A Two-Level Procedure for the Global Optimum Design of Composite Modular Structures-Application to the Design of an Aircraft Wing: Part 1: Theoretical Formulation

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Part 1: Theoretical Formulation

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Abstract This work concerns a two-level procedure for the global optimum design of composite modular structures. The case-study considered is the least weight design of a stiffened wing-box for an aircraft structure. The method is based on the use of the polar formalism and on a genetic algorithm. In the first level of the procedure, the optimal structure is designed as it was composed by a single equivalent layer, while a laminate realizing the optimal structure is found in the second level. The method is able to automatically find the optimal number of modules, no simplifying assumptions are used, and it can be easily generalized to other problems. The work is divided into two parts: the theoretical formulation in this first part, the genetic procedure and some numerical examples in the second one.

Keywords Laminates · Composite materials · Polar method · Genetic algorithms

1 Introduction

The design of modular systems is a difficult task whenever the number of modules is unknown. The difficulty increases even more when the modules can have different dimensions. Similar problems arise in several engineering domains. We consider here

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a case which can be viewed as paradigmatic: the design of a least weight wing-box girder, with an unknown number of stiffeners, that is to be realized by composite laminates.

Stiffened panels are largely used in many structural applications, mostly because they allow for a substantial weight saving. Of course, this point is of paramount importance especially in aircraft design, where an important reduction of the structural weight can be achieved if composite laminates are used in place of aluminum alloys. A drawback of such a choice is that the optimal design of the structure is much more cumbersome than that of a classical metallic structure. In fact, though the use of laminated structures is not a recent achievement in structural mechanics, up to now no general rules and methods exist for their optimal design, and engineers always use some simplifying assumptions or rules.

These assumptions are used on one side to obtain a short-cut to a possible solution, i.e., to eliminate from the true problem some particularly difficult points or properties to be obtained. On the other side, some of such rules are considered to prevent the final structure from some undesired phenomena, though this is never clearly and rigorously stated and proved. Unfortunately, for the most part, the use of such simple rules leads only to a *suboptimal solution*, i.e., to a solution which is not a real global optimal one. Two examples are the use of symmetric stacking sequences, a sufficient but not necessary condition for bending-extension uncoupling, and the use of balanced stacks to obtain orthotropic laminates in bending. When symmetric stacking sequences are used, the design is done using half of the layers, which means also half of the design variables. Once half of the stack has been designed, the other half is simply added, symmetrically with respect to the mid-plane, in order to obtain uncoupling. Of course, it is very difficult to obtain the lightest structure using a similar strategy.

The use of balanced stacks, on the other side, leads systematically to mechanically false solutions: whenever such a rule is used, bending orthotropy, a rather difficult property to be obtained, [1], is simply understated, assumed, but not really obtained, as in [2] or [3], sometimes ignored, like in [4] (about this topic, see [5] for more details). In aircraft structural design, some other rules are imposed to the design of laminated panels; see for instance [3]. None of them are mechanically well justified. Certainly, an appropriate mathematical formulation of the design process could take into account the mechanical and technological problems that such drastic, empirical rules want to prevent.

Many works exist on optimal design of stiffened composite laminates. For example, and without any ambition of exhaustiveness, we cite here the works of Butler and Williams [6], Wiggenraad et al. [7], Nagendra et al. [8], Kaletta and Wolf [9], Bisagni and Lanzi [10], but many other works on the topic can be found in the literature.

The research presented in this paper has been motivated by the following purpose: to show that an appropriate optimization procedure can lead to a substantial weight saving in the design of modular composite structures. The case that we have considered is that of a wing-box stiffened girder made of composite laminates. The objective of the optimal problem is to design the lightest structure, submitted to a constraint on the buckling load, which is a classical problem in aircraft structural engineering. The same procedure, however, can be applied to other problems and also other requirements, in the form of additional constraints to the optimum problem, can be taken into account.

The design procedure that we propose is inspired by a radical point of view: to design a modular composite laminated structure by a mathematically rigorous numerical optimization procedure that will not use any simplifying assumption. Only avoiding the use of *a priori* assumptions one can hope to obtain the true global optimum for a given problem: this is a key-point in our approach. The design process that we propose is, on one side, completely free, i.e., not submitted to restrictions, and on the other side completely automatic: the operator does not need to take any preliminary decision, for instance on the number of the layers or of the stiffeners, because the method will do that for him, in the best way. In fact, the approach presented in this paper can automatically optimize also the number of design variables during the iterations.

Actually our hope is twofold: first, to show that, if old design rules and *a priori* assumptions are abandoned in the design of structural laminates, interesting solutions can be obtained, especially in weight saving. Then that modern numerical methods allow such an approach and make it possible to substitute old simplifying and limiting assumptions with more rigorous requirements that can be included in the numerical procedure.

Being the work rather lengthy, its presentation is divided into two parts. In this first part, the theoretical formulation of the problem is detailed, along with a description of the solution procedure. In the second part, the characteristics of the genetic algorithm used to solve the problem are given and some numerical examples shown.

This first part is organized as follows: the mechanical problem considered in the study is introduced in Sect. 2 and the optimization strategy is explained in Sect. 3. The mathematical formulation of the minimum weight design problem is detailed in Sect. 4 and the problem of determining a suitable laminate is formulated in Sect. 5. Finally, Sect. 6 ends the paper with some concluding remarks and perspectives.

2 Description of the Structural Problem

The optimization procedure is applied to a classical long-range aircraft wing-box stiffened panel. Figure 1 shows the conceptual steps which lead to the construction of the approximate model of the wing-box section. In particular, we have considered the wing-box section located at the 60 % of the wing span, whose typical dimensions are shown in Fig. 1. These values represent specific dimensions for a long-range aircraft with a design range of about 9300 km, 350 passengers, two engines, cruising altitude between $\sim 7600 \div 10700$ m and Mach number of about 0.82. For more details, see [11].

The structure has a width w of 2610 mm, height h_b of 720 mm and a length L of 700 mm. The wing-box section represents a portion of the wing between two consecutive ribs. We consider the wing-box simply-supported on these ribs. Figure 1 shows also the loads acting on the structure in normal flight conditions: in this case, only on the upper panel there can be buckling phenomena. The whole wing-box is made of composite laminates composed of highly anisotropic unidirectional carbon-epoxy layers T300/5280, [12]. The material properties of the elementary layer are

