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A systematic review of uncertainty theory with the use of scientometrical method

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Abstract

Uncertainty theory is an area in axiomatic mathematics recently proposed by Professor Baoding Liu and aiming to deal with belief degrees. Retrieving 1004 journal articles from the Web of Science database between 2008 and 2019, and utilizing CiteSpace and Pajek software, we analyze the publications per year and by geographical distribution, productive scholars and their cooperation, key journals, highly cited articles and main paths of the field. In this way, seven key sub-fields of uncertainty theory and their research potential are derived. The results show the following: (1) The literature on uncertainty theory follows a linear growth trend, involves an extensive network of 1000 scholars worldwide and is published in 300 journals, indicating thus that uncertainty theory has become increasingly attractive, and its academic influence is gradually expanding. (2) Seven key sub-fields of uncertainty theory have clearly been identified, including the axiomatic system, uncertain programming, uncertain sets, uncertain logic, uncertain differential equations, uncertain risk analysis, and uncertain processes. Among them, uncertain differential equations and programming are the two main sub-fields with the largest numbers of published papers. Furthermore, for evaluating the research potential of sub-fields, maturity and recent attention indicators are calculated using the citations, total number of publications, quantity of most cited literature and half-life. Based on these indicators, uncertain processes shows the greatest development potential, and has remained a hot topic in recent years, being mainly concentrated on the uncertain renewal reward process, optimal control of discrete-time uncertain systems, and uncertain linear quadratic optimal control. Additionally, uncertain risk analysis is ranked second, and focuses on the analysis of expected losses, investment risk, and structural reliability of uncertain systems.

Keywords Uncertainty theory · Bibliometrics · CiteSpace · Key sub-fields · Research potential

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1 Introduction

Uncertainty theory, proposed by Professor Baoding Liu (Tsinghua University, China) in 2007, is relatively new but has become an increasingly popular branch of axiomatic mathematics used to deal with the "belief degree" Liu (2007). Although probability has been considered one of the most widely used theories among academics and practitioners with which to model uncertainty, and thus to explore the inherent law of random events, it may not be sufficient to solve all types of uncertainties, especially those related to human beings' personal beliefs (Liu, 2010a). For fulfilling this demand, uncertainty theory has been substantially developed lately to provide an advanced and versatile mathematical tool for modelling uncertainties and overcoming counter-intuitive results caused by probability and/or fuzzy set theories when they are preferred instead (see, (Liu, 2012c)).

Since its foundation year of 2007, uncertainty theory has increasingly attracted extensive attention among scholars. As a consequence, a large volume of research outputs has been published, and thus some remarkable achievements in both theory and practice have been reported. Representatively, among the numerous theoretical outputs, Liu's two books can be regarded as seminal in this field. The first version of his book entitled *Uncertainty Theory*, published in 2007, provides the axiomatic foundations needed for the most recent development of uncertainty theory. Since then, Liu has regularly extended and updated the 2007 version, with three more editions published in 2010, 2012 and 2015 (see, (Liu, 2010a, 2012a, 2015)). His second book, entitled *Theory and Practice of Uncertain Programming*, published in 2009 (Liu, 2009a), expounds some important mathematical concepts, discusses the design of algorithms, and provides some interesting and realistic applications of uncertain programming.

Although many academics have enthusiastically followed Professor Baoding Liu's research directions, no scholars have yet reviewed and presented comprehensively the developments and research trends of uncertainty theory. The majority of existing publications have just reviewed certain branches of uncertainty theory or fuzzy theory, such as uncertain linear regression models (Guo et al., 2011), uncertain sets (Yang & Gao, 2014), the uncertain portfolio selection (Huang, 2017) and other fuzzy subjects (Diaz-Curbelo et al., 2020; Ji et al., 2021; Verma et al., 2022). Therefore, this paper aims to systematically review and categorically present the developments of uncertainty theory in order to summarize the current achievements and promote the future developments, which can then be further used and explored in any subsequent deployments.

For an adequate analysis of existing literature, bibliometrics provides significant advantages when compared with alternative, more traditional literature review methods (e.g. (Broadus, 1987)). Mapping published records, it is now broadly acknowledged as a reliable method for evaluating detailed academic topics in information science, and thus has become one of the most important way of summarizing and predicting hot trends in research (Fahimnia et al., 2015). However, as a method it also has some critical limitations. In particular, the

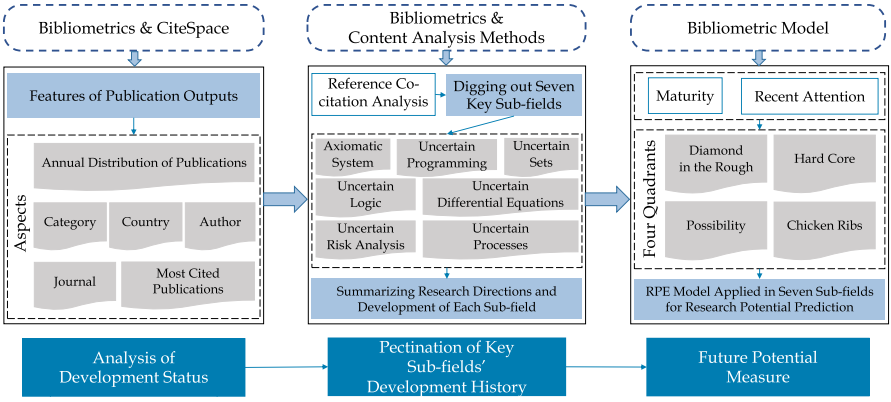

Fig. 1 Framework diagram of the paper: In particular, the three blue boxes at the bottom of the figure represent the three research directions of this paper. Analytically, this study searches carefully and systematically the development history and seven key sub-fields of uncertainty theory, and then evaluates future research potential of sub-fields. The three solid-line boxes connected by thick blue arrows in the upper part indicate the main research content at different stages, and the top dashed boxes show the corresponding research methods. Specifically, this paper first uses bibliometrics and CiteSpace software to reveal the development status of this field from six perspectives, and combines bibliometrics and content analysis methods based on the results of co-citation analysis to summarize the research characteristics and hot topics, and then uses the research potential evaluation model that is based on the maturity and recent attention indicators, and is divided into four potential quadrants to evaluate the future potential of key sub-fields

information extracted only reflects general, macro-level characteristics and/or trends in the area of interest, so that it might not be sufficient for investigating micro-level and more detailed characteristics. In order to overcome these important limitations, Pedrini and Ferri (2019) systematically reviewed articles on stakeholder management by adding qualitative content analysis to bibliometrics, and Luo et al. (2018) did something similar when reviewing the agri-food supply chain management literature and proposing some potential future directions for it. Following them, our paper uses CiteSpace software (Chen, 2017) to combine bibliometrics and content analysis effectively in order to provide a comprehensive overview of uncertainty theory from many different angles.

We chose the *Web of Science* (WoS) database as our source of literature, and selected "uncertainty theory" as the subject of retrieval, carried out on January 1, 2020. Further, the publications categories were limited to "article" and "review", and the time span was from January 1, 2008 to December 31, 2019. Moreover, we manually retrieved relevant data to establish a reference bank of 1,004 publications, of which 952 articles were derived from the *Science Citation Index* (SCI) and *Social Science Citation Index* (SSCI) databases, and 52 are documents from other databases.

Figure 1 provides the framework diagram of this paper. As a first comprehensive study, this article not only analyses the development status of uncertainty theory from six features of publication outputs, but also explores research contents, hot issues and future potential of seven sub-fields by means of bibliometrics and

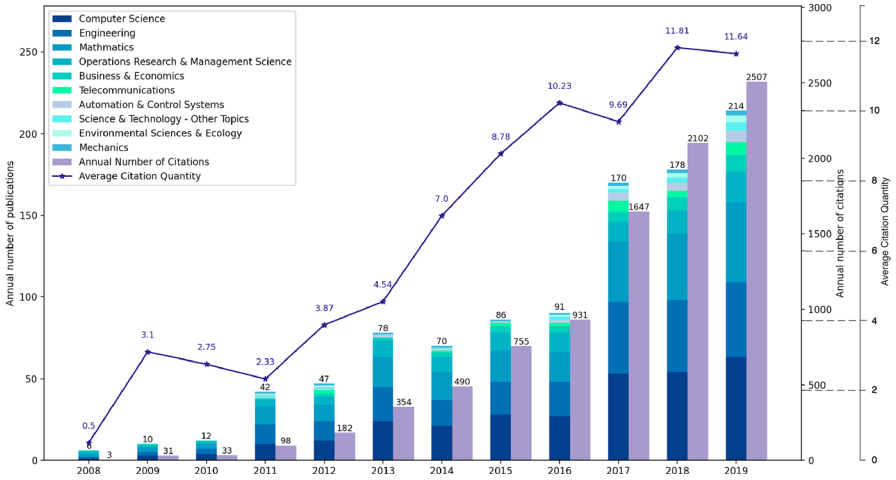

Fig. 2 Annual numbers of publications and citations on uncertainty theory from 2008 to 2019

content analysis methods. The remainder of the paper is structured as follows. Section 2 analyses the features of publication outputs, including the distribution of publications per year, the publications' category, country, author(s), and journal, and the most cited publications, and then elaborates the past and current developments of uncertainty theory facilitated by bibliometrics and the CS software. Section 3 employs the clustering results of co-citation analysis to identify seven key sub-fields of uncertainty theory, and then utilizes a specific content analysis method to sort out and summarize the research directions and development of each sub-field. The RPE model, which is developed based on two indicators, i.e., maturity and recent attention, is applied to uncertainty theory in Sect. 4 to assess and predict future trends for each sub-field. Finally, Sect. 5 concludes the whole discussion and reiterates the importance of our findings.

2 Features of publication outputs

In this section, we present the current status of uncertainty theory. In particular, we report the number of publications, categories, geographical distribution, leading scholars and their cooperation, productive journals and highly cited papers in the corresponding field. Using bibliometric methods and the CiteSpace software, we clearly depict the growth trajectory of uncertainty theory over the past twelve years, and illustrate the general outline of its development from various perspectives.

2.1 Annual distribution of publications

The distribution of publications and citations over time clearly reflects the overall development and can be used to gain an appreciation of the status of a research area in academia. In this direction, our paper employs a three-axis diagram (see

Fig. 2) to respectively describe the number of publications per year (stacked figure), citations (purple pillars), and average citation quantities (broken line). The coloured columns of different lengths indicate the numbers of publications for the top 10 categories per year, and the numbers of citations are provided by another single-coloured column per year. The average citation quantity is the annual number of citations divided by the publications.

As shown in Fig. 2, the number of publications has increased considerably year by year, especially during the past three years, i.e., 2017-2019, rising from less than 10 articles in the first year, more than 70 in 2013, to 170 by 2017, and exceeding 200 for the first time in 2019. This significant achievement reflects that, although the number of papers published per year on uncertainty theory is relatively small, which also justifies possible fluctuations, the overall trend is consistently increasing. In addition, we observe that more than half of the articles in this field were published between 2017 and 2019, revealing that this area of research has entered a period of important growth.

The number of citations shows a significant exponential growth trend, rising from a single digit in 2008 to 182 in 2012, 755 in 2015, well over 1,000 in 2017, and surpassing 2,000 in 2018. This reflects that an increasing number of scholars are actively involved in the developments of uncertainty theory, and 2011 and 2017 are the two key years with relatively higher numbers.

Over the course of time, concepts of uncertainty theory have gradually been applied in several areas of research, including but not limited to mathematics, computer science, engineering, operations research, management science, business and economics, environmental sciences, ecology, and telecommunications, which provides support to our endeavour to approach this as a multidisciplinary field of research. In Fig. 2, the top ten categories by published volume are marked with columns of different colours. The wider is the length of the column, the more literature the discipline has. As shown in Fig. 2, computer science has become the most prolific category, followed by engineering and mathematics. Computer science publishes 37.89% of the total number of papers in this field, utilizing numerical algorithms, numerical simulation, multi-objective genetic algorithms, and hybrid intelligent algorithms to solve the uncertain programming, optimal control, and graph problems. The category of engineering accounts for 20.66% of all publications, dealing with supply chain network design, vehicle routing problems, parallel machine scheduling, system reliability, risk analysis, and others. With 11.60% of all publications, the category of mathematics deals with complex uncertain variables, uncertain equations of optimality, uncertain random sequences, uncertain calculus, uncertainty distributions, and uncertain sets. Overall, uncertainty theory has successfully been proposed and developed in these three categories, where synergies appear in a straightforward manner, and thus significant achievements have been reported so far.

To sum up, we observe that uncertainty theory, although it might be considered as a relatively new and currently under development area of multidisciplinary (and axiomatic) mathematics, possess strong influence in academia. In this direction, not only the numbers of papers and citations have significantly been

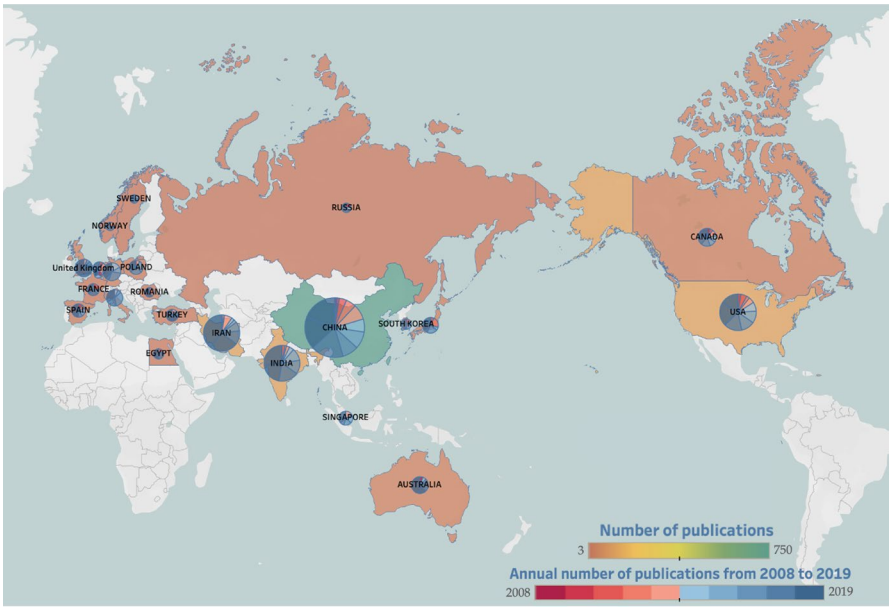

Fig. 3 The geographical distribution of uncertainty theory from 2008 to 2019

increased lately, but also the research areas and concepts that uncertainty theory has influenced is consistently being increased.

