Journal of Optimization Theory and Applications (2020) 186:1053–1055
https://doi.org/10.1007/s10957-020-01723-9

FORUM

A Note on "Optimal and Sub-Optimal Feedback Controls for Biogas Production"

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Received: 26 May 2020 / Accepted: 18 July 2020 / Published online: 7 August 2020

Abstract
We correct Proposition 3.1 of Ref. Haddon et al. (J Optim Theory Appl 183:642, 2019).

Keywords Optimal control · Chemostat model · Singular Arc · Sub-optimality · Infinite horizon

Mathematics Subject Classification 49K15 · 49N35 · 49N90 · 93B52

1 Introduction

In Ref. [1], Proposition 3.1 deals with the convergence of the discounted reward (16), the associated value function (17) and optimal trajectories, as the discount factor goes to 0. The proof of the Γ-convergence of the discounted reward is incorrect since, in general, this reward is not monotone with respect to the discount factor δ.

Communicated by Benoit Chachuat.

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2 The Correction

Proposition 3.1 can be revised as follows.

Proposition 2.1 *For all $\xi \in \mathcal{D}$ and for all $\delta > 0$, the suprema are attained,*

$$V_\delta(\xi) = \max_{\zeta(\cdot)} J_\delta(\zeta(\cdot)).$$

If the $\Gamma-$limit of $J_\delta(\cdot)$ exists as δ goes to 0,

$$J_0(\zeta(\cdot)) := \Gamma - \lim_{\delta \to 0} J_\delta(\zeta(\cdot)),$$

then the maxima converge, pointwise in ξ, to the maximum of the limit,

$$V_0(\xi) := \lim_{\delta \to 0} V_\delta(\xi) = \max_{\zeta(\cdot)} J_0(\zeta(\cdot)). \tag{1}$$

Furthermore, if $\zeta_\delta(\cdot)$ is an optimal trajectory, i.e. if $V_\delta(\xi) = J_\delta(\zeta_\delta(\cdot))$, and if $\zeta_\delta(\cdot)$ converges to $\zeta_0(\cdot)$ in $\mathcal{S}(\xi)$, then $\zeta_0(\cdot)$ is an optimal trajectory for (1) and

$$V_0(\xi) = J_0(\zeta_0(\cdot)) = \lim_{\delta \to 0} J_\delta(\zeta_\delta(\cdot)).$$

Proof To show that the suprema are attained, we show that the set of all forward trajectories of (3) of [1], with initial condition ξ, is compact for the topology on $W^{1,1}\big(0, \infty; \mathbb{R}^2, e^{-bt}dt\big)$ given in Definition 3.1 of [1].

For each $\xi \in \mathcal{D}$ we set

$$F_\xi(\zeta) := F\big(P_{\mathcal{L}(\xi)}(\zeta)\big),$$

where $P_{\mathcal{L}(\xi)}$ is the projection on the convex set $\mathcal{L}(\xi)$. Then, F_ξ has linear growth, so that we can define

$$c = \sup_{\zeta \in \mathrm{Dom}(F_\xi)} \frac{||F_\xi(\zeta)||}{||\zeta|| + 1},$$

where $||F_\xi(\zeta)|| := \sup_{\eta \in F_\xi(\zeta)} ||\eta||$. Note that F is upper semi-continuous and has compact non-empty convex images (such a map is known as a Marchaud map [2]). With this, the set $\mathcal{S}(\xi)$ is the set of absolutely continuous solutions of the differential inclusion

$$\dot{\zeta}(t) \in F_\xi(\zeta(t)), \qquad \zeta(0) = \xi.$$

We can therefore use [2, Theorem 3.5.2] to establish that $\mathcal{S}(\xi)$ is compact for the topology of $W^{1,1}\big(0, \infty; \mathbb{R}^2, e^{-bt}dt\big)$ for $b > c$, thereby proving the existence of optimal trajectories in $\mathcal{S}(\xi)$.

In addition, this allows us to show that the maxima converge to the maximum of the limit. Indeed, when the rewards $\Gamma-$converge, it is sufficient to show that there exists a countably compact set on which the suprema are attained for all δ [3, Theorem 7.4]. The set $\mathcal{S}(\xi)$ is clearly independent of δ and countably compact, since it is compact. Finally, the convergence of optimal trajectories can be shown with [3, Corollary 7.20]. □

3 Conclusions

The originally published proof of Proposition 3.1 of [1] was incorrect and we have revised here the result to obtain an accurate statement. However, we have not found reasonable assumptions that ensure the existence of the Γ-limit of $J_\delta(\cdot)$, when δ goes to 0, although it seems to be satisfied in our examples. We thus posit the Γ-convergence as a conjecture, that will be investigated in future research.

Acknowledgements The authors would like to thank Frédéric Bonnans for pointing out a problem with the originally published proof. The first and second authors were supported by FONDECYT grant 1160567 and 1160204, and by Basal Program CMM-AFB 170001 from CONICYT-Chile. The first author was supported by a doctoral fellowship CONICYT-PFCHA/Doctorado Nacional/2017-21170249. The third author was supported by the LabEx NUMEV incorporated into the I-Site MUSE.

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