



Guest Editorial: Discrete Geometry and Mathematical Morphology

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This topical issue of the Journal of Mathematical Imaging and Vision brings together a number of contributions addressing fundamental questions and recent trends in discrete geometry and mathematical morphology, with a focus on theoretical results and algorithmic development. These topics play an essential role in various fields related to digital images, such as image analysis, computer graphics, pattern recognition, shape modeling and computer vision.

Eight contributions have been selected for the topical issue, each advancing the state of the art in the respective fields.

A fundamental fact in the study of discrete and digital geometry is that concepts and definitions from geometry in continuous spaces do not always preserve their properties when applied in discrete spaces. Two contributions in this topical issue are focused on such challenges. In *Homotopic Affine Transformations in the 2D Cartesian Grid*, N. Passat, P. Ngo, Y. Kenmochi, and H. Talbot, study the problem of topology preservation under affine transformations in a discrete space. In *An alternative definition for digital convexity*, J.-O. Lachaud proposes *full convexity* as an alternative definition of digital convexity, thereby avoiding many problems related to its usual definitions, while preserving its desirable features.

An active area of research is to combine and unify methods from discrete geometry and mathematical morphology with convolutional neural networks and other advances in the field of machine learning. In *Learning grayscale mathematical morphology with smooth morphological layers*, R. Hermay, G. Tochon, E. Puybureau, A. Kirszenberg, and J. Angulo, explore the integration of mathematical morphology operations within convolutional neural network architectures. In *Towards a Sparsity Theory on Weighted Lattices*, N. Tsilivis, A. Tsiamis, and P. Maragos study issues of sparse representation in nonlinear vector spaces. The developed theoretical tools allow them to make structured

arguments about the pruning of a special class of neural networks, called morphological neural networks.

While initially conceived for binary and scalar-valued images, mathematical morphology has been successfully extended to vector-valued images using several approaches. A downside of some of such approaches is that they may introduce irregularities in the output image. In *Irregularity Index for Vector-Valued Morphological Operators*, M. E. Valle, S. Francisco, M. Aurélio Granero, and S. Velasco-Forero, propose a method for measuring the irregularity of a vector-valued morphological operator.

Inverse problems play an important role in many scientific applications. Two contributions in this topical issue focus on discrete inverse problems. In *On the reconstruction of 3-uniform hypergraphs from degree sequences of span-two*, G. Palma, A. Frosini, and S. Rinaldi, study a class of inverse problems on hypergraphs. In *Characterization of hv-convex sequences*, P. Dulio and A. Frosini study the problem of discrete tomography under a convexity constraint.

In mathematics, just as in any other scientific topic, connections between different fields of study often provide fruitful insights and novel approaches. In *Some equivalence relation between persistent homology and morphological dynamics*, N. Boutry, L. Najman, and T. Geraud highlight connections between mathematical morphology and topological data analysis, paving the way for further exploration of the relations between these two research fields.

We thank the authors for submitting their outstanding work to this special issue. We would also like to thank the reviewers for their invaluable contribution to this special issue.

The special issue was preceded by the International Conference on Discrete Geometry and Mathematical Morphology, hosted by the Centre for Image Analysis, Department of Information Technology, Uppsala University, Sweden, during May 24–27, 2021. The articles included in this topical issue have undergone rigorous peer-review according to the journal's high standards.

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