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Medical health resources allocation evaluation in public health emergencies by an improved ORESTE method with linguistic preference orderings

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Abstract
As an important major public health emergency, COVID-19 broke out more than two years. At present, China has entered the post-epidemic era. However, it is still necessary to study the medical health resource allocation in public health emergencies. Therefore, the evaluation of medical health resources allocation is important. Firstly, we use two kinds of linguistic preference orderings (LPOs) to represent experts' opinions when evaluating the medical health resources allocation in public health emergencies. Then, a novel ORESTE method with LPOs is developed to solve multiple criteria decision-making (MCDM) problems. Additionally, we apply the proposed ORESTE method to solve a practical MCDM problem involving the medical health resources allocation in public health emergencies. Finally, some comparative analyses among the proposed ORESTE method and some existing methods under a double hierarchy linguistic environment are set up, and some discussions are summarized to show the validity and applicability of the proposed novel ORESTE method.

Keywords Linguistic preference orderings · ORESTE · Multiple criteria decision-making · Medical health resources allocation · Public health emergencies

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1 Introduction

In December 2019, COVID-19 broke out, and the World Health Organization (WHO) classified the outbreak as a public health emergency of international concern because of the increasing number of infected people and the outbreak in many countries around the world. In the early stage of the epidemic, an important reason for the high mortality rate in Hubei province may be the scarcity of medical and health resources, especially beds and medical staff. Therefore, how rationally allocating medical and health resources is an important means to halt the development of the epidemic. According to this idea, almost all the medical resources in China began to be inclined to Hubei province, lots of provinces sent medical support teams, the huoshenshan Hospital and Leishenshan Hospital were built, etc. At present, China has entered the post-epidemic era. However, it is still necessary to study the medical health resources allocation based on public health emergencies, and to provide suggestions for China's emergency management in this field. Therefore, this paper is devoted to evaluating the medical health resources allocation methods, which consists of two important parts: One is to represent the evaluation information accurately and clearly, and the other one is to develop a more rational decision-making approach to respond to the medical health resources allocation evaluation in public health emergencies.

To deal with the first issue, experts usually prefer to use natural languages or linguistic terms to express their evaluations instead of some crisp numbers considering that the formers are more in line with people's habits of expressions and close to the real thoughts of people (Gou et al., 2019; Zadeh, 2012). Double hierarchy linguistic term set (DHLTS), as a popular complex linguistic expression model, was defined by Gou et al. (2017a), and it is formed by two hierarchy linguistic term sets (LTSs). In recent years, some extensions of DHLTS have been developed and the popular research fields consist of the aggregation operators (Liu et al., 2019), the measure methods (Zhang et al., 2023), the preference relations (Gou et al., 2018, 2019, 2021a), the decision-making methods (Gou et al., 2017a, 2021b; Krishankumar et al., 2019; Liu et al., 2018, 2019; Wang et al., 2020), etc. Additionally, some preference orderings were developed to express the evaluation of experts (Hervés-Beloso & Cruces, 2018; Zhang et al., 2018). However, the existing preference orderings can only reflect the orderings of alternatives but lack the research on the precise relationship between any two adjacent alternatives, As a result, they may lead to the misconception that the relationship between any two alternatives in decision-making process is equal. To make up for this shortcoming and let the original linguistic information be more accurately described, Gou et al. (2020) defined the concept of linguistic preference orderings (LPOs), which consist of two types of information: One is the preference order which is similar as the existing preference orderings, the other one is to add double hierarchy linguistic terms (DHLTs, the basic elements of DHLTS) to the preference ordering and use them to express the unbalanced relationship between any two adjacent alternatives. Especially, according to the habits of the experts, LPOs usually contain two kinds of forms, i.e., the LPOs in continuous form and the LPOs in decentralized form, respectively (Gou et al., 2020, 2021c). Considering that LPOs can fully express experts' assessments, it is used to describe experts' opinions when evaluating the medical health resources allocation during public health emergencies.

Additionally, the second issue is to develop a reasonable decision-making approach to obtain the final solution after getting the assessments of experts. When solving multiple criteria decision-making (MCDM) problems, there exist two kinds of decision-making methods, i.e., the utility value-based decision-making methods (Dong & Wan, 2016; Gou et al., 2017a, b; Krishankumar et al., 2019) and the outranking-based decision-making methods (Li et al., 2020; Liu et al., 2018; Wang et al., 2020). Firstly, the utility value-based methods mainly include aggregation-based methods and reference point-based methods (Liao et al., 2020). Under a double hierarchy linguistic environment, the aggregation function-based decision-making methods mainly consist of the MULTIMOORA method (Gou et al., 2017a), the AQM method (Gou et al., 2017b), the VIKOR method (Gou et al., 2021b) and the WASPAS method (Krishankumar et al., 2019), etc. However, the aggregation-based methods are limited in rationality due to the different measurements among criteria, and the reference point-based methods may be closest to the ideal one, but not always dominate others. Additionally, the outranking-based decision-making methods are based on pairwise comparisons of alternatives under each criterion, and these methods based on double hierarchy linguistic environment mainly include the LINMAP method (Li et al., 2020; Liu et al., 2018) and the ORESTE method (Wang et al., 2020), etc.

As one of the outranking-based decision-making methods in MCDM, the classical ORESTE method was first proposed by Roubens (1982), and it has been studied in amounts of fields such as web design firm selection (Adali & Tusisik, 2017), innovative design selection of shared cars (Wu & Liao, 2018) and assessment of traffic congestion (Wang et al., 2020), etc. The ORESTE method has two main advantages: (1) One is that it uses Besson's rank (Pastijn & Leysen, 1989) to make the decision instead of transforming the original information into crisp weights, so it can reduce the loss of original information to some extent; the other one is that the ORESTE method extends the relationships of alternatives $A_i (i = 1, 2, ..., m)$ to three forms, i.e., the preference relation, the indifference relation, and the incomparability relation. Therefore, it greatly increases the accuracy of the rank of alternatives.

However, it is obvious that the ORESTE method changes the original information when calculating the global preference score because Besson's ranks of original information and the importance of criteria are only the ranks but not the original information. Therefore, this paper develops an improved ORESTE method to deal with a practical MCDM problem involving the evaluation of the medical health resources allocation methods in public health emergencies in which the original information is used to obtain the global preference score instead of Besson's ranks.

The main innovation points of this paper are highlighted as follows:

(1) The evaluations of experts are expressed by LPOs, which are more in line with the real thoughts of experts. Then, the LPOs can be transformed into the related double hierarchy linguistic preference relations (DHLPRs) with complete consistencies equivalently.

(2) An improved ORESTE method is developed to deal with MCDM problems with LPOs and it can be used to avoid the situation where the loss of original information.
(3) We apply the proposed improved ORESTE method to deal with a practical MCDM problem involving the medical health resources allocation evaluation in public health emergencies.
(4) Some comparative analyses among the improved ORESTE method and several existing decision-making methods under a double hierarchy linguistic environment are set up, and the discussions are summarized to show the advantages and disadvantages of the improved ORESTE method.

The remainder of the paper is organized as follows: Sect. 2 reviews some related concepts of DHLTS, DHLPR, LPOs, and the classical ORESTE method. Section 3 develops an improved ORESTE method with LPOs in MCDM. Section 4 applies the improved ORESTE method to deal with a practical MCDM problem, and makes comparative analyses with existing methods. Some conclusions are summarized in Sect. 5.

2 Preliminaries

This section mainly reviews some related concepts including DHLTS, DHLPR, LPOs, and the classical ORESTE method.

2.1 Double hierarchy linguistic term set

Zadeh (2012) provided the concept of Computing with Words (CWW), which can be used to express and handle natural languages. Based on CWW, some complex linguistic representation models were developed (Herrera & Martínez, 2000; Rodríguez et al., 2012). However, there are some gaps in the existing linguistic representation models. For example, it is very difficult to express some more complex linguistic information such as "*only a little fast*" or "*between a little low and much high*". To solve these gaps, Gou et al. (2017a) defined the concept of DHLTS by splitting a complex linguistic term into two parts with the form of "adverb+adjective" and expressing them by different kinds of linguistic terms respectively.

Let $S=\{s_t|t=-\tau,...,-1,0,1,...,\tau\}$ and $O^t=\{o_k^t|k=-\varsigma,...,-1,0,1,...,\varsigma\}$ be the first hierarchy LTS and the second hierarchy LTS of linguistic term s_t in S, respectively. Gou et al. (2017a) proposed the concept of DHLTS S_O shown as follows:

$$S_O=\{s_{t<o_k^t>}|t=-\tau,...,-1,0,1,...,\tau;\ k=-\varsigma,...,-1,0,1,...,\varsigma\} \tag{1}$$

where the basic element $s_{t<o_k^t>}$ is called DHLT, and o_k^t expresses the second hierarchy linguistic term of the linguistic term s_t in S. For convenience, Gou et al., (2017a, 2017b) used a unified form to express all second hierarchy LTSs, and Eq. (1) can be rewritten by $S_O=\{s_{t<o_k>}|t=-\tau,...,-1,0,1,...,\tau;\ k=-\varsigma,...,-1,0,1,...,\varsigma\}$.

