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A Low Computational Complexity DOA Estimation using Sum/difference Pattern based on DNN

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RESEARCH

A low computational complexity DOA estimation using sum/difference pattern based on DNN

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Abstract

Tracking low-elevation targets over an uneven surface is challenging because of the complicated and volatile multipath signals. Multipath signals cause the amplitude and phase distortion of direct signal, which degrades the performance and generates mismatch between existing classical multipath signal and actual model. Machine learning-based methods are data-driven, they do not rely on prior assumptions about array geometries, and are expected to adapt better to array imperfections. The amplitude comparison Direction-of-Arrival (DOA) algorithm performs a few calculations, has a simple system structure, and is widely used. In this paper, an efficient DOA estimation approach based on Sum/Difference pattern is merged with deep neural network. Fully learn the potential features of the direct signal from the echo signal. In order to integrate more phase features, the covariance matrix is applied to the amplitude comparison algorithm, it can accommodate multiple snapshot signals instead of a single pulse automatically. The outputs of the deep neural network (DNN) are concatenated to reconstruct a covariance matrix for DOA estimation. Moreover, the superiority in computational complexity and generalization of proposed method are proved by simulation experiments compared with state-of-the-art physics-driven and data-driven methods. Field data sets acquired from a VHF array radar are carried out to verify the proposed method satisfies practicability in the severe multipath effect.

Keywords: DOA estimation; array imperfections; deep neural network; sum/difference pattern

1 Introduction

When radar tracks a low-elevation target over a ground or sea surface, the direct echo of the target, accomplished by multipath reflection signals, enters the radar receiver. The multipath reflection signals include the specular reflection signal and the diffuse reflection [1–4]. The multipath and direct waves of the target are within a beamwidth, which leads to distinguish the direct signal from the space, range, and doppler domains hardly. The main reason is that the multipath signal causes the phenomenon of beam split [5], and the ideal plane wave model evolves into a spherical wave model.

For the near-classic problem, many attempts have been made to reduce the multipath error in low-angle tracking, including improved monopulse technology [6,7] and advanced array signal processing [8–13]. In [6,7], the sum and difference signals of monopulse are no longer strictly in-phase or quadrature under multipath conditions. It wanders as the relative amplitudes

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and phases because of the complex reflection. Hence, complex and changeable multipath signals make the traditional amplitude comparison technology ineffective.

For the low-angle tracking problem, there are some super-resolution methods such as digital beamforming (DBF), multiple signal classification (MUSIC) [8–10], and maximum likelihood (ML) [11–13] methods which are advanced array signal processing approaches. The basic thought of the MUSIC is to use the orthogonality of the signal subspace and the noise subspace to estimate the target direction. ML is the most general estimation method for obtaining unknown non-random parameters, which can be regarded as a fit between the array output data and the actual array sampling data.

These physics-driven models establish a forward mapping relationship from the signal direction to the array output, and assume that this mapping is reversible. But when the radar position environment is relatively complex, the altitude of the ground reflector is not the same, the amplitude of the multipath signal received by each array element is not the same, and the phase does not meet the linear change. So the ideal far-field plane-wave model is difficult to describe

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the actual echo signal, and the mismatch of the signal model-driven greatly reduces the performance of the existing super-resolution algorithm.

Recently, deep learning technology is widely used in image, speech, emotion and other fields [14], and has made remarkable achievements in these areas. So many efforts devoted to apply the neural network-based algorithm to attach the issues of signal processing [15–21]. In [15,16], an efficient DOA estimation approach based on the support vector regression demonstrates that the performance is better than the MUSIC algorithm. In [17], a novel framework of end-to-end learning is proposed to solve the DOA estimation problem in Multiple-input-multiple-output (MIMO) communication systems, it can realize high-resolution DOA estimation compared with conventional methods. In [18], a multitask autoencoder (AE) and a series of parallel multilayer classifiers are proposed to realize spatial filtering. This classification method performs excellently in both generalization and array imperfection. In [19,20], the accuracy of DOA estimation is more biased towards the influence of phase error, therefore, a novel phase enhancement method is used to DOA estimation based on supervised DNN learning in complex multipath circumstance. In [21], an efficient deep convolution network (DCN) based spatial spectra recovery algorithm is proposed and applied to electromagnetics (EM) DOA estimation.