2.2 Cooperation analysis

Nowadays, collaborative research plays a progressively more important role in academia. This paper analyses the cooperative research in this field from the perspectives of countries, institutions and authors. In terms of research methodology, the degree of collaboration among them is calculated by mathematical formula, and the collaborative analysis results were obtained by using the author analysis function of the CiteSpace software.

Now collecting and counting the numbers of publications from various countries, we plot the geographical distribution in Fig. 3, where the colour and size of the pie chart are related to the amount of publications from each country in the field. The larger the pie chart and the greener the country's colour, the more publications there are in that country. In the pie chart, the number of articles published from 2008 to 2019 is highlighted by shading that changes from blue to red. As expected, China, the birthplace of uncertainty theory, produces the largest number of publications, accounting for about 70% of the total number of publications in the field. After this, the United States, Iran, and India have the highest numbers of documents, accounting for 7.47%, 7.37%, and 7.17%, respectively, followed by Germany, Canada, the U.K., Australia, Italy, and Japan.

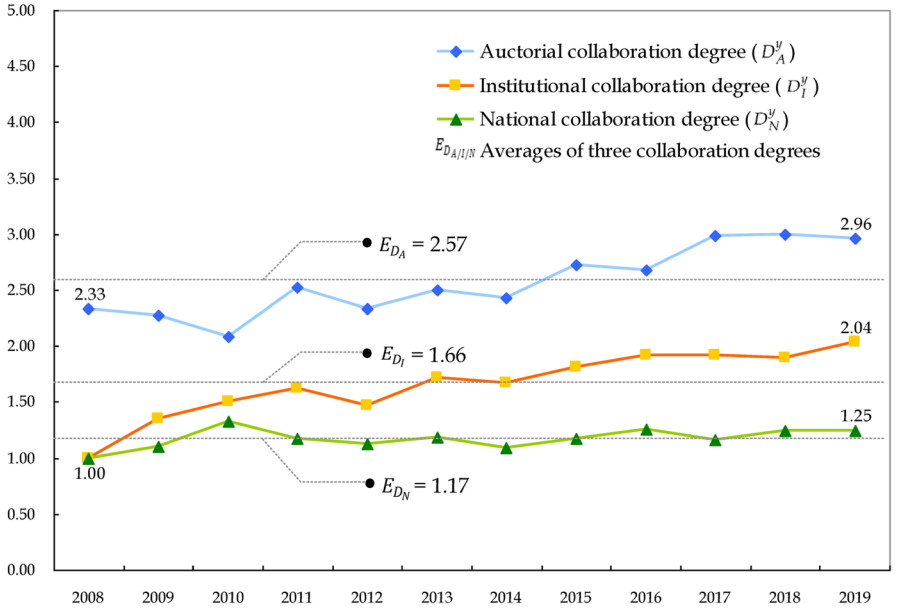

Fig. 4 The three scales of collaboration degree for uncertainty theory from 2008 to 2019

The data collected are from 738 institutions and 1,370 scholars, and around 50% of the institutions and 65% of the scholars are from China. Tsinghua University, with 85 papers, accounts for 8.47% of the total number of publications, followed by Nanjing University of Science Technology (66 papers), Beihang University (49 papers), Renmin University of China (46 papers), the University of the Chinese Academy of Sciences (40 papers), Tianjin University (40 papers), and Tongji University (40 papers).

To calculate the cooperative relations that exist among countries, institutions and scholars, this paper refers to Yu et al. (2016). The calculation formulas for the three are as follows: $D_A^y = \sum_{i=1}^{T_y} A_{i,y}/T_y$, $D_I^y = \sum_{i=1}^{T_y} I_{i,y}/T_y$, $D_N^y = \sum_{i=1}^{T_y} N_{i,y}/T_y$, where D_A^y, D_I^y, and D_N^y represent the degrees of cooperation among authors, institutions, and countries in year y, respectively; $A_{i,y}$, $I_{i,y}$ and $N_{i,y}$ are defined as the number of authors, institutions and countries of the i-th publication in year y; T_y is the total quantity of literature in year y; y ranges from 2008 to 2019.

The derived results are presented in a line chart, as shown in Fig. 4. E_{D_A}, E_{D_I} and E_{D_N} are the averages of the three cooperation degrees, calculated as $E_{D_{A/I/N}} = \sum_{y=2008}^{2019} D_{A/I/N}^y/12$. It can be seen from the figure that authors have the highest degree of cooperation, with an average of 2.57, followed by institutions with an average of 1.66. Although the degrees of cooperation between authors and institutions are not too high, the overall trend is growing. Moreover, the level of national cooperation has been hovering at a low level for many years, with an average value of only 1.17. Overall, even though academic cooperation in this field is gradually expanding in scale and scope, it is believed that it is still rather limited, and thus, there is important ground for further expansion and growth.

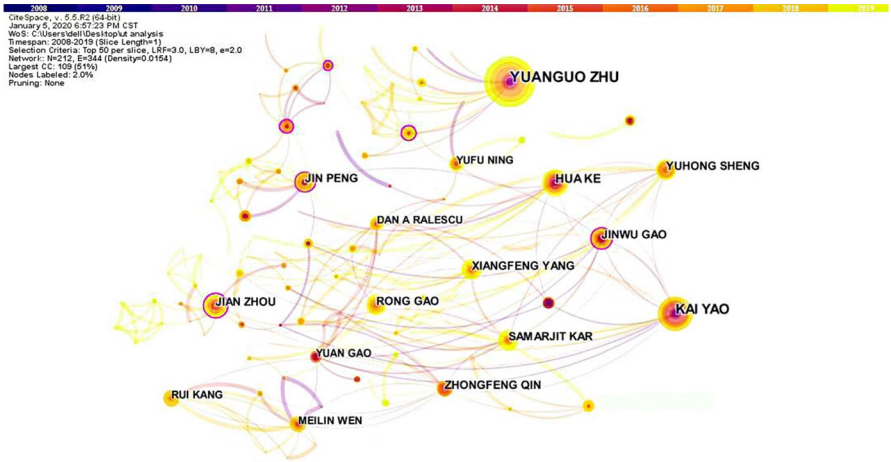

Fig. 5 The network of the productive authors on uncertainty theory from 2008 to 2019

After applying a general approach to look at the cooperation between countries, institutions and authors with respect to uncertainty theory, this paper now focuses on a more detailed explanation of the cooperation among authors. Since CiteSpace software can clearly show more detail about the length and intensity of collaborations between authors, this paper applies its author analysis function to further investigate author cooperation, and the results are presented in Fig. 5. Each colour zone of the gradient line above the knowledge graph represents a year from 2008 to 2019, corresponding to the multicoloured annual rings of the circular nodes, whose width reflects the amount of publications. Each node stands for a researcher, and the names of the 16 most productive authors are shown in the figure. The colour and thickness of the lines between the nodes of cooperative relationships in different regions indicate the time and closeness of the initial cooperation, respectively. As illustrated in Fig. 5, the joints between nodes are numerous, with different thicknesses, and colours which are mainly warm, that is, close to yellow. The lack of blue illustrates that, recently, most of the scholars have preferred to work in teams, and have worked closely with each other. Moreover, most of the high-yielding authors are from China, and all of the foreign productive scholars are in close relationships with them.

The above analysis verifies that Chinese scholars engaged in the developments of uncertainty theory have already established research teams domestically, while no similar teams are observed internationally. However, the degree of cooperation among authors, institutions and countries is gradually being increased, and consequently, international groups might also be established in the foreseen future.

2.3 Journal analysis

Each journal specializes in one or more areas of research and has its own research preferences. Through the analysis of the journals, we can identify which of them

Table 1 The 20 most productive journals on uncertainty theory from 2008 to 2019

Rank	Title	FY	IF	CS	Discipline	TP	PUT	PUT/TP	IY	LY
1	Fuzzy Optimization and Decision Making	2002	4.319	6.60	Computer Science, Artificial Intelligence (Q1); Operations Research & Management Science (Q1)	279	61	21.86%	2010	2019
2	International Journal of Uncertainty, Fuzziness and Knowlege-Based Systems	1996	1.375	2.30	Computer Science, Artificial Intelligence (Q4)	672	31	4.61%	2009	2019
3	Soft Computing	1997	3.050	4.70	Computer Science, Artificial Intelligence (Q2); Computer Science, Interdisciplinary Applications (Q2)	3777	82	2.17%	2009	2019
4	Journal of Ambient Intelligence and Humanized Computing	2010	4.594	3.80	Computer Science, Artificial Intelligence (Q1); Computer Science, Information Systems (Q1); Telecommunications (Q1)	983	21	2.14%	2017	2019
5	Journal of Intelligent Manufacturing	1990	4.311	8.10	Computer Science, Artificial Intelligence (Q1); Engineering, Manufacturing (Q1)	1319	25	1.90%	2012	2017
6	Journal of Intelligent & Fuzzy Systems	1993	1.851	2.60	Computer Science, Artificial Intelligence (Q3)	4894	91	1.86%	2013	2019
7	IEEE Transactions on Fuzzy Systems	1993	9.518	16.20	Computer Science, Artificial Intelligence (Q1); Engineering, Electrical & Electronic (Q1)	1714	24	1.40%	2012	2019
8	Journal of Industrial and Management Optimization	2005	1.366	1.80	Engineering, Multidisciplinary (Q3); Mathematics, Interdisciplinary Applications (Q3); Operations Research & Management Science (Q3)	901	11	1.22%	2015	2019
9	European Journal of Control	1995	1.540	3.30	Automation & Control Systems (Q3)	518	6	1.16%	2017	2019
10	Iranian Journal of Fuzzy Systems	2004	2.276	3.20	Mathematics (Q1); Mathematics, Applied (Q1)	611	6	0.98%	2013	2018
11	International Journal of General Systems	1974	1.671	4.10	Computer Science, Theory & Methods (Q2)	522	5	0.96%	2011	2018
12	International Journal of Machine Learning and Cybernetics	2010	3.753	6.00	Computer Science, Artificial Intelligence (Q2)	966	9	0.93%	2013	2019
13	International Journal of Fuzzy Systems	2004	4.406	5.80	Automation & Control Systems (Q1); Computer Science, Artificial Intelligence (Q1)	998	6	0.60%	2011	2019
14	Computers & Industrial Engineering	1976	4.135	6.60	Computer Science, Interdisciplinary Applications (Q1); Engineering, Industrial (Q1)	3864	20	0.52%	2011	2019

Table 1 (continued)

Rank	Title	FY	IF	CS	Discipline	TP	PUT	PUT/TP	IY	LY
15	Engineering Optimization	1974	2.165	4.20	Engineering, Multidisciplinary (Q2); Operations Research & Management Science (Q2)	1185	5	0.42%	2011	2019
16	Journal of the Operational Research Society	1950	2.175	3.50	Management (Q3); Operations Research & Management Science (Q2)	2000	8	0.40%	2014	2019
17	Applied Soft Computing	2001	5.472	10.20	Computer Science, Artificial Intelligence (Q1); Computer Science, Interdisciplinary Applications (Q1)	5435	20	0.37%	2012	2018
18	Insurance Mathematics & Economics	1982	1.359	2.60	Economics (Q3); Mathematics, Interdisciplinary Applications (Q3); Social Sciences, Mathematical Methods (Q3); Statistics & Probability (Q2)	1419	5	0.35%	2013	2017
19	Information Sciences	1968	5.910	11.30	Computer Science, Information Systems (Q1)	6819	21	0.31%	2008	2019
20	Applied Mathematical Modelling	1976	3.633	6.60	Engineering, Multidisciplinary (Q1); Mathematics, Interdisciplinary Applications (Q1); Mechanics (Q1)	5977	16	0.27%	2012	2019

FY: the year the journal was founded; IF: Impact Factor; CS: CiteScore, which measures the average number of citations of all publications in the journal; TP: Total Publications; PUT: Publications on Uncertainty Theory; PUT/TP: the number of publications on uncertainty theory as a percentage of the total publications; IY: the year in which the journal first published literature on uncertainty theory; LY: the year in which the journal last published literature on uncertainty theory

are keen on publishing papers in uncertainty theory, and provide support to promote the great contributions of uncertainty theory to the scientific community and practice.

The top 20 of the 305 journals that have published papers related to uncertainty theory, and closely related fields, are reported in Table 1, including the name of the journal, the founded year (FY), the impact factor (IF), the CiteScore (CS), the discipline, the total number of publications (TP), the publications on uncertainty theory (PUT), the number of publications on uncertainty theory as a percentage of the total publications (PUT/TP), the initial year (IY) and the last year (LY). Among these, the values of the impact factor, CiteScore, and discipline refer to the latest 2019 data, and the initial and last year represent the years in which articles on uncertainty theory were first and last published during the time frame of 2008 to 2019. The journals in the table are ranked in descending order of the PUT/TP measure. In order to understand the attention paid to, degree of emphasis on, and continuity of uncertainty theory in various journals, four indicators, TP, PUT, IY and LY, are used in quantitative analysis.