To facilitate the calculations of DHLTs when introducing the additively consistent DHLPR and comparing any two DHLTs, Gou et al., (2017a, 2017b) proposed two equivalent transformation functions in which the subscript (t, k) of the DHLT $s_{t<o_k>}$ which expresses the equivalent information to the membership degree γ can be transformed to each other by functions f and f^{-1}:

$$f : [-\tau, \tau] \times [-\varsigma, \varsigma] \rightarrow [0, 1], f(t, k) = \frac{k + (\tau + t)\varsigma}{2\varsigma\tau} = \gamma \tag{2}$$

$$\begin{aligned} f^{-1} &: [0, 1] \rightarrow [-\tau, \tau] \times [-\varsigma, \varsigma], f^{-1}(\gamma) = [2\tau\gamma - \tau] < o_{\varsigma(2\tau\gamma-\tau-[2\tau\gamma-\tau])} > \\ &= [2\tau\gamma - \tau] + 1 < o_{\varsigma((2\tau\gamma-\tau-[2\tau\gamma-\tau])-1)} > \end{aligned} \tag{3}$$

2.2 Double hierarchy linguistic preference relation

Considering that the pairwise comparison methods are more accurate than some common forms of evaluations, and the main advantage of pairwise comparison is that experts only need to focus exclusively on two alternatives at a time when expressing their preferences (Chiclana et al., 2009). Therefore, Gou et al. (2021a) proposed the concept of DHLPR. Firstly, the operational laws of DHLTs only used to check whether a DHLPR is of acceptable consistency should be given. Suppose that $s_{t<o_k>}$, $s_{t^1<o_{k1}>}$ and $s_{t^2<o_{k2}>}$ are three DHLTs, and $\lambda(0 \leq \lambda \leq 1)$ is a real number. Then, $s_{t^1<o_{k1}>} \oplus s_{t^2<o_{k2}>} = s_{t^1+t^2<o_{k1+k2}>}$, if $t^1 + t^2 \leq \tau, k^1 + k^2 \leq \varsigma$; and $\lambda s_{t<o_k>} = s_{\lambda t<o_{\lambda k}>}$.

In an MCDM problem under a double hierarchy linguistic environment, let $A = \{A_1, A_2, ..., A_m\}$ be a fixed set of alternatives, the experts evaluate alternatives by pairwise comparisons and provide their preference information. Then, the concept of DHLPR is defined as follows:

Definition 1 (Gou et al., 2021a) Let S_O be a DHLTS. A DHLPR R is presented by a $m \times m$ matrix $R = (r_{ij})_{m\times m}$, where each element $r_{ij} \in S_O\ (i,j = 1, 2, ..., m)$ is a DHLT, indicating the degree of the alternative A_i over A_j. For all $i,j = 1, 2, ..., m$, $r_{ij}(i < j)$ satisfies the conditions $r_{ij} + r_{ji} = s_{0<o_0>}$ and $r_{ii} = s_{0<o_0>}$.

Additionally, based on the function f, if a DHLPR $R = (r_{ij})_{m\times m}$ satisfies

$$f(r_{ij}) = f(r_{i\rho}) + f(r_{\rho j}) - 0.5 \quad (i, j, \rho = 1, 2, ..., m, i \neq j) \tag{4}$$

then, it can be called an additively consistent DHLPR (Gou et al., 2020).

Based on Eq. (4), the following theorem was developed to get the additively consistent DHLPR.

Theorem 1 (Gou et al., 2020) *Let* $R = (r_{ij})_{m\times m}$ *be a DHLPR. If* $f(\overline{r}_{ij}) = \frac{1}{m}(\bigoplus_{\rho=1}^{m}(f(r_{i\rho}) + f(r_{\rho j}) - 0.5))$ *for all* $i, j, \rho = 1, 2, ..., m, i \neq j$, *then* $\overline{R} = (\overline{r}_{ij})_{m\times m}$ *is an additively consistent DHLPR.*

Example 1 Let $S = \{s_{t<o_k>} | t = -4, \ldots, 4; k = -4, \ldots, 4\}$ be a DHLTS, one DHLPR can be established as:

$$R = (r_{ij})_{4\times4} = \begin{pmatrix} s_{0<o_0>} & s_{-1<o_1>} & s_{0<o_2>} & s_{1<o_{-2}>} \\ s_{1<o_{-1}>} & s_{0<o_0>} & s_{1<o_{-1}>} & s_{2<o_1>} \\ s_{0<o_{-2}>} & s_{-1<o_1>} & s_{0<o_0>} & s_{1<o_1>} \\ s_{-1<o_2>} & s_{-2<o_{-1}>} & s_{-1<o_{-1}>} & s_{0<o_0>} \end{pmatrix}.$$

Using Eq. (4), additively consistent DHLPR $\overline{R} = (\overline{r}_{ij})_{4\times4}$ of R can be obtained:

$$\overline{R} = (\overline{r}_{ij})_{4\times4} = \begin{pmatrix} s_{0<o_0>} & s_{-1<o_{0.5}>} & s_{0<o_{0.25}>} & s_{1<o_{0.25}>} \\ s_{1<o_{0.5}>} & s_{0<o_0>} & s_{1<o_{-0.25}>} & s_{2<o_{-0.25}>} \\ s_{0<o_{-0.25}>} & s_{-1<o_{0.25}>} & s_{0<o_0>} & s_{1<o_0>} \\ s_{-1<o_{-0.25}>} & s_{-2<o_{0.25}>} & s_{-1<o_0>} & s_{0<o_0>} \end{pmatrix}.$$

2.3 Linguistic preference orderings

In decision-making processes, sometimes experts prefer to represent their assessments using preference orderings. To express these assessments clearly and accurately, Gou et al. (2020) defined two kinds of LPOs, i.e., the LPO in continuous form and the LPO in decentralized form, respectively. In this instance, the continuous form uses the LPO to rank all alternatives in a continuous manner, while the decentralized form provides the relationship between any two alternatives, which is then used to create a preference ranking set.

2.3.1 The LPO in continuous form

Let $S_O = \{s_{t_{<o_k>}} | t = -\tau, ..., -1, 0, 1, ..., \tau; k = -\varsigma, ..., -1, 0, 1, ..., \varsigma\}$ be a DHLTS. Suppose that an expert provides his/her assessments about all alternatives by a LPO denoted by:

$$LPO' = \left\{ A_{\sigma(1)} \overset{s^{(\sigma(1),\sigma(2))}_{t<o_k>}}{>} A_{\sigma(2)} \overset{s^{(\sigma(2),\sigma(3))}_{t<o_k>}}{>} \dots \overset{s^{(\sigma(m-1),\sigma(m))}_{t<o_k>}}{>} A_{\sigma(m)} \right\} \tag{5}$$

in which all alternatives are concluded in a preference ordering and there is a relationship between any two adjacent alternatives. Then it is called the LPO in continuous form, where $\bigoplus_{i=1}^{m-1} s^{(\sigma(i),\sigma(i+1))}_{t<o_k>} \le s_{\tau<o_\varsigma>}$, $A_{\sigma(i)}(i = 1, 2, ..., m)$ denotes the i - th largest alternative, and the linguistic preference information $s^{(\sigma(i),\sigma(i+1))}_{t<o_k>}(i = 1, 2, ..., m-1)$ is a DHLT that means the degree of the i - th largest alternative is better than the $(i + 1)$ - th largest alternative (Gou et al., 2020).

For example, suppose that a set of alternatives is $A = \{A_1, A_2, A_3, A_4\}$, and a LPO in continuous form may be denoted as $LPO' = \left\{ A_2 \overset{alittlehigh}{>} A_3 \overset{verymuchhigh}{>} A_1 \overset{onlyalittlehigh}{>} A_4 \right\}$, this LPO means that the ordering of alternatives is $A_2 > A_3 > A_1 > A_4$ and $A_{\sigma(1)} = A_2$,$A_{\sigma(2)} = A_3, A_{\sigma(3)} = A_1, A_{\sigma(4)} = A_4$. Additionally, A_2 is a little higher than A_3, A_3 is very much higher than A_1, and A_1 is absolutely higher than A_4.

2.3.2 The LPO in decentralized form

Furthermore, sometimes experts may only prefer to provide partial pairwise comparisons between any two alternatives. In this case, Gou et al. (2020) defined the concept of LPO in a decentralized form:

$$LPO''^{a} = \left\{ A_i \overset{s^{ij}_{t<o_k>}}{>} A_j | s^{ij}_{t<o_k>} \in S_O, \quad i,j = 1,2,...,m; i \neq j \right\} \tag{6}$$

where $s^{ij}_{t<o_k>}$ expresses the relationship between two alternatives A_i and A_j $(i,j = 1,2,...,m; i \neq j)$.

For example, suppose that a set of alternatives $A = \{A_1, A_2, A_3, A_4\}$, and an expert provides his/her LPO in decentralized form, which is denoted as $LPO'' = \left\{ A_2 \overset{alittlehigh}{>} A_3, A_2 \overset{verymuchhigh}{>} A_1, A_4 \overset{onlyalittlehigh}{>} A_3 \right\}$. The LPO'' should include all alternatives, and there should be some sort of relationship—direct or indirect—between them. We can see that, the relationships between A_1 and A_3, between A_1 and A_4, and between A_2 and A_4 are indirect, which can be obtained by a method proposed in follows, to build the preference relationship matrix and retrieve the preference relationships between all alternatives.

