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Preface to Asen L. Dontchev Memorial Special Issue

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Asen L. Dontchev was born on June 19, 1948, in Pleven, Bulgaria, where he completed his high school education. Recognition of his mathematical talent inspired him to study at the Warsaw University of Technology; he completed his Ph.D. in control sciences in 1974. Upon returning to Bulgaria, Asen joined the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences, where he became a full professor in 1987. In 1989, Asen started a new chapter in a new country at a time of great change in Eastern Europe. He first accepted a visiting professorship at the University of Florida, where he started a collaboration with William Hager, and then in 1990, he accepted an editor position at Mathematical Reviews, where he served more than 30 years. In 2000 he became an adjunct professor at the University of Michigan, and from 2007 to 2009 he served as an Analysis Program Director in the National Science Foundation's Division of Mathematical Sciences.

During his distinguished research career, Asen made substantial contributions to analysis and optimization through more than 140 papers and six books. His last book,

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Lectures on Variational Analysis (2022), was published after his death on September 16, 2021. The early part of his career focused on sensitivity and error analysis of approximations of optimal control problems. His series of works on singular perturbations in optimal control (mostly co-authored with Vladimir Veliov) led to a new and influential understanding of limit problems. His work on constrained best approximation, and the justification of the Newton method for such problems, originates from this time period. Newton-type methods for nonsmooth and generalized equations were a favorite topic for his entire life.

Asen's visit to the University of Florida resulted in a paper with William Hager on Lipschitzian stability in nonlinear control and optimization that provided powerful machinery for analysis of stability properties of generalized equations such as those associated with the first-order optimality conditions at a local minimizer. This machinery, and the underlying ideas, have had a substantial impact on the optimization community, and resulted in numerous contributions by Asen and coauthors on the stability and error analysis of discrete approximations in optimal control. Asen's investigations in this period also resulted in a book (1993) with Tullio Zolezzi entitled *Well-Posed Optimization Problems*. Asen's works on inverse mappings and implicit function theorems for set-valued mappings also provide important contributions. His collaboration with Terry Rockafellar on fundamental questions of variational analysis was especially productive and culminated in the book *Implicit Functions and Solution Mappings: A View from Variational Analysis* (2009), which explored various regularity properties of set-valued mappings, implicit function theorems, the radius of preservation of regularity under disturbances, and applications in numerical analysis. The abstract notion of metric subregularity, a term coined by Asen and Terry in 2004 to describe a weaker regularity concept found in the literature at that time, was systematically investigated by Asen and subsequent collaborators and became a powerful tool in approximation theory for optimization problems. Asen used this theoretical foundation when he devoted a part of his final years to justification and improvements of the model predictive control method, which became the final chapter of his last book.

Asen provided outstanding services to the scientific community. He served on the editorial boards of numerous important journals, including SIAM Journal on Optimization, SIAM Journal on Control and Optimization, Computational Optimization and Applications, Journal of Mathematical Analysis and Applications, Journal of Optimization Theory and Applications, Journal of Dynamical and Control Systems, Open Applied Mathematics Journal, and Journal of Industrial and Management Optimization. Throughout his career, Asen mentored several generations of mathematicians. He was an excellent lecturer and possessed a remarkable ability to disseminate his knowledge and inspire young researchers. Due to his openness and friendliness, he had a large number of scientific contacts in universities all over the world and partnered with more than 60 coauthors. Asen loved all outdoor activities and was an avid fisherman. He was happiest and most relaxed on the water in his boat.

The 18 papers in this special issue, with 45 different authors, are a tribute to the memory of Asen L. Dontchev. The papers are by his former students and colleagues who have been influenced by his work, teaching, mentoring, and generous friendship.

A. The first group of papers are related to dynamics and optimal control, focusing on analysis of numerical methods and underlying properties such as solution stability and consistency of approximations.

In [4], Z. Artstein examines order reduction of linear stationary singularly perturbed systems. In contrast to the traditional approaches, the system is not split into slow and fast variables. Instead, a new order reduction framework is developed which may improve the efficiency of computations.

Paper [5] by R. Baier and E. Farkhi presents an extension of the Filippov stability theorem for the solutions of differential inclusions under weaker conditions than Lipschitz continuity of the right-hand side. The result strengthens existing theory and is illustrated by examples.

