A Multi-Objective Evolutionary Algorithm Based on Mixed Encoding for Community Detection

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A Multi-Objective Evolutionary Algorithm Based on Mixed Encoding for Community Detection

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Abstract

Community structure is one of the most significant features of complex networks and community detection is a crucial method to analyze community structure. Existing representations in community detection are inflexible and easily generate invalid solutions. To address the drawbacks, this paper proposed a multi-objective evolutionary algorithm based on mixed encoding (MOGAME). The algorithm combines the locus-based representation and labels-based representation, which can avoid generating invalid solution and improve the performance. Extensive experiments on both synthetic and real-word networks show that the proposed algorithm performs better than the existing algorithms with respect to accuracy and stability.

Keywords Complex network · Multi-objective evolutionary · Mixed encoding ·Community detection

1 Introduction

Many complex systems in real life can be represented by complex networks, such as traffic networks, social networks, neural networks and biological networks [1-4]. Community structure is one of great significance in the study of complex networks. The characteristic of the structure is that the internal nodes of the community are closely connected, and the external nodes are sparsely connected [5,6]. Community detection uses the information of network topology to find out the community structure, which is helpful for studying the modules, functions and evolution of the entire network in the way of dividing and conquering [7]. Therefore, it is meaningful to detect community in networks. For example, community detection helps to find proteins with similar biological functions in protein-structure networks and helps to find people with the same hobbies in social networks. In the last decade, many researchers have been proposed a great number of community detection algorithm [8-12]. From the perspective of the number of optimization objective, the community detection optimization single objective and multi-objectives. Tasign used genetic algorithm to optimize the modularity function proposed in [13] to discover the community structure (GA-NET) [14]. Pizzuti defined the function named community score, which is able to describe the situation of the community structure [15]. Blondel proposed a fast modularity optimization algorithm based on hierarchical clustering to obtain the best partition [16]. Cai proposed a greedy discrete particle swarm optimization framework for detecting communities in large-scale networks, which use a greedy strategy to get better solution and redesign the status update rules according to the network topology [17]. Sun pro-posed a two-stage community detection method based on label propagation (TS-LPA), which uses the idea of expanding neighborhoods to measure the centrality of nodes [18].

Although the single objective optimization algorithms mentioned above can find community structures, they cannot meet the actual requirements. In real-world applications, many factors need to be taken into consideration when detecting community structure. These factors usually conflict with each other. Therefore, some researchers model the community detection problem as multi-objective optimization problem [19-23]. Pizzuti used the NSGA-II framework to apply to community detection (MOGA-NET) [24]. Gong put forward a decomposition-based community detection meth-od for improving the diversity of the population and accelerating the convergence of the solution [25]. Ji combined ant colony optimization method on the basis of MOEA/D-NET
(MOCD-ACO) [26].

However, the representations of the multi-objective algorithms, either are highly con-strained by net-work

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topology that caused inconvenience to evolutionary operator, or have a lot of randomness that easily generate invalid solution. To address this problem, this paper proposes a multi-objective evolutionary algorithm denoted as MOGAME. It combines locus-based representation and labels-based representation [27,28], which not only make full use of network topology information but also make solution flexibly apply to evolutionary operations. The experimental results show that MOGAME has a good performance than other algorithms. Details of MOGAME are described in section 3. Section 4 conducts comprehensive experiments to examine the efficacy of the pro-posed algorithm. The experimental results with respect to two evaluation criteria are analyzed. Finally, the concluding remarks are drawn in Section 5.

2 Relate work

2.1 Formulation

Community detection is a process of dividing network nodes into different partitions according to the connection density of network nodes. The links between nodes in the same partitions (internal link density) need to be as dense as possible, and the links in different partitions should be sparse enough [29,30]. Give an undirected network denoted as $G(V,E)$, where V and E are the sets of nodes and edges, respectively. The adjacency matrix is used to represent the information of the network. The community structure of network can be considered as $P=\{P_1,P_2,...P_i,...P_s\}$, which divided network into s parts. P_i is a community of the network. For example, Figure 1 denote a complex net-work with 11 nodes. The network is divided into two community. The first community is composed of 6 nodes $\{3,6,8,9,10,11\}$.

The second community contains nodes $\{1,2,4,5,7\}$.