Clearly, the LPOs can express linguistic information more completely and correctly. However, considering the special structure of these two kinds of LPOs, it is difficult to make correct calculations among them via the existing methods. Therefore, Gou et al. (2020) developed a useful method to transform the LPOs into the corresponding completely consistent DHLPRs.

In this transformation process, a model is established to calculate unknown preference information of the DHLPR $R = (r_{ij})_{m \times m}$:

$$\begin{cases} f\left(r_{\sigma(1)\sigma(3)}\right) = \frac{1}{m}\left(\overset{m}{\underset{\rho=1}{\oplus}} \left(f\left(r_{\sigma(1)\rho}\right) + f\left(r_{\rho\sigma(3)}\right) - 0.5\right)\right) \\ \ldots \\ f\left(r_{\sigma(1)\sigma(m)}\right) = \frac{1}{m}\left(\overset{m}{\underset{\rho=1}{\oplus}} \left(f\left(r_{\sigma(1)\rho}\right) + f\left(r_{\rho\sigma(m)}\right) - 0.5\right)\right) \\ f\left(r_{\sigma(2)\sigma(4)}\right) = \frac{1}{m}\left(\overset{m}{\underset{\rho=1}{\oplus}} \left(f\left(r_{\sigma(2)\rho}\right) + f\left(r_{\rho\sigma(4)}\right) - 0.5\right)\right) \\ \ldots \\ f\left(r_{\sigma(2)\sigma(m)}\right) = \frac{1}{m}\left(\overset{m}{\underset{\rho=1}{\oplus}} \left(f\left(r_{\sigma(2)\rho}\right) + f\left(r_{\rho\sigma(m)}\right) - 0.5\right)\right) \\ \ldots \\ f\left(r_{\sigma(m-2)\sigma(m)}\right) = \frac{1}{m}\left(\overset{m}{\underset{\rho=1}{\oplus}} \left(f\left(r_{\sigma(m-2)\rho}\right) + f\left(r_{\rho\sigma(m)}\right) - 0.5\right)\right) \end{cases} \tag{7}$$

In Eq. (7), Eq. (4) and Theorem 1 are utilized to obtain the Missing elements included in the given LPO. For example, the $r_{\sigma(1)\sigma(3)}$ can be obtained by $f\left(r_{\sigma(1)\sigma(3)}\right) = \frac{1}{m}\left(\bigoplus_{\rho=1}^{m}\left(f\left(r_{\sigma(1)\rho}\right) + f\left(r_{\rho\sigma(3)}\right) - 0.5\right)\right)$, and the remaining elements can be obtained using a similar track and formula.

2.4 The classical ORESTE method

The classical ORESTE method is to solve the MCDM problem based on general ranking (Pastijn & Leysen, 1989). The approach, which is a standard rank-preference relational decision algorithm, may combine the importance ranking of an attribute with the score ranking of each alternative under that attribute to determine the preference score value for each alternative. The classic ORESTE method consists of two stages: the first stage produces the weak ranking between the alternatives; the second stage obtains the PIR framework between the alternatives; and finally, the strong ranking between the alternatives can be produced based on the weak ranking and the PIR framework.

Firstly, the MCDM problem is described as: The set of alternatives is $A = \{A_1, A_2, ..., A_m\}$, and the set of criteria is $C = \{C_1, C_2, ..., C_n\}$. Let δ_j and $\delta_j(A_i)$ be the importance degree of the criterion C_j and the merit of the alternative A_i with respect to the criterion C_j, respectively. Both of them are expressed by Besson's ranks (Pastijn & Leysen, 1989). Then, the classical ORESTE method is shown as follows:

Step 1 Building weak rankings between alternatives

(1) The global preference score $G(\alpha_{ij})$ of the alternative A_i with respect to the criterion C_j is obtained:

$$G(\alpha_{ij}) = \sqrt{\varepsilon(\delta_j)^2 + (1-\varepsilon)(\delta_j(A_i))^2} \tag{8}$$

where ε $(0 \le \varepsilon \le 1)$ expresses the parameter measuring the ranking importance of criteria and alternatives, and is given by the decision makers. α_{ij} is the action of the alternative A_i with respect to the criterion C_j, $G(\alpha_{ij})$ indicates the alternative A_i's overall preference score according to the C_j criterion.

(2) Identify the global weak ranking $\delta(\alpha_{ij})$, which is denoted by Besson's ranks and determined by the values of $G(\alpha_{ij})$ $(i = 1, 2, ..., m; j = 1, 2, ..., n)$. For any $i, p = 1, 2, ..., m$ and $j, q = 1, 2, ..., n$, if $G(\alpha_{ij}) > G(\alpha_{pq})$, then $\delta(\alpha_{ij}) > \delta(\alpha_{pq})$; if $G(\alpha_{ij}) = G(\alpha_{pq})$, then $\delta(\alpha_{ij}) = \delta(\alpha_{pq})$.

(3) Obtain the weak ranking of every alternative A_i:

$$\Delta(A_i) = \sum_{j=1}^{n} \delta(\alpha_{ij}) \tag{9}$$

Step 2 Building the PIR structure between alternatives

(1) In fact, two alternatives may have different performances with respect to different criteria when they have the same global weak rankings. Thus, it is necessary to calculate the preference intensity of any two alternatives, which is an index to distinguish the incomparability relation or the indifference relation between two alternatives. The most prominent characteristic of the ORESTE method is to make further conflict analysis based on the preference intensities. Then, the average preference intensity between the two alternatives A_i and A_p is obtained by

$$F(A_i, A_p) = \frac{\sum_{j=1}^{n} \max\{\delta(\alpha_{pj}) - \delta(\delta_{ij}), 0\}}{(m-1)n^2} \tag{10}$$

Then, the net preference intensity between these two alternatives A_i and A_p is calculated by

$$\Delta F(A_i, A_p) = F(A_i, A_p) - F(A_p, A_i) \tag{11}$$

(B) Establish the PIR structure. The relationship between any two alternatives can be divided into three types: preference (A_iPA_p), indifference (A_iIA_p), and incomparability (A_iRA_p) between two alternatives A_i and A_p. The conflict analysis is obtained according to the following rules shown in Fig. 1.

In this structure, σ, η and ρ are three different parameters to determine the PIR relationships between two alternatives. Generally, their values are given based on the following rules:

$$\sigma < \frac{1}{(m-1)n}, \quad \rho > \frac{(n-2)}{4}, \quad \eta < \frac{\lambda}{2(m-1)} \tag{12}$$

where λ expresses the maximal rank difference between two alternatives (Pastijn & Leysen, 1989).

(C) By the global weak ranking and the PIR structure, the strong ranking of alternatives is obtained.

3 An improved ORESTE method with LPOs in MCDM

The traditional ORESTE approach has the following drawbacks: (1) the decision matrix under consideration includes limited details. (2) Information is lost during computation of the overall preference score, weak preferences, and PIR. (3) The preference conflict degree is expressed subjectively in the ORESTE approach. (4)

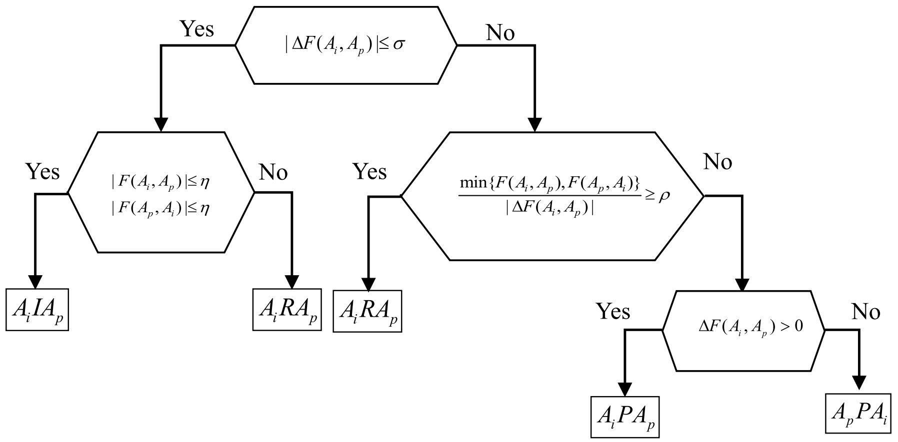

Fig. 1 The conflict analysis rules

The extended double hierarchy linguistic information cannot be handled by the classical ORESTE approach.

To deal with the above shortcomings, this section enhances the traditional ORESTE method and suggests an improved ORESTE method to handle the MCDM problems with LPOs. The improved method can make the original information more accurate and richer and avoid the loss of original information, and it is used to solve a real-world MCDM problem involving healthcare resource allocation assessment in public health emergencies.

Firstly, the MCDM method with LPOs can be described as follows: Let $A = \{A_1, A_2, ..., A_m\}$ be a set of alternatives, and $C = \{C_1, C_2, ..., C_n\}$ be a set of criteria, where $w = (w_1, w_2, ..., w_n)^T$ is the weight vector of criteria, and all of them are expressed by DHLTs, denoted by $w_j = s^j_{\phi<o_\varphi>} (j = 1, 2, ..., n)$. The invited experts evaluate all alternatives with respect to each criterion and provide their evaluations using LPOs, and the assessments establish an original decision matrix $ODM = (LPO_j)$ $(j = 1, 2, ..., n)$ shown as follows (Table 1).