In Table 1, the PUT/TP of *Fuzzy Optimization and Decision Making* (*FODM*) is the highest, at close to 22%, and far ahead of all the other journals, showing that *FODM* is the leading journal in uncertainty theory. Specializing in modelling and computing under uncertainty, *FODM* is dedicated to the theoretical formulation and application of fuzzy optimization and decision making under uncertainty. As a relatively new journal founded only in 2002, *FODM*'s impact factor grew rapidly to 4.319 in 2019, and the JCR partition results of the WoS database in 2019 demonstrate that both the Artificial Intelligence category of Computer Science and the Operations Research & Management Science category of *FODM* are located in the Q1 region. As a journal of high quality with a continuously increasing impact factor, *FODM* continues to invest considerable resources in providing substantial impetus to uncertainty theory (Yao & Liu, 2020; Ye & Liu, 2022).

Although articles on uncertainty theory account for only about 2% of its total publications, the *Journal of Intelligent & Fuzzy Systems* (*JIFS*), as the most proficient in fuzzy logic, intelligent systems and web-based applications, is still the journal with the largest amount of literature on uncertainty theory, with 91 articles. This journal focuses on theoretical exploration of uncertainty theory, such as the solution and stability of uncertain differential equations, multi-class entropy, and the theory of uncertain random variables, among others. *Soft Computing*, which has 82 publications on uncertainty theory, and is ranked third in terms of percentage, focuses primarily on the application of uncertainty theory. In particular, in recent years, this journal has published applications of uncertain programming, such as vehicle path and facility location, uncertain portfolio selection, and laws of uncertain variables. Its publications on uncertainty theory as a percentage of its total publications is only 2.17%, proving that uncertainty theory is just one of its research directions.

In addition, some other high impact journals, such as *IEEE Transactions on Fuzzy Systems (TFS)*, *Information Sciences*, and *Applied Soft Computing*, also pay attention to uncertainty theory. As the top journal in the area of fuzzy theory and logic, *TFS*, whose impact factor, CiteScore, and PUT are 9.518, 16.20, and 24 respectively, also has a definite interest in uncertainty theory, most recently preferring to explore

the reliability of uncertain systems, the random renewal reward process of uncertainty, and its application.

Since it might not be enough to consider a pool of just 1,004 papers, it is also necessary to analyse the references therein, which include classical concepts, theorems, methodologies, and high-academic-value research results, and constitute the academic foundation of this field. By checking them, we can further understand which journals publish more high-quality literature and have greater academic influence on uncertainty theory. Based on statistics on the information of reference journals, *FODM* have the most literature on uncertainty theory, followed by the *Journal of Uncertain Systems* (*JUS*), the *European Journal of Operational Research* (*EJOR*)[1] and *Soft Computing*. It is worth noting that *JUS* has not yet been included in the WoS database, and for that reason, it has not been included in Table 1. In fact, *JUS* was founded in 2007, and has already published 52 articles on uncertainty theory, accounting for 13.94% of the journal's total publication volume, which is only lower than *FODM*'s figure. Hence, its contribution to this field is not inferior to those of other journals.

2.4 Productive author analysis

Table 2 reports the 20 most productive authors in the field of uncertainty theory in descending order of publication volume, and related information including their name, institution, country, TP, total citations (TC), total citations per publication (TC/TP), *h*-index and citation thresholds. The country is based on the location of the author's institution, and not the author's actual nationality, and specifically, the institution at which the author has most recently been located. The top three authors are Yuanguo Zhu, Kai Yao and Jin Peng. Although Yuanguo Zhu has published the most papers on uncertainty theory, his average citation frequency and *h*-index are only of a medium level among the 20 most high-yielding authors. As a scholar at Nanjing University of Science and Technology, more than half of his research results are devoted to the exploration of optimal control in uncertain systems, including the most cited paper (Zhu, 2010). Among other topics, he pays special attention to the linear quadratic optimal control, parameter optimal control, and bang-bang optimal control of uncertain systems. Kai Yao, who is currently located at the university of the Chinese Academy of Sciences, is a leading scholar on uncertain stochastic renewal reward processes, and uncertain differential equations. Specifically, the former research direction is mainly applied research, while the latter focuses on uncertain partial differential equations, uncertain differential equations with jumps, and the stability and solution methods of original uncertain differential equations. Yao not only has the highest total number of citations and the highest *h*-index, but also possesses more than 20 published citations, which is also the highest among these authors. Jin Peng is a scholar in the Institute of Uncertain Systems at Huanggang

[1] For EJOR, its publications on uncertainty theory (8 papers) as a percentage of its total publications is only 0.10%. This percentage is too low to appear in Table 1. But when counting the references therein, the proportion of *EJOR* ranked third, so it is mentioned here.

Table 2 The 20 most productive authors on uncertainty theory from 2008 to 2019

Rank	Author	Institution	Country	TP	TC	TC/TP	*H*	> 100	> 50	> 20	> 10
1	Zhu Yuanguo	Nanjing Univ Sci & Technol	China	53	566	10.68	11	1	1	5	13
2	Yao Kai	Univ Chinese Acad Sci	China	40	968	24.20	20	1	3	20	30
3	Peng Jin	Huanggang Normal Univ	China	29	340	11.72	12		3	10	13
4	Gao Jinwu	Renmin Univ China	China	26	582	22.38	12		3	10	12
5	Ke Hua	Tongji Univ	China	25	272	10.88	10			4	10
7	Ralescu Dan A	Univ Cincinnati	USA	23	436	18.96	12		2	8	12
6	Gao Rong	Hebei Univ Technol	China	23	141	6.13	7			2	6
8	Yang Xiangfeng	Univ Int Business & Econ	China	23	323	14.04	10		1	5	10
9	Sheng Yuhong	Xinjiang Univ	China	22	200	8.70	8			3	7
10	Ning Yufu	Shandong Youth Univ Polit Sci	China	20	146	7.30	8			1	7
11	Qin Zhongfeng	Beihang Univ	China	20	299	14.95	11		1	5	11
12	Wen Meilin	Beihang Univ	China	19	210	11.05	10			2	8
13	Zhang Bo	Zhongnan Univ Econ & Law	China	18	244	13.56	11			5	11
14	Kang Rui	Beihang Univ	China	18	194	10.78	9			2	7
15	Gao Yuan	Beijing Jiaotong Univ	China	18	599	33.28	13	1	4	10	14
16	Zhou Jian	Shanghai Univ	China	17	175	10.29	7			2	5
17	Kar Samarjit	Natl Inst Technol	India	17	277	16.29	9		1	5	8
18	Chen Xiaowei	Nankai Univ	China	17	728	42.82	11	2	4	10	12
19	Liu Baoding	Tsinghua Univ	China	16	446	27.88	11	1	2	6	11
20	Huang Xiaoxia	Univ Sci & Technol Beijing	China	16	328	20.50	11		1	8	11

TP: total publications; TC: total citations; TC/TP: total citations per publication; *h*: *h*-index; > 100, > 50, > 20, > 10, > 5: Number of papers with more than 100, 50, 20, 10, and 5 citations

Normal University, a professional institution specializing in uncertainty theory. Peng's research interests are in uncertain programming, uncertain network optimization, uncertain risk analysis, intelligent algorithms and uncertain graphs.

From Table 2, it can be seen that Xiaowei Chen, who is located at Nankai University, is the only one of the 20 most productive authors to have published two papers with more than 100 citations, one of which is in collaboration with Yao, and the other with Liu on the existence and uniqueness theorems of uncertain differential equations (Chen & Liu, 2010). Chen concentrates on the uncertainty in finance. His average citation rate is the highest in this field, and his number of published citations, above 50, is equal first with that of Yuan Gao, who has the second-highest average number of citations and *h*-index. Gao is currently a scholar at Beijing Jiaotong University, spending a lot of effort studying the problem of vehicle paths and facility location using uncertain programming. In his paper whose citations exceed 100, Gao (2011a) gave the uncertainty distribution of the shortest path length and the solution and effective algorithms of the alpha-shortest path and the shortest path in the uncertain network, with the help of the operating law of uncertainty theory.[2] In Table 2, Professor Liu appears not to be very high in the list with respect to the number of published papers, but the citations of each of his articles are relatively high, so that his TC/TP is in the leading position, verifying his leadership and great contribution to this area.

Among the 20 most productive scholars, we report that only two are not Chinese. Dan A. Ralescu from the University of Cincinnati in the United States, who has done extensive research on fuzzy and random phenomena, published his first paper on uncertainty theory in 2010, extending the relevant definitions, theorems, calculation formulas and solution methods for uncertain differential equations, uncertain sets, uncertain risk analysis, uncertain finance, and uncertain vertex colouring problems. Samarjit Kar, from the National Institute of Technology Durgapur in India, focuses mainly on the solid transport and postman problems in uncertain programming, and has also studied portfolio selection in finance. Finally, it is worth noting that three of the 20 most productive authors are from the same institution - Beihang University - reflecting the high level of attention and support that institution provides to uncertainty theory.

2.5 Most-cited publications

Table 3 lists the 15 most-cited journal publications on uncertainty theory from 2008 to 2019, including detailed information such as the title, author, publication year, journal, category, TC, total citations per year (TC/Year), number of authors (NA), number of institutions (NI) and number of references (NR). There are three papers with more than 120 citations. Zhu (2010) is ranked first and is cited by the most productive scholars of Table 2. It obtained the optimal principle of uncertain optimal control through Behrman's optimal principle and dynamic programming, which

[2] As a senior academic at Tsinghua University, and the founder of uncertainty theory, Liu is more inclined to write books. His books on uncertainty theory are numerous and of considerable quality.

Table 3 The 15 most-cited publications on uncertainty theory from 2008 to 2019

Rank	Title	Author	Year	Journal	Category	TC	TC/Year	NA	NI	NR
1	Uncertain Optimal Control with Application to a Portfolio Selection Model	Zhu Yuanguo	2010	Cybernetics and Systems	Computer Science	193	19.30	1	1	13
2	Existence and Uniqueness Theorem for Uncertain Differential Equations	Chen Xiaowei, Liu Baoding	2010	Fuzzy Optimization and Decision Making	Computer Science, Operations Research & Management Science	187	18.70	2	1	16
3	A Numerical Method for Solving Uncertain Differential Equations	Yao Kai, Chen Xiaowei	2013	Journal of Intelligent & Fuzzy Systems	Computer Science	129	18.43	2	2	21
4	Uncertain Random Variables: a Mixture of Uncertainty and Randomness	Liu Yuhan	2013	Soft Computing	Computer Science	104	14.86	1	1	31
5	Shortest Path Problem with Uncertain Arc Lengths	Gao Yuan	2011	Computers & Mathematics with Applications	Mathematics	101	11.22	1	1	23
6	Uncertain Term Structure Model of Interest Rate	Chen Xiaowei, Gao Jinwu	2013	Soft Computing	Computer Science	94	13.43	2	2	27
7	Uncertain Models for Single Facility Location Problems on Networks	Gao Yuan	2012	Applied Mathematical Modelling	Engineering, Mathematics, Mechanics	85	10.63	1	1	16
8	Uncertain Random Programming with Applications	Liu Yuhan	2013	Fuzzy Optimization and Decision Making	Computer Science, Operations Research & Management Science	83	11.86	1	1	21
9	On the Convergence of Uncertain Sequences	You Cuilian	2009	Mathematical and Computer Modelling	Computer Science, Mathematics	81	7.36	1	1	19
10	Some Stability Theorems of Uncertain Differential Equation	Yao Kai, Gao Jinwu, Gao Yuan	2013	Fuzzy Optimization and Decision Making	Computer Science, Operations Research & Management Science	79	11.29	3	2	16

Table 3 (continued)

Rank	Title	Author	Year	Journal	Category	TC	TC/Year	NA	NI	NR
11	Linear-Quadratic Uncertain Differential Game with Application to Resource Extraction Problem	Yang Xiangfeng, Gao Jinwu	2016	IEEE Transactions on Fuzzy Systems	Computer Science, Engineering	72	18.00	2	2	60
12	Group Multi-Criteria Supplier Selection Using Combined Grey Systems Theory and Uncertainty Theory	Memon Muhammad Saad, Lee Young Hae, Mari Sonia Irshad	2015	Expert Systems with Applications	Computer Science, Engineering, Operations Research & Management Science	68	13.60	3	1	50
13	On Liu's Inference Rule for Uncertain Systems	Gao Xin, Gao Yuan, Ralescu Dan A	2010	International Journal of Uncertainty Fuzziness and Knowledge-based Systems	Computer Science	68	6.80	3	3	13
14	Uncertain Alternating Renewal Process and its Application	Yao Kai, Li Xiang	2012	IEEE Transactions on Fuzzy Systems	Computer Science, Engineering	65	8.13	2	2	28
15	Single-Period Inventory Problem under Uncertain Environment	Qin Zhongfeng, Kar Samarjit	2013	Applied Mathematics and Computation	Mathematics	60	8.57	2	2	36

TC: total citations; TC/Year: total citations per year; NA: number of authors; NI: number of institutions; NR: number of references

provided a theoretical foundation for subsequent research on uncertain optimal control and its applications. The second most highly cited paper (Chen & Liu, 2010) breathtakingly expounded some solutions to linear uncertain differential equations, and proved their existence and uniqueness theorem. The third most highly cited paper, Yao and Chen (2013), innovatively proposed a numerical scheme for handling uncertain differential equations with high efficiency, which essentially solved each alpha-path and generated an inverse uncertainty distribution of the solution. It supplemented the method proposed in the second paper mentioned above, and provided insights for follow-up research.