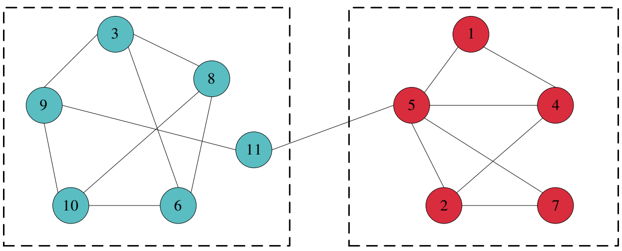


Fig.1 Schematic diagram of the community structure

Researchers define the community detection problem as a multi-objective optimization problem to get better community structure. In this study, we minimize two objective functions, ratio cut (RC) [31] and kernel K-means (KKM) [32]. Because they can reflect the overall situation of the community structure. Two objective functions are formalized as follows:

$$min\begin{cases} RC=\sum_{i=1}^{s}\frac{L(P_i,\overline{P}_i)}{|P_i|} \\ KKM=2(n-s)-\sum_{i=1}^{s}\frac{L(P_i,P_i)}{|P_i|} \end{cases} \tag{1}$$

In formula (1), n is total the number of nodes in the network and s denotes the number of communities in a solution(individual). $L(P_i,P_i)=\sum_{i,j\in P_i}A_{i,j}$ represents the sum of node degrees within community. $L(P_i,\overline{P}_i)=\sum_{i\in P_i,j\notin P_i}A_{i,j}$ is the sum of node degrees between nodes in the community and nodes out of the community. As mentioned in [33], KKM is a decreasing function measured the density of links within the community, but RC is an increasing function. Therefore, KKM and RC conflict with each other, which meets the requirements for community structure.

2.2 Multi-objective optimization

Multi-objective optimization is a way to solve optimization problems [34]. When two or more conflicting objectives need to be optimized, it can successfully find a set of solutions. These solutions satisfy to the theory of Pareto Optimal. For example, vehicle speed and time are two contradictory objectives. At this time, multi-objective optimization provides decision-makers with options. Figure 2 is the flowchart of the multi-objective evolutionary algorithm based on non-dominant.

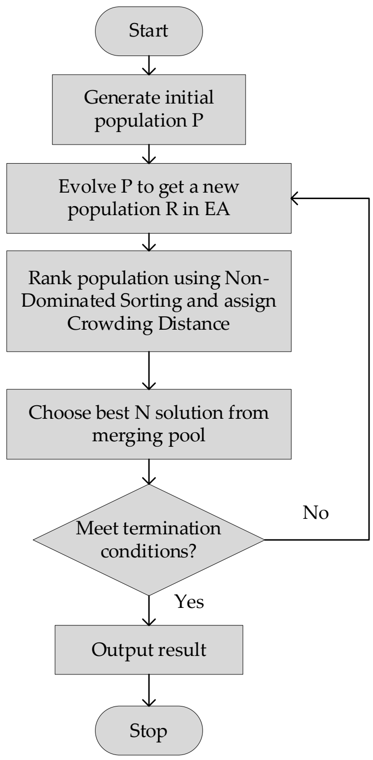


Fig.2 Multi-objective evolutionary algorithm

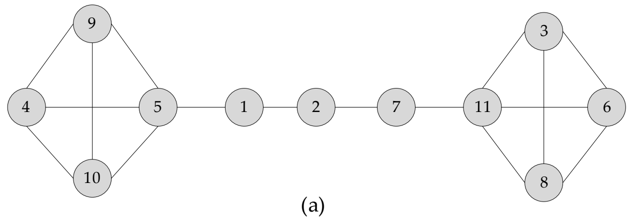
2.3 Exiting representation

The evolutionary algorithm applied to community detection usually needs to encode a genotype for the network. There are two popular representations: labels-based representation and locus-based representation [35]. Labels-based representation is an integer vector ($X = x_1, x_2, \ldots, x_i, \ldots, x_n$), where n is the number of nodes. x_i denotes the label of community to which node i belongs. Unlike labels-based representation, x_i is adjacent node of node i in locus-based representation t induces a division of the network into connected subgraphs. Figure 3 (a)Shows a network with 11 nodes, (b)Locus-based represent of the network (a), (c) Corresponding graph division into three connected components, (d) Labels-based representation of network (a).

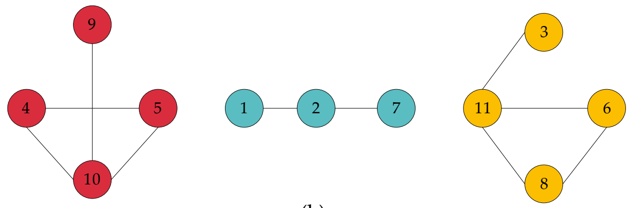
3 The proposed algorithm

3.1 Mixed representation

In multi-objective community detection, the representation has a leading role in the evolution, and affects the convergence of algorithm and quality of community di-vision. However, locus-based representation causes inconvenience to evolutionary progress, because it often takes into account the con-strains of network topology [35]. Labels-based representation is not restricted by the network topology and thus has great flexibility [36]. In this study, MOGAME utilizes the complementary advantages of locus-based representation and labels-based representation to design a mixed representation. Three steps are taken to generate population: firstly, locus-based representation is used firstly. Secondly, the individual is decoded. Finally, labels-based representation is used for coding again. As shown in Figure 3, Figure 3(b) is locus-based representation of network Figure 3(a). According to Figure 3(b), three subgraphs can be decomposed, visualized by different colors in Figure 3(c). Figure 3(d) is labels-based representation on the basis of subgraphs obtained in previous step and node has the same label in the same subgraph.