3.1 Some basic concepts for the ORESTE method with LPOs in MCDM

Before giving the ORESTE method with LPOs in MCDM, some basic concepts are introduced as follows:

I. Considering that the structure of LPOs cannot make decision correctly, Gou et al. (2020) proposed a useful way to transform all LPOs into the corresponding completely consistent DHLPRs $R_j = (r^j_{ip})_{m \times m}$ $(j = 1, 2, ..., n)$ via Eq. (7). Therefore, Table 2 is transformed into the following form.
II. Based on the addition operation of DHLTs (Gou et al., 2017a), i.e., $s_{t^1<o_{k^1}>} \oplus s_{t^2<o_{k^2}>} = s_{t^1+t^2<o_{k^1+k^2}>}$, all DHLTs of each row in a DHLPR are

Table 1 The original decision matrix provided by experts

	C_1	C_2	…	C_n
LPOs	LPO_1	LPO_2	…	LPO_n

Table 2 The assessments with the form of DHLPRs transformed from LPOs

	C_1	C_2	…	C_n
LPOs	R_1	R_2	…	R_n

aggregated into a collective DHLT to express the degree of that the related alternative is better than the remaining alternatives. Thus, the overall assessment of an alternative with respect to each criterion is obtained by

$$OE(A_i)_j = \frac{1}{m} \overset{m}{\underset{p=1}{\oplus}} r^j_{ip} \tag{13}$$

Then, all of them establish a decision matrix:

$$DM = \begin{array}{c} \\ A_1 \\ A_2 \\ \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{pmatrix} OE(A_1)_1 & OE(A_1)_2 & \dots & OE(A_1)_n \\ OE(A_2)_1 & OE(A_2)_2 & \dots & OE(A_2)_n \\ \dots & \dots & \dots & \dots \\ OE(A_m)_1 & OE(A_m)_2 & \dots & OE(A_m)_n \end{pmatrix} \end{array}$$

III. For any two DHLTs $s_{\phi_1<o_{\varphi_1}>}$ and $s_{\phi_2<o_{\varphi_2}>}$, we can compare them based on the function f:

(1) If $f(s_{\phi_1<o_{\varphi_1}>}) > f(s_{\phi_2<o_{\varphi_2}>})$, then $s_{\phi_1<o_{\varphi_1}>} > s_{\phi_2<o_{\varphi_2}>}$;
(2) If $f(s_{\phi_1<o_{\varphi_1}>}) = f(s_{\phi_2<o_{\varphi_2}>})$, then $s_{\phi_1<o_{\varphi_1}>} = s_{\phi_2<o_{\varphi_2}>}$.

IV. For any two DHLTs $s_{\phi_1<o_{\varphi_1}>}$ and $s_{\phi_2<o_{\varphi_2}>}$, the distance between them is calculated as

$$d(s_{\phi_1<o_{\varphi_1}>}, s_{\phi_2<o_{\varphi_2}>}) = |f(s_{\phi_1<o_{\varphi_1}>}) - f(s_{\phi_2<o_{\varphi_2}>})| \tag{14}$$

***Example* 2.** Let $S_O = \{s_{t<o_k>} | t = -4, \dots, 4;\ k = -4, \dots, 4\}$ be a DHLTS,$A = \{A_1, A_2, A_3, A_4\}$ be a set of alternatives, and $C = \{C_1, C_2, C_3\}$ be a set of criteria. The LPOs provided by experts are shown in Table 3.

Then, by Eq. (7), all LPOs in Table 3 are transformed into three DHLPRs $R_j = (r^j_{ip})_{m\times m}$ $(j = 1, 2, 3)$ shown in Table 4.

Based on Eq. (13), the decision matrix DM is established:

$$DM = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{pmatrix} s_{-0.25<o_{-1}>} & s_{2<o_{-0.5}>} & s_{2<o_{-2.25}>} \\ s_{-1.25<o_0>} & s_{-2<o_{1.5}>} & s_{-1<o_{1.75}>} \\ s_{1.75<o_0>} & s_{1<o_{-1.5}>} & s_{-1<o_{-0.25}>} \\ s_{-0.25<o_1>} & s_{-1<o_{0.5}>} & s_{0<o_{0.75}>} \end{pmatrix} \end{array}$$

3.2 The improved ORESTE method with LPOs in MCDM

The process of the ORESTE method with LPOs in MCDM is shown as follows:

Step 1. Building weak rankings between alternatives

Firstly, it is necessary to calculate the double hierarchy linguistic global preference score (DHLGPS) LP_{ij} of the alternative A_i with respect to the criterion C_j.

For the decision matrix DM, we determine the maximum DHLT $OE(A_i)_{j+}$ and the minimum DHLT $OE(A_i)_{j-}$ of alternatives A_i with respect to each criterion C_j, respectively:

$$OE(A_i)_{j+} = \begin{cases} \max\limits_{i=1,2,\dots,m} \{OE(A_i)_j\}, & \textit{for the benefit criterion } C_j \\ \min\limits_{i=1,2,\dots,m} \{OE(A_i)_j\}, & \textit{for the } \cos t \textit{ criterion } C_j \end{cases} \tag{15}$$

$$OE(A_i)_{j-} = \begin{cases} \min\limits_{i=1,2,\dots,m} \{OE(A_i)_j\}, & \textit{for the benefit criterion } C_j \\ \max\limits_{i=1,2,\dots,m} \{OE(A_i)_j\}, & \textit{for the } \cos t \textit{ criterion } C_j \end{cases} \tag{16}$$

Then, the weight of the most important criterion and the most unimportant criterion satisfy

$$w^+ = \max_{j=1,2,\dots,n} \{w_j\} = \max_{j=1,2,\dots,n} \{s^j_{\phi<o_\varphi>}\} \tag{17}$$

$$w^- = \min_{j=1,2,\dots,n} \{w_j\} = \min_{j=1,2,\dots,n} \{s^j_{\phi<o_\varphi>}\} \tag{18}$$

When calculating the DHLGPS, it is necessary to consider two kinds of information, i.e., the decision-making information of experts and the weight information of criteria. Based on Eq. (14), we can obtain the normalized values of them respectively:

Table 3 The LPOs provided by experts

	C_1	C_2	C_3
LPOs	$\{A_1 \overset{s_{1<o_{-1}>}}{>} A_2, A_3 \overset{s_{2<o_1>}}{>} A_1, A_4 \overset{s_{1<o_1>}}{>} A_2\}$	$\{A_1 \overset{s_{1<o_1>}}{>} A_3 \overset{s_{2<o_{-2}>}}{>} A_4 \overset{s_{1<o_{-1}>}}{>} A_2\}$	$\{A_1 \overset{s_{2<o_{-3}>}}{>} A_4 \overset{s_{1<o_{-1}>}}{>} A_2 \overset{s_{0<o_2>}}{>} A_3\}$

Table 4 The completely consistent DHLPRs

	C_1	C_2	C_n
DHLPRs	$\begin{pmatrix} s_{0<o_0>} & s_{1<o_{-1}>} & s_{-2<o_{-1}>} & s_{0<o_{-2}>} \\ s_{-1<o_1>} & s_{0<o_0>} & s_{-3<o_0>} & s_{-1<o_{-1}>} \\ s_{2<o_1>} & s_{3<o_0>} & s_{0<o_0>} & s_{2<o_{-1}>} \\ s_{0<o_2>} & s_{1<o_1>} & s_{-2<o_1>} & s_{0<o_0>} \end{pmatrix}$	$\begin{pmatrix} s_{0<o_0>} & s_{4<o_{-2}>} & s_{1<o_1>} & s_{3<o_{-1}>} \\ s_{-4<o_2>} & s_{0<o_0>} & s_{-3<o_3>} & s_{-1<o_1>} \\ s_{-1<o_{-1}>} & s_{3<o_{-3}>} & s_{0<o_0>} & s_{2<o_{-2}>} \\ s_{-3<o_1>} & s_{1<o_{-1}>} & s_{-2<o_2>} & s_{0<o_0>} \end{pmatrix}$	$\begin{pmatrix} s_{0<o_0>} & s_{3<o_{-4}>} & s_{3<o_{-2}>} & s_{2<o_{-3}>} \\ s_{-3<o_4>} & s_{0<o_0>} & s_{0<o_2>} & s_{-1<o_1>} \\ s_{-3<o_2>} & s_{0<o_{-2}>} & s_{0<o_0>} & s_{-1<o_{-1}>} \\ s_{-2<o_3>} & s_{1<o_{-1}>} & s_{1<o_1>} & s_{0<o_0>} \end{pmatrix}$

$$\overline{OE}(A_i)_j = \frac{d(OE(A_i)_{j+}, OE(A_i)_j)}{d(OE(A_i)_{j+}, OE(A_i)_{j-})} \tag{19}$$

$$\overline{w}_j = \frac{d(w^+, w_j)}{d(w^+, w^-)} \tag{20}$$

Then according to Eq. (13), the DHLGPS LP_{ij} of the alternative A_i with respect to the criterion C_j is developed by

$$LP_{ij} = \left(\zeta\left(\overline{OE}(A_i)_j\right)^2 + (1-\zeta)\left(\overline{w}_j\right)^2\right)^{1/2} \tag{21}$$

where ζ $(0 \le \zeta \le 1)$ is the parameter to measure the relative importance between $\overline{OE}(A_i)_j$ and $\overline{w}_j$. Without loss of generality, we let $\zeta = 0.5$. Clearly, there is $LP_{ij} \in [0, 1]$.