In spite of the great achievements of data-driven algorithms above, there is little study to apply Sum/Difference method to deep neural network for DOA estimation. WI It is well known that the computation complexity of amplitude comparison algorithm is very small. Recent advanced phased array radar systems employ digital beamforming in place of the monopulse comparator network to form the analog sum and difference beams [22–24]. DBF allows digital monopulse beamforming, which is independent form the sum beamforming, can be achieved.

Due to the fact that disturbance caused by the multipath signal, the phase of the array element no longer satisfies the linear distribution [25]. We propose the Sum/Difference method merging deep neural network which utilizes error curve to derive DOA estimation inversely. Compared with existing DL-based DOA estimation schemes, this work has taken less run time to realize the same performance and generalization of super-resolution after training. And the covariance matrix with ample phase features of echo signals are used in the amplitude comparison algorithm, which is not limited to a single pulse.

The rest of the paper is organized as follows, Section 2 describes the mathematical formulation of DOA estimation and derives the mathematical formulation of

Sum/Difference amplitude comparison. While Section 3 represents a baseline framework of DNN in detail. In Section 4, the proposed method is compared with physics-driven algorithm in terms of performance and computational complexity. In Section 5, the performance of the proposed method is validated by feild data. Section 6 gives the conclusion of this paper.

Notations: $(\cdot)^{\mathrm{T}}$ denotes the transpose operation, $(\cdot)^{\mathrm{H}}$ denotes Hermitian transpose operation, $\|\cdot\|$ denotes the norm of the vector, $j = \sqrt{-1}$ represents the imaginary unit, tr[·] is the trace of the matrix.

2 Mathematical Formulation

2.1 Signal model

The classical multipath signal model is illustrated in Fig.1. As shown above, the signal received by the array includes direct signal and multipath reflection signals which are specular reflection signals and diffuse reflection signals. θ_d represents the incident angle of direct wave, and θ_i are the incident angle of a multipath reflected waves.

Assume that K narrowband far-field signals $s_k(t), k = 1, \dots, K$ impinge onto an M-element uniform linear array(ULA) from drections $\boldsymbol{\theta} = \{\theta_1, \theta_2, \dots, \theta_K\}$. Then the received data of array is given by

$$\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{a}(\theta_k) s_k(t) + \boldsymbol{n}(t)$$
(1)

where $\boldsymbol{x}(t)$ is the $M \times 1$ vectors

$$\boldsymbol{x}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^{\mathrm{T}}$$
 (2)

where $\boldsymbol{n}(t)$ represents the independent Gaussian white noise with zero mean. $\boldsymbol{a}(\theta_k)$ denotes the steering vector corresponding to θ_k , which is presented as

$$\boldsymbol{a}(\theta) = \left[e^{-j(M-1)\frac{\pi}{\lambda}dsin(\theta)}, e^{-j(M-3)\frac{\pi}{\lambda}dsin(\theta)}, \cdots, e^{j(M-3)\frac{\pi}{\lambda}dsin(\theta)}, e^{j(M-1)\frac{\pi}{\lambda}dsin(\theta)}\right]^{\mathrm{T}} (3)$$

In the matrix notation, the vector of the received signals $\boldsymbol{x}(t)$ can be expressed compactly as

$$\boldsymbol{x}(t) = \boldsymbol{A}(\theta)\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{4}$$

where

$$\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_K)]$$
(5)

and $\boldsymbol{s}(t)$ is the $M \times 1$ vectors of the signals

$$\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]$$
(6)