(a)



(b)

i	1	2	3	4	5	6	7	8	9	10	11
x_i	7	7	11	9	4	11	1	6	10	5	8

(c)

i	1	2	3	4	5	6	7	8	9	10	11
x_i	2	2	3	1	1	3	2	3	1	1	3

(d)

Fig.3 (a) An example network with 11 nodes (b) Locus-based representation (c) The network division into three connected components. (d) Labels-based representation.

Algorithm 1 Mixed representation

Input: $G = (V, E)$: the graph of a complex network; A : Adjacency matrix of a complex network; n : the number of genes in each chromosome; s : the number of communities in network.

Output: Initial population;

while population size is not satisfied:
  encode individual $X = \{x_1, x_2, \ldots, x_n\}$ in locus-based representation, where gene is the node and the corresponding allele is any adjacent nodes of the node.
  decode individual $X = \{x_1, x_2, \ldots, x_n\}$ to generate a partition $P = \{P_1, P_2, \ldots P_s\}$ of the Graph G
  encode a new individual in labels-based representation according partitions, where nodes in the same community assigned identical label.
end while
return Initial population

3.2 Variation operators

The crossover operator is an important part of the evolutionary algorithm. It is not suitable for one-point crossover and two-point crossover to be applied in community detection. Therefore, MOGAME algorithm adopts the two-way crossover used in [37] to improve communication of information between individuals. As for mutation operator, the random mutation is used in MOGAME. An individual is randomly chosen in population. Corresponding allele of a gene randomly chosen in the individual is changed to the community label of any adjacent nodes.

In the initialization phase, the network is encoded with locus-based representation, then the network is decoded some partitions, finally partitions used to encode again in labels-based representation. In evolutionary phase adopts tournament selection, two-way crossover and random mutation method. Finally, the optimized solution set is obtained.

3.3 MOGAME algorithm

Algorithm 2 MOGAME

Input: *f1, f2*: fitness function;
Output: The partition of the network;

```
Generate population by mixed representation.
   While number of iterations is not satisfied:
   Decode each individual of the population to generate a partition.
   Evaluate the fitness values by using f1, f2.
   Sort each individual according to nondomination rank and give a rank.
   Select individual to create offspring by using the crossover operator and mutation operator.
   Combine the parents and offspring, and partition them into fronts finally.
   Select pop individuals on the lower front, and create the next population by variation operators.
End while
Select the individual with maximum MNI and Q from the Pareto front
Return the individual after decoding (the partition of the network)
```

4 Experiments

In this study, we verify the performance of our algorithm on both synthetic data set [38] and real network data set [39]. MOGAME is compared with two famous MOEAS in community detection (GA-Net and MOGA-Net), and two state-of-the-art algorithms (TSLPA and MOCDACO). The algorithms are coded in python and all experiments are conducted on a PC with Intel(R) Core (TM) i5-8250 CPU and 10.0G RAM running 64 bits windows 10.

4.1 evaluate metrics

Modularity(Q) is a commonly used measure to evaluate the structural strength of network communities. Normalized mutual information (NMI) is an external measure to estimate the similarity between real result and the result of the algorithm [40][41].

Modularity Q function is defined as

$$Q=\frac{1}{2m}\sum_{i,j}\left(A_{ij}-\frac{k_i k_j}{2m}\right)\delta(P_i,P_j) \tag{2}$$

where m represent the number of edges. A_{ij} is the element of the adjacency matrix *A* of the network. $A_{ij}=1$ means that node *i* connects node *j*. $A_{ij}=0$ means the edge is no existence between node *i* and node *j*. k_i is the degree of node *i*. $\delta(P_i,P_j)$ is used to judge whether node *i* and node *j* belong to the same community. The larger of the value of *Q*, the stronger of the community structure.

NMI function is defined as

$$NMI(T,P)=\frac{-2\sum_{i=1}^{M_T}\sum_{j=1}^{M_P}M_{i,j}\log\frac{M_{ij}n}{M_{i.}M_{.j}}}{\sum_{i=1}^{M_T}M_{i.}\log\frac{M_{i}}{n}+\sum_{j=1}^{M_P}M_{.j}\log\frac{M_{.j}}{n}} \tag{3}$$

where T is the true partition and P is obtained partition by the algorithm. M_T is denoted the number of real communities and M_P is the number of found communities; M_{ij} is the number of nodes sharing in common by community i in real T and community j in P; $M_{i.}$ is the sum of row *i* and $M_{.j}$ is the sum of column j in confusion matrix *M*. The larger of the value of *NMI*, the better the partition obtained.

