



Floating-point numbers round-off error analysis by constraint programming

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Abstract

Floating-point numbers are used in many applications to perform computations, often without the user's knowledge. The mathematical models of these applications use real numbers that are often not representable on a computer. Indeed, a finite binary representation is not sufficient to represent the continuous and infinite set of real numbers. The problem is that computing with floating-point numbers often introduces a rounding error compared to its equivalent over real numbers. Knowing the order of magnitude of this error is essential in order to correctly understand the behaviour of a program. Many error analysis tools calculate an over-approximation of the errors. These over-approximations are often too coarse to effectively assess the impact of the error on the behaviour of the program. Other tools calculate an under-approximation of the maximum error, i.e., the largest possible error in absolute value. These under-approximations are either incorrect or unreachable. In this thesis, we propose a constraint system capable of capturing and reasoning about the error produced by a program that performs computations with floating-point numbers. We also propose an algorithm to search for the maximum error. For this purpose, our algorithm computes both a rigorous over-approximation and a rigorous under-approximation of the maximum error. An over-approximation is obtained from the constraint system for the errors, while a reachable under-approximation is produced using a generate-and-test procedure and a local search. Our algorithm is the first to combine both an over-approximation and an under-approximation of the error. Our methods are implemented in a solver, called FERa. Performance on a set of common problems is competitive: the rigorous enclosure produced is accurate and compares well with other state-of-the-art tools.

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