Then the spatial correlation matrix of $\boldsymbol{x}(t)$ can be expressed as

$$\boldsymbol{R} = \mathbb{E}\left[\boldsymbol{x}(t)\boldsymbol{x}(t)^{\mathrm{H}}\right] = \boldsymbol{A}(\theta)\boldsymbol{S}\boldsymbol{A}(\theta)^{\mathrm{H}} + \sigma^{2}\boldsymbol{I} \qquad (7)$$

with

$$\boldsymbol{R} = \boldsymbol{U}_{S} \boldsymbol{\Sigma}_{S} \boldsymbol{U}_{S}^{\mathrm{H}} + \boldsymbol{U}_{N} \boldsymbol{\Sigma}_{N} \boldsymbol{U}_{N}^{\mathrm{H}}$$

$$\tag{8}$$

Where $\mathbf{S} = \mathbb{E} \left[\mathbf{s}(t) \mathbf{s}^{\mathrm{H}}(t) \right]$ represents the signal covariance matrix and it must be nonnegative definite, σ^2 is the power of the noise. U_S and U_N represent the signal subspace and noise subspace, respectively. When the elevation of a target is smaller than one beamwidth, and there are unknown array imperfections, i.e., gain/pahse errors between different elements in a complicated terrain environment, the disturbed received vector data can be expressed as in [25,26]

$$\boldsymbol{x}(t) = \boldsymbol{\Gamma} \odot \boldsymbol{a}(\theta) \boldsymbol{s}(t) + \boldsymbol{n}(t)$$
(9)

where $\mathbf{\Gamma} = [\tau_1, \tau_2, \cdots, \tau_M]^{\mathrm{T}}$ is the set of disturbance coefficient. Different elements correspond to different perturbation coefficients caused by rugged terrain. These interferences of multipath signals cause deviations to the array responding function $\mathbf{a}(\theta)$, and the matching mapping relation between signal directions and array outputs does not hold any longer.

Hence, the task of DOA estimation in a multipath environment is transformed into the task of DOA estimation with unknown array imperfections. When we obtain the sampled data $\boldsymbol{x}(t)$, the DOA of the direct signal can be estimated by following classic physicsdriven methods, which can be expressed as

$$\hat{\theta}_{DBF} = \arg \max \boldsymbol{a}^{\mathrm{H}}(\theta) \boldsymbol{R} \boldsymbol{a}(\theta)$$
 (10)

$$\hat{\theta}_{MUSIC} = \arg\min \, \boldsymbol{a}^{\mathrm{H}}(\theta) \boldsymbol{U}_{N} \boldsymbol{U}_{N}^{H} \boldsymbol{a}(\theta)$$
(11)

$$\hat{\theta}_{ML} = \arg\max tr[\boldsymbol{P}_{A(\theta)}\boldsymbol{R}] \tag{12}$$

with

$$\boldsymbol{P}_{A(\theta)} = \boldsymbol{A}(\theta) \left[\boldsymbol{A}^{\mathrm{H}}(\theta) \boldsymbol{A}(\theta) \right]^{-1} \boldsymbol{A}^{\mathrm{H}}(\theta)$$
(13)

where $P_{A(\theta)}$, spanned by the columns of $A(\theta)$, denotes the signal projection space.

2.2 Sum/Difference Pattern

As illustrated in Fig.2, in a classical monopulse system, two slightly overlapping beams squinted at a certain angle ϕ are formed to illuminate the targets. In the receiver, the sum and difference beams are synthesized, which is shown in Fig.3. Assume that there is a small off-axis angle θ between a target and the antenna system axis 0. The beam pattern of the sum and difference beams can be represented as

$$G_{\Sigma}(\theta) = f(\theta + \phi) + (f(\theta - \phi))$$
(14)

and

$$G_{\Delta}(\theta) = f(\theta + \phi) - (f(\theta - \phi)) \tag{15}$$

repectively, where $f(\theta)$ is the beam pattern.