4.2 Experiments on Synthetic Network

In this paper, we mainly experiment on GN extended benchmark network and LFR benchmark network. GN network has 128 nodes partitioned by four communities of 32 nodes. The average of degree of each node is 16. Unlike the GN network, the node degree and community size of the LFR benchmark network obey an exponential distribution. The fuzzy of both benchmark networks can be control by μ. The fuzzier the network is, the cluster structure more difficult to identify [38]. Figure 4 and Figure 5 display the maximum MNI values obtained by five algorithms on GN network and LFR network, respectively. It can be seen that MOGAME can accurately find out the true partition after $\mu > 3.5$. The value of *NMI* is sustained above 0.92 when μ is less than 3.5. Obviously, whether on GN network or LFR network, MOGAME is more stable and accurate than the other algorithms

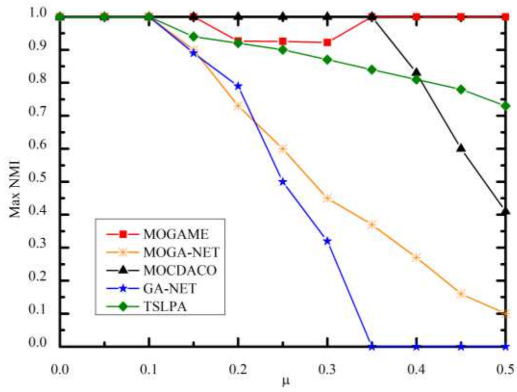


Fig.4 GN benchmark network test results

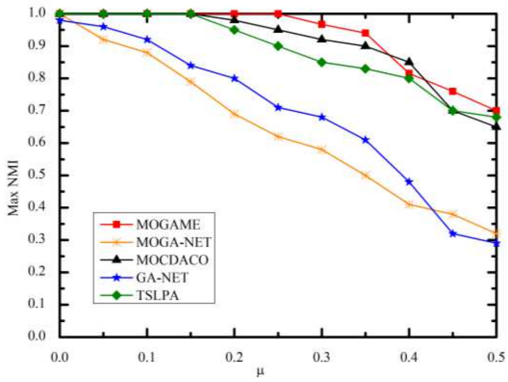


Fig.5 LFR benchmark network test results

4.3 Experiments on real network

In the section, four kinds of real-world networks are used to test our algorithm [40-42]. Zachary's Karate Club is a real social network constructed by observing American college karate clubs; Bottlenose Dolphins is a network formed by observing the exchanges of 62 dolphins living in the Strait of New Zealand; Krebs' books network on American politics is a network of political books complied by V. Krebs [26]; Football network is based on the American college football league. Table 1 shows the details of three real networks.

In this study, we compare MOGAME with MOGA-NET、MOCDACO、GA-NET and TSLPA. The experimental results are averaged over 30 independent runs. It can be seen from the experimental results in Table 2 that MOGAME has better performance to divide the community structure, because MOGAME adopted the mixed representation to create better initialization population during the initialization process, so that it can accurately determine the community to which the node belongs.

Table 1 Description of the real network data set

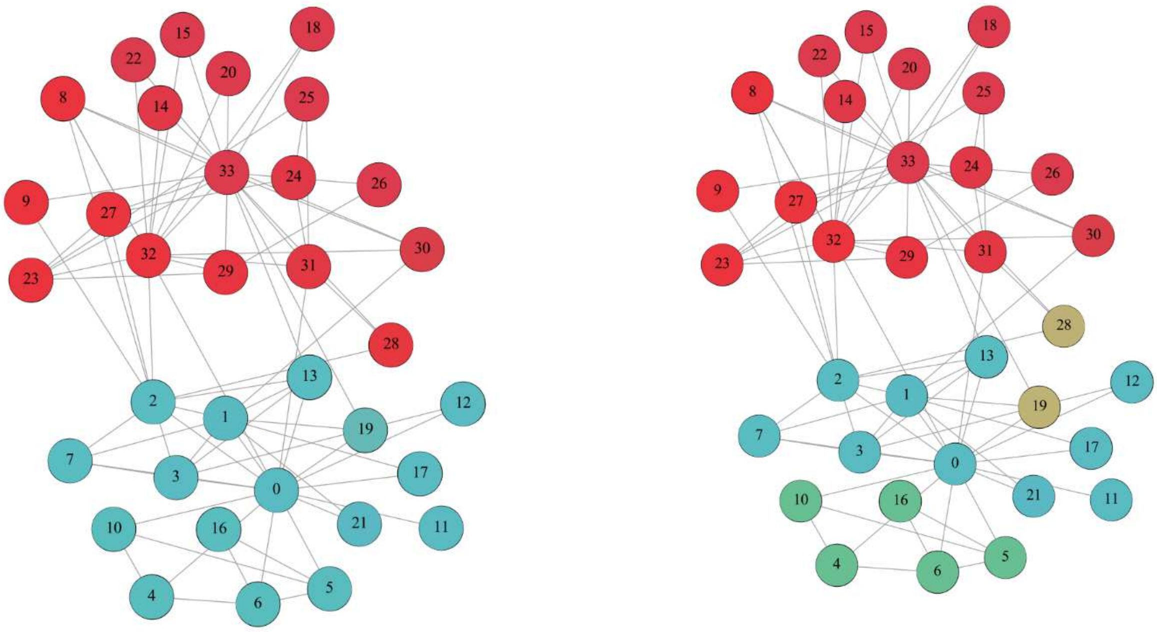
Dataset	Number of nodes	Number of edges	Number of real communities
Karate	34	78	2
Dolphins	62	159	2
Books	105	440	3
Football	115	613	12