Then, based on LP_{ij} $(i = 1, 2, ..., m; j = 1, 2, ..., n)$, the double hierarchy linguistic preference score of alternatives A_i is obtained by

$$LP_i = \frac{1}{n}\sum_{j=1}^{n} LP_{ij} \tag{22}$$

Obviously, $LP_i \in [0, 1]$. By putting the LP_i in increasing order, we can determine the weak ranking of A_i, denoted by $wr(A_i)$. Then, P and I relations of A_i and A_p($i, p = 1, 2, ..., m, i \ne p$)are obtained by the following rules:

(1) If $LP_i > LP_p$, then $wr(A_p) > wr(A_i)$, and denoted by A_pPA_i;
(2) If $LP_i = LP_p$, then $wr(A_p) = wr(A_i)$, and denoted by A_pIA_i.

Clearly, to some extent, the weak ranking of alternatives is very useful to determine the rank of all alternatives. However, there exist three obvious flaws: Firstly, even though we can obtain $wr(A_p) = wr(A_i)$, sometimes their performances are significantly different with respect to some criteria. Secondly, in the above rules, the incomparability (R) relation of alternatives is not considered. Finally, under a double hierarchy linguistic decision-making environment, the crisp preference scores transformed by Eq. (21) and Eq. (22) may lead to the loss of information to some extent. Therefore, it is necessary to improve the weak ranking P and I relations and develop the PIR structure by proposing the concepts of the double hierarchy linguistic preference intensities (DHLPIs) between alternatives A_i and A_p.

Step 2 Building the PIR structure between alternatives

Under a double hierarchy linguistic decision-making environment, the DHLPI used to determine the degree of dominance of alternative A_i over A_p can be obtained on the basis of the DHLGPS.

Definition 2 Let LP_{ij} and LP_{pj} be the DHLGPS of A_i and A_p with respect to criterion C_j, respectively. Then, the DHLPI of A_i over A_p with respect to criterion C_j is defined as.

$$U_j(A_i, A_p) = \max\{LP_{pj} - LP_{ij}, 0\} \tag{23}$$

Then, the double hierarchy linguistic average preference intensity (DHLAPI) of A_i over A_p is got as

$$U(A_i, A_p) = \frac{1}{n}\sum_{j=1}^{n} U_j(A_i, A_p) = \frac{1}{n}\sum_{j=1}^{n} \max\{LP_{pj} - LP_{ij}, 0\} \tag{24}$$

and the double hierarchy linguistic net preference intensity (DHLNPI) of A_i over A_p is obtained as

$$\Delta U(A_i, A_p) = U(A_i, A_p) - U(A_p, A_i) \tag{25}$$

Based on the classical ORESTE method (Pastijn & Leysen, 1989), the rules of conflict analysis under double hierarchy linguistic environment can be given as follows:

(1) When the DHLNPI of A_i over A_p is large enough, then the preference (P) relation between them is confirmed; Otherwise, the indifference (I) relation and incomparability (R) relation can be identified, respectively.
(2) When the DHLNPI of A_i over A_p is close to 0 and their DHLPIs with respect to all criteria are also close to 0, then the I relation is identified.
(3) When the DHLNPI of A_i over A_p is close to 0 and their DHLPIs with respect to all criteria are very large, then the R relation is established.

Similarly, it is necessary to define three thresholds to distinguish the PIR relations between any two alternatives. Suppose that μ is the indifference threshold, which is used to identify I and R relations with respect to every criterion; The preference threshold, denoted as ν, is defined to identify P relation; The incomparability threshold, denoted as o, is used to differentiate I and R relations. Specially, these three thresholds can be developed from the DHLGPSs. Based on Wu and Liao (2018), we can obtain that the indifference threshold $\mu \in [0, \frac{\sqrt{2}}{8\tau}]$, and the preference threshold $\nu = \frac{\mu}{n}$. Specially, if n is odd, then the incomparability threshold $o = \frac{(n+2)\mu}{2n}$; if n is even, then the incomparability threshold $o = \frac{\mu}{2}$.

Based on these three thresholds μ, ν and o, the rules for identifying the PIR relations are as follows:

(1) If $|\Delta U(A_i, A_p)| \geq \nu$, then

A) If $\Delta U(A_i, A_p) > 0$, then $A_i \; P \; A_p$;
B) If $\Delta U(A_p, A_i) > 0$, then $A_p \; P \; A_i$.

(2) If $|\Delta U(A_i, A_p)| < \nu$, then

A) If $U(A_i, A_p) < o$ and $U(A_p, A_i) < o$, then $A_i\ I\ A_p$;
B) If $U(A_i, A_p) \geq o$ or $U(A_p, A_i) \geq o$, then $A_i\ R\ A_p$.

In summary, according to the above analyses about the weak ranking and PIR relations, the specific steps of improved ORESTE method with LPOs in MCDM can be established as follows:

Algorithm 1. The improved ORESTE method with LPOs in MCDM

Step 1. The information on the evaluation of experts' preferences for each alternative under each criterion C_j is gathered and represented using LPOs, $LPO_j(j = 1, 2, ..., n)$. The weight vector of criteria is $w = (w_1, w_2, ..., w_n)^T$, and all of the weights are expressed by DHLTs, denoted by $w_j = s^j_{\phi<o_\varphi>}(j = 1, 2, ..., n)$. Then, the original decision matrix ODM is established.
Step 2. Transform all LPOs $LPO_j(j = 1, 2, \ldots, n)$ into the corresponding completely consistent DHLPRs $R_j = (r^j_{ip})_{m\times m}$ $(j = 1, 2, ..., n)$. Based on Eq. (13), establish the decision matrix $DM = (OE(A_i)_j)_{m\times n}$.
Step 3. Obtain the maximum DHLT $OE(A_i)_{j+}$ and the minimum DHLT $OE(A_i)_{j-}$ of all alternatives A_i $(i = 1, 2, ..., m)$ with respect to each criterion C_j respectively via Eq. (15) and Eq. (16). Then, calculate the weight of the most important criterion and the weight of the most unimportant criterion respectively based on Eq. (17) and Eq. (18).
Step 4. Calculate the DHLPGPS LP_{ij} of the alternative A_i with respect to the criterion C_j based on Eq. (21), and obtain the double hierarchy linguistic preference score LP_i of the alternative A_i via Eq. (22).
Step 5. Determine the weak ranking $wr(A_i)$ of each alternative A_i.
Step 6. Obtain the DHLPI $U_j(A_i, A_p)$ of A_i over A_p under criterion C_j by Eq. (23), the DHLAPI $U(A_i, A_p)$ of A_i over A_p by Eq. (24) and the DHLNPI $\Delta U(A_i, A_p)$ of A_i over A_p by Eq. (25).
Step 7. Determine the indifference threshold μ, the preference threshold ν, and the incomparability threshold o.
Step 8. Obtain the strong ranking of all alternatives according the given rules.

To understand the proposed ORESTE method better, one figure can be drawn as follows (Fig. 2).

4 The application of the improved ORESTE method with LPOs in MCDM

In public health emergencies, sometimes it is far from enough for some hospital departments to undertake the medical work. Therefore, it is effective to break down the internal divisions of hospitals and centralize the medical staff, beds, and medical supplies in each department for the admission and treatment of patients. Additionally, due to the difference in the number of infected people between regions and people's rush to big hospitals, the health system in the region faces different pressure in receiving and treating, resulting in the situation of supply and demand contradiction. Therefore, the reasonable allocation of medical health resources in the regional health system is very necessary for epidemic response. Finally, the delivery of temporary hospitals has largely alleviated the problem of beds being too tight and patients being unable to be treated, the pressure on front-line medical staff can be alleviated by transferring medical staff from different provinces and cities.