Highly cited articles initiate innovative theoretical research and thus offer new concepts, properties, theorems and methodologies, providing solid ground for subsequent theoretical research and practical applications. About one third of the literature in Table 3 is related to uncertain differential equations, which verifies the high interest of scholars in exploring theorems, stability, numerical methods and applications of uncertain differential equations. Furthermore, the other highly cited papers in Table 3 are related to some other classical concepts of this field, such as uncertain random variables and programming, and uncertain alternating renewal processes.

It is also worth mentioning that, in addition to the highly cited references from the WoS database that are listed above, there are some others, available in Scopus, that have not been included in Table 3. The first article, written by Liu (2012c), dealt with the meanings and situations of uncertainty, the difference between uncertain variables and uncertain sets, and between uncertainty, fuzziness and probability. It is highly appreciated by scholars in this domain and has 282 citations. Prior to this paper, the author wrote two papers with significant theoretical innovations, in 2009 and 2011, respectively. Liu (2009b) proposed concepts such as the product measure axiom and a new uncertain calculus, while Liu (2011) expanded the work around the uncertain logic of human language, and has obtained 90 citations so far. All three articles are published in the *JUS*, which verifies the strong academic influence of, and support offered by, this journal in the field of uncertainty theory.

2.6 Main path analysis

Through the main path analysis (Hummon & Dereian, 1989), publications indicating the main knowledge evolution directions in a field are effectively identified, as well as the knowledge flow with special evolutionary significance and distinctive characteristics. After the citation network is drawn by CiteSpace, the relevant data is imported into Pajek software to execute the algorithm of search path count to extract the main paths of uncertainty theory. As shown in Fig. 6, the development skeleton of uncertainty theory is represented by seven main paths composed of 53 key literature according to the research direction and chronological order, which are marked with seven rainbow colors respectively. Each colored node signifies a main path document, and the connected gray arrow line reflects the citation relationship between the documents. The name of the first author, the year of publication and the serial number sorted by local citation score (LCS) of these publications are marked in Fig. 6, whose more detailed information is summarized in Table 4 to better grasp

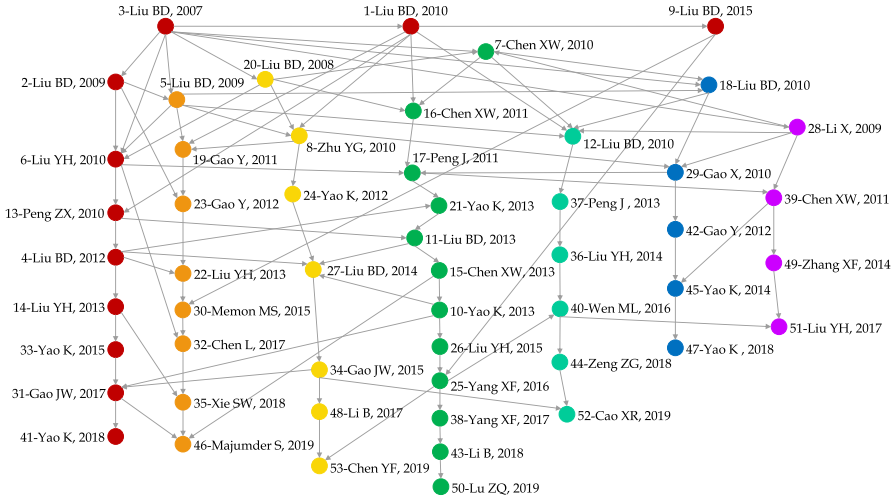

Fig. 6 The main path of uncertainty theory from 2008 to 2019

the evolution of research directions in this field. Table 4 lists the serial number, title, author, journal and year of the main path literature in descending order of LCS, where LCS shows the citations of a document in the data set input to the CiteSpace software.

Analysis of the research contents and development trends of the main path literature indicates that the book on uncertainty theory written by Professor Liu opens a new era in this field, and has been continuously updated over the years to keep the content up to date, gaining high citations and wide recognition. Based on the enrichment of basic theories, scholars in this domain gradually explore in-depth towards diversified research directions, and form seven main paths as shown in Fig. 6. The red one reveals that the field of uncertainty theory attaches great importance to the development of basic theories and the construction of axiomatic systems since the initial stage, such as the concepts and theorems about uncertain sequences and uncertain variables. As two research directions developed earlier, the orange path receives higher citations and attention than the yellow path, and accumulates more publications about employing uncertain programming to solve the shortest route, facility location, supply chain decision-making and other practical problems. Uncertain optimal control and uncertain renewal process are the research hotspots of the yellow path, whose research achievements are greatly promoted by Yuanguo Zhu and Kai Yao respectively. With the largest number of main path literature, the green path clearly displays the great attention and continuous exploration of uncertain differential equations by researchers, and shifts from expanding fundamental theorems, formulas and numerical solutions to the application in realistic problems. For example, the variants such as uncertain differential games and uncertain partial differential equations are utilized to address practical solutions for the resource extraction and heat conduction, and uncertain structure and currency models, option pricing formulas and portfolio selection models are established to optimize uncertain

Table 4 The 53 main path literature of uncertainty theory from 2008 to 2019

Rank	Title	Author	Journal	Year	LCS
1	Uncertainty Theory: A Branch of Mathematics for Modeling Human Uncertainty	Liu Baoding	–	2010	370
2	Some Research Problems in Uncertainty Theory	Liu Baoding	Journal of Uncertain Systems	2009	306
3	Uncertainty Theory	Liu Baoding	–	2007	224
4	Why is There a Need for Uncertainty Theory	Liu Baoding	Journal of Uncertain Systems	2012	210
5	Theory and Practice of Uncertain Programming	Liu Baoding	–	2009	206
6	Expected Value of Function of Uncertain Variables	Liu Yuhan, Ha Minghu	Journal of Uncertain Systems	2010	173
7	Existence and Uniqueness Theorem for Uncertain Differential Equations	Chen Xiaowei, Liu Baoding	Fuzzy Optimization and Decision Making	2010	170
8	Uncertain Optimal Control with Application to a Portfolio Selection Model	Zhu Yuanguo	Cybernetics and Systems	2010	165
9	Uncertainty Theory	Liu Baoding	–	2015	141
10	A Numerical Method for Solving Uncertain Differential Equations	Yao Kai, Chen Xiaowei	Journal of Intelligent & Fuzzy Systems	2013	139
11	Toward Uncertain Finance Theory	Liu Baoding	Journal of Uncertainty Analysis and Applications	2013	131
12	Uncertain Risk Analysis and Uncertain Reliability Analysis	Liu Baoding	Journal of Uncertain Systems	2010	123
13	A Suffcient and Necessary Condition of Uncertainty Distribution	Peng Zixiong, Iwamura Kakuzo	Journal of Interdisciplinary Mathematics	2010	122
14	Uncertain Random Variables: A Mixture of Uncertainty and Randomness	Liu Yuhan	SoftComputing	2013	111
15	Uncertain Term Structure Model of Interest Rate	Chen Xiaowei, Gao Jinwu	Soft Computing	2013	110
16	American Option Pricing Formula for Uncertain Financial Market	Chen Xiaowei	International Journal of Operations Research	2011	109
17	A New Option Pricing Model for Stocks in Uncertainty Markets	Peng Jin, Yao Kai	International Journal of Operations Research	2011	108

Table 4 (continued)

Rank	Title	Author	Journal	Year	LCS
18	Uncertain Set Theory and Uncertain Inference Rule with Application to Uncertain Control	Liu Baoding	Journal of Uncertain Systems	2010	107
19	Shortest Path Problem with Uncertain Arc Lengths	Gao Yuan	Computers & Mathematics with Applications	2011	102
20	Fuzzy Process, Hybrid Process and Uncertain Process	Liu Baoding	Journal of Uncertain Systems	2008	101
21	Some Stability Theorems of Uncertain Differential Equation	Yao Kai, Gao Jinwu, Gao Yuan	Fuzzy Optimization and Decision Making	2013	100
22	Uncertain Random Programming with Applications	Liu Yuhan	Fuzzy Optimization and Decision Making	2013	99
23	Uncertain Models for Single Facility Location Problems on Networks	Gao Yuan	Applied Mathematical Modelling	2012	98
24	Uncertain Alternating Renewal Process and Its Application	Yao Kai, Li Xiang	IEEE Transactions on Fuzzy Systems	2012	91
25	Linear-Quadratic Uncertain Differential Game with Application to Resource Extraction Problem	Yang Xiangfeng, Gao Jinwu	IEEE Transactions on Fuzzy Systems	2016	87
26	Uncertain Currency Model and Currency Option Pricing	Liu Yuhan, Chen Xiaowei, Ralescu Dan A.	International Journal of Intelligent Systems	2015	79
27	Uncertainty Distribution and Independence of Uncertain Processes	Liu Baoding	Fuzzy Optimization and Decision Making	2014	78
28	Hybrid Logic and Uncertain Logic	Li Xiang, Liu Baoding	Journal of Uncertain Systems	2009	77
29	On Liu's Inference Rule for Uncertain Systems	Gao Xin, Gao Yuan, Ralescu Dan A.	International Journal of Uncertainty Fuzziness and Knowledge-Based Systems	2010	73
30	Group Multi-Criteria Supplier Selection Using Combined Grey Systems Theory and Uncertainty Theory	Memon Muhammad Saad, Lee Young Hae, Mari Sonia Irshad	Expert Systems with Applications	2015	72

Table 4 (continued)

Rank	Title	Author	Journal	Year	LCS
31	Uncertain Shapley Value of Coalitional Game with Application to Supply Chain Alliance	Gao Jinwu, Yang Xiangfeng, Liu Di	Applied Soft Computing	2017	47
32	Pricing and Effort Decisions for a Supply Chain with Uncertain Information	Chen Lin, Peng Jin, Liu Zhibing, Zhao Ruiqing	International Journal of Production Research	2017	44
33	A Formula to Calculate the Variance of Uncertain Variable	Yao Kai	Soft Computing	2015	41
34	Some Concepts and Theorems of Uncertain Random Process	Gao Jinwu, Yao Kai	International Journal of Intelligent Systems	2015	39
35	Multi-Objective Active Distribution Networks Expansion Planning by Scenario-Based Stochastic Programming Considering Uncertain and Random Weight of Network	Xie Shiwei, Hu Zhijian, Zhou Daming, Li Yan, Kong Shunfei, Lin Weiwei, Zheng Yunfei	Applied Energy	2018	37
36	Risk Index in Uncertain Random Risk Analysis	Liu Yuhan, Ralescu Dan A.	International Journal of Uncertainty Fuzziness and Knowledge-Based Systems	2014	36
37	Risk Metrics of Loss Function for Uncertain System	Peng Jin	Fuzzy Optimization and Decision Making	2013	33
38	Uncertain Partial Differential Equation with Application to Heat Conduction	Yang Xiangfeng, Yao Kai	Fuzzy Optimization and Decision Making	2017	32
39	A Note on Truth Value in Uncertain Logic	Chen Xiaowei, Ralescu Dan A.	Expert Systems with Applications	2011	29
40	Reliability Analysis in Uncertain Random System	Wen Meilin, Kang Rui	Fuzzy Optimization and Decision Making	2016	28
41	Uncertain Regression Analysis: An Approach for Imprecise Observations	Yao Kai, Liu Baoding	Soft Computing	2018	26
42	Uncertain Inference Control for Balancing an Inverted Pendulum	Gao Yuan	Fuzzy Optimization and Decision Making	2012	25

Table 4 (continued)

Rank	Title	Author	Journal	Year	LCS
43	Multi-Period Portfolio Selection Problem under Uncertain Environment with Bankruptcy Constraint	Li Bo, Zhu Yuanguo, Sun Yufei, Aw Grace, Teo Kok Lay	Applied Mathematical Modelling	2018	17
44	Uncertainty Theory as a Basis for Belief Reliability	Zeng Zhiguo, Kang Rui, Wen Meilin, Zio Enrico	Information Sciences	2018	16
45	Sine Entropy of Uncertain Set and Its Applications	Yao Kai	Applied Soft Computing	2014	15
46	Uncertain Multi-Objective Multi-Item Fixed Charge Solid Transportation Problem with Budget Constraint	Majumder Saibal, Kundu Pradip, Kar Samarjit, Pal Tandra	Soft Computing	2019	14
47	Uncertain Statistical Inference Models with Imprecise Observations	Yao Kai	IEEE Transactions on Fuzzy Systems	2018	13
48	Parametric Optimal Control for Uncertain Linear Quadratic Models	Li Bo, Zhu Yuanguo	Applied Soft Computing	2017	11
49	A Modified Uncertain Entailment Model	Zhang Xingfang, Li Lingqiang, Meng Guangwu	Journal of Intelligent & Fuzzy Systems	2014	9
50	Numerical Approach for Solution to an Uncertain Fractional Differential Equation	Lu Ziciiang, Zhu Yuanguo	Applied Mathematics and Computation	2019	8
51	Uncertain Random Logic and Uncertain Random Entailment	Liu Yuhan, Yao Kai	Journal of Ambient Intelligence and Humanized Computing	2017	6
52	Reliability Analysis of Discrete Time Series-Parallel Systems with Uncertain Parameters	Cao Xuerui, Hu Linmin, Li Zhenzhen	Journal of Ambient Intelligence and Humanized Computing	2019	5
53	Indefinite LQ Optimal Control with Cross Term for Discrete-Time Uncertain Systems	Chen Yuefen, Zhu Yuanguo, Li Bo	Mathematical Methods in the Applied Sciences	2019	2

LCS: local citation score, is the citations of the literature in the input data set

financial means. Besides, the cyan path explores the risk index, risk metrics and reliability analysis of uncertain systems, and obtains increasing attention in recent years. The two paths about uncertain sets and uncertain logic have the least theoretical output and are relatively slow in development. The detailed introduction of these seven research directions will be explained separately in the next section.