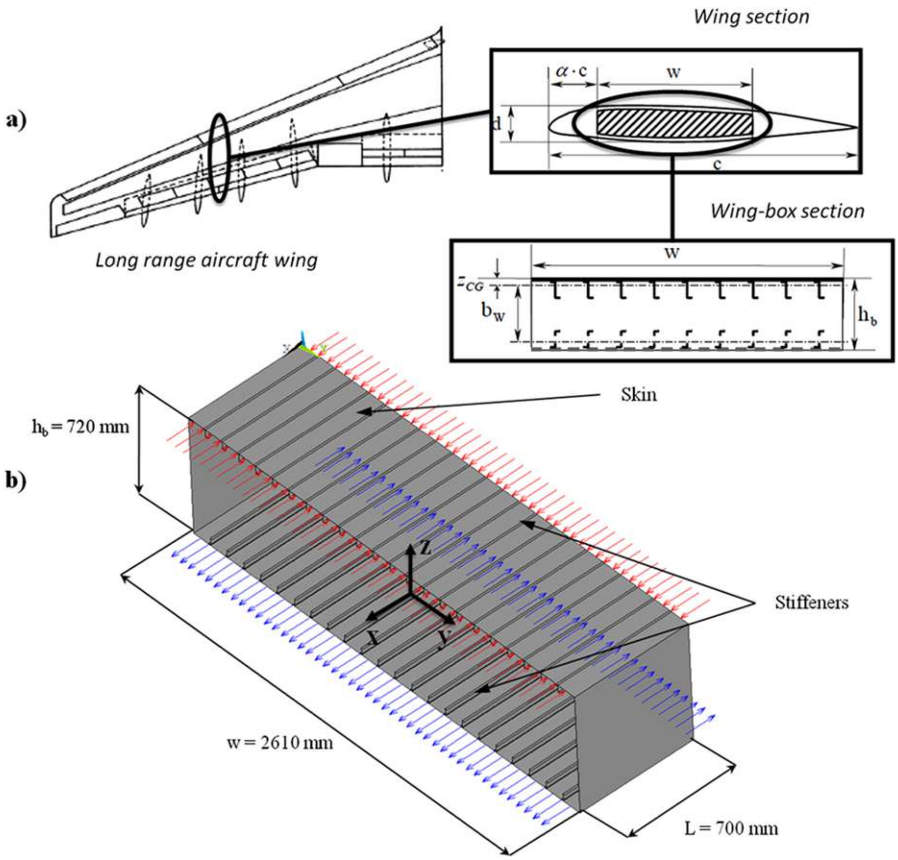


Fig. 1 (**a**) Conceptual phases which lead to the construction of the wing-box model (**b**) Structure of the wing-box stiffened panel and applied loads

Table 1 Technical moduli and polar parameters for unidirectional plies of carbon-epoxy T300/5208

Technical moduli		Polar parameters	
Young's modulus E_1 [MPa]	181000	T_0 [MPa]	26880
Young's modulus E_2 [MPa]	10300	T_1 [MPa]	24744
Shear modulus G_{12} [MPa]	7170	R_0 [MPa]	19710
Poisson's ratio ν_{12}	0.28	R_1 [MPa]	21433
Density ρ [kg/m^3]	1580	Φ_0	0
Ply thickness t_{ply} [mm]	0.125	Φ_1	0

shown in Table 1. Both upper and lower panels have Z-shaped stiffeners with equal flanges. The core and the flanges of each stiffener have the same thickness.

As previously said, no simplifying hypotheses are made for the panel: indeed each stiffener can be different from any other, in terms of geometrical and mechanical behavior, but, and this is very important, for evident mechanical reasons we impose that each plate composing the structure is orthotropic both in bending and in extension, and with the orthotropy axes aligned with the axes of the wing-box. Indeed, about the geometry of the structure, we only assume that, for constructive reasons, the wing-box section is symmetric with respect to the global $x - y$ plane, as shown in Fig. 1.

3 The Optimization Strategy

The optimal design of a stiffened wing-box girder made of composite laminates is an hard task, if no simplifying assumptions are used. Actually, such a problem, like many other similar problems in structural engineering, has some peculiarities and the optimization strategy must take into account all of them:

– the structure is a mechanical system composed by modules. Actually, there are two types of modules in this system: the modules of the first type are the stiffeners. All the stiffeners are modules because they have the same function and geometry, but not necessarily the same dimensions and mechanical properties. In fact, in the most general case each stiffener can be different from another one, because it can have different dimensions, number, and orientation of the plies, and hence different mechanical properties. The modules of the second type are the layers: All the layers, composing each part of the structure, are identical, but each member of the structure (stiffeners, skins) can be composed by a different number of layers that normally are differently oriented;
– the design process must be able to completely determine the *optimal configuration of each module* and their *optimal number*; this point is of a particular importance whenever the objective is the least weight, because dimensions and number of modules greatly affect the final weight of the structure;
– the design process must be able to take into account all the *mechanical prescriptions imposed to the structure*, without using simplifying assumptions; namely, it must be possible to take into account general properties concerning the elastic symmetries, like orthotropy for both bending and extension behavior, uncoupling and so on; this can be done effectively by a proper choice of the anisotropy representation;
– the design procedure must be able to *handle the direction of anisotropy*, namely the orientation of the orthotropy axes, without imposing particular stacking sequences and/or orientation angles that automatically fix the anisotropy direction in a particular direction, like cross-ply, angle-ply, or balanced quasi-isotropic sequences;
– all the *constraints imposed to the problem*, of mechanical or technological nature, of the inequality or equality type, must be effectively handled by the procedure;
– the numerical tool used for the solution of the optimum problem must be able to handle, at the same time, design variables of different nature: *continuous*, *discrete* or *grouped* variables, these last being a sort of pointers that when chosen in a list, imply the automatic choice of a set of variables; it is typically the case of materials, whose choice in a set of technical materials that are at the disposal of the designer implies automatically the choice of all the physical properties of the material to be used;
– the numerical strategy used to solve the optimum problem must be able to handle effectively *highly nonconvex problems*, both in the objective function and in the constraints.

Another point is very important when the design concerns composite laminated structures: There is not a bijective correspondence between the elastic properties of the laminate and the stacking sequence; see, for instance, [13]: the same mechanical behavior in bending, coupling, or extension can be obtained by several different laminates, all composed of the same identical plies but not necessarily by the same number of plies or with the same orientations.

Regardless of the layers number, the mechanical properties of the laminate are determined only by a restricted set of overall mechanical parameters. There are different possible choices for these mechanical parameters, but in all the cases they are at most 18 parameters in the framework of the Classical Laminated Plates Theory (CLPT), though they are not completely independent.

All the above points have suggested us the optimization strategy to be used to tackle structural problems like the one considered in this paper. In particular, they have inspired us in the choice of the general organization of the procedure, of the mathematical formulation, of the mechanical parameters and of the numerical algorithm.

For what concerns the general organization of the procedure, we have adopted a *two-level strategy*: the problem of finding the lightest stiffened wing-box, composed of identical layers of a chosen material, is split into two different but linked optimum problems:

- *First level*: at this stage, *we consider each part of the structure, skins, and stiffeners, as composed of a single equivalent homogeneous layer*; the problem of finding the least weight structure with the imposed constraints is formulated and solved. The output of this step is hence the geometry of the structure, i.e., the number and dimensions of stiffeners and skins, in particular their number of layers, and its mechanical properties, i.e., the components of the stiffness tensors of the skin and of each stiffener. Hence, this is the step where the true optimal design of the structure is done, in terms of its overall properties.
- *Second level*: during this phase, *we look for one stacking sequence giving the optimal overall properties found during the first step*, and this for all the parts of the structure, i.e., for the skins and for each stiffener. At this stage, the design variables are the layer orientations and we can add some requirements concerning different aspects. For instance, more constraints on the elastic behavior can be added for different reasons, or the orientations of the layers can be restricted to a set of possible values and so on. This is possible because the fact that several laminates share the same overall elastic behaviour leaves us a large panel of possibilities in terms of suitable laminates, and this panel rapidly increases with the number of the plies.