The digital weight w_s for sum beamforming in a fully digitized array radar is given as

$$\boldsymbol{w}_s = \frac{\boldsymbol{a}(\phi) + \boldsymbol{a}(-\phi)}{\| \boldsymbol{a}(\phi) + \boldsymbol{a}(-\phi) \|}$$
(16)

In response to the difference beamforming with a monopulse comparator in conventional phased array radars, the equivalent conventional difference taper w_d for DBF can be expressed as

$$\boldsymbol{w}_{d} = \frac{\boldsymbol{a}(\phi) - \boldsymbol{a}(-\phi)}{\| \boldsymbol{a}(\phi) - \boldsymbol{a}(-\phi) \|}$$
(17)

To realize monopulse angle estimation in a phased array radar, the antenna elements should be positioned in a symmetric fashion around the antenna center. The element antenna positons d_m in this arrangement can be expressed as

$$\left\{ d_1, d_2, \cdots, d_{\frac{M}{2}}, d_{\frac{M}{2}+1}, \cdots, d_{M-1}, d_M \right\}$$

$$= \left\{ -d_1, -d_2, \cdots, -d_{\frac{M}{2}}, d_{\frac{M}{2}}, \cdots, d_2, d_1 \right\}$$
(18)

The array factor of such a linear symmetric array is given in (19) and (20) for sum and difference pattern, respectively.

$$F_{\Sigma}(\theta) = \sum_{m=1}^{M} w_s(m) e^{-j\frac{2\pi}{\lambda} d_m sin(\theta)}$$
(19)

$$F_{\Delta}(\theta) = \sum_{m=1}^{M} w_{\rm d}(m) e^{-j\frac{2\pi}{\lambda} d_m \sin(\theta)}$$
(20)

where $w_s(m)$ and $w_d(m)$ represented as the weights for the Sum/Difference patterns which is characterized by an even $(w_s(m) = w_s(-m))$ and odd $(w_d(m) = -w_d(-m))$. Then the classical monopulse ratio of Sum/Difference can be presented as

$$\frac{\Delta}{\Sigma} = \frac{F_{\Delta}(\theta)}{F_{\Sigma}(\theta)} \tag{21}$$

In practical application scenario, the covariance matrix can only be estimated using K snapshots, which contains the phase information of multiple snapshots signals. While the traditional Sum/Difference algorithm only applies to a single pulse. In order to construct the covariance matrix in the amplitude comparison ratio of the Sum/Difference algorithm. Then, a new amplitude comparison ratio can be defined as follows

$$\chi(\theta) = \left(\frac{\Delta}{\Sigma}\right)^2 = \frac{F_{\Delta}(\theta)^{\mathrm{H}} F_{\Delta}(\theta)}{F_{\Sigma}(\theta)^{\mathrm{H}} F_{\Sigma}(\theta)} = \frac{\boldsymbol{w}_{\mathrm{d}}^{\mathrm{H}} \boldsymbol{R} \boldsymbol{w}_{\mathrm{d}}}{\boldsymbol{w}_{\mathrm{s}}^{\mathrm{H}} \boldsymbol{R} \boldsymbol{w}_{\mathrm{s}}} \quad (22)$$

The amplitude comparison ratio $\chi(\theta)$ is integrated with covariance matrix, which the received data of array not only in monopulse but also exist in multiple snapshots. Generally, the network learning model extract features fully from covariance matrix to construct nonlinear mapping. In this way, we break the limitation of monopulse tracking with classical Sum/Difference algorithm, so that it can be applied to multiple snapshot signals, and can be fused to neural networks to extract more abstract features of signal direction.

A 24-element uniform linear array with half-wavelength element spacing and the $\lambda = 1$ m. The beamwidth of the sum pattern is $\theta_{3dB} = 4.23^{\circ}$. The beam pattern are shown in Fig.4, and the sum pattern and difference pattern are presented in Fig.5. The error curves response to amplitude comparison ratio is shown in Fig.6. This is approximately a curved opening upward parabola instead of the oddly symmetric classical error curve generated by the conventional Sum/Difference algorithm.

3 Proposed method

The neural network can fit nonlinear object with a high accuracy, it has robust nonlinear mapping, and it is easy to implement. It is an effective method to optimize the traditional Sum/Difference algorithm by using the neural network method. Fig.7 shows a representative structure of DNN, which learns the non-linear mapping between the input and output. $I = [I_1, I_2, \cdots, I_{M(M-1)/2}]^T$ represents the input vector, $O = [O_1, O_2, \cdots, O_{M(M-1)/2}]^T$ represents the output vector of the output layer. The number of input and output neurons depends on array elements M.