On the whole, according to the knowledge evolution path of uncertainty theory, it can be seen that its research has gradually expanded from basic theory to more diversified, targeted and specialized application-oriented achievements, especially the development of useful tools and effective solutions to practical problems. Consistent with the previous conclusions, several productive authors play a crucial role in leading and promoting different main paths, as well as the *FODM* and *JUS*. Uncertainty theory is increasingly widely recognized by scholars, and will be applied to more practical scenarios while accumulating abundant theoretical outcomes of various paths.

3 Discussion of development history

In this section, seven key sub-fields of uncertainty theory, derived using co-citation analysis and main path analysis, are presented, namely the axiomatic system, uncertain differential equations, uncertain processes, uncertain programming, uncertain logic, uncertain sets and uncertain risk analysis. At the same time, by referring to half-life, quantity of publications and other indexes, and combining these with the specific content analysis method, we analyse the hot topics and research characteristics of each sub-field in detail.

3.1 Co-citation analysis

Co-citation means that two publications are cited by the same article, while frequent co-citations indicate that the two have common, related research topics (Small, 1973). Co-citation analysis[3] can identify the core theme of a research field by allowing the investigation of the literature in each cluster, after the relevant literature has been grouped based on similarity of content. Therefore, co-citation analysis here can highlight the status of uncertainty theory, and the clustering results could help scholars to explore research characteristics, current directions, and hot topics in the key sub-fields.

We apply the CiteSpace software to conduct a co-citation analysis of the literature on uncertainty theory, and preliminarily obtain 11 clustering topics. The highly correlated clusters are then combined to give seven key sub-fields of uncertainty theory with reference to the outcomes of main path analysis, namely, the axiomatic system, uncertain differential equations, uncertain processes, uncertain programming, uncertain logic, uncertain sets and uncertain risk analysis. Next, the relative proportion

[3] This is also known as *reference co-citation analysis*.

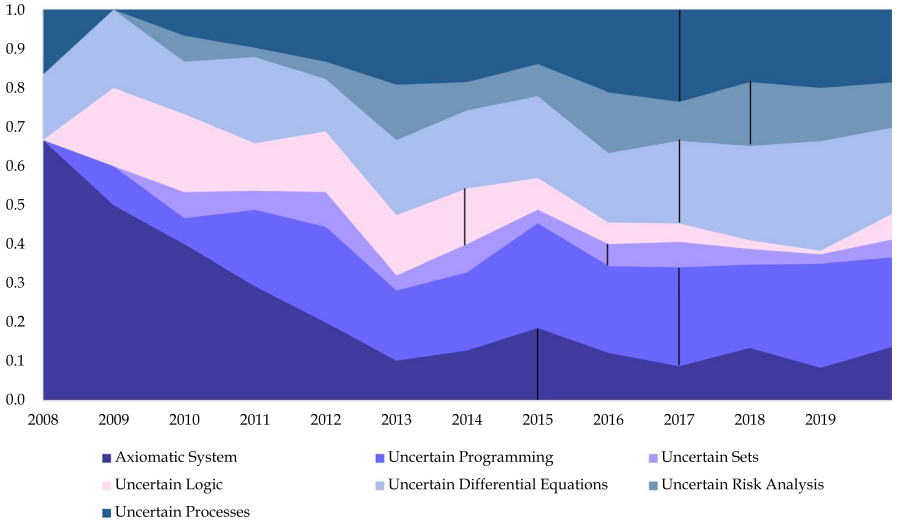

Fig. 7 The proportion of the literature in each sub-field of uncertainty theory from 2008 to 2019

and half-life of the number of publications per year in each sub-field from 2008 to 2019 are calculated, and the results are plotted as an accumulation diagram, shown in Fig. 7. In this figure, each sub-field corresponds to an irregular ribbon of a different colour, and the area of the ribbon shown in each year represents the volume of literature in the sub-field in that year as a proportion of the total volume of literature on uncertainty theory in the same year. For example, in 2009, there is no band of uncertain risk analysis and uncertain processes, indicating that the amount of publications of those two sub-fields in that year was zero. It is worth noting that the area of the colour band in a given year is not in direct proportion to the number of papers. In fact, with the increasing number of papers on uncertainty theory year by year, the amount of literature in each sub-field has increased more or less. For instance, the proportion of the literature on the axiomatic system in 2009 was much higher than that in 2017, but in fact, 15 articles were published in 2017, three times more than in 2009.

The half-life was originally used to describe the index of a journal (Bernal, 1959). In this paper, it is extended to indicate the time period over which the latest 50% of publications in the sub-field have been published, which is the time interval to the right of the solid black line in Fig. 7. If the position of the solid black line is further to the right, the half-life of the sub-field is shorter, meaning that most of the literature in this sub-domain has been published recently, and the rate of growth in papers has increased significantly. For example, the solid black line of the uncertain sets sub-field is located in 2016, indicating that half of the existing articles in this sub-field have appeared since 2016 and this topic has a half-life of four years.

Figure 7 clearly shows the starting year and duration of each sub-field. According to the change in the relative proportion over time and the half-life of the literature in each sub-field, we can figure out the evolution of scholars' research emphasis in

uncertainty theory. Therefore, combining the visualization results presented in the figure with the large amount of literature, in the following sections we will introduce the chronological order in which the sub-fields began to evolve.

3.2 Sub-fields of uncertainty theory

3.2.1 Axiomatic system

At the beginning of the establishment of uncertainty theory, Liu advocated setting up a complete axiomatic system, equivalent to a theoretical framework, or collection of theories, which systematically proposes various basic mathematical concepts, theorems, formulas, methods, etc. The uncertain measure, variables and uncertainty distribution are the three cornerstones of the axiomatic system (Liu, 2012c).

Mathematically speaking, uncertainty theory is essentially a substitute theory of measurement, so the uncertain measure is one of the most fundamental and key components of the axiomatic system. The uncertain measure is interpreted as the individual's confidence in the possibility of an uncertain event, which depends on their personal cognition degree about this event. In order to rationally deal with confidence levels and give an axiomatic definition of the uncertain measure, Liu (2007) proposed four axioms: the normality, monotonicity, self-duality, and countable subadditivity axioms. Among them, the self-duality axiom is actually the application of the law of conservation of truth in uncertainty theory, which ensures that uncertainty theory is consistent with the law of intermediate exclusion and the law of contradiction. In 2009, Liu (2009b) defined the measurement of product uncertainty, resulting in the fifth axiom of uncertainty theory. Moreover, Gao (2009), Liu (2010a), Zhang (2011), Peng and Iwamura (2012), and Liu (2013c) also actively supplemented and extended the relevant properties, theorems and structural features of the uncertain measure.

As a basic concept of uncertainty theory, an uncertain variable is, roughly speaking, the measurable function on the uncertain space, which was proposed by Liu (2007) to represent the quantity of uncertainty. Later, Liu (2009b) put forward the concept of the independence of uncertain variables, and a new concept of uncertain random variables (Liu, 2013). In addition, with regards the variance (Yao, 2015a), covariance (Gao et al., 2019a), correlation coefficient (Zhao et al., 2018), moment (Sheng & Kar, 2015), basic inequality (Liu & Xu, 2010; Tian, 2011), and variational inequality (Chen & Zhu, 2015) of uncertain variables, many scholars are actively exploring their definitions and calculation methods. Meanwhile, the definitions, properties and principles of cross-entropy (Chen et al., 2012; Gao et al., 2018), triangular entropy (Ning et al., 2015), partial entropy (Ahmadzade et al., 2017a), sine entropy (Yao et al., 2013a), quadratic entropy (Dai, 2018), maximum entropy (Chen & Dai, 2011), and relative entropy (Zhou et al., 2015, 2016; Sheng et al., 2017b) have been proposed in several papers. Furthermore, the convergence concept of uncertain sequences and their interrelation (You, 2009; Wu & Xia, 2012; Chen et al., 2014, 2016; Ahmadzade et al., 2017b), and the limit theorem (Wang et al., 2018, 2018), have also been discussed in many publications.

As carriers of the incomplete information of uncertain variables, uncertainty distributions were proposed by Liu (2007) in order to describe uncertain variables. Peng and Iwamura (2010) proved sufficient and necessary conditions of uncertainty distributions. At the beginning, uncertainty distributions were divided into five types, namely the linear, zigzag, normal, lognormal, and discrete uncertainty distributions. Later, Liu (2010a) introduced the inverse uncertainty distributions of four of the uncertainty distributions, with the exception of the discrete uncertainty distribution, which increased the number of types of uncertainty distributions to nine, and then in another work he verified the sufficient and necessary conditions of these inverse uncertainty distributions (Liu, 2013d). More importantly, Liu (2010a) also proposed a measure inversion theorem, which can derive the uncertain measure from the uncertainty distribution of the corresponding uncertain variables.

However, as the theoretical content has gradually been enriched, the research space has become narrower and attention paid to this area has eventually declined. From Fig. 7, it can clearly be seen that the proportion of documents on the axiomatic system, being obviously large at first, shows a gradual downward trend, reflecting a relative decline of scholars' interest in this sub-field. Besides, the topic has a half-life of 5 years, which is long compared to other sub-fields, also supporting this view. The reason for this situation is that the axiomatic system of uncertainty theory has been established, leaving scholars only needing to check for gaps and improve the completeness, which gives authors little room for innovation and fewer development opportunities. With the further enrichment of the related content, the accumulation of literature on axiomatic system has slowed down.

3.2.2 Uncertain differential equations

Uncertain differential equations belong to the family of ordinary differential equations driven by the normative process and are a key component of uncertainty theory. As shown in Fig. 7, the proportion of papers falling into the sub-field of uncertain differential equations has remained at a high level since 2008 and the rate of paper publication is accelerating. As the focus of numerous scholars, the topic of uncertain differential equations has a half-life of only three years. Although this half-life ranks second, its value is almost the same as that of uncertain programming, and research on the topic has been relatively well deepened and refined. Overall, this sub-field is the only one to have attained a high level of attention and strong performance throughout the entire time interval, and thus it might be sensible to expect more breakthroughs in this area in due course.

After Liu (2008) proposed this new class of differential equations, he defined the concept of stability in the following year, in (Liu, 2009b). Yao et al. (2013b) proved some stability theorems on the basis of Liu's study, and later other scholars expanded the work by looking at the stability of different types. Chen and Liu (2010) first proved the existence and uniqueness theorems of solutions to uncertain differential equations under the conditions of linear growth and Lipschitz continuity, and Gao (2012a) then verified this theorem under local conditions. In terms of solutions to uncertain differential equations, Chen and Liu (2010) obtained analytic solutions of linear uncertain differential equations, and Liu (2012) and Yao (2013)

Table 5 Summary of research on uncertain differential equations

Rank	Name	PY	Introduction	Theoretical Research	Application	References
1	Uncertain Differential Equation	2008	A type of differential equation driven by the canonical process	Existence and uniqueness theorem; numerical methods; stability; necessary and sufficient conditions; continuous dependence theorems; differential mean value theorems; expected value of solution	Modelling the susceptible-infected-susceptible epidemic; establishing a two-factor term structure model to value bond prices; uncertain finance	Liu (2008); Yao and Chen (2013); Gao and Yao (2014); Yang and Ralescu (2015); Wang et al. (2015); Gao (2016); Li et al. (2017b); Zhang et al. (2017); Chen and Gao (2018); Chen and Li (2018); Ji and Zhou (2018); Zhang et al. (2019a)
2	Linear Uncertain Differential Equation	2010	Where the function of the uncertain differential equation is continuous	Stability; necessary and sufficient conditions	The resource extraction problem	Chen and Liu (2010); Chen and Gao (2013); Sheng and Wang (2014)
3	Backward Uncertain Differential Equation	2013	Where the uncertain process in the uncertain differential equation has the final value	Existence and uniqueness of solutions; stability; relationship between different types of stability	Waiting for development	Ge and Zhu (2013); Fei (2014); Wang and Ning (2017a); Shi and Sheng (2019)
4	Uncertain Differential Equation with Jumps	2008	Adding an uncertain renewal process to an uncertain differential equation	Stability; sufficient conditions with unique solution; relationship between different types of stability	Modelling steep drifts embedded in uncertain dynamic systems; uncertain finance	Liu (2008); Yao(2015d); Ji and Ke (2016); Ma et al. (2017b); Gao (2019); Liu (2019)
5	Multi-Dimensional Uncertain Differential Equation	2013	Where the uncertain process in the uncertain differential equation is in a multi-dimensional uncertain state	Sufficient condition with unique solution; stability; relationship between different types of stability	Explaining the internal characteristics of uncertain singular systems	Ge and Zhu (2013); Ji and Zhou (2015); Su et al. (2016); Feng et al. (2018); Shu and Zhu (2018); Sheng and Shi (2019)
6	Uncertain Fractional Differential Equation	2015	Two kinds of Riemann-Liouville type and Caputo type	Solutions; alpha path; numerical methods; expected value of solution	Solving uncertain impulsive differential systems of fractional order; modelling complex systems with discrete-time features and memory effects	Zhu (2015); Stamov and Stamova (2018); Lu and Zhu (2019, 2020); Wang and Ralescu (2021)

Table 5 (continued)

