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ERRATUM

Strong Completeness and Limited Canonicity for PDL

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Our paper Renardel de Lavalette et al. (2008) "Strong completeness and limited canonicity for PDL" contains three unfortunate mistakes that we would like to correct.

1. First of all, Lemma 1 on the equivalence of saturated and maximal consistent sets for PDL_ω is not original, contrary to what we stated in the paper. In fact, it has been proved before as Corollary 9.3.6 on p. 222 of Goldblatt (1993) and as Corollary 3.10 in Segerberg (1994).
2. Theorem 1 of Renardel de Lavalette et al. (2008) on strong completeness of PDL_ω is not really new. In Goldblatt (1982, 1993) and Segerberg (1994), strong completeness has been proved for several infinitary modal logics, and in Goldblatt (1987) completeness for first-order dynamic logic has been proved. All these proofs follow essentially the same pattern: first it is shown in a Lindenbaum Lemma that each consistent set is contained in a maximal consistent set, then a canonical model is constructed from maximal consistent sets and a Truth Lemma is proved. In all

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cases, the proof of the Lindenbaum Lemma requires additional effort, since the logics in question are not compact. This additional effort can be summarized by the slogan *saturated sets are maximal consistent*, and this is done explicitly in Goldblatt (1993) and Segerberg (1994).

In PDL, formulas and programs are defined with mutual recursion, since test programs A? for arbitrary formulas A are allowed. This is unlike the logics treated in the completeness proofs in Goldblatt (1982, 1987, 1993) and Segerberg (1994), where the recursion is nested: first the modalities/programs are defined, and then the formulas using the modalities or programs. The structure of the Truth Lemma reflects this. First, some property is proved for modalities/programs. This property is then used in the proof of the following formula property: a formula holds in a maximal consistent set in the canonical model iff it is an element of that set. For PDL_ω, the property for programs and the formula property have to be proved via simultaneous induction. This requires some adaptation of the proofs mentioned above, but the adaptation is rather straightforward, as Professor Goldblatt has kindly shown us (in private correspondence).

Thus, contrary to our remark on p. 70 that the proof in Goldblatt (1982) "does not transfer to PDL", there is an extension of Goldblatt's methods to PDL. In particular, Theorem 13.12 on first-order dynamic logic in Goldblatt (1987) straightforwardly leads to the Lindenbaum Lemma for PDL_ω. Therefore, we no longer claim any priority regarding the proof of strong completeness of PDL_ω. Indeed, the proof of Theorem 1 in Renardel de Lavalette et al. (2008) turns out to be a rather short argument that can be obtained by stripping down other, more general proofs to the bare essentials for PDL.

3. Finally, the notion of derivable sequent used in Lemma 1 of Renardel de Lavalette et al. (2008) does not correspond to Definition 4. This can be remedied by the following adaptation of Definition 4. A sequent $\Gamma \vdash \varphi$ is derivable iff it is the root of some derivation tree; a derivation tree is a well-founded tree (i.e. with all branches finite), with leaves labeled with axioms, and non-leaves labeled with sequents that are the conclusion of a rule with the labels of the children as premises. This yields an equivalent notion of derivability, corresponding to the definition in Mirkowska (1981).

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