It is worth noting that this kind of strategy has already been used in the past, with various approaches to the first and second level, by some of the coauthors, [14, 15], or by other scientists on other problems; see, for example, the classical excellent work of Foldager et al. [13].

For what concerns the mathematical formulation, this will be detailed, for both the first and second step, in the next section. Nevertheless, we can see immediately that during the first stage, the design of the thickness of the different parts of the structure must be done using discrete variables, with a step equal to the thickness of the material layer used for the fabrication of the structure. Of course, this responds to a technological need, and moreover, this will give us also another result: the number of layers to be used during the second phase design.

Concerning the mechanical parameters, we describe the elastic behavior of each structural part by a set of elastic invariants, the *polar parameters*, originally introduced by Verchery [16]. Through these quantities, it is possible to express the classical **A**, **B**, **D** tensors which describe the behavior of the laminate in an effective way, especially for design problems. Moreover, being the polar parameters tensor invariants, they are frame-independent. In addition, they allows for the most effective invariant description of elastic symmetries and for easily expressing the bounds on the design of the material of both elastic and geometric nature; see [17]. We will describe the role of these parameters in Sect. 4. For more details on the polar formalism, the reader is addressed to [18].

Other tensor representations can of course be employed in the description of anisotropic problems, namely we could use the so-called parameters of Tsai and Pagano, [19], or the classical Cartesian representation. Nevertheless, these methods make use of frame-dependent quantities, and in anisotropic design problems, where properties depend upon the frame, the use of frame-independent parameters constitutes a true advantage. In a sense, this is very similar to what is often done in physics problems, when invariant quantities are very used, like first integrals.

About the numerical procedure, we have used a genetic algorithm, coupled to a finite element code in the first level problem. The reasons of this choice, the details on the special genetic algorithm that we have used and the description of the numerical finite element model used to simulate the mechanical behavior of the structure, are the topics of the second part of this work, whereto we address the reader.

The structural problem considered in this paper mainly concerns, through the constraint on the buckling load, the bending behavior of the different laminates that compose the structure. Nevertheless, we have also imposed a condition on the extension behaviour of the laminates: each laminate is required to be *quasi-homogeneous*. Quasihomogeneity is a property first introduced by Kandil and Verchery, [20]: a laminate is quasi-homogeneous when its extension and bending behaviors are uncoupled and identical in each direction, [21]. In this way, only the extension tensor **A** has to be designed, the bending one, **D**, is automatically obtained. So, the choice of using quasi-homogeneous sequences, a mechanical assumption, has two direct mathematical consequences on the optimum problem: it reduces to only six the elastic parameters to be designed for the laminate, and transforms the problem from the design of the bending tensor to that of the extension tensor, much easier to be done. Another mathematical consequence, important for a correct definition of the constraints to be imposed to the optimum problem, as specified below, is the fact that with quasi-homogeneity the interdependency of the elastic parameters of extension and bending is complete. Finally, it must be noticed that this choice does not diminish the generality of the approach under a mechanical point of view, because for the bending behavior, the fundamental one for this kind of problems, no restrictions are given and all the situations are still possible.

Another important point is constituted by the *feasibility conditions*: during the first step, an anisotropic equivalent layer is designed and, just like for any other elastic material, some bounds are to be imposed to the search, in order to obtain elastic parameters that satisfy physical existence conditions. Nevertheless, this is not sufficient because the fictitious homogeneous anisotropic material designed during the first step, is not really fabricated. In fact, the optimal mechanical properties obtained as results of the first level problem are realized in practice using composite laminates, that in general are different for the skin and for the stiffeners. So, in the second level problem, a laminate having the overall elastic properties optimized in the first step is looked for, and this is done for the skin and for each stiffener.

Recently, Vannucci [17] has shown that laminates constitute a sort of *restricted elastic class*: the elastic bounds valid for a homogeneous anisotropic material can never be attained by a laminate composed by the same material. This happens because the stacking sequence imposes some links among the different elastic moduli of the extension or of the bending tensor, links that shrink the existence domain of the elastic moduli of the tensor. Such links are of geometrical nature, because they depends on the geometry of the stack, i.e., upon the orientation angle and the position of each layer in the stacking sequence. As the fictitious material object of the first design level will be, in the second level, realized by a laminate, in order to obtain a feasible laminate, the *geometric constraints* on the feasibility of the laminate are to be imposed directly to the optimum problem of the first level, otherwise one could get an optimum elastic tensor that cannot be obtained using a laminate of the same material.

To this purpose, it is worth noting also the importance of the quasi-homogeneity requirement: the bounds for the elastic moduli of the extension and bending tensors taken together are not known, and it should be impossible to specify them correctly in the first step problem. The assumption of quasi-homogeneity allows for considering in the first phase a fictitious material that has the same properties for bending and extension, for which the same geometric bounds are valid for both the tensors and hence are mathematically correct.

After these considerations, we pass to detail and mathematically formulate the two different steps of the procedure.

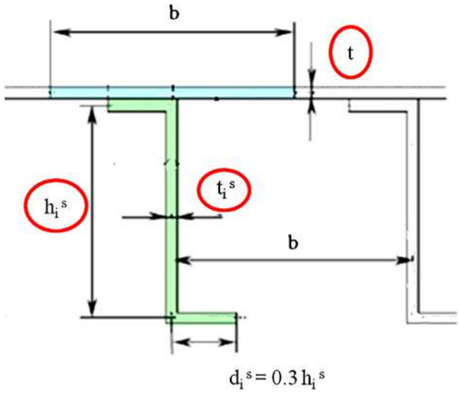
4 Mathematical Formulation of the First Level Problem

The overall characteristics of the optimal structure are to be designed during this phase. For the problem at hand, this means that in this phase the optimal values of the following parameters are to be determined:

- the number of the stiffeners;
- the thickness, and hence the number of layers, of the skin and of each stiffener;
- the geometrical dimensions of each stiffener;
- the mechanical properties of the skin and of each stiffener.

It is worth noting the peculiarity of this structural optimization problem: unlike classical optimum problems of structural engineering, where the only design variables are the geometrical dimensions of the structure, in this case we need also determine the optimal number of the modules and their mechanical characteristics, besides their dimensions. We recall, in fact, that in the most general situation, the stiffeners share the same form but can have different dimensions and mechanical properties.

Fig. 2 Geometrical design variables of the wing-box stiffened panel



The objective is to minimize the weight of the wing-box; we have already specified that this must be done satisfying on one side the constraint on the buckling load, and on the other side the geometric bounds for the elastic moduli.