Rank	Name	PY	Introduction	Theoretical Research	Application	References
7	Multifactor Uncertain Differential Equation	2015	A type of uncertain differential equation driven by multiple canonical Liu processes	Stability; relationship between different types of stability	Waiting for development	Li et al. (2015); Zhang et al. (2016); Ma et al. (2017a); Sheng et al. (2017a); Sheng and Shi (2018)
8	Uncertain Delay Differential Equation	2010	Used to handle phenomena related to current and previous states of the system	Stability; relationship between different types of stability; existence and uniqueness theorem	Uncertain time-delay systems; uncertain multivariable feedback systems with multiple time delays	Barbacioru (2010); Wang and Ning (2017c); Jia and Sheng (2019); Wang and Ning (2019)
9	Uncertain Partial Differential Equation	2017	A type of partial differential equation driven by Liu processes	Analytical solutions; inverse uncertainty distribution; numerical methods	Internet Public Opinion issues; modelling the age structure of the population	Yang and Yao (2017); Sheng et al. (2017); Gao and Fu (2017)
10	Uncertain Heat Equation	2017	As an application of an uncertain partial differential equation, the noise of a heat source is described by a Liu process	Existence and uniqueness of solutions; alpha path; solving formulas; uncertainty distribution and inverse uncertainty distribution of solutions; numerical methods; expected value of solution	Uncertain adaptive boundary control	Yang and Yao (2017); Yang and Ni (2017); Yang (2018, 2019); Yang and Ni (2019)
11	Uncertain Wave Equation	2016	A second-order partial differential equation driven by Liu processes for modelling wave phenomena with uncertain noises	Existence and uniqueness of solutions; inverse uncertainty distribution; stability	The Cauchy initial problem and the infinite half-boundary problem	Gao (2017); Gao and Ralescu (2019); Gao et al. (2019c)
12	High-Order Uncertain Differential Equation	2016	Used to model differentiable uncertain systems with high-order differentials	Stability theorems	Modelling differentiable uncertain systems with higher-order differentials	Yao (2016); Sheng (2017)

Table 5 (continued)

Rank	Name	PY	Introduction	Theoretical Research	Application	References
13	Uncertain Spring Vibration Equation	2017	Describing the vibration of an object subjected to a spring and a time-varying external force whose noise is displayed by a Liu process	Alpha path; numerical methods; existence and uniqueness of solutions; inverse uncertainty distribution; stability; expected value of solution	Uncertain optimal control	Jia and Dai (2017); Jia et al. (2018); Jia and Yang (2018); Jia and Ni (2019); Jia et al. (2019)

PY: the year in which the uncertain differential equation was first proposed

developed a series of analytic methods for solving some special classes of nonlinear uncertain differential equations. Then, as more and more researchers have joined this sub-field, the types of uncertain differential equations have been expanded greatly. In this paper, we summarize the existing uncertain differential equations in Table 5, including name, proposed year, introduction, theoretical research, application and related references. Of these indicators, "introduction" provides a simple generalization of the question "what is it", and "theoretical research" and "application" divide the paper's research content into two parts: theory and application. Table 5 reports not only papers related to the expected value (Wang et al., 2015; Jia et al., 2018; Ji & Zhou, 2018), but also to the existence and unique- ness theorem (Chen & Liu, 2010; Jia et al., 2019) and sufficient and necessary conditions (Yao, 2015d) of the solutions to these types of uncertain differential equation, and to the continuous dependence theorem (Gao & Yao, 2014; Zhang et al., 2019a), differential mean value theorems (Chen & Li, 2018), alpha path (Yang, 2018; Jia et al., 2018; Lu & Zhu, 2019), chain rules (Chirima et al., 2019), and numerical algorithms (Yao & Chen, 2013; Yang & Ralescu, 2015; Gao, 2016; Zhang et al., 2017; Gao & Fu, 2017). Besides, a lot of researchers have discussed diverse concepts, sufficient conditions and correlations for different types of uncertain differential equations, such as almost certain stability (Liu et al., 2014; Wang & Ning, 2019b), p-th moment stability (Sheng & Wang, 2014; Ma et al., 2017b), mean stability (Zhang et al., 2016; Feng et al., 2018; Gao, 2019; Sheng & Shi, 2019), measure stability (Sheng, 2017; Shu & Zhu, 2018; Sheng & Shi, 2018), exponential stability (Wang & Ning, 2017a; Liu, 2019), distribution stability (Ma et al., 2017a; Jia & Sheng, 2019) and inverse distribution stability (Yang et al., 2017).

Among the applications of uncertain differential equations, the most widespread and successful are in finance. Since Liu (2009b) first introduced uncertain differential equations into the field of finance, many scholars have followed on from that research. Up to now, portfolio selection and optimization problems have been the main research directions of uncertainty theory in finance. In this work, the improved or original expected variance model (Zhang et al., 2015), neural network model (Nazemi et al., 2015; Omidi et al., 2017), mean risk model (Huang, 2011; Zhai & Bai, 2017), mean-risk-skewness model (Zhai et al., 2018), mean-chance model (Huang & Zhao, 2014a), mean variance model (Qin, 2015; Li et al., 2017a; Zhai & Bai, 2018), mean-variance-skewness model (Chen et al., 2018, 2019), and mean semi-variance model (Chen et al., 2017b) are the tools used to conduct research on portfolio selection or optimization against the background of the securities industry, and great importance is attached to the practical availability of results.

In addition, three families of uncertain financial models are considered in the corresponding literature, namely the uncertain stock, interest rate, and currency models. Liu (2009b) first proposed an uncertain stock model and a European option price formula. Later, Yao (2015b) proved the no-arbitrage theorem of this uncertain stock model, Chen (2011) derived the American option price formula, and Sun and Chen (2015) developed the Asian option price formula. Besides these, other scholars have pioneered or improved those families of uncertain models with mean regression (Yao, 2012; Sun & Su, 2017; Shi et al., 2017; Sun et al., 2018a; Tian et al., 2019; Yao & Qin, 2021), periodic dividends or jumps (Yu, 2012; Chen et al., 2013;

Ji & Zhou, 2014), and uncertain currency models with floating interest rates (Liu et al., 2015a; Wang & Ning, 2017b; Ji & Wu, 2017; Wang & Chen, 2019) to derive European, American and Asian option pricing formulas. The uncertain interest rate model is generally used for the valuation of zero-coupon bonds (Jiao & Yao, 2015; Sun et al., 2018b) and the calculation of some upper and lower limits (Zhang et al., 2016; Mehrdoust & Najafi, 2019). About 59% of papers on uncertain differential equations were published between 2017 and 2019, which indicated that the research in this sub-field has received high attention recently.

3.2.3 Uncertain processes

An uncertain process is essentially a series of variables indexed by time. It is often used to model uncertainty over time, and this is why uncertain processes were established by Liu (2008). Liu not only initialized the independent increment process (Liu, 2008), but also introduced an extreme value theorem (Liu, 2013b), and put forward the concepts of uncertainty distributions, inverse uncertainty distributions and independence to describe the uncertain process (Liu, 2014). Over the years, more and more uncertain processes have been discovered, that is, the half a standard process (Gao, 2011b), stable independent increment process (Chen, 2012), enlightenment process (Di Caprio et al., 2014), outline of the process (Yao, 2015c), strongly monotonic uncertain process (You & Xiang, 2018), insurance risk process (Yao & Qin, 2015; Yao & Zhou, 2018b), complex Liu process (Qin & Wen, 2015), finite variation process (Chen, 2015), renewal incentive process (Yao & Qin, 2015), uncertain stochastic process (Gao & Yao, 2015), uncertain renewal process, and so on. Moreover, authors have delved into the concepts, properties, variances, analytical functions, changes, sample paths, uncertain integrals, independence, and continuity of these uncertain processes.

Among these uncertain processes, the most representative one is the uncertain renewal process, in which events occur continuously and independently at uncertain times. In addition to defining the uncertain renewal process (Liu, 2008), Liu (2010a) also put forward the concept of the uncertain renewal reward process and the theorem used to determine the long-term reward rate. Yao and Li (2012) defined the concept of the uncertain alternative renewal process and proved the uncertain alternative renewal theorem for determining availability. Zhang et al. (2013) proposed an uncertain delayed renewal process and a basic delayed renewal theorem. These three types of uncertain renewal processes have been extended in different directions (Yao & Gao, 2015; Yao & Zhou, 2016, 2018a; Yao, 2019), and the theoretical outcomes are exploited to determine systems' block replacement strategies (Zhang & Guo, 2014; Ke & Yao, 2016), the optimal age replacement strategy for parts (Zhang & Guo, 2013) and the optimal replacement time for equipment (Yao & Ralescu, 2013).

In addition, uncertain optimal control has attracted scholars' attention in recent years, and is also a hot topic in this sub-field. The research objects of optimal control are uncertain explosion control (Xu & Zhu, 2012), multi-dimensional case control (Deng & Zhu, 2012; Chen & Li, 2015; Deng et al., 2018), control problems with jumps (Deng & Zhu, 2013; Deng & Chen, 2016; Deng & Shen, 2019), linear quadratic equation optimal control problems (Chen & Zhu, 2016; Li & Zhu, 2017;

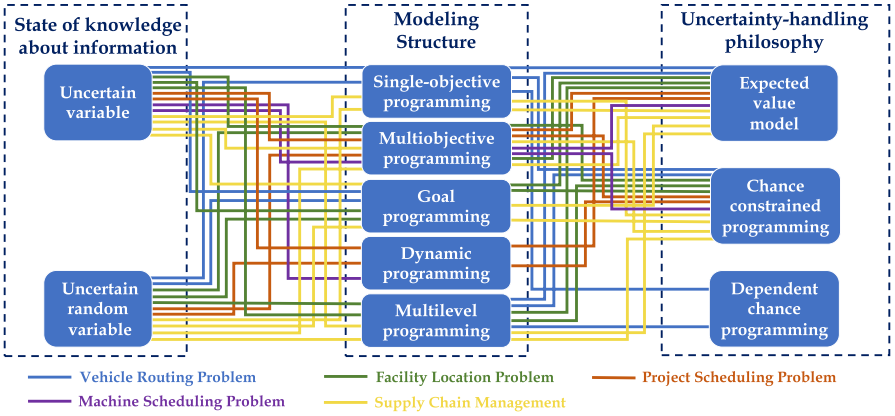

Fig. 8 Summary of Research Content of Uncertain Programming

Sheng et al., 2018; Li & Zhu, 2018; Chen et al., 2019), discrete-time control problems (Chen & Zhu, 2018a, b; Chen et al., 2019), nonlinear uncertain control (Ding et al., 2017), switched systems (Yan & Zhu, 2015; Yan et al., 2017), singular systems (Shu & Zhu, 2017a, b) and uncertain dynamic systems (Deng & Chen, 2017). Scholars have derived optimal equations, recursive equations, optimal principles, analytical expressions, sufficient and necessary conditions and well-posedness for these research objects, and have founded a continuous-time model, bang-bang optimal control model, optimistic value model, and optimal value model, so as to make use of uncertainty theory to settle practical problems.

Although the study of uncertain processes began in 2008, the volume of publications in the first three years was extremely low, and scholars did not focus on this sub-field until 2011. So far, the total amount and proportion of papers published in 2019 mean that it ranks third after uncertain differential equations and uncertain programming. In addition, the half-life of this sub-field is 3 years, which is only inferior to uncertain risk analysis, indicates that scholars are also paying great attention to it.

3.2.4 Uncertain programming

Uncertain programming has attracted extensive attention from many researchers since its inception in Liu (2009a). There are now more than 230 articles on uncertain programming, covering a wide range of disciplines. As shown in Fig. 7, the proportion of uncertain programming with a half-life of 3 years has gradually been expanded, having the second top ranking in 2019, which proves that scholars are very interested in researching this sub-field, and that there are likely to be more research outcomes on uncertain programming in the near future.

The main content of uncertain programming can be summarized in Fig. 8, where it is decomposed into three perspectives: the state of knowledge about information, modelling structure and uncertainty-handling philosophy (Liu, 2009a). Each perspective belongs to a column, and each column lists different types of perspective.

Moreover, choosing one type from each column to carry out three-three combination forms a certain type of uncertain programming. For example, choosing the uncertain random variable topic on the leftmost column, goal programming on the middle column, and chance-constrained programming on the rightmost column, we can get uncertain random chance-constrained goal programming. The general framework of single-objective programming and dynamic programming was established (Liu, 2009a). As an extension, multi-objective programming and goal programming were developed by Liu and Chen (2015), and multilevel programming was proposed by Liu and Yao (2015) in the same year for modelling a decentralized decision-making system with uncertain factors.

Furthermore, in the application of uncertain programming, hot research areas include the vehicle routing, facility location, project scheduling, machine scheduling, and supply chain management problems. These are shown in Fig. 8 and are represented by five different colours. The more broken lines there are of a given colour in the figure, the more kinds of uncertain programming are utilized in the application direction represented by the colour. Obviously, supply chain management is the most frequent context in which the field of uncertain programming is applied. At the same time, by observing the number of occurrences of each type of uncertain programming after connecting the lines, it can be found that the most used types of uncertain programming are uncertain expected value multi-objective programming and uncertain chance-constrained multi-objective programming.

However, the theoretical approaches to uncertain programming used in the application to each category of practical problem are different. Specifically, for the vehicle routing problem, scholars focus on effective path choice (Zhang & Peng, 2012; Huang et al., 2015, 2016; Wang et al., 2018) and transportation network design (Chen & Xu, 2012; Zhang et al., 2016; Chen et al., 2017a; Dalman, 2018). In terms of the facility location problem, authors discuss centre location of P-hub (Gao & Qin, 2016), location of single facility (Gao, 2012c), and facility location-allocation (Wen et al., 2014; Zhang et al., 2017, 2018). Literature on the project-scheduling problem is differentiated due to the different classifications and conditions of the projects. For example, the project classifications include R & D projects (Huang & Zhao, 2014b) and robust logistics projects (Ke et al., 2015); its conditions consist of uncertain activity duration times (Zhang & Chen, 2012; Ding & Zhu, 2015), expert estimates as project parameters (Huang & Zhao, 2016), and resource-constrained projects (Ma et al., 2016; Ke & Zhao, 2017). Scholars have looked at machine-scheduling problems with uncertain processing times and costs (Shen & Zhu, 2016), batch delivery (Shen & Zhu, 2019b), work deterioration and learning effect (Shen, 2019), parallel machines (Ning et al., 2017; Shen & Zhu, 2019a) and single machines (Shen & Zhu, 2018). The research on supply chain management focuses on aggregate production planning (Ning et al., 2013, 2019), the multi-product newsboy problem (Ding, 2013; Ding & Gao, 2014), single-period or multi-cycle inventory problems (Qin & Kar, 2013; Gao et al., 2013), and the pricing decision problem (Zhong et al., 2017; Chen et al., 2017; Cheng et al., 2018) of general, closed-loop, and dual-channel supply chains, involving network revenue management (Mou & Wang, 2014), manufacturing (Chien et al., 2018),

remanufacturing (Wen et al., 2015), furniture (Yang et al., 2017), fresh (Ning et al., 2019), e-commerce (Ke et al., 2018) and other industries.