Table 2 Description of the real network data set

Dataset	Index	MOGAME	MOGA-NET	MOCDACO	GA-NET	TSLPA
Karate	NMI_{max}	**1.0000**	0.8385	1.0000	0.8990	1.0000
	NMI_{avg}	**1.0000**	0.8381	1.0000	0.8029	1.0000
	Q_{max}	**0.4696**	0.4150	0.4197	0.4112	0.4185
	Q_{avg}	**0.4696**	0.4147	0.4197	0.3768	0.4127
	NMI_{max}	**1.0000**	0.9918	1.0000	0.6725	0.9210

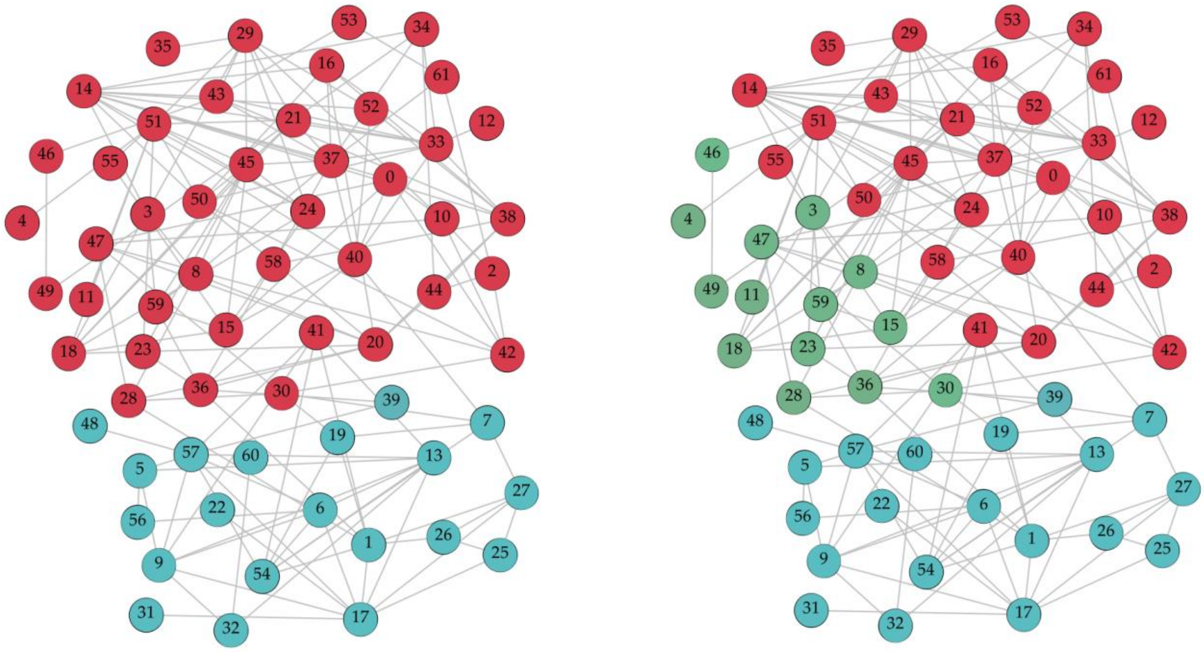
Dolphins	NMI_{avg}	0.8235	0.9875	1.0000	0.6369	0.9135
	Q_{max}	**0.5432**	0.5144	0.5258	0.4664	0.5212
	Q_{avg}	**0.5346**	0.5110	0.5230	0.4441	0.5104
Books	NMI_{max}	0.5548	0.5270	0.6240	0.4648	0.5075
	NMI_{avg}	0.4920	0.4872	0.5972	0.4462	0.4821
	Q_{max}	**0.5235**	0.5184	0.5208	0.5005	0.5033
	Q_{avg}	**0.5214**	0.5042	0.5194	0.4895	0.4915
Football	NMI_{max}	0.9270	0.7894	0.9374	0.8842	0.9010
	NMI_{avg}	**0.9268**	0.7863	0.9268	0.8556	0.8942
	Q_{max}	**0.6131**	0.5194	0.5999	0.5803	0.5810
	Q_{avg}	**0.6092**	0.5163	0.5886	0.5619	0.5795

The community detection results of our method on Karate data set, Dolphin data set, Krebs' books data set, and Football data set are shown in Figure 6, Figure 7, Figure 8, and Figure 9, respectively. NMI=1 means that community structure obtained by algorithm is consistent with the real partition of the network, so MOGAME can detect the real partition in Karate and Dolphin. In Krebs' books dataset, MOGAME performs better than MOGA-NET、GA-NET and TSLPA. As for Football data set, except for ultra-small communities that are difficult to find, other communities with obvious community structures can be detected by MOGAME.