As said above, for mechanical reasons, we impose also two other requirements: the fictitious material to be designed in this phase must be orthotropic and with the orthotropy axes parallel to the wing-box axes. Actually, we will see that this two last conditions are quite simple to be obtained in this phase, and will not give additional constraints, while they will be an important part of the second level problem.

4.1 Geometrical Design Variables

Before specifying the mathematical formulation, we introduce the design variables; these are of two types: *geometrical* and *mechanical*. For what concerns the geometrical design variables, they are shown in Fig. 2 and are:

- the number of stiffeners N;
- the thickness of each stiffener $t_i^S,\ i = 1, \ldots, N$;
- the height of each stiffener h_i^S $(i = 1, \ldots, N)$;
- the thickness of the skin t.

All these variables are discrete valued; the ranges of their variation, along with their steps, are shown in Table 2. As previously said, the step of the thickness is equal to the thickness of the carbon-epoxy T300/5208 layers, the material chosen for the structure, see Table 1.

For technological reasons, the width of the flange of each stiffener, d_i^S $(i = 1, \ldots, N)$, is not a design variable and depends on the height of the stiffener as shown in Fig. 2. The stiffeners are automatically equispaced, with a step b which depends on the number of stiffeners through the following relation:

$$b = \frac{w}{N+1}, \tag{1}$$

where w is the width of the whole wing-box section.

An important point to be remarked: the dimension of the design space, i.e., the number of the design variables, depends on the number of modules, the stiffeners,

Table 2 Design variables for the first optimization problem

Design variable	Type	Lower bound	Upper bound	Step
N	discrete	18	23	1
t_i^S [mm]	discrete	2.0	5.0	0.125
h_i^S [mm]	discrete	40.0	90.0	0.5
$(R_{0K}^{A*})_i^S$ [MPa]	continuous	−19710	19710	–
$(R_1^{A*})_i^S$ [MPa]	continuous	0.0	21433	–
t [mm]	discrete	2.0	5.0	0.125
(R_{0K}^{A*}) [MPa]	continuous	−19710	19710	–
(R_1^{A*}) [MPa]	continuous	0.0	21433	–

and must be optimally determined by the procedure. The determination of the optimal value of the other module, the number of the layers, is implicitly done determining the optimal value of the thicknesses.

4.2 Mechanical Design Variables

For what concerns the mechanical variables, we have already said in Sect. 3 that we use the *polar formalism*, which gives a representation of any planar tensor by means of a complete set of independent invariants. These invariants are called *polar parameters* and a great advantage in the design of anisotropic structures is that they are directly linked to the different symmetries of the tensor, [16, 18].

By the polar formalism, the reduced stiffness tensor $\mathbf{Q}$ of a layer is expressed as:

$$\begin{aligned}
Q_{xxxx} &= T_0 + 2T_1 + R_0 \cos 4\Phi_0 + 4R_1 \cos 2\Phi_1,\\
Q_{xxyy} &= -T_0 + 2T_1 - R_0 \cos 4\Phi_0,\\
Q_{xxxy} &= R_0 \sin 4\Phi_0 + 2R_1 \sin 2\Phi_1,\\
Q_{yyyy} &= T_0 + 2T_1 + R_0 \cos 4\Phi_0 - 4R_1 \cos 2\Phi_1,\\
Q_{yyxy} &= -R_0 \sin 4\Phi_0 + 2R_1 \sin 2\Phi_1,\\
Q_{xxyy} &= T_0 - R_0 \cos 4\Phi_0.
\end{aligned} \tag{2}$$

In (2), T_0, T_1, R_0, R_1, $\Phi_0 - \Phi_1$ are the *polar tensor invariants*. T_0 and T_1 are the moduli related to the isotropic part of the tensor, R_0 and R_1 are the moduli related to the anisotropic one, while Φ_0 and Φ_1 are the polar angles. The polar parameters of the material used in the numerical examples presented in the second part of this research are given in Table 1. The same (2) apply to any other fourth-rank tensor having the elastic tensor symmetries. For more details on the properties of polar parameters, see [18].

The CLPT gives the constitutive law of a laminate, linking the internal actions to the deformations of the laminate:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix}. \tag{3}$$

In (3), **N**, **M**, $\boldsymbol{\epsilon}$, and $\boldsymbol{\kappa}$ are second-rank symmetric plane tensors; **N** represents the membrane forces, **M** the bending moments, $\boldsymbol{\epsilon}$ the in-plane strains and $\boldsymbol{\kappa}$ the out-of-plane curvatures. The CLPT gives also the composition laws of these three tensors for a laminate composed of n plies. Such laws depend on the mechanical properties of the layers, on their thickness, orientation, and position:

$$\begin{aligned} \mathbf{A} &= \sum_{j=1}^{n} \mathbf{Q}_j(\delta_j)(z_j - z_{j-1}), \\ \mathbf{B} &= \frac{1}{2}\sum_{j=1}^{n} \mathbf{Q}_j(\delta_j)\left(z_j^2 - z_{j-1}^2\right), \\ \mathbf{D} &= \frac{1}{3}\sum_{j=1}^{n} \mathbf{Q}_j(\delta_j)\left(z_j^3 - z_{j-1}^3\right). \end{aligned} \tag{4}$$

The previous laws are independent from the tensor representation and hence apply also to the polar formalism. Some standard passages give then the polar parameters of **A**, **B,** and **D**:

$$\begin{aligned} T_0^A, T_0^B, T_0^D &= \frac{1}{m}\sum_{j=1}^{n} T_{0j}\left(z_j^m - z_{j-1}^m\right), \\ T_1^A, T_1^B, T_1^D &= \frac{1}{m}\sum_{j=1}^{n} T_{1j}\left(z_j^m - z_{j-1}^m\right), \\ R_0^A e^{4i\Phi_0^A}, R_0^B e^{4i\Phi_0^B}, R_0^D e^{4i\Phi_0^D} &= \frac{1}{m}\sum_{j=1}^{n} R_{0j} e^{4i(\Phi_{0j}+\delta_j)}\left(z_j^m - z_{j-1}^m\right), \\ R_1^A e^{2i\Phi_1^A}, R_1^B e^{2i\Phi_1^B}, R_1^D e^{2i\Phi_1^D} &= \frac{1}{m}\sum_{j=1}^{n} R_{1j} e^{2i(\Phi_{1j}+\delta_j)}\left(z_j^m - z_{j-1}^m\right), \end{aligned} \tag{5}$$

where T_0^A, T_1^A, R_0^A, R_1^A, Φ_0^A, and Φ_1^A are the polar components of tensor **A**, T_0^B, T_1^B, R_0^B, R_1^B, Φ_0^B, and Φ_1^B are the polar components of tensor **B**, and T_0^D, T_1^D, R_0^D, R_1^D, Φ_0^D, and Φ_1^D are the polar components of tensor **D**. In (5), $m = 1, 2, 3$ for the extensional, coupling and bending stiffness tensor, respectively. T_{0j}, T_{1j}, R_{0j}, R_{1j}, Φ_{0j}, and Φ_{1j} are the polar parameters of the reduced stiffness tensor of the jth lamina; δ_j is the jth ply's orientation measured with respect to the global frame of the laminate, while z_j and z_{j-1} are the z coordinates of the top and bottom of the jth layer's surfaces.