Paper [9] by A. Corella, N. Jork and V. Veliov investigates the property of strong metric subregularity of the mapping associated with the first-order (Pontryagin type) optimality conditions for affine optimal control problems arising in parabolic partial differential equations. Several subregularity and stability results are obtained that provide a foundation for error analysis of Tikhonov's regularization and discretization methods.

In [10], G. Eichfelder, L. Grüne and L. Krügel extend the model predictive control method to multi-objective control problems for non-linear ordinary differential equations with control and terminal constraints. The proposed algorithm ensures asymptotic stability of the feedback control together with performance estimates for the cost criteria.

In [14], W. W. Hager gives an extension of the Switch Point Algorithm [2] for solving optimal control problems whose solutions are singular and/or bang-bang with a finite number of switches. The novelty is the presence of initial and terminal constraints on the state, which substantially complicates the numerical solution of the problem.

Paper [15] by Y. Kaya and H. Maurer considers the problem of finding an optimal solution over the Pareto front in a multiobjective optimal control problem. The algorithm for optimal control problems with two objectives incorporates Chebyshev scalarization, the concept of essential interval weights, and a bisection method.

Paper [17] by B. Martens presents error estimates for Runge-Kutta schemes applied to optimal control problems subject to differential-algebraic equations. A key point is that in the Runge-Kutta discretization, the algebraic subsystem is treated as a mixed state-control constraint with the algebraic variables regarded as controls.

B. As mentioned earlier in the preface, the properties of metric regularity and subregularity of set-valued mappings play an important role in the numerical analysis of optimization methods. The following set of papers present extended versions of this property and the associated analysis.

Paper [7] by J. Camacho, M. López and J. Parra introduces the new notions of robust and continuous metric subregularity of mappings associated with finite systems of linear inequalities. The radii of these types of regularity are always positive (in contrast to the standard radius of subregularity) and are characterized and estimated.

In [11], H. Gfrerer and A. Kruger extend a previous work by the authors and Asen Dontchev on the radius of subregularity by considering set valued mappings in Banach/Asplund spaces and considering various classes of perturbations.

In [13], S. Guo, H.-D. Qi and L. Zhang establish conditions for strong regularity and stability of local solutions to the Euclidean distance matrix optimization problem. The results extend known characterizations of these properties for semidefinite programming problems.

C. The next set of papers presents general algorithms and particular applications.

S. Adly, H. Attouch, and M. Le in [1] develop first-order optimization algorithms based on a temporal discretization of a damped dynamics involving dry friction. The function to be minimized could be either differentiable, or convex and potentially nonsmooth. Applications and comparisons are made using the Lasso problem and nonsmooth d.c. programming.

In [3], F. Aragón-Artacho, Y. Malitsky and M. Tam propose and analyze algorithms for finding equilibria of finite sums of maximal monotone operators and coercive functions. The algorithms are of forward-backward type and are suitable for a distributed decentralized implementation in ring networks.

In [6], R. Boţ, C. Ernö and M. Sedlmayer develop a gradient/proximal algorithm OGAProx for solving convex-concave saddle point problems, which are smooth in one variable and nonsmooth in the other. Relative to the function values, the convergence rates for convex-concave, convex-strongly concave, and strongly convex-strongly concave functions are $\mathcal{O}(1/K)$, $\mathcal{O}(1/K^2)$, and $\mathcal{O}(\theta^K)$ respectively, with $\theta < 1$.

In [8], J. Christiansen, B. Dandurand, A. Eberhard and F. Oliveira analyze the application of progressive hedging to stochastic mixed-integer programming problems. The theory shows that convergence to a feasible solution should be expected; moreover, insights into the convergence of proximal-point-like methods in the presence of integer variables is provided.

Paper [12] by H. Gfrerer, M. Mandlmayr, J. Outrata and J. Valdman proposes a conceptual semismooth Newton-type method for finding an equilibrium point of a set-valued mapping. Superlinear convergence is proved under conditions formulated in terms of a reasonably defined derivative of the mapping, and is demonstrated on a static contact problem with Coulomb friction.