In addition, the majority of uncertain programming problems obtain solutions by exploiting hybrid intelligent algorithms and genetic algorithms. However, different types of application problems improve the genetic algorithms based on different methods according to the actual situations (Sadjadi et al., 2012; Huang et al., 2016). Hybrid intelligent algorithms often combine genetic algorithms with Monte Carlo simulations (Xu et al., 2011), simplex algorithms (Wen et al., 2015), uncertain simulations (Zhang et al., 2018), 99 methods (Zhang et al., 2011; Zhang & Chen, 2012), and cell automation (Huang et al., 2016), among others.

3.2.5 Uncertain logic

Uncertain logic is a generalization of mathematical logic that takes advantage of uncertainty theory to process uncertain information and knowledge, and is an effective method for extracting language summaries from a collection of original data (Liu, 2015). Uncertain logic comprises uncertain propositional logic and uncertain predicate logic, the key to which is the calculation of the true value of uncertain propositions. The uncertain entailment was proposed by Liu (2009c) to use the maximum uncertain principle to calculate the true value of an uncertain proposition when other uncertain propositions are given. In practical examination, uncertain logic and uncertain entailment are closely linked in the research of uncertainty theory.

Li and Liu (2009) put forward uncertain logic within the framework of uncertainty theory, and investigated basic characteristics such as the laws of intermediate exclusion, contradiction and truth conservation. Two years later, as an application, Liu (2011) used uncertain quantifiers, uncertain subjects and uncertain predicates to deal with the uncertain logic of human language. In the same year, Chen and Ralescu (2011) designed a numerical method for calculating truth values of uncertain propositions, which was named the Chen-Ralescu theorem. On this basis, Liu and Yao (2017) proposed an uncertain stochastic logic in 2017 and derived a formula for calculating truth values of uncertain stochastic propositions. Furthermore, for uncertain predicate logic, Zhang and Li (2014) explored the semantic study of uncertain first-order predicate logic, and introduced the concepts of the uncertain predicate proposition, uncertain predicate formula, and degree of uncertain truth. Articles about uncertain entailment inquire into the validity and improvement of the uncertain entailment model (Zhang et al., 2014), the uncertain random entailment model (Liu & Yao, 2017) and the computation of the true value of its resolution principles (Yang et al., 2018).

There are 65 articles published in this sub-field, which is only higher than the number on uncertain sets. As shown in Fig. 7, the proportion of papers on uncertain logic shows a declining trend. The half-life is 6 years, which is relatively long, indicates that scholars' enthusiasm for this sub-field has decreased in recent years. As a result, uncertain logic, as a sub-field with a declining publication volume, is facing the threat of being eliminated in due course.

3.2.6 Uncertain sets

An uncertain set is a set-valued function on uncertain space, associated with which there are two basic tools: the membership function and the uncertainty distribution. In general, scholars first determine the membership function of the uncertain set, convert the membership function into an uncertainty distribution, and then perform arithmetic operations on the uncertain set (Liu, 2010a). The uncertain set was first proposed by Liu (2010c) to model "unclear concepts" that are essentially sets but do not clearly describe boundaries. Liu (2012b) also defined the concept and nature of the independence of the membership function and the uncertain set, and provided the operation laws of an uncertain set. Then, targeting the weakness of the definition of the independence of uncertain sets, Liu (2013a) proposed a stronger definition and discussed related mathematical properties. Yang and Gao (2015) presented the moment and central moment of an uncertain set, gave some formulas for calculating them through membership functions, and systematically reviewed uncertain sets (Yang & Gao, 2014). Later, since log entropy could not measure the uncertainty associated with all uncertain sets, researchers expansively supplemented the definition, properties and calculating criteria of the sine entropy (Yao, 2014), relative entropy (Yao & Ke, 2014), quadratic entropy (Wang & Ha, 2013), elliptic entropy (Gao & Ralescu, 2018), radical entropy (Peng & Li, 2013) and triangular entropy (Lu & Wang, 2013) of uncertain sets. The total order uncertain set, conditional uncertain set, distance measurement between different types of uncertain sets, and related inequalities have also been the subject of publications.

In addition, uncertain inference is the process of using uncertain set theory to derive results from human knowledge or evidence, an idea originally raised by Liu (2010c). For uncertain inference, Gao et al. (2010) investigated Liu's inference rules and their expressions, with multiple precedents and if-then rules. Other scholars have focused on the inference modelling (Yao, 2018b), programming and practicality of uncertain inference controllers (Gao, 2012b).

The hot content of this sub-field is mainly divided into uncertain sets and uncertain inference. These two areas' share of the literature has remained at the lowest level for many years, and the total number of related articles is 46. Therefore, it can be seen that this sub-field has not been fully explored. According to the first article published in 2010, and the half-life of 4 years, it is derived that uncertain sets belongs to a new sub-field, which will gradually receive more scholars' attention in the future.

3.2.7 Uncertain risk analysis

In uncertainty theory, the risk is defined as the unexpected loss plus the uncertain loss measure, the risk index is defined as the uncertain measure of the specific loss, and the reliability index is defined as the uncertain measure of the operating system. Thus, risk and reliability have the same root, and the analysis of both is strongly correlated. Uncertain risk analysis is a tool used to quantify the risk in uncertainty theory. Similarly, uncertain reliability analysis is also a means to deal with system reliability through uncertainty theory.

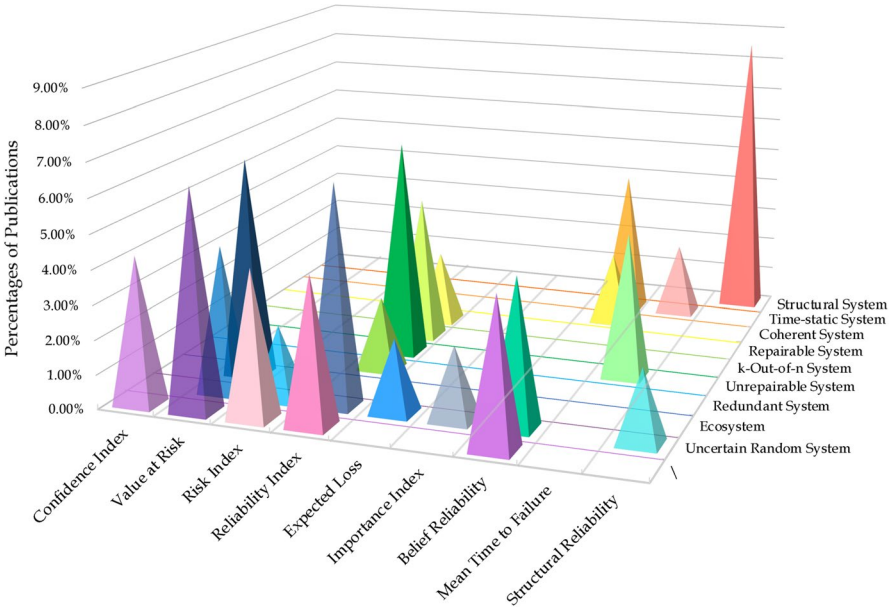

Fig. 9 The percentages of publications on various topics within uncertain risk analysis

When uncertain risk analysis and uncertain reliability analysis were proposed, Liu (2010b) defined the risk and reliability indices, and proved the risk and reliability index theorem. Three years later, Peng (2013) proposed the concept of value-at-risk as an alternative to the risk index. Other scholars have expanded the risk and reliability indicators further and then applied them. Figure 9 lists common indicators and applied systems in uncertain risk analysis and uncertain reliability analysis, and counts the ratios of documents with different combinations of indicators and systems to the total publications in this sub-field. The backslash indicates that the content of the document does not refer to a system. In Fig. 9, the proportion of the combination of structural reliability and structural systems has the highest share of the total publications, at 8.51%, followed by combinations of value-at-risk and backslash, value-at-risk and ecosystems, the reliability index and uncertain random systems,and the reliability index and the unrepairable system. For uncertain risk analysis, scholars have not only investigated the concepts and corresponding theorems of indicators such as value-at-risk (Peng, 2013; Liu & Ralescu, 2017) and the risk index (Liu & Ralescu, 2014), but have also studied uncertain risk assessment (Zhou et al., 2017; Zhang et al., 2018a), loss function risk measurement of uncertain systems (Peng, 2013), and the uncertain risk measure and comparison rules (Li & Peng, 2012), among others. Various systems, and the alpha most reliable paths, products and equipment in systems are all research objects in uncertain reliability analysis, and authors have discussed the mathematical models (Liu et al., 2015b; Zu et al., 2018; Hu et al., 2019), reliability indicators and their calculation formulas (Zeng et al., 2017; Liu et al.,

2018; Gao et al., 2018), and numerical algorithms or quantitative means (Wang et al., 2017; Zeng et al., 2018; Zhang et al., 2018b, 2019b) of these objects.

These analyses all together constitute the hot content of this sub-field and have received increasing attention from 2010 onwards. Although the share of this sub-field as a proportion of the total field put it in only fourth place in 2019, that is in the middle of the ranking, its number of publications has shown a steady rising trend, and its two-year half-life is the shortest, indicating that the attention paid to it might increase prominently in the future.

4 Measurement of research potential

The above analysis and elaboration have revealed the research contents and development status of the seven key sub-fields corresponding to the clusters of uncertainty theory obtained from the co-citation analysis, which also roughly analyzes the future development of several sub-fields based on the number of publications and half-life. However, the research potential of these sub-fields has not yet been accurately measured, which may make it difficult for researchers to correctly judge the value of the sub-fields and predict the hot trends of the future. Therefore, it is necessary to use relevant data on the number of publications, citations and half-life and a mature research potential evaluation model (Zhou et al., 2022) to reliably measure the sub-fields of uncertainty theory, so as to provide an effective reference and inspiration for scholars engaged in this field.

4.1 Exposition: research potential evaluation model

As a convenient approach for assessing the relative level of research potential, the proposed research potential evaluation model is inspired by the importance-performance analysis (IPA), which is a widely accepted tool in quality control (Martilla & James, 1977). Thus, the IPA model selects importance and performance as two dimensions to form a two-dimensional matrix, which divides quality attributes into four categories: "keep up the good work", "possible overkill", "low priority", and "concentrate here". Although multiple factors are involved in the research potential evaluation, the two indicators, maturity and recent attention (RA), cover several fundamental dimensions of research potential, such as citations, number of publication, duration, and publication density.

Maturity indicates the number of highly cited publications as a proportion of the total volume of publications in a particular sub-field, and reflects how much work in the sub-field is widely recognized. The calculation formula is as follows:

$$Maturity = \frac{number\ of\ highly\ cited\ papers}{total\ number\ of\ papers}$$

The statistics on the quantity of highly cited publications refer to Lotka's law (Allison et al., 1976), which means that the threshold for a highly cited paper is given by $N = 0.749 \times \sqrt{N_{\max}}$, where $N_{\max}$ is the number of citations of the most frequently

Table 6 Some indicators of the seven sub-fields

Rank	Sub-fields	TC/TP	D	R	NH	RA	M
1	Axiomatic System	16.328	12	5	63	0.272	0.460
2	Uncertain Programming	12.874	11	3	114	0.390	0.496
3	Uncertain Sets	6.804	10	4	13	0.170	0.283
4	Uncertain Logic	8.138	11	6	35	0.123	0.538
5	Uncertain Differential Equations	17.869	12	3	96	0.496	0.432
6	Uncertain Risk Analysis	6.517	10	2	26	0.326	0.224
7	Uncertain Processes	15.139	12	3	65	0.421	0.348

TC: total citations; TP: total publications; D: duration; R: recency; NH: number of highly cited paper; RA: recent attention; M: maturity

cited paper in this topic. All documents with a score higher than N are regarded as highly cited papers, and are thus included in the statistics. The maturity ranges from zero to one. Without considering other factors, this paper assumes that the lower is a topic's maturity, the higher is its research potential.

RA is used to detect cutting-edge research and predict possible future research directions, and its formula is:

$$\mathrm{RA} = \frac{TC}{TP \times D} \times \frac{1}{R},$$

where TC is total citations, TP is total publications, D is the time span of publications and R is recency. The smaller is the half-life, the higher is the recency. If there is rapid growth in the volume of literature in a topic in the short term, this research field may soon become a research hotspot. Recency can be used to judge the degree of attention and future trends of a research field. The higher the RA is, the more attention is paid to that research subject, and the more likely it is to see explosive development.