As already said, in this work we use quasi-homogeneous, orthotropic laminates for both the skin and the stiffeners of the wing-box section. Introducing the *normalized stiffness tensors*, defined as

$$\mathbf{A}^* = \frac{1}{h_{lam}}\mathbf{A}, \qquad \mathbf{B}^* = \frac{2}{h_{lam}^2}\mathbf{B}, \qquad \mathbf{D}^* = \frac{12}{h_{lam}^3}\mathbf{D}, \tag{6}$$

the conditions on general elastic properties specified above, i.e., quasi-homogeneity and orthotropy, are mathematically expressed by:

$$\begin{aligned} \mathbf{B}^* &= \mathbf{O} \quad \text{uncoupling condition,} \\ \mathbf{A}^* &= \mathbf{D}^* \quad \text{homogeneity condition,} \\ \Phi_0^{A^*} - \Phi_1^{A^*} &= K^{A^*}\frac{\pi}{4} \quad \text{orthotropy condition.} \end{aligned} \tag{7}$$

If the first two conditions of (7) are satisfied, the laminate is said to be *quasi-homogeneous*; see [21]. In (6), h_{lam} is the total thickness of the laminate, while in the third of (7) $\Phi_0^{A^*}$ and $\Phi_1^{A^*}$ are the polar angles of the tensor $\mathbf{A}^*$. The invariant K^{A^*} determines the type of ordinary orthotropy (see [18, 22]), and it can assume only the values 0 or 1. Vannucci [23] has shown the importance of this material invariant parameter in some problems of optimal design; namely, if a solution is optimal for $K = 0$, it is normally antioptimal for $K = 1$ and inversely, and in some cases, the change of K can lead to a loss of uniqueness of the solution of an optimum problem. To be remarked that the second and third of (7) give also bending orthotropy.

A simple result of the polar formalism is that, for the general case of laminates with identical layers, the isotropic moduli $T_0^{A^*}$ and $T_1^{A^*}$ are equal to those of the elementary layer, T_0 and T_1, respectively; see [21]. $T_0^{A^*}$ and $T_1^{A^*}$ are hence fixed by the choice of the material of the layers, so they are no more design variables: the polar formalism allows for easily eliminating some redundant mechanical variables from a design problem of a laminate composed of identical layers.

From the third condition of (7), we get

$$\begin{aligned} \cos 4\Phi_0^{A^*} &= (-1)^{K^{A^*}} \cos 4\Phi_1^{A^*}, \\ \sin 4\Phi_0^{A^*} &= (-1)^{K^{A^*}} \sin 4\Phi_1^{A^*}, \end{aligned} \tag{8}$$

relations that can be used in (2), valid for any fourth-order elasticity-like tensor, so for tensor $\mathbf{A}^*$ too. Therefore, introducing the quantity $R_{0K}^{A^*} = (-1)^{K^{A^*}} R_0^{A^*}$ (see also [24]) thanks to quasi-homogeneity and to the polar formalism, we are reduced to only 3 mechanical design variables for each laminate: the polar parameters $R_{0K}^{A^*}$, $R_1^{A^*}$, concerning the anisotropic part, and the polar angle $\Phi_1^{A^*}$ that represents the direction of the orthotropy axis. A theoretical remark: $R_{0K}^{A^*}$ is still a tensor invariant, because it is a combination of two distinct tensor invariants, K^{A^*} and $R_0^{A^*}$.

As said previously, we must introduce in this phase the geometric bounds for the design of the laminate that will be done during the next second level problem. Such bounds can be written independently for tensors $\mathbf{A}^*$ or $\mathbf{D}^*$, and are of course the same for the case of quasi-homogeneous laminates. They can be written using the well-known *lamination parameters*, introduced by Tsai and Hahn, [12], and a wide discussion about the geometric bounds is given in [25]. The expression of these bounds using the polar formalism can be found in [17], and for the case of an orthotropic tensor $\mathbf{A}^*$ they are (the quantities without the index A^* refer to the layer)

$$\begin{cases} -R_0 \leq R_{0K}^{A^*} \leq R_0, \\ 0 \leq R_1^{A^*} \leq R_1, \\ 2\left(\frac{R_1^{A^*}}{R_1}\right)^2 - 1 \leq (-1)^K \frac{R_{0K}^{A^*}}{R_0}. \end{cases} \tag{9}$$

These constraints are to be considered for the optimal design of every laminate composing the skin or the stiffeners of the wing-box section.

4.3 Mathematical Formulation of the First Level Problem

As said previously, the goal of the global structural optimization is to find a minimum-weight wing-box configuration respecting the buckling and geometric constraints. To state the optimum problem in a standard form, we first reorder the design variables according to the following scheme (the apex S stands for stiffeners, the quantities without this apex are referred to the skin):

– the vector $\mathbf{x}$ collects the following design variables, concerning the overall structure and the skin:

$$\mathbf{x} = \left\{ \begin{array}{c} x_1 = N \\ x_2 = t \\ x_3 = R_{0K}^{A^*} \\ x_4 = R_1^{A^*} \end{array} \right\}; \tag{10}$$

– each one of the vectors $\mathbf{y}^i$ collects the design variables of the ith stiffener, $i = 1, \ldots, N$:

$$\mathbf{y}^i = \left\{ \begin{array}{c} y_1 = h_i^S \\ y_2 = t_i^S \\ y_3 = (R_{0K}^{A^*})_i^S \\ y_4 = (R_1^{A^*})_i^S \end{array} \right\}. \tag{11}$$

Then we introduce the following functions:

– the objective function W, expressing the overall weight of the structure:

$$W = W\left(\mathbf{x}, \mathbf{y}^i\right); \tag{12}$$

– the function for expressing the constraint on the critical buckling load:

$$f\left(\mathbf{x}, \mathbf{y}^i\right) = p_{ref} - p_{cr}\left(\mathbf{x}, \mathbf{y}^i\right); \tag{13}$$

– the functions for expressing the five geometric constraints (9) on the polar parameters of the skin:

$$g_1(x_3) = -x_3 - R_0; \tag{14}$$

$$g_2(x_3) = x_3 - R_0; \tag{15}$$

$$g_3(x_4) = -x_4; \tag{16}$$

$$g_4(x_4) = x_4 - R_1; \tag{17}$$

$$g_5(x_3, x_4) = 2\left(\frac{x_4}{R_1}\right)^2 - (-1)^K \frac{x_3}{R_0}; \tag{18}$$

– the functions for expressing the five geometric constraints (9) on the polar parameters of the ith stiffener, with $i = 1, \ldots, N$:

$$h_1^i\left(y_3^i\right) = -y_3^i - R_0; \tag{19}$$

$$h_2^i\left(y_3^i\right) = y_3^i - R_0; \tag{20}$$

$$h_3^i\left(y_4^i\right) = -y_4^i; \tag{21}$$

$$h_4^i\left(y_4^i\right) = y_4^i - R_1; \tag{22}$$

$$h_5^i\left(y_3^i, y_4^i\right) = 2\left(\frac{y_4^i}{R_1}\right)^2 - (-1)^K \frac{y_3^i}{R_0}. \tag{23}$$

In (13), p_{ref} is a limiting value for the critical buckling load of the structure, p_{cr}. The parameter K in (18) and (23) fixes the orthotropy shape of the material, and it is equal to 0 for the carbon-epoxy T300/5208, which has $\Phi_0 = \Phi_1$; see Table 1 and the third of (7).