According to the medical health resources allocation methods mentioned above, four medical health resources allocation methods are summarized as alternatives: A_1: the integration of medical resources within the hospital; A_2: the integration of medical resources within health systems; A_3: building of new hospitals and increasing the supply of beds; A_4: relieving the pressure of front-line medical personnel, and transferring medical personnel from provinces and cities. Additionally, six criteria are also summarized including the diversity of causes of events (C_1), the difference in distribution (C_2), the pervasiveness of communication (C_3); the complexity of hazards (C_4); the comprehensiveness of governance (C_5), and the

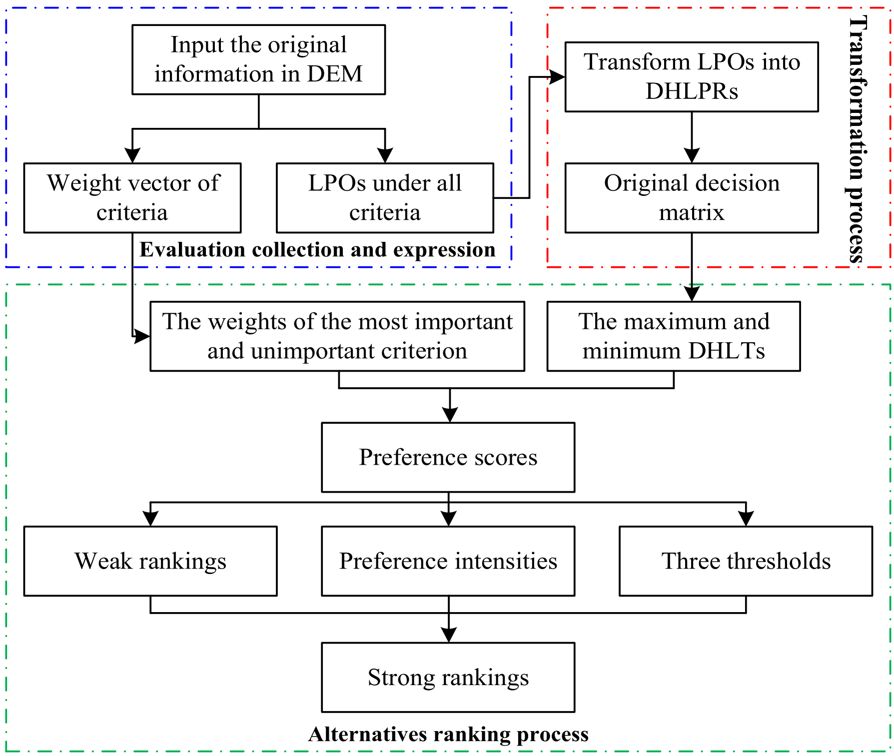

Fig. 2 The process of the ORESTE method with LPOs in MCDM

diversity of species (C_6). Let $S_O = \{s_{t<o_k>} | t = -4, ..., 4; k = -4, ..., 4\}$ be a DHLTS, where $S = \{s_{-4} = extremely\ bad,\ s_{-3} = very\ bad, s_{-2} = bad, s_{-1} = slightly\ bad, s_0 = medium, s_1 = slightly\ good,\ s_2 = good,\ s_3 = very\ good,\ s_4 = extremely\ good\}$

$$O = \{o_{-4} = far\ from, o_{-3} = scarcely, o_{-2} = only\ a\ little,\ o_{-1} = a\ little, o_0 = just\ right, o_1 = much, o_2 = very\ much, o_3 = extremely\ much,\ o_4 = entirely\}.$$

Then, let $w = (s_{1<o_2>}, s_{2<o_1>}, s_{-1<o_1>}, s_{-2<o_2>}, s_{0<o_1>}, s_{-1<o_{-1}>})^T$ be the weight vector of the above criteria, and some experts are invited to evaluate these four alternatives according to these six criteria and S_O, and provide their assessments shown in Table 5.

Clearly, this is a practical MCDM problem. We can apply the proposed ORESTE method to solve it.

4.1 Solve this MCDM problem by the proposed ORESTE method

Step 1. Transform all LPOs $LPO_j (j = 1, 2, \ldots, 6)$ into the corresponding completely consistent DHLPRs $R_j = (r_{ip}^j)_{4\times4}\,(j = 1, 2, ..., 6)$ shown in Table 6.

Based on Eq. (13), the decision matrix $DM = (OE(A_i)_j)_{4\times6}$ is established.

$$DM = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{pmatrix} s_{0.25<o_{0.5}>} & s_{2<o_{-0.5}>} & s_{2<o_{-2.25}>} & s_{0<o_{-0.25}>} & s_{1.75<o_{-1}>} & s_{1.25<o_{-0.75}>} \\ s_{-0.75<o_{-0.5}>} & s_{-2<o_{1.5}>} & s_{-1<o_{1.75}>} & s_{0<o_{0.75}>} & s_{-0.25<o_2>} & s_{-1.75<o_{0.25}>} \\ s_{1.25<o_{-1.5}>} & s_{1<o_{-1.5}>} & s_{-1<o_{-0.25}>} & s_{-2<o_{0.75}>} & s_{-1.25<o_{-1}>} & s_{1.25<o_{2.25}>} \\ s_{-0.75<o_{1.5}>} & s_{-1<o_{0.5}>} & s_{0<o_{0.75}>} & s_{2<o_{-1.25}>} & s_{-0.25<o_0>} & s_{-0.75<o_{-1.75}>} \end{pmatrix} \end{array}$$

Table 5 The evaluation information with LPOs

	C_1	C_2	C_3
LPOs	$\{A_1 \overset{s_{1<o_1>}}{>} A_2, A_3 \overset{s_{1<o_{-2}>}}{>} A_1, A_4 \overset{s_{0<o_2>}}{>} A_2\}$	$\{A_1 \overset{s_{1<o_1>}}{>} A_3 \overset{s_{2<o_{-2}>}}{>} A_4 \overset{s_{1<o_{-1}>}}{>} A_2\}$	$\{A_1 \overset{s_{2<o_{-3}>}}{>} A_4 \overset{s_{1<o_{-1}>}}{>} A_2 \overset{s_{0<o_2>}}{>} A_3\}$
	C_4	C_5	C_6
LPOs	$\{A_1 \overset{s_{2<o_{-1}>}}{>} A_3, A_2 \overset{s_{0<o_1>}}{>} A_1, A_4 \overset{s_{2<o_{-2}>}}{>} A_2\}$	$\{A_1 \overset{s_{2<o_{-3}>}}{>} A_2 \overset{s_{0<o_2>}}{>} A_4 \overset{s_{1<o_1>}}{>} A_3\}$	$\{A_3 \overset{s_{0<o_3>}}{>} A_1 \overset{s_{2<o_1>}}{>} A_4 \overset{s_{1<o_{-2}>}}{>} A_2\}$

Table 6 The transformed completely consistent DHLPRs form LPOs

	C_1	C_2
DHLPRs	$R_1=\begin{pmatrix} s_{0<o_0>} & s_{1<o_1>} & s_{-1<o_2>} & s_{1<o_{-1}>} \\ s_{-1<o_{-1}>} & s_{0<o_0>} & s_{-2<o_1>} & s_{0<o_{-2}>} \\ s_{1<o_{-2}>} & s_{2<o_{-1}>} & s_{0<o_0>} & s_{2<o_{-3}>} \\ s_{-1<o_1>} & s_{0<o_2>} & s_{-2<o_3>} & s_{0<o_0>} \end{pmatrix}$	$R_2=\begin{pmatrix} s_{0<o_0>} & s_{4<o_{-2}>} & s_{1<o_1>} & s_{3<o_{-1}>} \\ s_{-4<o_2>} & s_{0<o_0>} & s_{-3<o_3>} & s_{-1<o_1>} \\ s_{-1<o_{-1}>} & s_{3<o_{-3}>} & s_{0<o_0>} & s_{2<o_{-2}>} \\ s_{-3<o_1>} & s_{1<o_{-1}>} & s_{-2<o_2>} & s_{0<o_0>} \end{pmatrix}$
	C_3	C_4
DHLPRs	$R_3=\begin{pmatrix} s_{0<o_0>} & s_{3<o_{-4}>} & s_{3<o_{-2}>} & s_{2<o_{-3}>} \\ s_{-3<o_4>} & s_{0<o_0>} & s_{0<o_2>} & s_{-1<o_1>} \\ s_{-3<o_2>} & s_{0<o_{-2}>} & s_{0<o_0>} & s_{-1<o_{-1}>} \\ s_{-2<o_3>} & s_{1<o_{-1}>} & s_{1<o_1>} & s_{0<o_0>} \end{pmatrix}$	$R_4=\begin{pmatrix} s_{0<o_0>} & s_{0<o_{-1}>} & s_{2<o_{-1}>} & s_{-2<o_1>} \\ s_{0<o_1>} & s_{0<o_0>} & s_{2<o_0>} & s_{-2<o_2>} \\ s_{-2<o_1>} & s_{-2<o_0>} & s_{0<o_0>} & s_{-4<o_2>} \\ s_{2<o_{-1}>} & s_{2<o_{-2}>} & s_{4<o_{-2}>} & s_{0<o_0>} \end{pmatrix}$
	C_5	C_6
DHLPRs	$R_5=\begin{pmatrix} s_{0<o_0>} & s_{2<o_{-3}>} & s_{3<o_0>} & s_{2<o_{-1}>} \\ s_{-2<o_3>} & s_{0<o_0>} & s_{1<o_3>} & s_{0<o_2>} \\ s_{-3<o_0>} & s_{-1<o_{-3}>} & s_{0<o_0>} & s_{-1<o_{-1}>} \\ s_{-2<o_1>} & s_{0<o_{-2}>} & s_{1<o_1>} & s_{0<o_0>} \end{pmatrix}$	$R_6=\begin{pmatrix} s_{0<o_0>} & s_{3<o_{-1}>} & s_{0<o_{-3}>} & s_{2<o_1>} \\ s_{-3<o_1>} & s_{0<o_0>} & s_{-3<o_{-2}>} & s_{-1<o_2>} \\ s_{0<o_3>} & s_{3<o_2>} & s_{0<o_0>} & s_{2<o_4>} \\ s_{-2<o_{-1}>} & s_{1<o_{-2}>} & s_{-2<o_{-4}>} & s_{0<o_0>} \end{pmatrix}$

Step 2. Based on Eq. (15) and Eq. (16), calculate the maximum DHLT $OE(A_i)_{j+}$ and the minimum DHLT $OE(A_i)_{j-}$ of all alternatives A_i $(i=1,2,...,6)$ with respect to each criterion C_j, respectively. The results are shown in Table 7. Additionally, based on Eqs. (17) and (18), calculate the weight $w^+ = s_{2<o_1>}$ of the most important criterion and the weight $w^- = s_{-2<o_2>}$ of the most unimportant criterion respectively.