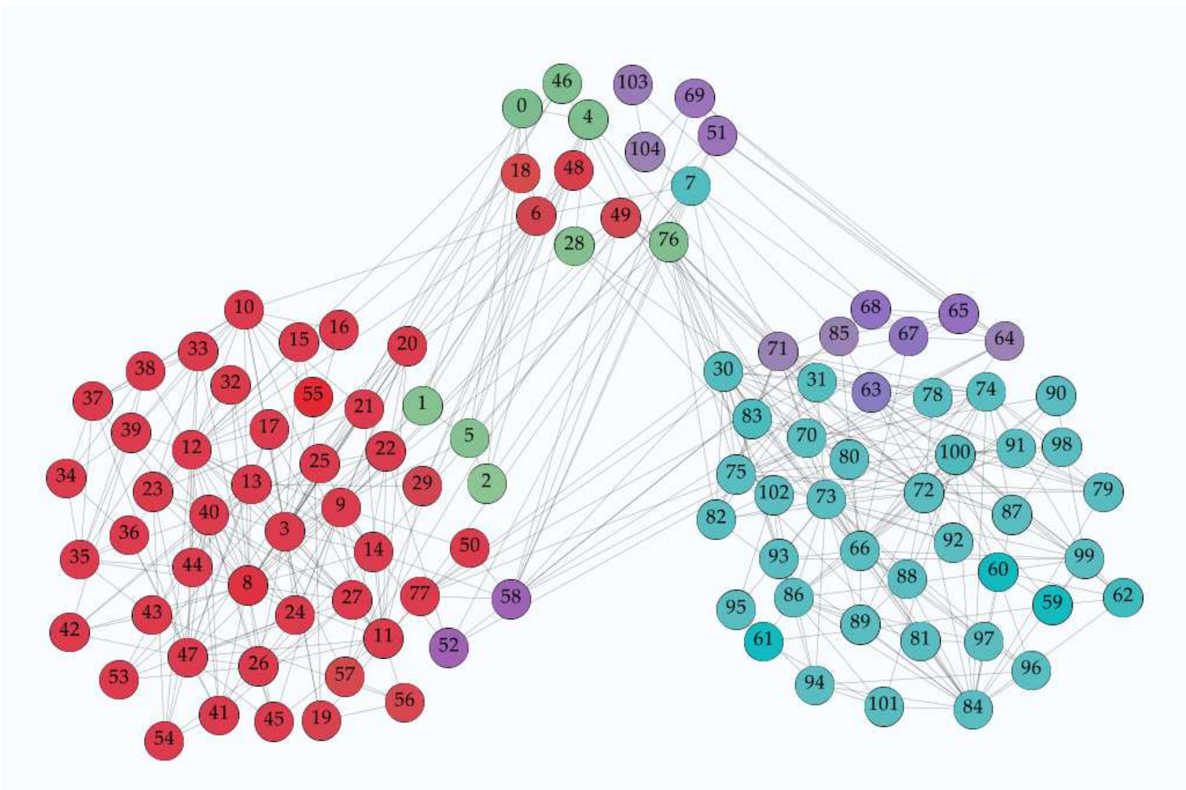
(**a**) NMI=1 (**b**)Q = 0.4696

Fig.6 Real structures found by MOGAME. (a) Karate network with maximum NMI (b) Karate network with maximum modularity