3.1 Array data preprocessing

In order to reduce the variability of the DNN input, which is influenced significantly by uncertain signal waveforms.

Therefore, we compute the array covariance matrix \boldsymbol{R} and reformulate the off-diagonal upper-right matrix elements of real and imaginary components as an input vector to the DNN, i.e.

$$\bar{z} = [R_{1,2}, R_{1,3}, \cdots, R_{1,M}, R_{2,3}, \cdots, R_{1,M}, \cdots, R_{M-1,M}]$$
(23)

where $\mathbf{R}_{i,j}$ represents the (i,j)th element of \mathbf{R} . Then the concatenated real/imaginary parts of \bar{z} is finally obtained according to

$$\hat{\boldsymbol{z}} = [\operatorname{Real}(\bar{\boldsymbol{z}}), \operatorname{Imag}(\bar{\boldsymbol{z}})]^{\mathrm{T}}$$
 (24)

where $\text{Real}(\cdot)$ and $\text{Imag}(\cdot)$ represent the real and image part of a complex-valued entity, repectively.

In general, data preprocessing has a huge impact on feature engineering. Similarly, training and test data also need to be preprocessed before being sent to the network for training or testing, which alleviates the problem of the vanishing gradient or the exploding gradient. Hence, Z-score normalization is a means to normalize the features of each dimension to the same value interval and eliminate the correlation between different features, that can be formulated as follows

$$\boldsymbol{z} = \frac{\hat{\boldsymbol{z}} - \boldsymbol{\mu}}{\sigma} \tag{25}$$

where (μ, σ) represent the statistical mean and standard deviation of \hat{z} .

3.2 DNN-based learning

The neural network is composed of three fully connected hidden layers, each with neuron numbers of 1024. The linear unit is utilized for the output layer, while the rectified linear unit (ReLU) is employed in the hidden layers to make sure a non-linear process. For an input z, the output of the p^{th} dense layer can be described as

$$\boldsymbol{o}^{p} = \begin{cases} \operatorname{ReLU}(\boldsymbol{W}^{p} \times \boldsymbol{o}^{(p-1)} + \boldsymbol{b}^{p}, & p = 1, 2, 3\\ \boldsymbol{W}^{p} \times \boldsymbol{o}^{(p-1)} + \boldsymbol{b}^{p}, & p = 4 \end{cases}$$
(26)

with

$$\operatorname{ReLU}(x) = \begin{cases} x & , x \ge 0\\ 0 & , x < 0 \end{cases}$$
(27)

where \boldsymbol{W}^p and \boldsymbol{b}^p represent the weight matrix and bias vector corresponding to the p^{th} layer.

After the network architecture is built, all parameters of the network should be adjusted and made fast convergence. All elements of the weight matrices and bias vectors are initialized randomly with every layer of neurons. Training the DNN and learning hidden features are aiming to minimize the difference between the output and the desired value.

The purpose of the training phase info contained in \mathbf{R} is to find the corresponding parameters which minimize the sum of the difference between the output and the desired value. The weights of network adjusted by using the least mean square error(MSE) made fast convergence of the algorithm to obtain better network parameters, i.e.

$$(\boldsymbol{W}, \boldsymbol{b}) = \arg\min \ loss \tag{28}$$

where

$$loss = \frac{1}{B} \sum_{b=1}^{B} \|\boldsymbol{z} - \boldsymbol{z}'\|^2$$
(29)

where B represents batch size. z' denotes the expected values of z. After we define the loss function, the adaptive Moment Estimate (Adam) strategy is adopted to calculate network parameters that affect training process and the output.

Back-propagation is imposed in the training phase to minimize loss and tune the parameters of the DNN. And it is commonly adopted via gradient descent optimization algorithm to adjust the weight of neurons by calculating the gradient of the loss function

$$\alpha_{new} = \alpha_{old} - \eta \frac{\partial loss}{\partial \alpha} \tag{30}$$

where α represents a specific tunable parameter and η is a self-defined positive learning rate smaller than 1.

