reference price, the regulator needs to invest NIS 1049895. This is feasible under the budget and is equivalent to NIS 21.000 for each item. Although the reference price is raised by the retailer to NIS 30, the consumers only pay a subsidized unit price (sp) of NIS 9.000, meaning that they receive a real discount of 42.3%. Here, the regulator not only pays the gap between the discounted price and the unit price of NIS 15.616 proposed when the regulator does not support consumers at all (i.e., $s = 1$), but he also contributes a significant additional amount that becomes an increase in the retailer's profit. Specifically, the retailer's profit per unit of time is NIS 198503.50 instead of NIS 74533.65. The amount per unit paid by the manufacturer (NIS 21.00) is more than three times larger than the absolute real discount (i.e., NIS 6.616).

Figure 12 presents the subsidy factor, which is the ratio between the amount paid by the regulator to subsidize each item (in NIS) and the real discount obtained by the consumer (in NIS). For all possible real discounts, the subsidy factor is greater than 1. That means that

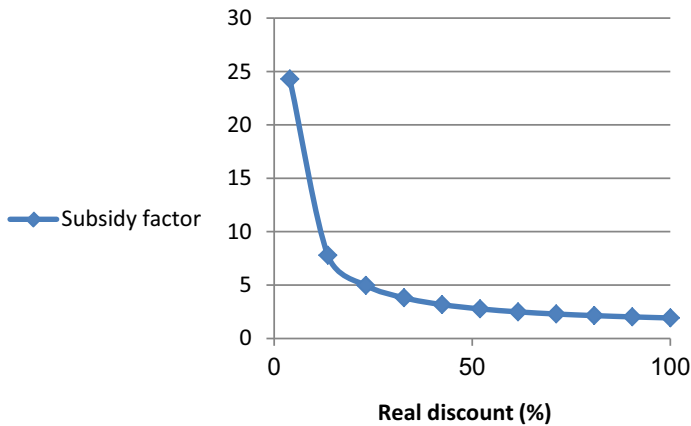


Fig. 12 Subsidy factor for different discount ratios

in order for consumers to benefit from a real discount, the regulator has to invest at least the amount saved. Interestingly, the greater the real discount (in %), the smaller the subsidy factor. The lower bound of the subsidy factor is 1.92. In general, the lower bound is $1/s^0$.

Figure 13 presents the equilibrium profit per unit time that the retailer gains given the subsidy ratio chosen by the regulator. We observe that the profit increases with greater subsidies (i.e., smaller values of s), which implies that among multiple equilibria, the retailer would favor the one that is characterized by the highest possible subsidy. Thus it is likely that both supply-chain parties would prefer this equilibrium, as the regulator is motivated by the welfare of consumers.

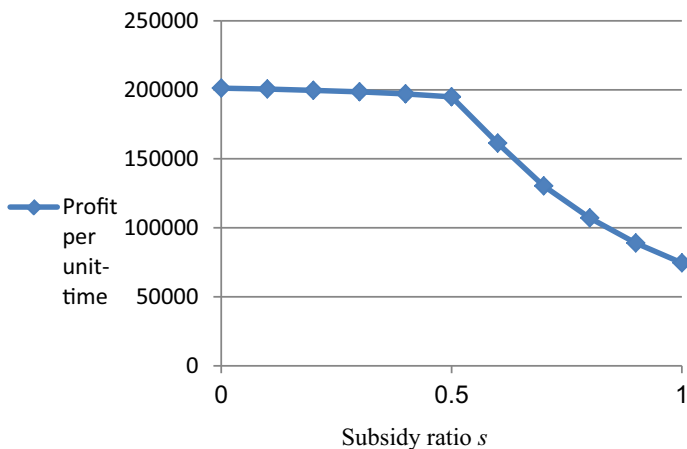


Fig. 13 Retailer's equilibrium profit per unit time $\pi_{Firm}^*(T)$ for different values of the subsidy ratio s

6 Conclusions

In this paper, we addressed price regulation under panic buying in a supply chain consisting of one manufacturer and one retailer. Regulation is attempted by the manufacturer who, driven by a corporate social responsibility objective, allocates some limited budget to control the retail price. Panic buying is considered to apply to products for which the demand increases as the quantity on the shelf decreases. We used a game-theoretic approach to model the interaction within the supply chain and we characterized the equilibrium prices and subsidies.