Step 3. Based on Eq. (21), and let $\zeta = 0.5$, calculate the DHLGPS LP_{ij} of the alternative A_i with respect to the criterion C_j, and establish the matrix LP:

$$LP = \left(LP_{ij}\right)_{4\times 6} = \begin{pmatrix} 0.2466 & 0 & 0.5657 & 0.7906 & 0.3771 & 0.6771 \\ 0.7211 & 0.7071 & 0.8 & 0.7693 & 0.4786 & 0.9672 \\ 0.1414 & 0.2525 & 0.9055 & 1 & 0.8014 & 0.66 \\ 0.5245 & 0.5556 & 0.6671 & 0.7071 & 0.5589 & 0.896 \end{pmatrix}$$

Then, based on Eq. (22), obtain the double hierarchy linguistic preference score of the alternative A_i: $LP_1 = 0.4429$,$LP_2 = 0.7406$, $LP_3 = 0.6268$, and $LP_4 = 0.6515$.

Step 4. Determine the weak ranking of all alternatives: $A_1 > A_3 > A_4 > A_2$.

Table 7 The maximum and minimum DHLTs of all alternatives with respect to each criterion

	C_1	C_2	C_3	C_4	C_5	C_6
$OE(A_i)_{j+}$	$s_{1.25<o_{-1.5}>}$	$s_{2<o_{-0.5}>}$	$s_{2<o_{-2.25}>}$	$s_{2<o_{-1.25}>}$	$s_{1.75<o_{-1}>}$	$s_{1.25<o_{2.25}>}$
$OE(A_i)_{j-}$	$s_{-0.75<o_{-0.5}>}$	$s_{-2<o_{1.5}>}$	$s_{-1<o_{-0.25}>}$	$s_{-2<o_{0.75}>}$	$s_{-1.25<o_{-1}>}$	$s_{-1.75<o_{0.25}>}$

Table 8 Three kinds of preference intensities

	C_1	C_2	C_3	C_4	C_5	C_6	$U(A_i, A_p)$	$\Delta U(A_i, A_p)$
$U_j(A_1, A_2)$	0.4745	0.7071	0.2343	0	0.1014	0.2901	$U(A_1, A_2) = 0.3012$	$\Delta U(A_1, A_2) = 0.2977$
$U_j(A_1, A_3)$	0	0.2525	0.3399	0.2094	0.4243	0	$U(A_1, A_3) = 0.2043$	$\Delta U(A_1, A_3) = 0.1840$
$U_j(A_1, A_4)$	0.2779	0.5556	0.1014	0	0.1818	0.2189	$U(A_1, A_4) = 0.2226$	$\Delta U(A_1, A_4) = 0.2087$
$U_j(A_2, A_3)$	0	0	0.1055	0.2307	0.3228	0	$U(A_2, A_3) = 0.1098$	$\Delta U(A_2, A_3) = -0.1138$
$U_j(A_2, A_4)$	0	0	0	0	0.0803	0	$U(A_2, A_4) = 0.0134$	$\Delta U(A_2, A_4) = -0.0890$
$U_j(A_3, A_4)$	0.3831	0.3030	0	0	0	0.2361	$U(A_3, A_4) = 0.1537$	$\Delta U(A_3, A_4) = 0.0247$
$U_j(A_2, A_1)$	0	0	0	0.0213	0	0	$U(A_2, A_1) = 0.0035$	$\Delta U(A_1, A_2) = -0.2977$
$U_j(A_3, A_1)$	0.1052	0	0	0	0	0.0172	$U(A_3, A_1) = 0.0204$	$\Delta U(A_1, A_3) = -0.1840$
$U_j(A_4, A_1)$	0	0	0	0.0835	0	0	$U(A_4, A_1) = 0.0139$	$\Delta U(A_1, A_4) = -0.2087$
$U_j(A_3, A_2)$	0.5797	0.4546	0	0	0	0.3073	$U(A_3, A_2) = 0.2236$	$\Delta U(A_2, A_3) = 0.1138$
$U_j(A_4, A_2)$	0.1966	0.1515	0.1329	0.0622	0	0.0712	$U(A_4, A_2) = 0.1024$	$\Delta U(A_2, A_4) = 0.0890$
$U_j(A_4, A_3)$	0	0	0.2385	0.2929	0.2425	0	$U(A_4, A_3) = 0.1290$	$\Delta U(A_3, A_4) = -0.0247$

Step 5. By Eqs. (23)–(25), the DHLPI $U_j(A_i, A_p)$ of A_i over A_p with respect to each criterion C_j, the DHLAPI $U(A_i, A_p)$ and the DHLNPI $\Delta U(A_i, A_p)$ of A_i over A_p are obtained and shown in Table 8.
Step 6. Based on Wu and Liao (2018), let the indifference threshold $\mu = 0.05$, then the preference threshold is $\nu = \frac{\mu}{6} = 0.0083$, and the incomparability threshold is $o = \frac{\mu}{2} = 0.025$.
Step 7. According to all the preference intensities and thresholds, as well as the rules for identifying the PIR relations given in the improved ORESTE method, a strong ranking of all alternatives is obtained: $A_1 > A_3 > A_4 > A_2$.

4.2 Solve this MCDM problem by the classical ORESTE method

Based on the classical ORESTE method, we can solve this MCDM problem.

Firstly, all the global preference scores $G(\alpha_{ij})$ $(i = 1, 2, 3, 4; j = 1, 2, ..., 6)$ are obtained, and all of them establish the following decision matrix:

$$G = \left(G(\alpha_{ij})\right)_{4\times6} = \begin{pmatrix} 2 & 1 & 2.9155 & 4.7434 & 2.2361 & 3.8079 \\ 3.1623 & 2.9155 & 3.5355 & 4.4721 & 2.5495 & 4.5277 \\ 1.5811 & 1.5811 & 4 & 5.0990 & 2.5355 & 3.6056 \\ 2.5495 & 2.2361 & 3.1623 & 4.3012 & 3 & 4.1231 \end{pmatrix}$$

Then, obtain the global weak ranking and establish the following matrix:

Table 9 Three kinds of preference intensities

	C_1	C_2	C_3	C_4	C_5	C_6	$F(A_i, A_p)$	$\Delta F(A_i, A_p)$
$F_j(A_1, A_2)$	2	3	2	0	1	2	$F(A_1, A_2) = 0.0926$	$\Delta F(A_1, A_2) = 0.0833$
$F_j(A_1, A_3)$	0	1	3	1	3	0	$F(A_1, A_3) = 0.0741$	$\Delta F(A_1, A_3) = 0.0556$
$F_j(A_1, A_4)$	1	2	1	0	2	1	$F(A_1, A_4) = 0.0648$	$\Delta F(A_1, A_4) = 0.0463$
$F_j(A_2, A_3)$	0	0	1	2	2	0	$F(A_2, A_3) = 0.0463$	$\Delta F(A_2, A_3) = -0.0278$
$F_j(A_2, A_4)$	0	0	0	0	1	0	$F(A_2, A_4) = 0.0093$	$\Delta F(A_2, A_4) = -0.0278$
$F_j(A_3, A_4)$	2	1	0	0	0	2	$F(A_2, A_3) = 0.0463$	$\Delta F(A_3, A_4) = -0.0093$
$F_j(A_2, A_1)$	0	0	0	1	0	0	$F(A_2, A_1) = 0.0093$	$\Delta F(A_2, A_1) = -0.0833$
$F_j(A_3, A_1)$	1	0	0	0	0	1	$F(A_3, A_1) = 0.0185$	$\Delta F(A_3, A_1) = -0.0556$
$F_j(A_4, A_1)$	0	0	0	2	0	0	$F(A_4, A_1) = 0.0185$	$\Delta F(A_4, A_1) = -0.0463$
$F_j(A_3, A_2)$	3	2	0	0	0	3	$F(A_3, A_2) = 0.0741$	$\Delta F(A_3, A_2) = 0.0278$
$F_j(A_4, A_2)$	0	1	1	1	0	1	$F(A_4, A_2) = 0.0370$	$\Delta F(A_4, A_2) = 0.0278$
$F_j(A_4, A_3)$	0	0	2	3	1	0	$F(A_4, A_3) = 0.0556$	$\Delta F(A_4, A_3) = 0.0093$

$$\Upsilon = \left(\delta(\alpha_{ij})\right)_{4\times6} = \begin{pmatrix} 2 & 1 & 1 & 3 & 1 & 2 \\ 4 & 4 & 3 & 2 & 2 & 4 \\ 1 & 2 & 2 & 4 & 4 & 1 \\ 3 & 3 & 4 & 1 & 3 & 3 \end{pmatrix}$$

Based on Υ, the global weak ranking of every alternative A_i is got: $\Delta(A_1) = 10$, $\Delta(A_2) = 19$, $\Delta(A_3) = 16$ and $\Delta(A_4) = 15$. Therefore, the weak ranking of all alternatives is obtained as $A_1 > A_4 > A_3 > A_2$.