In [16], Z. Luo and L. Qi propose necessary and/or sufficient optimality conditions for Tucker low-rank tensor optimization problems. Low-rank tensor

optimization naturally arise when taking into account structures or features in the high-dimensional multi-way datasets.

In [18], R. T. Rockafellar analyzes the convergence of a variable-metric extension of the proximal point method. In a generic sense, the convergence is Q-linear at a rate that depends in a simple way on the modulus of metric subregularity of the mapping at the solution.

The paper [19] of C. Vitt, D. Dentcheva, A. Ruszczyński and N. Sandberg is motivated by the design of radiation therapy for the treatment of cancer. The optimization model must allow the safe delivery of radiation to a tumor while avoiding damage to vital organs. This leads to the development of risk-averse stochastic optimization problems along with efficient and robust solution algorithms. The paper includes numerical experiments with clinical data and comparisons to solutions currently used by the hospital.

The editors of this special issue are collaborators and friends of Asen for many decades, and we use this opportunity to express our deep appreciation to him as an esteemed scientist, mentor and dear friend.

References

1. Adly, S., Attouch, H., Le, M.: First order inertial optimization algorithms with threshold effects associated with dry friction. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00509-9
2. Aghaee, M., Hager, W.: The switch point algorithm. SIAM J. Control. Optim. **59**, 2570–2593 (2021). https://doi.org/10.1137/21M1393315
3. Aragón-Artacho, F., Malitsky, Y., Tam, M.: Distributed forward-backward methods for ring networks. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-022-00400-z
4. Artstein, Z.: Comput. Optim. Appl. (2022). Linear singularly perturbed systems without slow-fast split. https://doi.org/10.1007/s10589-022-00412-9
5. Baier, R., Farkhi, E.: A Filippov approximation theorem for strengthened one-sided Lipschitz differential inclusions. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00517-9
6. Boţ, R., Ernö, C., Sedlmayer, M.: An accelerated minimax algorithm for convex-concave saddle point problems with nonsmooth coupling function. Comput. Optim. Appl. (2022). https://doi.org/10.1007/s10589-022-00378-8
7. Camacho, J., López, M., Parra, J.: Robust and continuous metric subregularity for linear inequality systems. Comput. Optim. Appl. (2022). https://doi.org/10.1007/s10589-022-00437-0
8. Christiansen, J., Dandurand, B., Eberhard, A., Oliveira, F.: A study of progressive hedging for stochastic integer programming. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00532-w
9. Corella, A., Jork, N., Veliov, V.: On the solution stability of parabolic optimal control problems. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00473-4
10. Eichfelder, G., Grüne, L., Krügel, L.: Relaxed dissipativity assumptions and a simplified algorithm for multiobjective MPC. Comput. Optim. Appl. (2022). https://doi.org/10.1007/s10589-022-00398-4
11. Gfrerer, H., Kruger, A.: Radius theorems for subregularity in infinite dimensions. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-022-00431-6
12. Gfrerer, H., Mandlmayr, M., Outrata, J., Valdman, J.: On the SCD semismooth Newton method for generalized equations with application to a class of static contact problems with Coulomb friction. Comput. Optim. Appl. (2022). https://doi.org/10.1007/s10589-022-00429-0
13. Guo, S., Qi, H.-D., Zhang, L.: Perturbation analysis of the euclidean distance matrix optimization problem and its numerical implications. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00505-z

14. Hager, W.: Extension of switch point algorithm to boundary-value problems. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00530-y
15. Kaya, C., Maurer, H.: Optimization over the Pareto front of nonconvex multi-objective optimal control problems. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00535-7
16. Luo, Z., Qi, L.: Optimality conditions for Tucker low-rank tensor optimization. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00465-4
17. Martens, B.: Error estimates for Runge-Kutta schemes of optimal control problems with index 1 DAEs. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00484-1
18. Rockafellar, R.: Generic linear convergence through metric subregularity in a variable-metric extension of the proximal point algorithm. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00494-z
19. Vitt, C., Dentcheva, D., Ruszczyński, A., Sandberg, N.: The deepest event cuts in risk-averse optimization with application to radiation therapy design. Comput. Optim. Appl. (2023). https://doi.org/10.1007/s10589-023-00531-x