We showed that the retailer has strong leverage during the panic-buying period over consumers and over a manufacturer with a social objective. As a result, the manufacturer is unable to regulate price hikes during a panic through subsidization of the retailer, because the wholesale price does not affect the retailer's pricing policy. The only way for the regulator to keep the price under control is to provide direct discounting (e.g., with coupons and various types of rebate) to consumers. Even this method can be inefficient, as the retailer will increase her price when discounting is offered by the manufacturer. Nevertheless, if the financial commitment of the manufacturer is high enough, then there is a threshold discount ratio such that when the manufacturer offers a discount above this threshold, he is able to successfully regulate the price that consumers pay to purchase the product. That is to say, there is a minimal budget that the regulator would need to allocate in order for subsidization to make a real difference to consumers. In any case, the retailer will take advantage of panic buying, which allows her to drive up prices and thereby yield higher profits.

Through the numerical examples, we demonstrated that the length of the scarcity period is an influential factor. It actually determines whether a monetary benefit exists or not. In particular, setting a short selling period can make the firm prefer a shutdown (i.e., the retailer prefers not to sell at all). On the other hand, the retailer clearly exploits her monopolistic power when the selling period increases. In such a case, she keeps emptying her entire inventory while increasing the price per product unit, and accordingly, the profit. Further, traveling costs affect the price consumers pay for a product. We illustrated this by assuming a triangular probability density function of consumer locations, which permits flexibility in positioning the peak at which most consumers are located. We showed that under a given setting, the position of the peak may notably alter the total unit price and the traveling distance.

Several avenues for future research could extend the current study:

- (i) It would be worthwhile considering other factors that affect demand, e.g., the quality and perishability of the scarce product.
- (ii) We found that the selling period that the manufacturer estimates to be sufficient to respond to panic buying (by restoring the supplies) is an influential factor. The regulator may reconsider his policy of providing such information to the retailer. Consequently, selecting the best information policy for the regulator is an important area of study.
- (iii) Our model showed that under some scenarios, the retailer gains higher profits by having leftovers at the end of the selling period. Developing more complicated regulations to prevent such an outcome would be important for consumer welfare.
- (iv) Exploring the question of whether competition, which is known to be favorable for many regular products, is favorable in the case of panic buying could have important implications. The traveling cost, which affects the overall price that consumers pay for a product, is expected to play an important role in this scenario, as consumers may choose to travel a longer distance to buy a cheaper product.

Appendix A: Proof of Theorem 3

In the case where $s = 1$, objective (12) gives rise to the optimal price for this subsidy ratio, i.e., $p^*(s = 1)$. If we replace $p(N - I(p, s, T))$ in (12) with $ps(N - I(p, s, T))$, the revised objective (12) results in the same optimal subsidized price $sp = p^*(s = 1)$. When s keeps decreasing, maintaining $sp = p^*(s = 1)$ is only possible by increasing p (a more rigorous proof can be obtained by differentiating $sp(s)$ with respect to s and equating it to 0). Yet, since p cannot exceed the non-subsidized price, α^{-1} , there exists a point s^0 such that, for $0 \leq s < s^0$, $p^* = \alpha^{-1}$. $\square$