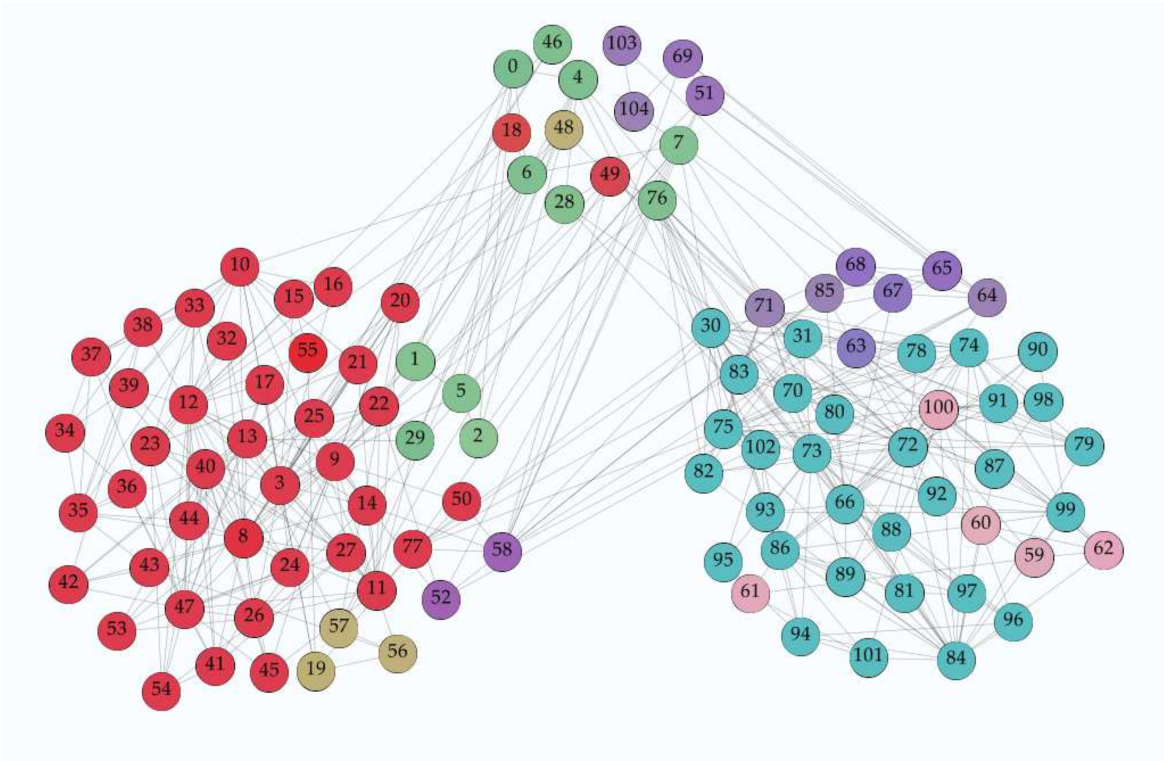


(**a**) NMI=1 (**b**) Q=0.5432

Fig.7 Real structures found by MOGAME. (a) Dolphin network with maximum NMI (b) Dolphin network with maximum modularity Q