Finally, the problem can be stated in the standard form:

$$\begin{cases} \min\ W(\mathbf{x}, \mathbf{y}^1, \ldots, \mathbf{y}^N), \\ \text{s.t.} \quad f(\mathbf{x}, \mathbf{y}^i, \ldots, \mathbf{y}^N) \leq 0, \\ \qquad g_j(\mathbf{x}) \leq 0, \quad j = 1, \ldots, 5, \\ \qquad h_l^i(\mathbf{y}^i) \leq 0, \quad i = 1, \ldots, N,\ l = 1, \ldots, 5. \end{cases} \tag{24}$$

Problem (24) is nonlinear, in terms of both the geometrical and mechanical variables. Its nonlinearity is given not only by the objective function and the geometrical constraints like those in (18) and (23), but, in a stronger way, by the constraint on the value of the buckling load, $p_{cr} \geq p_{ref}$. The value of the buckling load can be computed analytically if it has a theoretical expression, which happens for some particularly simple structures, like beams or plates of simple form. Unfortunately, no analytical solution is known for the buckling load of a structure as complicated as the one considered in this research; see Fig. 1. Hence, for the solution of problem (24) we need a tool for the numerical evaluation of p_{cr}. To this purpose, the structure, a *continuum*, is *discretized in finite elements* and the computation of p_{cr} is done using the well-known technique of the finite elements method.

From a mathematical point of view, the transformation of a continuum, i.e., of a body having infinite degrees of freedom, into a discrete structure that has a finite number of degrees of freedom, transforms the search of the buckling load into a classical algebraic problem: p_{cr} is the smallest eigenvalue λ of

$$[K]\{u\} = \lambda\{u\}; \tag{25}$$

$[K]$ is the stiffness matrix of the discretized structure; it is symmetric, positive definite, and its dimension is equal to the number of the degrees of freedom of the structure; $\{u\}$ is the vector of the state variables of the problem which, in the classical formulation of a finite element method, are physically the displacements, i.e., the degrees of freedom of the discrete structure. In our case, as it will be specified in the second part of this research, the discretization of the structure leads to a model having some hundreds of thousands of degrees of freedom. The solution of the Laplace's equation for a matrix having a so great dimension is clearly a nonlinear problem whose solution can be obtained only numerically.

As already said, we impose that the fictitious material designed in this first step must be orthotropic, with the axes of orthotropy aligned with the axes of the structure, and uncoupled. All these properties are easily obtained in this phase. In fact, the tensor $\mathbf{A}^*$ is computed, during this phase, using condition (8) in (2). This automatically ensures the orthotropy of the tensor, which is unique for bending and extension by the quasi-homogeneity assumption. Bending-extension coupling is simply ignored during this phase: the fictitious material being homogeneous, though anisotropic, coupling simply does not exist in this computation phase. Finally, we fix the orthotropy direction, for both the skin and the stiffener laminates, simply posing

$$\Phi_1^{A^*} = \left(\Phi_1^{A^*}\right)_i^S = 0, \quad \forall i = 1, \ldots, N, \tag{26}$$

which means that the principal orthotropy axis of each laminate composing the structure is aligned with the global x axis of the whole wing-box section, Fig. 1. In this way, we eliminate from the problem a mechanical design variable for each laminate.

Finally, the dimension of the design space, i.e., the number of design variables, and the number of constraint equations depend on the number N of the stiffeners. In particular, the total number of design variables is $4N + 4$ (there are in fact 4 variables for each stiffener, 3 variables for the skin and the number of stiffeners, N), while the total number of constraint equations is $5N + 6$: the buckling constraint, 5 constraints for the skin, and finally 5 constraints for each stiffener; see the second, third, and fourth of (24), respectively. Nevertheless, though the number of constraints is variable, each constraint added by the addition of a module depends only on the unknowns concerning that module, not on the other ones too; see again the fourth of (24).

The numerical method used to solve an optimization problem, which includes the number of design variables among the unknowns to be determined, is detailed in the second part of this work.

5 Mathematical Formulation of the Second Level Problem

The second problem of the optimization procedure concerns the design of the laminates. Of course, this second problem depends upon the results of the first one, because the laminates to be designed must own the optimal elastic properties and thickness obtained as results of the first level design problem.

It is to be highlighted that in our approach, that wants to be completely general, hence not using special stacking sequences nor orientations, also general elastic properties are concerned by the design problem, in particular quasi-homogeneity and orthotropy.

Several authors have considered different laminate design problems (rather complete but not exhaustive reviews on the state of the art can be found in [26–28]). Some papers on the general elastic properties of the laminates are the classical works of Werren and Norris, [29], on isotropy, those of Fukunaga, [30], Paradies, [31], Vannucci and Verchery, [32], still on isotropy, of Caprino and Crivelli-Visconti, [33], of Grédiac, [34] and of Valot and Vannucci, [1], on orthotropy, and finally those of Vannucci and Verchery, [21, 35], on uncoupling and quasi-homogeneity.

Vannucci [22] has considered the problem of designing the general elastic properties of a laminate. In that work, a general approach based on polar tensor invariants was proposed: no simplifying hypotheses nor special stacks or orientations are used, hence the method allows for finding a general solution to a given problem. This approach was applied in other works and extended in [36] to the constrained optimization of laminated plates and in [24] to the optimal design of laminates with given elastic moduli.

By this approach, the design of a laminate having some elastic properties is reduced to an unconstrained minimum problem. Mathematically, the technique is very simple: in the space of the polar parameters, a target tensor is fixed in some way. In the problem that we consider in this paper, the target is fixed by the optimal values of the polar parameters of the laminate, obtained as results of the first level problem. The tensors of the laminate to be designed must be the same of the target. Hence, the problem is reduced to the minimization of a distance between tensors in the 18-dimensional space of the polar parameters describing tensors **A**, **B,** and **D**. Actually, this is a typical inverse problem and the same approach can be extended to other mechanical properties, concerning other tensors, and hence other polar parameters; see, for instance, [37] or [38].

Therefore, the key-point of this phase is the construction of the distance function, objective of the minimum problem. This function drives the search for a quasi-homogeneous, orthotropic laminate, having the optimal elastic polar moduli issued from the first step. The design variables of this second level problem are the layer orientations δ_j (see (5)), and the optimization process has to be repeated for the laminates of each stiffener and of the skin.