3.3 DOA estimation

In this subsection, the scheme of proposed method is shown in Fig.8. The whole learning framework is divided into training stage and testing stage, in which the training part completes the extraction of direct signal phase features and eliminates the phase features of multipath signals. The testing stage decodes the extracted features, and finally reconstructs the covariance matrix to achieve DOA estimation by amplitude comparison ratio method. The steps are summarised as follows

Step 1) The covariance matrix \boldsymbol{R} and the amplitude comparison ratio $\boldsymbol{\chi}$ are designed by radar parameters according to mathematical formulation in Section 2.

- Step 2) The off-diagonal upper-right real and imaginary components of covariance matrix \boldsymbol{R} of training and testing data are transformed into a vector $\hat{\boldsymbol{z}}$, and after Z-score normalization is \boldsymbol{z} .
- Step 3) The z of training are fed to the DNN to learn the potential features, the network parameters are adjusted inversely by minimizing the loss function. Finally, the optimal network parameters (W, b) are obtained.
- Step 4) The z of testing are fed to the DNN with optimal parameters to extract the features.
- Step 5) The outputs of DNN are concatenated to reconstruct the covariance matrix, combined with χ for DOA estimation.

4 Results and Discussion

In this section, we describe the implementation of the proposed Sum/Difference algorithm for DOA estimation based on DNN learning. All the experiments were conducted on a computer with Intel i7-10700 CPU 2.90 GHz. The generation and preprocessing of training data and testing data are run in Matlab2020a, and construct the DNN network and complete the training by Python3.7.

4.1 DOA estimation results

In this subsection, we compare the performance and computation complexity of proposed method with DBF, ML, MUSIC and physics-driven Sum/Difference algorithm.

We consider a 24-element uniform linear array (ULA) with half-wavelength inter-element spacing, the $\lambda = 1$ m. The 3dB beamwidth is about 4.2°. As is wellknown, low-angle measurement is the tricky problem, considering the directions of signals impinging from the spatial scope $[0.1^\circ : 0.1^\circ : 3^\circ]$, which the range of angle is small than 3dB beamwidth. The SNR of the training data and test data is both 0dB. And the number of snapshots is 20. The amplitude error and phase error are $\pm 25\%$. Computer simulation generates 90000 batches which are fed to the neural network for training. At the same time, we randomly generate 3000 samples for test verification. We randomly sampled from 0° to 3° . Owing to the existence of interference from amplitude-phase error, the instability of the traditional algorithm is visible.

As shown in Fig.9 and Fig.10, we give 100 results of DOA estimation. Fig.9 shows the estimated results of DBF, ML, MUSIC and Sum/Difference algorithm based on physics-driven model, respectively. Fig.10 gives the corresponding the DOA estimation error.

The DOA estimation and estimation error after DNN training is shown in Fig.11 and Fig.12. It is observed that fluctuations of DOA estimation are unavoidable based physics-driven model. The proposed

method is compared with classical amplitude comparison method, improving accuracy of measurement to a large extent, especially in unpredictable scenarios. And Sum/Difference algorithm based deep neural networks can achieve the same effect as the super-resolution algorithm. Moreover, after DNN training, the performance of DBF, ML, MUSIC and proposed model are about the same, the error is only about 0.119°.

To assess the computation complexity of the proposed Sum/Difference with DNN, we count the rum time of different methods in Table 1. But the table denotes the proposed model has tiny run time than others, executing 3000 samples only takes 0.116s is nearly a hundredfold faster. These results verified the classical algorithms fail to obtain valid DOA estimation when there are perturbations in echo signals, the proposed method shows superiority in computation complexity and satisfactory recovery performance to defects.

4.2 RMSE versus SNR

In order to verify the performance of the proposed method, we investigate the DOA estimation accuracy with SNR. The SNR of signals fed to DNN is varied from -5dB to 5dB with a step of 1dB. To demonstrate the outstanding performance generalization of the proposed method to SNR.

we consider two scenarios about SNR, in scenario 1, the SNR of test data and training data is kept in sync. The amplitude and phase error are $\pm 25\%$. Due to the amount of data, the angle range we consider is between 1° and 2°. As shown in Fig.13, when the SNR is 0dB, the angular measurement accuracy of Sum/Difference decreases from 0.14° to 0.04° compared with after DNN. Hence, we can observe that the proposed method show more superior DOA estimation accuaracy over conventional Sum/Difference algorithm, and our proposed methed repeat the same precision of DBF, ML, and MUSIC algorithm after training. But the crucial advantage is that the computation complexity is greatly improved.