Appendix B: Proof of Proposition 1

- (a) For the case of $0 \leq \eta_1 < 1$ and $T_A(p, s) \leq T$, $I(p, s, T) = 0$. Under $\frac{S_1^{\max}}{N} \geq \alpha^{-1}$, the optimal response for the manufacturer (see (20.2)) is $s = 0$. The retailer's objective (12) increases with price p due to the fact that $s = 0$; this leads to markup pricing, as stated in this proposition (we assume that $p^*(s) = \alpha^{-1}$ is the maximal retail price). When $0 \leq \eta_1 < 1$ and $T_A(p, s) > T$ (and $s = 0$), the retailer's objective function is maximized by selecting the maximum possible price, as shown in Step 3 of Theorem 1.
- (b) We divide the condition of $\frac{S_1^{\max}}{N} < \alpha^{-1}$ into two cases. The first is when $(1 - s^0)\alpha^{-1}N < S_1^{\max} < \alpha^{-1}N$ and the second is $0 \leq S_1^{\max} \leq (1 - s^0)\alpha^{-1}N$. According to (9.1), $1 - \frac{S_1^{\max}}{pN} \leq s \leq 1$. For the first case, we conclude that $0 \leq s \leq s^0$ and accordingly: $p^* = \alpha^{-1}$ (see condition (3) of Theorem 3) and $s^* = 1 - \frac{S_1^{\max}}{N\alpha^{-1}}$ (see Theorem 2). For the second case, we conclude that $s^0 < s \leq 1$ and accordingly: $p^* = s^{-1}p^*(s = 1)$ (see condition (3) of Theorem 3) and $s^* = 1 - \frac{S_1^{\max}}{pN}$ (see Theorem 2). This equilibrium is identical to $p^* = \left(\frac{p^*(s=1)N}{p^*(s=1)N - S_1^{\max}}\right)p^*(s = 1)$ and $s^* = \frac{p^*(s=1)N - S_1^{\max}}{p^*(s=1)N} = 1 - \frac{S_1^{\max}}{p^*(s=1)N}$.
- (c) For $\eta_1 > 1$, the regulator's optimal response is not to subsidize, $s = 1$; hence, there is no regulation, regardless of the retail price p (see Theorem 2), while the equilibrium price is given by Theorem 1.
- (d) In the case where $\eta_1 = 1$, according to Theorem 2, the regulator's strategy is $s \in [0, 1]$ depending on the amount that he has determined he can spend on subsidization, i.e., there exists ρ , $0 \leq \rho \leq 1$, such that $Np(\rho)(1 - \rho) \leq$ and $s \in [\rho, 1]$. The corresponding equilibrium price is given by Theorem 1. $\square$

Appendix C: Proof of Proposition 2

In the special case where the demand rate weakly depends on inventory level, the FOC may be approximated by:

$$\begin{aligned}
 & -\beta(1 - \alpha sp)(\lambda_0 - \lambda_0^{\min})T + p\beta\alpha s(\lambda_0 - \lambda_0^{\min})T(1 + \beta(1 - \alpha sp)(\lambda_0 - \lambda_0^{\min})T) \\
 & + \frac{h}{\beta(1 - \alpha sp)(\lambda_0 - \lambda_0^{\min})}(\beta\alpha s(\lambda_0 - \lambda_0^{\min})T(1 + \beta(1 - \alpha sp)(\lambda_0 - \lambda_0^{\min})T))
 \end{aligned}$$

$$\begin{aligned}
& - \frac{\alpha sh}{\beta(1 - \alpha sp)^2(\lambda_0 - \lambda_0^{\min})} (\beta(1 - \alpha sp)(\lambda_0 - \lambda_0^{\min})T) = 0 \\
& - (1 - \alpha sp) + p\alpha s(1 + \beta(1 - \alpha sp)(\lambda_0 - \lambda_0^{\min})T) + \alpha shT = 0 \\
& - (1 - \alpha sp) + p\alpha s + p\alpha s\beta(1 - \alpha sp)(\lambda_0 - \lambda_0^{\min})T + \alpha shT = 0. \\
& - (\alpha s)^2\beta(\lambda_0 - \lambda_0^{\min})Tp^2 + (2\alpha s + \alpha s\beta(\lambda_0 - \lambda_0^{\min})T)p + \alpha shT - 1 = 0
\end{aligned}$$