To construct the distance function in this case, we recall that we need to find a stacking sequence which satisfies the conditions of (7), and that has the optimal polar parameters found in the first step, $\hat{K}^{A^*}$, $\hat{R}_0^{A^*}$, and $\hat{R}_1^{A^*}$. The relation among the polar parameters $\hat{R}_0^{A^*}$ and $\hat{K}^{A^*}$, and the polar quantity $\hat{R}_{0K}^{A^*}$ is, of course,

$$\hat{R}_0^{A^*} = \left|\hat{R}_{0K}^{A^*}\right|, \qquad \hat{K}^{A^*} = \begin{cases} 0 & \text{if } \hat{R}_{0K}^{A^*} > 0, \\ 1 & \text{if } \hat{R}_{0K}^{A^*} < 0. \end{cases} \tag{27}$$

In addition, we need to orient the orthotropy axes, imposing

$$\Phi_1^{A^*} = \hat{\Phi}_1^{A^*}; \tag{28}$$

in our case $\hat{\Phi}_1^{A^*} = 0$, which means that the principal orthotropy axis of each laminate has to be aligned with the x axis of the whole structure. Unlike in other more common approaches, where the orthotropy and its direction are normally imposed choosing particular sequences that automatically place the orthotropy axes in a direction, normally aligned with the axes of the laminate, with the polar formalism orthotropy and its direction are imposed by simple independent conditions, and any direction different from the axes of the laminate can be easily imposed, simply choosing an angle different from zero for $\hat{\Phi}_1^{A^*}$.

We remind that from the first level problem we know also the thickness of the skin and of the stiffener laminates. Being each laminate thickness a multiple of that of the elementary ply, the number of the laminate plies is also known.

Considering all these points, the tensor distance, objective function of the second level problem, can be stated for each laminate of the skin and of the stiffeners as:

$$\begin{aligned}
&\min_{\boldsymbol{\delta}} F(\boldsymbol{\delta}) = \sum_{j=1}^{6} f_j(\boldsymbol{\delta}) \quad \text{with:} \\
&f_1(\boldsymbol{\delta}) = \left(\frac{\|\mathbf{B}^*\|}{\|\mathbf{Q}\|}\right)^2, \qquad f_2(\boldsymbol{\delta}) = \left(\frac{\|\mathbf{C}\|}{\|\mathbf{Q}\|}\right)^2, \\
&f_3(\boldsymbol{\delta}) = \left(\frac{\Phi_0^{A^*} - \Phi_1^{A^*} - \hat{K}^{A^*}\frac{\pi}{4}}{\frac{\pi}{4}}\right)^2, \qquad f_4(\boldsymbol{\delta}) = \left(\frac{R_0^{A^*} - \hat{R}_0^{A^*}}{\hat{R}_0^{A^*}}\right)^2, \\
&f_5(\boldsymbol{\delta}) = \left(\frac{R_1^{A^*} - \hat{R}_1^{A^*}}{\hat{R}_1^{A^*}}\right)^2, \qquad f_6(\boldsymbol{\delta}) = \left(\frac{\Phi_1^{A^*} - \hat{\Phi}_1^{A^*}}{\frac{\pi}{4}}\right)^2.
\end{aligned} \tag{29}$$

In (29), $\boldsymbol{\delta}$ is the vector of layer orientations, while $f_j(\boldsymbol{\delta})$ is the jth partial term of the objective function, $j = 1, \ldots, 6$. The terms $f_1(\boldsymbol{\delta})$ and $f_2(\boldsymbol{\delta})$ are related to the quasi-homogeneity conditions, while the third one, $f_3(\boldsymbol{\delta})$, is linked to the orthotropy condition; see (7). The function $f_3(\boldsymbol{\delta})$ takes also into account the prescribed value $\hat{K}^{A^*}$ of parameter K^{A^*} issued from the first optimization phase. The terms $f_4(\boldsymbol{\delta})$ and $f_5(\boldsymbol{\delta})$ correspond to the prescribed optimal values $\hat{R}_0^{A^*}$ and $\hat{R}_1^{A^*}$ of the polar moduli $R_0^{A^*}$ and $R_1^{A^*}$. The term $f_6(\boldsymbol{\delta})$ corresponds to the imposed direction of orthotropy of the laminate: $\Phi_1^{A^*} = \hat{\Phi}_1^{A^*} = 0$. Finally, $\|\mathbf{B}^*\|$ is the norm of the homogenized coupling tensor and $\|\mathbf{C}\|$ is the norm of the homogeneity tensor.

The function defined in (29) is actually the square of a dimensionless tensor distance. In fact, we have normalized all the terms, which allows for all the terms to have a similar weight in the function. The tensor norms have been transformed in dimensionless quantities dividing them by the normalization factor $\|\mathbf{Q}\|$, that is, the norm of the layer reduced stiffness tensor. All the norms have been computed using the tensor norm proposed by Kandil and Verchery [20]; see also [22]. The normalization factor of the orthotropy requirement is assumed equal to $\frac{\pi}{4}$, while for the anisotropy parameters of tensor $\mathbf{A}^*$, it is equal to the corresponding target polar parameter.

The quadratic form of (29) is a nondimensional, positive semidefinite function of the polar parameters of the laminate. It depends on all the mechanical and geometrical properties of the laminate, i.e., stacking sequence, ply orientations, material, and thickness of the plies. In addition, the objective function of (29) is nonconvex in the space of layer orientations, since the polar parameters of the laminate depend upon circular functions of the orientations, as reported in (5).

A true advantage of formulation (29) is that the global minima of the function are zero valued. This is important for the numerical search strategy, because the knowledge of the value of the global minima is useful on one side to stop the numerical search, and on the other side to ensure that the solution so found is really a global minimum.

Finally, we remark that unlike the first problem, this second problem is an unconstrained problem with a known number of design variables, but the objective is still a highly nonconvex function; a simple glance at (5) is sufficient to realize this. For what concerns the nature of the design variables, the operator is free to chose continuous, discrete equally stepped variables or variables whose possible values belong to a defined set; actually, such a choice is mostly a practical, technological choice.

6 Concluding Remarks

The optimization procedure presented in this paper is characterized by several points that make it an innovative, effective, general method for the design of composite stiffened panels. Our motivation was to create a general procedure for the optimization of modular systems, with the number of the modules that belongs to the set of the design variables and without using special assumptions to get some results. The numerical method is, however, a fundamental part of the procedure, because it is thanks to an appropriate numerical tool that the simultaneous optimization of the number of the modules and of their characteristics is possible. The details about the numerical method are given in the second part of this work, to which we address the reader also for a general perspective and remarks on the procedure.