In actual situations, the SNR parameters of the test data samples do not completely match the samples of the training data. That is why in scenario 2, we adjust the SNR range of the test data to make the SNR mismatched between the training data and test data. The simulations are the same as scenario 1, only the SNR range of test data is changed from -6.5dB to 6.5dB with a step of 1dB. Fully ensure that the SNR of the training sample and the test sample are completely mismatched, which brings about the SNR parameters always maintain a difference of 0.5dB.

As is shown in Fig.14, the root mean square error (RMSE) of the DOA estimation of the proposed method compared with the physics-driven DBF, ML

and MUSIC algorithm. It can be seen that compared to the existing physics-drive algorithm, the proposed method improves the classic Sum/Difference algorithm, which can achieve the best DOA estimation effect, and reach the performance of the trained super-resolution algorithm. And the time is greatly discounted, which can benefit the computer's computing speed.

4.3 Complexity Analysis

In this subsection, the complexity of the proposed method is investigated. We compute the complexity in steps as show in Table 2. Specifically, for the comparison, we evaluate the complexity for each approach as shown in Table 3. The DBF algorithm needs about $O\{QM^2 + QM\}$. For MUSIC algorithm, it involves covariance matrix, eigenvalue decomposition (EVD) operator, and the orthogonality function [8, 9] between the signal subspace and the noise subspace (OFSN), the complexity of MU-SIC is about $O\left\{M^3 + 2QM^2 - QM\right\}$, where Q is the number of interested interval of airspace. The traditional ML applies the covariance matrix and projection matrix, so the ML method needs about $O\left\{QM^3 + 2QM^2 + 8QM\right\}$. The proposed method employs the weight vectors of sum/difference pattern, hence, it has lower complexity i.e., $O\{2LM\}$. Hence, the proposed method achieves DOA estimation performance at the lower computational efficiency.

5 Real data validation

In this section, the performance of the proposed method is verified by the feild data under the background of complex terrain position, which located hills, rivers and trees. In practice, the data received by the radar is processed offline, all tracks are correlated in azimuth, distance, time, and altitude, then parse the track corrected heading data for DOA estimation. The geographical environment of climate, terrain undulations and different azimuth are the key factors that affect the excellent characteristics of neural network learning.

As depicted in Fig.15 and Fig.17, we selected $(60^{\circ}-90^{\circ})$ data from the same sector for training, and tried to avoid the environment and other nonnegligible factors that seriously affect the training process. There are a total of 161 routes in this sector, we randomly have selected 2 routes as test data, and all the rest are used for training. The red traces are training data, the blue traces are test data.

The first test track flew radially from east to west, the distance slowly changed from 161km to 72km, and the elevation angle gradually increased from 2.2° to

 5.9° . Figure 16 gives the DOA estimation errors, comparing the results of the proposed method with classical Sum/Difference, DBF, ML and MUSIC, the classical algorithm is more sensitive to the complexity of terrain. In Table 4, the results of Track 1 DOA estimation in terms of RMSE are calculated, The angle measurement accuracy of the Sum/Difference method decreases from 0.637° to 0.151° .

The second track has a total of 80 traces, the target flew from 161km to 62km, and the elevation angle ranges from 1.32° to 6.9° . Fig,18 depicts the estimation error of Track 2. You can come out the uncertainty of classical algorithms for fluctuations, while the proposed algorithm is more stable. The RMSE of the proposed algorithm decreases from 1.370° to 0.238° in Table 4.

The experimental results proved that the proposed algorithm not only improves the measurement accuracy of the traditional Sum/Difference algorithms, but also the angle measurement can reach the performance of the super-resolution algorithm after training, but the computational complexity is far less than that of the super-resolution algorithm. Therefore, in the actual position, we can consider the Sum/Difference algorithm after DNN training to improve the computational complexity.