Additionally, three kinds of preference intensities are obtained and shown in Table 9.

Finally, similar to Wu and Liao (2018), let $\sigma = 0.03$, $\eta = 0.15$ and $\rho = 2$. According to the rules given in the classical ORESTE method, the strong ranking is obtained: $A_1\, P A_4\, I A_3\, R A_2$, i.e., $A_1 > A_4 = A_3 \sim A_2$, in which $>$, $=$ and $\sim$ means preference, indifference and incomparability, respectively.

4.3 Solve this MCDM problem by weighted aggregating operator

For this MCDM problem, we can also solve by the following weighted aggregating (WA) operator. Gou et al. (2019) define a double hierarchy fuzzy linguistic weighted averaging operator (DHLWA):

$$DHLWA_i(OE(A_i)_1, OE(A_i)_2, ..., OE(A_i)_3) = \bigoplus_{j=1}^{n} w_j OE(A_i)_j \tag{26}$$

Since all of the weight information collected in this study is expressed using double hierarchy linguistic terms, firstly, it is necessary to normalize the original weight vector of criteria $w = (w_1, w_2, ..., w_n)^T$. Based on the function f, obtain the normalized weight vector of criteria $\overline{w} = (\overline{w}_1, \overline{w}_2, ..., \overline{w}_n)^T$, where

$$\overline{w}_j = \frac{f(w_j)}{\sum_{j=1}^{n} f(w_j)} \tag{27}$$

Additionally, a WA operator for aggregating all DHLTs of each alternative A_i can be developed:

Table 10 The aggregation results and rank of alternatives

	A_1	A_2	A_3	A_4	Ranking
$DHLWA_i$	$s_{1<o_{0.81}>}$	$s_{-1<o_{0.99}>}$	$s_{0<o_{0.66}>}$	$s_{0<o_{-0.79}>}$	$A_1 > A_3 > A_4 > A_2$
$f(DHLWA_i)$	0.6502	0.4060	0.5205	0.4754	

$$DHLWA_i(OE(A_i)_1, OE(A_i)_2, ..., OE(A_i)_3) = \bigoplus_{j=1}^{n} \overline{w}_j OE(A_i)_j \tag{28}$$

Finally, the final rank of alternatives by the size of the aggregation results of alternatives is obtained.

By the above steps, the weight vector of the criteria $\overline{w} = (0.2245, 0.2551, 0.1327, 0.102, 0.1735, 0.1122)^T$ is obtained. Then, the aggregation results and the rank of alternatives are calculated and shown in Table 10.

4.4 Solve this MCDM problem by some existing methods

(1) Firstly, by the weight vector of criteria obtained by Eq. (27), all the transformed completely consistent DHLPRs shown in Table 6 are aggregated into an overall DHLPR $R' = (r'_{ip})_{4\times4}$, where $r'_{ip} = \bigoplus_{j=1}^{n} \overline{w}_j r'^{j}_{ip}$:

$$R' = (r'_{ip})_{4\times4} = \begin{pmatrix} s_{0<o_0>} & s_{2<o_{1.47}>} & s_{1<o_{2.09}>} & s_{1<o_{2.76}>} \\ s_{-2<o_{0.73}>} & s_{0<o_0>} & s_{-1<o_{2.71}>} & s_{-1<o_{2.04}>} \\ s_{-1<o_{0.17}>} & s_{1<o_{1.83}>} & s_{0<o_0>} & s_{0<o_{3.06}>} \\ s_{-1<o_{1.60}>} & s_{1<o_{-1.73}>} & s_{0<o_{1.38}>} & s_{0<o_0>} \end{pmatrix}$$

Then, the overall value (denoted by $OV(A_i)$) of each alternative is obtained by aggregating all elements of each row in R': $OV(A_1) = 2.6974$; $OV(A_2) = 1.6712$; $OV(A_3) = 2.1582$ and $OV(A_4) = 2.0533$. By ranking the overall values in decreasing order, the final rank of alternatives is obtained: $A_1 > A_3 > A_4 > A_2$.

(B) Based on some existing MCDM methods under a double hierarchy linguistic environment, such as the MULTIMOORA method (), the TOPSIS method (Gou et al., 2018), the LINMAP method (Liu et al., 2018a) and the PROMETHEE method (Liu et al., 2019), this MCDM problem is solved and the results are shown in Table 11.

Table 11 The rankings of alternatives based on some existing methods

	Decision-making methods			
	MULTIMOORA	TOPSIS	LINMAP	PROMETHEE
Ranking	$A_1 > A_4 > A_3 > A_2$	$A_1 > A_4 > A_3 > A_2$	$A_1 > A_3 > A_4 > A_2$	$A_1 > A_3 > A_4 > A_2$

4.5 Comparative analysis

According to the decision results based on the proposed ORESTE method and some existing methods, some comparative analyses are summarized as follows:

(1) As we discussed in Introduction, it is very difficult to aggregate the LPOs only based on some aggregation methods. Therefore, based on the transformation methods given by Gou et al., (2020), every LPO can be transformed into the corresponding completely consistent DHLPR. Then, we can use the proposed ORESTE method or some existing methods to solve this MCDM problem conveniently.

(2) Based on all the methods shown above, it is clear that the rankings of alternatives are not exactly the same. Some reasons are summarized as follows: firstly, before calculating these three kinds of preference intensities in the classical ORESTE method, one matrix is established using the ranking number of each alternative under each criterion, as well as the weight information is also changed based on this method. Therefore, the original decision information and weight information are changed during the decision process, which is the main reason why we only obtain the strong ranking $A_1 > A_4 = A_3 \sim A_2$ and cannot get detailed results. Additionally, the WA method can be used to obtain the decision result directly based on the aggregation method. However, this method neglects the relationship between any two alternatives. Similarly, the first method discussed in Sect. 4.5 has the same characteristic. Furthermore, for the PROMETHEE method, considering that the indifference threshold and strict threshold are very subjective, it is possible that the rank of alternatives will be changed if we use different thresholds. Similarly, by transforming decision-making information into a preference relation that only consists of the elements 0 and 1, and then the decision result can be obtained, so the LINMAP method still has flaws considering the original information is changed.

(3) According to the above comparations, some advantages are summarized: firstly, the LPOs and the corresponding transformed completely consistent DHLPRs are equal. Therefore, the decision-making result is more correct considering that the original information is not changed. Secondly, the decision-making result based on the classical ORESTE method exists lots of incomparability relations among the alternatives because the decision information is transformed by Besson's ranks. Therefore, the proposed ORESTE method makes a special promotion at this point. Finally, The proposed ORESTE method can be used to deal with practical MCDM problems, it is a very useful method in public health emergencies.

(4) According to the solving process and decision-making results of this real MCDM problem, some analyses can be summarized as follows: firstly, using the proposed ORESTE method, *the integration of medical resources within the hospital* should be given priority in the rational allocation of medical and health resources in the regional health system for epidemic response. The capacity of medical services can be significantly increased by actively promoting the integration and sharing of medical resources, which will also boost equity and accessibility of medical and health care. Secondly, the medical health resources allocation methods, Building new hospitals and increasing the supply of beds, Relieving

the pressure of front-line medical personnel, and transferring medical personnel from provinces and cities, assist in easing the burden on the distribution of medical supplies and enable reasonable judgments to be made in light of various circumstances in various places. Finally, the integration of medical resources within health systems is the final method to be considered. Therefore, to optimize the medical health resources allocation under the public health emergency, the medical resources within the hospital are one of the most important elements, and it is necessary to integrate them as soon as possible. Furthermore, by improving the above policies one by one, studying and strengthening epidemic prevention and control, our country can establish and improve major epidemic prevention and control measures in terms of institutions and mechanisms, improve the national public health emergency management system, and enhance our ability to respond to major public health emergencies.

5 Conclusions

In this paper, to evaluate the medical health resources allocation under the public health emergency, we first use two kinds of LPOs to describe experts' opinions during the process of evaluation. Then, a novel ORESTE method with LPOs in MCDM is developed. Additionally, we apply the proposed ORESTE method to deal with a practical MCDM problem involving the evaluation of the medical health resources allocation in public health emergencies. Finally, some comparative analyses among the proposed ORESTE method and some existing decision-making methods under a double hierarchy linguistic environment are set up.

The contributions of this paper are summarized as follows: Firstly, the transformation process for transforming LPOs into the corresponding completely consistent DHLPRs can keep the original information completely. Secondly, the proposed ORESTE method is developed to deal with MCDM problems with LPOs, which can avoid the loss of original information. Finally, the proposed improved ORESTE method can be used to solve a practical MCDM problem involving the evaluation of the medical health resources allocation in public health emergencies.

In future studies, we will devote ourselves to the research of the method for dealing with LPOs more correctly. Meanwhile, it is a good research field to highlight the proposed ORESTE method and minimize the indifference and incomparability relations and make the rank of alternatives more precise. Finally, we will study the applications of LPOs in large-scale group decision-making or large-scale alternatives decision-making problems.

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