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PREFACE

Foreword

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The idea of this special issue was conceived during the workshop on "Discontinuous Galerkin methods for Partial Differential Equations," co-organized with Dominik Schötzau, that took place at the Banff International Research Station, Canada, in November 25–30, 2007. Just as was done last century in [7], and more recently, in [6] and [10], we gather here some of the main recent developments of this increasingly popular method.

The papers in this special issue reflect the capability of the method to handle a wide variety of problems. Indeed, we have papers devoted to second-order elliptic equations, to fluid flow problems, to fourth-order elliptic equations, to the wave and the Maxwell equations, and to a variety of nonlinear problems arising in geophysics, semiconductor device simulation and mathematical biology. Let us briefly describe them.

The papers concerning second-order elliptic problems are the following. In [1], iterative and preconditioning techniques are presented for interior penalty methods, symmetric and non-symmetric, using piecewise linear approximations. In [2], an a posteriori error analysis of an over-penalized symmetric interior penalty method is presented. In [16], a novel approach, namely, the weighted-residual formulation of the discontinuous Galerkin methods, is used to derive a-posteriori error estimates for all known discontinuous Galerkin methods in mixed form. In [13], the widespread belief that the non-symmetric interior penalty method and the Baumann-Oden method converge with optimal order for odd-polynomial approximations is shown, experimentally, to be false.

On fluid flow, we have the following works. In [15], a comparative study of a widely used continuous Galerkin method and a recently developed discontinuous Galerkin method for shallow water equations is carried out. It is demonstrated that the latter is more efficient.

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In [9], equal-order LDG methods for the Navier-Stokes are introduced and analyzed, thus completing a series of papers devoted to this topic. Finally, in [14], a discontinuous Galerkin method for the linearized magneto-hydrodynamic equations is introduced and analyzed.

There are two papers on fourth-order problems. Indeed, in [8], the first hybridizable discontinuous Galerkin method for the biharmonic equation is proposed and theoretically studied. In [18], LDG methods for the difficult equations for surface diffusion and Willmore flow of graphs are proposed. Stability results are proven and numerical results illustrating the performance of the method are shown.

In [12], the analysis of a fully discrete symmetric interior penalty discontinuous Galerkin method for the wave equation is presented. In [3], two eigensolvers using nonconforming finite element approximations and another using an interior penalty type discontinuous Galerkin method are considered. It is shown that they are all free of spurious eigenmodes. Finally, in [4], it is shown how discontinuous Galerkin methods for Maxwell equations may generate spurious solutions on general irregular meshes. It is then shown how to remedy this unfortunate situation.

Finally, in [5], a method is introduced and proven to be optimally convergent for numerically solving the two-dimensional coupled problem of time-dependent incompressible Navier-Stokes equations with the Darcy equations. In [17], a multiscale LDG method to simulate the one-dimensional stationary Schrödinger-Poisson problem in a resonant tunneling diode. In [11], a fully discrete discontinuous Galerkin method for the two-dimensional Keller-Segel chemotaxis model is considered. Last but not least, in [19], a combination of a discontinuous Galerkin space discretization with Strang type symmetrical operator splitting technique is used to simulate developmental biology phenomena modeled by nonlinear reaction-diffusion systems.

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