6 Conclusion

In this paper, a novel Sum/Difference model for DOA estimation using supervised DNN is proposed to address DOA estimation problem of amplitude phase imperfections on radar. The proposed method is free from ineluctable imperfections of traditional Sum/Difference algorithm, and obtains DOA estimation with higher precisions than the most widely studied parametric method of DBF, ML and MUSIC. Besides, through DNN learning network, Sum/Difference algorithm reduces the time complexity compared to the super-resolution algorithm. The results of computer simulation and comparative analysis of field data show the validity of the proposed method, which is superior to classic physics-driven methods in estimation accuracy and state-of-the art data-driven methods of computational complexity.

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Abbreviations

DOA:Direction of arrival; DNN:Deep neural network;SNR: Signal-to-noise ratio; DBF:Digital beamforming; MUSIC:Multiple signal classification; ML:Maximum likelihood; MIMO:Multiple-input-multiple-output; DCN:deep convolution network; EM:electromagnetics; Adam:adaptive Moment Estimate; EVD:Eigenvalue decomposition; OFSN:Orthogonality function between the signal subspace and the noise subspace;

Availability of data and materials

Not applicable.

Ethics approval and consent to participate

Not applicable.

Competing interests

The authors declare that they have no competing interests.

Consent for publication

Not applicable.

Authors' contributions

Saiqin Xu found the idea and wrote this article, Baixiao Chen and Houhong Xiang put forward some constructive suggestions for revision.

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Figures

Figure 1 Classical signal model

Figure 2 Two beam patterns of monopulse system

Figure 3 Sum and difference beam patterns of monopulse system

Figure 4 Beam pattern

Figure 5 Sum/Difference pattern

Figure 6 Error curve of Sum/Difference pattern

Figure 7 Architecture of DNN

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Figure 15 Track 1

Figure 16 DOA estimation errors of Track 1

Figure 17 Track 2

Figure 18 DOA estimation errors of Track 2

 Table 1 Computation Complexity

Approach	Run Time(s)
DBF	5.187
ML	22.547
SSMUSIC	8.936
Sum/Difference	0.113
Trained DBF	5.260
Trained ML	22.569
Trained SSMUSIC	9.012
Trained Sum/Difference	0.116

Table 2 Computation Complexity

Steps	Complexity
Perform EVD	$O\left\{M^3\right\}$
Execute OFSN	$O\left\{2\dot{M}^{2}-M\right\}$
Construct $oldsymbol{P}$	$O\left\{2M^2 + 8M\right\}$
Calculate PR	$O\{M^3\}$

 $\label{eq:table 3} Table \ 3 \ Computation \ Complexity$

Approach	Complexity
DBF	$O\left\{2QM^2\right\}$
ML	$O\left\{QM^{3}+2QM^{2}+8QM\right\}$
MUSIC	$\hat{O}\left\{M^{3}+2QM^{2}-QM\right\}$
Sum/Difference	$O\left\{2LM\right\}$

Table 4 RMSE of DOA estimation

Approach	Track 1	Track 2
Approach	RMSE(°)	RMSE(°)
DBF	0.451	0.522
ML	0.276	0.429
MUSIC	0.312	0.460
Sum/Difference	0.637	1.370
Trained DBF	0.146	0.241
Trained ML	0.152	0.238
Trained MUSIC	0.152	0.237
Trained Sum/Difference	0.151	0.238

Figures



Figure 1

Classical signal model





Two beam patterns of monopulse system





Sum and difference beam patterns of monopulse system





Beam pattern





Sum/Difference pattern





Error curve of Sum/Difference pattern



Figure 7

Architecture of DNN



Figure 8

Framework of the proposed method



Figure 9

DOA estimates before DNN





DOA estimation errors before DNN



Figure 11

DOA estimates after DNN





DOA estimation errors after DNN





RMSE of estimation versus matched SNR





RMSE of estimation versus mismatched SNR



Figure 15

Track 1



Figure 16

DOA estimation errors of Track 1



Figure 17

Track 2



Figure 18

DOA estimation errors of Track 2