Observing the horizontal and vertical axes, it is obvious that both of them are related to the citations, but with different emphases. RA reflects the overall condition of the sub-field in recent time, while maturity values highly cited papers more, which represents the top and most widely accepted portion of the research. Similar to IPA, the research potential evaluation model is also a four-quadrant matrix by which different research sub-fields are categorized according to the overall average of RA and maturity. As shown in Fig. 9, the four quadrants are "Diamond in the Rough", "Hard Core", "Possibility", and "Chicken Ribs". According to the different characteristics of the four quadrants, their research potential can be ranked. Figure 9 assigns different rainbow gradients to each quadrant according to whether their potential is high or low. Red indicates the highest research potential, while purple indicates the lowest, so the research potential ranking of the quadrants is "Diamond in the Rough" > "Hard Core" > "Possibility" > "Chicken Ribs" . In addition, the dotted arrows in the figure represent possible future transitions between quadrants. This research potential evaluation model offers an effective

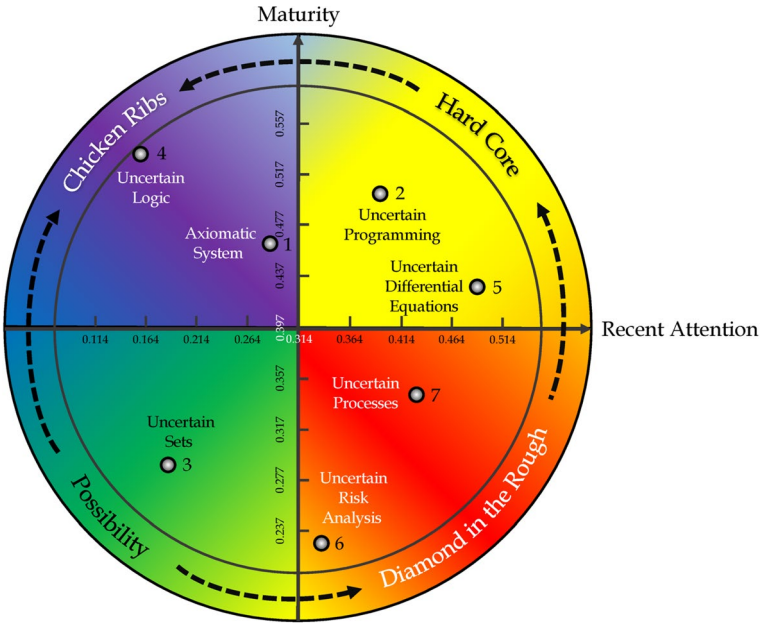

Fig. 10 Prediction

method for identifying future trends in sub-fields, and scientific and reasonable guidance for the selection of research topics.

4.2 Prediction: the model applied to uncertainty theory

The relevant data from the co-citation analysis of uncertainty theory are calculated, and the numerical results of some indicators are shown in Table 6. The distribution of the seven sub-fields in the research potential assessment model is shown in Fig. 10. While there is only one sub-field in the "Possibility" quadrant, each of the other three quadrants has two sub-fields. Combined with the results presented in Fig. 10 and the reading of the literature, we make the following inferences about the research potential of these key sub-fields, with the discussion divided into the four quadrants:

(1) As the quadrant with the highest research potential, "Diamond in the Rough" has a high amount of recent attention and a low maturity, which indicates that, although the research topic has not been fully explored, it has aroused widespread attention and may be an emerging frontier in the future. Uncertain risk analysis and uncertain processes are located in this quadrant, revealing that they have the highest research potential among the sub-fields. Comparing the two, we find that, although they are both in "Diamond in the Rough", the recent attention paid to uncertain processes is much higher, manifesting in it having built up more outstanding achievements, better participation and stronger research potential for scholars. As this

"Diamond in the Rough" topic is developing and has a continuously increasing number of highly cited works of literature, it will likely transfer to "Hard Core". Owing to the fact that the literature growth rate for the uncertain processes topic is faster than that for the uncertain risk analysis topic, we predict that the former will become a mainstream sub-field of uncertainty theory more quickly and be promoted to the "Hard Core" quadrant. In the last two years, more attention has been paid to the uncertain renewal reward process (Yao & Zhou, 2018a; Yao, 2019), the optimal control of the discrete-time uncertain system (Chen & Zhu, 2018a, b; Chen et al., 2019) and uncertain linear quadratic optimal control with jump (Deng & Shen, 2019; Chen et al., 2019, 2019). As one of the latest sub-fields to emerge, with the shortest half-life, uncertain risk analysis is a relatively new research direction of uncertainty theory, and the amount of published literature on the topic is increasing year by year. In recent years, papers in this sub-field have mainly focused on expected loss (Liu & Ralescu, 2018), investment risk analysis (Li et al., 2019), and structural reliability analysis of uncertain systems (Zhai & Zhang, 2019; Zhang et al., 2019b), among others. Since uncertain risks are highly valued, scholars are bound to carry out more in-depth exploration and analysis of uncertain risks. Therefore, the number of references with high citation frequencies in this field will increase significantly and it has great development prospects.

(2) The "Hard Core" quadrant, second only to "Diamond in the Rough", represents the mainstream of the research in this field and is regarded as a positive and powerful pillar of uncertainty theory, with a high amount of recent attention and a high maturity. The sub-fields in this quadrant are uncertain programming and uncertain differential equations, which are highly thought of, have good development trends, and are seeing steadily rising amounts of literature. In recent years, the research content of uncertain programming has tended towards the applications related to the sustainable facility location problem (Wang et al., 2019), multiobjective integrated multiproject scheduling problem (Hematian et al., 2019), parallel-machine scheduling problem (Shen & Zhu, 2019a) and optimal pricing decision for supply chains (Ma et al., 2019). As the only sub-field with consistently strong performance, both theoretical and applied research of uncertain differential equations have been highly valued by researchers and have made remarkable achievements. In particular, research on the uncertain heat equation (Yang, 2018, 2019; Yang & Ni, 2019), stability analysis of uncertain spring vibration equation (Jia et al., 2018; Jia & Yang, 2018; Jia & Ni, 2019; Jia et al., 2019), uncertain wave equation (Gao & Ralescu, 2019; Gao et al., 2019c), uncertain delay differential equation (Jia & Sheng, 2019; Wang & Ning, 2019, 2019b), multi-period polynomial portfolio selection problem (Chen et al., 2018; Zhang, 2019; Chen et al., 2019; Xue et al., 2019) and option pricing formula (Zhang et al., 2019; Yang et al., 2019; Lu et al., 2019) has not yet abated the scholars' enthusiasm.

It can be said that uncertain differential equations are the best-developed sub-field and the one that has attracted the most attention in this field, even exceeding uncertain programming. Therefore, the research potential of uncertain differential equations is stronger. High maturity corresponds to low development potential, and topics in the "Hard Core" quadrant may move to the "Chicken Ribs" quadrant later. However, uncertain programming and uncertain differential equations pose no

worries in this regard. Since they have continued to attract a high level of attention and maintained strong growth in recent years, and involve diverse research topics with room for improvement, it is reasonable to speculate that both will continue to offer good research potential and even see some breakthroughs.

(3) The themes in the "Possibility" quadrant, with low RA and low maturity, represent new but unpopular sub-fields or sub-fields with a long history. Uncertain sets, the only topic in this quadrant, belong to the former category. Since its emergence in 2010, this sub-field has produced a low amount of literature and has had a low RA, and has never seen major progress. Nevertheless, just as the name "Possibility" suggests, there is the chance of diversity and uncertainty in its future, and the possibility of it shifting to one of the other three quadrants. The research content of this sub-field has changed significantly in the past two years, no longer being limited purely to theoretical exploration, with uncertain sets used to solve practical problems, such as in Yao (2014), who utilized sine relative entropy of uncertain set to portfolio selection, Gao and Ralescu (2018), who handled portfolio clustering with elliptic entropy of uncertain set, and Sun et al. (2019), who applied uncertain sets to analyse user emotion in a restaurant recommendation system. Given the increasing annual amount of literature and its half-life of four, there is cause for optimism that numerous scholars will join in the study of uncertain sets and strengthen the accumulation of results in the future. This sub-field is most likely to transform into a "Diamond in the Rough".

(4) "Chicken Ribs" topics, which rank bottom for research potential, have low recent attention and high maturity. For sub-fields in this quadrant, it will be relatively difficult to make great progress in the future, and there is no opportunity for them to move into other quadrants. The axiomatic system and uncertain logic belong to this quadrant, but they are not similar. The axiomatic system, as an early research direction in this field, is gradually declining in terms of the attention it receives. The research content mainly concerns the theoretical extension of complex uncertain variables (Nath & Tripathy, 2019; Gao et al., 2019a), uncertain graph (Rosyida et al., 2018; Chen et al., 2019; Gao et al., 2019), and the like. In view of the fact that the axiomatic system of uncertainty theory has been successfully established and the content is roughly complete, future researchers will only need to supplement and strengthen the axiomatic system, so they will not put too much energy into this sub-field. However, uncertain logic is an unpopular sub-field with a relatively narrow research scope, and its high maturity is mainly due to the low total number of publications. Unlike axiomatic systems, uncertain logic may be phased out because of the unpopularity of the subject matter. At the same time, uncertain logic has a higher maturity and lower recent attention, so it can be concluded that its research potential is weaker.

In summary, the research potential of the seven key sub-fields can be ranked as follows:uncertain processes > uncertain risk analysis > uncertain differential equations > uncertain programming > uncertain sets > axiomatic system > uncertain logic . Not only is the research potential evaluation of sub-fields, the RPE model can also measure the potential of more detailed topics, and then more specifically analyze the future trends of the domain.

5 Conclusion

Uncertainty theory has unique advantages in dealing with human uncertainty and belief degrees, and has gained widespread attention and development since its birth. As the review of uncertainty theory, this paper applies bibliometrics and content analysis methods to analysis the development status of this field and the main research directions of seven sub-fields in detail, and utilizes the *research potential evaluation* (RPE) model to assess the future potential space and development trends of each sub-field. The research significance and innovations of this paper are listed as follows: As a first comprehensive study, this paper not only provides a systematic overview of uncertainty theory from multiple angles, but also clusters its research directions and hot topics. Second, implementing an innovative research methodology that combines co-citation analysis based on bibliometrics with content analysis methods, seven key sub-fields are derived, and then their characteristics and main contents are elaborated. Third, this study employs the RPE model to measure the potential of each sub-field from the point of view of maturity and recent attention, and reveals hot issues and future development of sub-fields. This research provides practical and useful guidance to scholars aiming to choose suitable research topics that are worth investigating, and thereby further promote the development of uncertainty theory. The main important and inspiring conclusions of this paper are summarized in Table 7.

Looking into the future, uncertain risk and reliability analysis, uncertain optimal control and uncertain finance may be the main research hotspots of uncertainty theory according to the outcomes of the above analysis. Considering the uncertain fluctuations in the financial market and human uncertainty in the decision making process and experts' estimations, uncertainty theory can be employed to deal with the background risk of portfolio selection and propose practically useful option pricing formulas. Besides, in the era of intelligence based on big data, uncertain regression analysis, uncertain time series analysis, uncertain differential equation and uncertain hypothesis test of uncertainty theory can be integrated with big data to improve or innovate the big data technology, as well as overcoming the defects of historical data. For example, scholars utilize uncertain statistics to track and analyze the big data related to pandemics (e.g., COVID-19), and establish the uncertain growth model to assist in solving practical problems. Such big data technology based on uncertainty theory could be applied to uncertain rumour spread, uncertain chemical reaction, uncertain software reliability, among many others.

All in all, uncertainty theory, as a new tool for dealing with uncertainty, is a tree full of power and value bursting out of the land of mathematics. The axiomatic system topic has a certain scale, has seen considerable academic achievements and gained wide attention, which provides a fertile soil and solid foundation for the field. Uncertainty theory has been recognised broadly and propagated by many distinguished Chinese scholars to many universities and research communities around the Globe. Even though the different branches of the tree are growing differently, the whole is gradually being strengthened and advanced. It

Table 7 Summary of main conclusions

Domain Feature	Satus	Trends
Number of publications	Increasing numbers of papers and citations; more than half (562 papers) published between 2017 and 2019	A significant exponential growth trend
Application categories	The top three categories: computer science (37.89%), mathematics (20.66%) and engineering (11.60%)	The expanding range of categories
Academic cooperation	Spreading from China to more than 50 countries, especially in the United States, India and Iran; the cooperation among more than 1,000 scholars and 700 institutions	Establishing a stable and efficient transnational group in the future
Journal analysis	More than 300 journals, including EJOR, IEEE TFS, INS, RESS, MSSP, IEEE TIE, CIE; the highest percentage of literature of FODM (21.86%)	More attention and promotion of high- quality classical journals
Main path	Evolution from fundamental theory to applied research of seven paths, with special emphasis on uncertain differential equations	More diversified and specialized achie- vements to solve practical problems
Sub-field Name	Status & Trends	Hot and specific issues
Axiomatic system	A gradual downward proportion of publications; fewer development opportunities	The theoretical extension of complex uncertain variables, uncertain graph, and the like
Uncertain differential equations	The only one to have held a high level of attention and strong performance; the second largest number of papers	Stability analysis of uncertain spring vibration equation and uncertain delay differential equation, multi-period polynomial portfolio selection problem
Uncertain processes	Low attention between 2008 and 2010; the third largest volume of publications; the second-ranked half-life (3 years); being paid great attention recently	Uncertain random renewal reward process, optimal control of the discrete-time uncertain system, uncertain linear quadratic optimal control with jump
Uncertain programming	The highest total number of publications (230 articles); receiving a high degree of attention	Sustainable facility location-allocation problem, multiobjective integrated multiproject scheduling, parallel- machine scheduling, optimal pricing decision for supply chains
Uncertain logic	A declining publication volume; the longest half-life (6 years); facing the threat of being eliminated	A relatively narrow and unpopular research scope

Table 7 (continued)

Sub-field Name	Status & Trends	Hot and specific issues
Uncertain sets	Not being fully paid attention to; many possibilities for its future development	Application of emotion analysis and portfolio selection
Uncertain risk analysis	The shortest half-life; receiving increasing attention from 2010 onwards	Expected loss and structural reliability analysis of uncertain systems

is believed that, in the future, uncertainty theory will eventually grow into a lush towering tree and bear rich fruits of thought.

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