The solution of the above equation is given in (24). $\square$

References

- Chernonog, T. (2020). Inventory and marketing policy in a supply chain of a perishable product. *International Journal of Production Economics*, 291, 259–274.
- Coady, D., Grosh, M., & Hoddinott, J. (2004). *Targeting of transfers in developing countries: Review of lessons and experience*. World Bank.
- Davis, L. W. (2014). The economic cost of global fuel subsidies. *American Economic Review*, 104(5), 581–585.
- Fox, E. J., Montgomery, A. L., & Lodish, M. L. (2004). Consumer shopping and spending across retail formats. *The Journal of Business*, 77(S2), S25–S60.
- Gu, X., Ieromonachou, P., & Li, Z. (2019). Subsidising an electric vehicle supply chain with imperfect information. *International Journal of Production Economics*, 211, 82–97.
- Le Blanc, D. (2008). A framework for analyzing tariffs and subsidies in water provision to urban households in developing countries. *UN Department of Economic and Social Affairs (DESA) Working Papers*, No. 63, UN, New York. <https://doi.org/10.18356/bfc5bf00-en>.
- Li, G., Zheng, H., Sethi, S. P., & Guan, X. (2020). Inducing downstream information sharing via manufacturer information acquisition and retailer subsidy. *Decision Sciences*, 51(3), 691–719.
- Liu, X., Li, J., Wu, J., & Zhang, G. (2017). Coordination of supply chain with a dominant retailer under government price regulation by revenue sharing contracts. *Annals of Operations Research*, 257(1–2), 587–612.
- Oruc, R. (2015). *The effects of product scarcity on consumer behavior: A meta-analysis*. Doctoral dissertation, Europa-Universität Viadrina, Frankfurt. Retrieved from 28 April, 2015, <https://opus4.kobv.de/opus4-euv/frontdoor/index/index/docId/180>.
- Post, S. (2011). Rumours trigger panic buying of salt across mainland. *South China Morning Post*, 18 March, 2011. <http://www.scmp.com/article/741250/rumours-trigger-panic-buying-salt-across-mainland-hk>.
- Saurwein, F. (2011). Regulatory choice for alternative modes of regulation: How context matters. *Law & Policy*, 33(3), 334–366.
- Schweiger, H., & Stepanov, A. (2018). European Bank for reconstruction and development, Working Paper No. 224 Prepared in December 2018.
- Senden, L. A. J. (2005). Soft law, self-regulation and co-regulation in European law: Where do they meet? *Electronic Journal of Comparative Law*, 9(1), 1–27.
- Tsao, Y. C., Raj, P. V. R. P., & Yu, V. (2018). Product substitution in different weights and brands considering customer segmentation and panic buying behavior. *Industrial Market Management*, 77, 209–220.
- Van Schooten, H., & Verschuuren, J. (2008). *International governance and law: State regulation and non-state law* (p. 2008). Edward Elgar.
- Verhallen, T. M. M. (1982). Scarcity and consumer choice behavior. *Journal of Economic Psychology*, 2(4), 299–322.
- Verhallen, T. M. M., & Robben, H. S. J. (1994). Scarcity and preference: An experiment on unavailability and product evaluation. *Journal of Economic Psychology*, 15(2), 315–331.
- Worchel, S., Lee, J., & Adewole, A. (1975). Effects of supply and demand on ratings of object value. *Journal of Personality and Social Psychology*, 32(5), 906–914.
- World Bank. (2020). Food security and covid-19. *World Bank Brief*, September 2020. <https://www.worldbank.org/en/topic/agriculture/brief/food-security-and-covid-19>.
- Yoon, J., Narasimhan, R., & Kim, M. K. (2018). Retailers sourcing strategy under consumer stockpiling in anticipation of supply disruptions. *International Journal of Production Research*, 56(10), 3615–3635.
- Zheng, L. (2020). We're entering the age of corporate social justice. *Harvard Business Review*, 15 June 2020.
- Zheng, R., Shou, B., & Yang, J. (2020). Supply disruption management under consumer panic buying and social learning effects, *OMEGA* (in press).

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