Nevertheless, it is worth noting a fundamental point, already introduced in Sect. 3: The correspondence between an elastic tensor and a laminate is not bijective. This is extremely important, because it renders the two-level approach feasible and effective. In fact, at the first level, we can consider the structure as it was formed by a fictitious single layer, while the second level concerns the other properties to be designed, just because the mechanical parameters are not uniquely determined by the stacking sequence. For instance, in our case, this allows us for using quasi-homogeneous laminates: at the first level, this assumption lets us consider only an elastic tensor to be designed, at the second level this property has to be obtained, but this would be, generally speaking, impossible to be done if only one sequence should give the elastic tensor found at the first level. On the contrary, because several laminates share the same elastic behavior obtained at the first step, we can look for one of them which is also quasi-homogeneous.

Of course, in the same way, we could add at the second step other requirements to select, among the possible laminates, one having some other additional properties, for instance on the laminar strength or something else. Nevertheless, there is not any guarantee of finding a laminate satisfying all the requirements: Mathematical conditions ensuring that a given optimal design problem for a laminate has at least one solution are in general unknown. Anyway, the condition for not having an overdetermined problem is

$$n - 1 \geq n_r, \tag{30}$$

where n is the number of layers and n_r the number of requirements imposed to the search of the layer. Actually, one layer must be subtracted because the requirements have to be frame independent, and eventual conditions on the direction of the anisotropy are not to be considered; in our case, this is the condition given by $f_6(\boldsymbol{\delta})$ in (29). For the problem considered here, we have 7 requirements: 3 for getting $\hat{K}^{A^*}$, $\hat{R}_0^{A^*}$ and $\hat{R}_1^{A^*}$, 2 for imposing that $\mathbf{D}^* - \mathbf{A}^* = \mathbf{O}$ and 2 for having $\mathbf{B} = \mathbf{O}$; see [21]. Hence, we can hope to obtain a solution if, for the skin and for each one of the stiffeners, we obtain a number of layers not less than 8. For this reason, but also for technological reasons, we have put, for the ply thickness, a lower bound of 2 mm that gives a minimum of 16 layers of T300/5208 carbon-epoxy; see Table 2.

The proposed approach appears to be very flexible and applicable to various problems of structural engineering. Moreover, the procedure has a high level of versatility: more constraints could be easily added to the optimization problem, e.g., constraints on the strength, yielding or delamination of the laminates which compose the structure, without reducing the power and the robustness of the proposed approach. This is a substantial part of the future developments that we intend to study.

Some final remarks to end this part. The structural problem considered here, namely the one concerning the first level of the procedure, is actually one of the oldest structural optimum problems. In fact, the first to study a problem of this type was Lagrange, in 1770, [39]. He considered the case of a column subjected to a tip compressive load; the objective was to design the lightest column able to withstand a given load without buckling, which is just the problem that we have considered at the first level. He gave an erroneous result, subsequently corrected by Clausen in 1851 [40]. All along the last century, several other authors considered the same or a closely similar problem. The dual of the problem originally considered by Lagrange has also been treated: to maximize the buckling load for a column composed by a fixed amount of matter and charged by a compressive force at its top. A rather complete bibliography on this topic can be found in the classical book from Banichuk [41].

The problem that we have considered in this paper, however, is slightly different from the classical ones considered since Lagrange. In fact, the constraint on the minimum buckling load is not the only one; see (24). The geometrical constraints, namely those in the third of (9), are particularly important. They change, of course, the problem and its dual too. To the best knowledge of the authors, it is the first time that a similar problem has been formulated in the form given in this paper, and the formulation of its dual is still an open problem.

There are at least two other reasons that render the problem considered in this paper different from those, more classical, recalled hereon. In fact, normally the authors consider the case of the optimal shape of the structure, and look for a function defining the best form to be given to it. In our problem, the shape is known and the dimensions are to be determined along with the number of the modules, the stiffeners. In some sense, the number of modules changes the shape, but the changes are not continuous, because the number of modules is an integer.

The second reason, is the fact that our structure is anisotropic, while normally isotropic structures are considered. Hence, in our problem we need at the same time to optimize geometrical and mechanical quantities (in our case, we have chosen the polar invariants to represent the physical properties of the structure). Hence, we deal with a problem which is at the same time mechanical and geometrical, for its design variables.

The anisotropic nature of the problem, which in particular enters directly, though not explicitly, in the definition of the buckling constraint, is important also for another reason. In fact, we have already said that the first level problem is nonlinear; this is easy to be understood, simply considering the objective function and the geometrical constraints. About the buckling constraint, we have already recalled that it is impossible to be explicitly specified: the buckling load can be computed only by a numerical approach. Nevertheless, it depends upon the stiffness of the structure, which in turn depends on the mechanical and geometrical variables. It is well known, for instance, that stiffness is a nonconvex function of the orientation of the anisotropy. So, it is likely that the buckling load is a nonconvex function of the design variables.

A sensitivity analysis of the problems would greatly help in understanding the role played by the different variables. Unfortunately, it cannot be done analytically, of course, and a numerical procedure should be used. Nevertheless, to perform such an analysis on a finite element model like the one considered by us, cfr. the second part of this work, is a very long work, and we did not do it. An interesting strategy should probably be the use of automatic differentiation, which, however, needs some specific finite element codes, [42]. This is a possible subject for future investigations.

Finally, for what concerns the second level problem, it is always strongly nonconvex. To the best knowledge of the authors, its dual is not known. More generally, in laminate design duality is an unexplored domain: no dual methods are known in this field. In the reference book on laminated composite design and optimization, [43], the word duality is never employed.

In addition, the solution is almost never unique, nor isolate. This is still an open mathematical problem in laminates design. Actually, no rules are known up to day to state if a problem like (29) has a solution and if it is unique or not. Such problems are, in fact, constituted by a sum of optimization subproblems that are not independent and that are, in some cases, compatible. In other words, if the number of layers, i.e., of unknowns, is sufficiently large, a problem of this type will have at least one solution. In this case, all the subobjectives that compose the objective function are compatible. On the contrary, if the number of unknowns is not sufficiently large, the subobjectives become incompatible and the global problem becomes a multiobjective one without any mechanical meaning nor interest.

The minimum number of unknowns, i.e., of layers, to ensure the existence of the solution to a given problem of the type (29) is not known; of course, it depends upon the type of subobjectives composing the global objective function. An attempt to give a numerical answer to such kind of questions have been proposed by the authors, [44]: the general problem is stated in a slightly different manner from (29), including the number of layers among the design variables. Then the problem is formulated in a way similar to that used to solve the first level problem, by the genetic algorithm BIANCA with variable length chromosomes; see the second part of this work. The result is a laminate with the least number of layers satisfying the imposed requirements. Nevertheless, this is just a numerical approach, and a general rigorous theoretical study of the conditions for a laminate design problem like that in (29) have a solution is still lacking.

What we have observed in all the cases that we have solved is that when the solution exists, it is not unique nor isolate. Actually, there exist some functional relations among the solutions that allows to change solutions changing with continuity some of the design variables. Unfortunately, it is possible, in general, to express analytically such relations only in very elementary cases, [45], while in some other cases, very simple too, a graphical representation of the locus of all the solutions has been found numerically, [1].

All this points have also influenced the choice of the numerical procedure used for solving the problem described in this paper. These aspects are considered in the second part of this work.

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