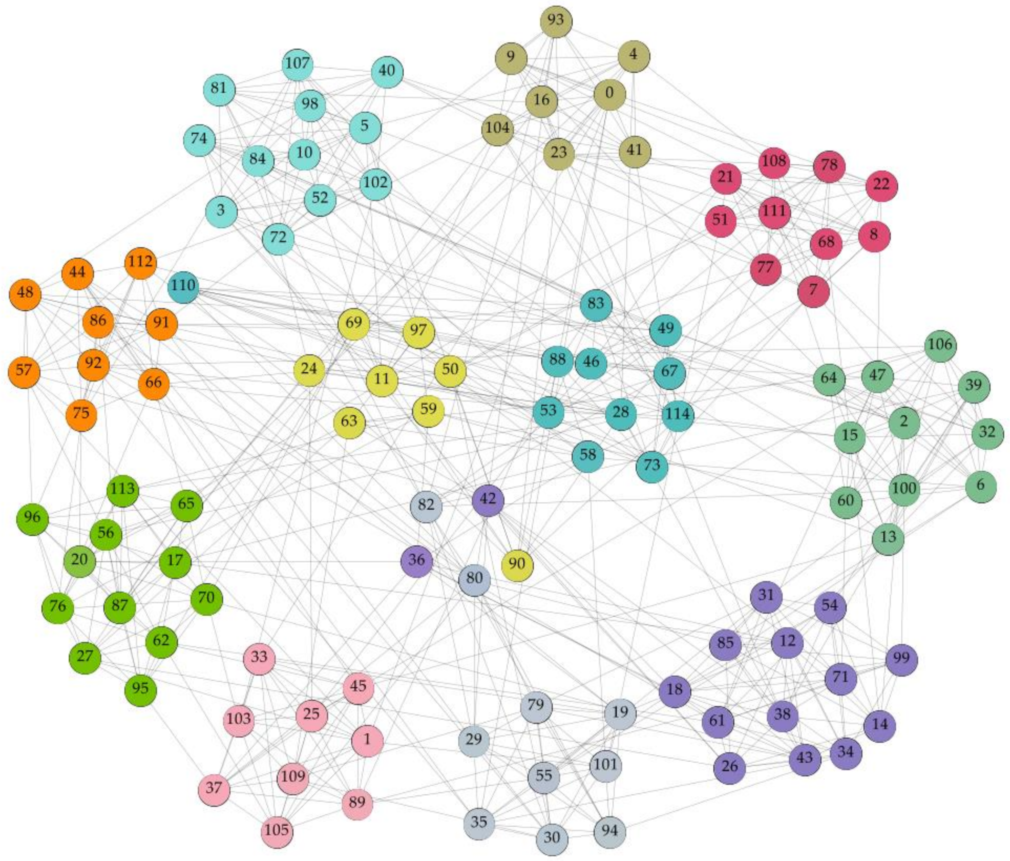


(a) NMI=1

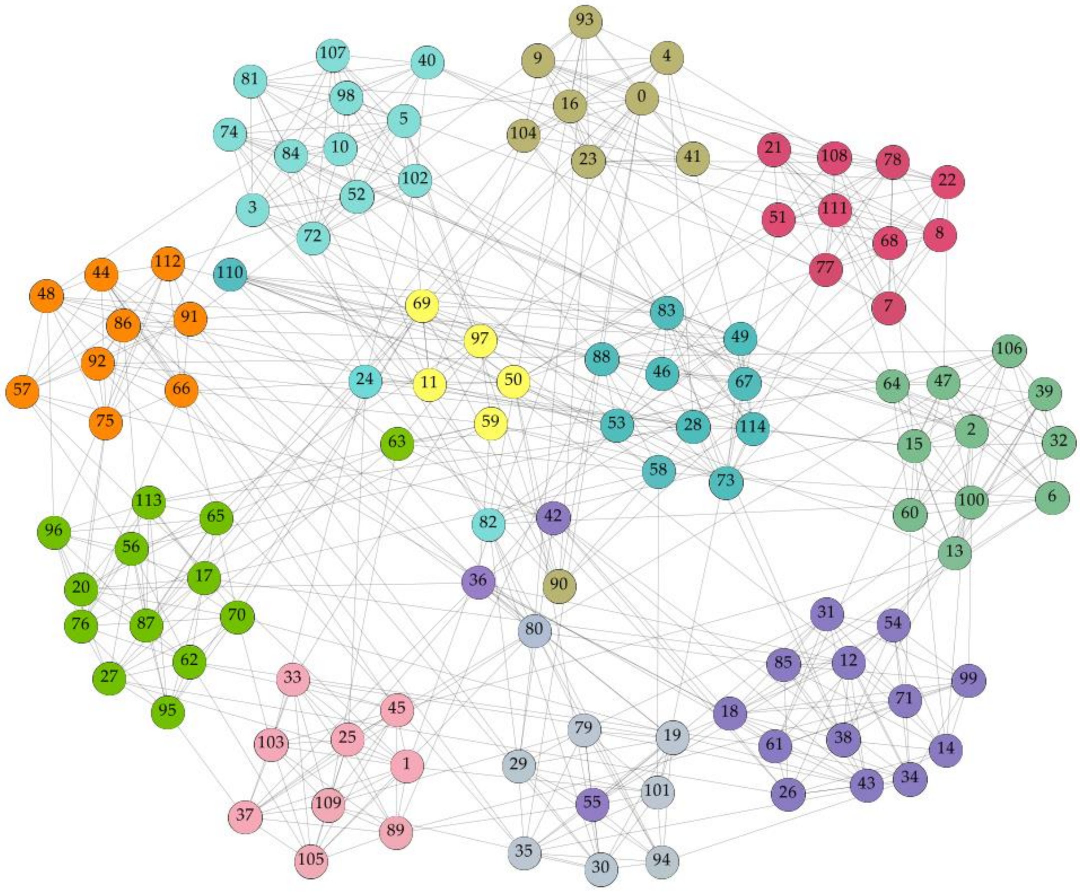


(**b**) Q=0.5235

Fig.8 Real structures found by MOGAME. (a) Krebs' books network with maximum NMI (b) Krebs' books network with maximum modularity Q.



(a) NMI=0.9268



(b) Q=0.6131

Fig.9 Real structures found by MOGAME. (a) Football network with maximum NMI (b) Football network with maximum modularity Q.

5 Conclusions

In this study, a new multi-objective community detection algorithm (MOGAME) has been proposed. In the initialization phase, the population is preprocessed using mixed representation, which combine locus-based representation and labels-based representation. Then, evolutionary operators (selection, crossover, and mutation) based on the mixed representation are applied iteratively to obtain a set of Pareto optimal solutions. We test our algorithm on both synthetic and real-world networks. The experimental results show that the proposed algorithm performs better than or competitive with existing algorithms including GA-NET, TSLPA, MOGA-NET and MOCDACO. In the next work, MOGAME will be apply to detect community in large scale networks and signed social networks. In addition, local search for the objectives be introduced to develop performance of the algorithm.

Compliance with Ethical Standards statements

Ethical approval This paper does not contain any studies with human participants or animals performed by any of the authors.

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Conflict of interest The authors declare that they have no conflict of interest.

Informed Consent The authors declare that they give informed consent of this paper.

Code availability Codes are available and could be shared at any time when readers ask. They can send a mail on the address displayed in the top of the article. Codes are programmed under Python language.

Authorship contributions

All authors contributed to the study conception and design. Experiments, data collection and written were performed by Simin Yang. The conceptualization and methodology were performed by Wenhong Wei. The reviewing and editing were performed by Yuhui Zhang. The all authors read and approved the final manuscript.

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