Order-dependent sampling control of uncertain fractional-order neural networks system

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Research Article

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Posted Date: October 31st, 2022

DOI: https://doi.org/10.21203/rs.3.rs-2200961/v1

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Abstract. The asymptotic stability of the fractional-order neural networks system with uncertainty by sampled-data controller is addressed in the article. First, considering the influence of uncertainty and fractional-order on the system, a new sampled-data controller is designed with alterable sampling period. In the light of the input delay approach, the fractional system is simulated by the delay system. The main purpose of the method presented is to design a sampled-data controller, which the closed-loop fractional-order system can guarantee the asymptotic stability. Then, the fractional-order Razumishin theorem and linear matrix inequalities (LMIs) are utilized to derive the stable conditions. A stability conditions are presented in the form of LMIs on the new delay-dependent and order-dependent. Furthermore, the sampling controller can be acquired to promise the stability and stabilization for fractional-order system. A numerical example is gotten to demonstrate the effectiveness and advantages for the provided method.

Keywords: Fractional-order systems, neural networks, sampled-data control, Lyapunov function, stability.

1. Problem statement and preliminary

Neural network (NNs) is a classic complicated system made of the interconnected neurons, which has been focused by many scholars. An artificial NNs is made up of artificial neurons or nodes in the modern science. The NNs is a network system consisting of interlinked nodes, which simulates the function of neurons in the brain. It is widely applied to help people solve various matter, for instance, image processing, pattern recognition, convex optimization, robot manipulator in [1]-[6], etc. It should be noted that these adhibitions have an significant relationship with their interior dynamics and the dynamical behavior of NNs, such as stability, multi-stability, synchronization, etc. Nowdays, NNs have attracted numerous attention and become a related research field.

With the development of society, fractional calculus is introduced to replace integer order with some non-integer order. In recent years, the calculus and the control systems of fractional-order (FO) systems have attracted much attention from researchers in control theory. It is worth mentioning that the order of FO calculus is not integer order, which can overcome the disadvantage that the differential equation model of integer order can not accurately describe the complex system. And fractional derivatives can better describe the dynamic behavior of neurons. Fractional derivatives have non-local characteristics, and FO has more accurate memory and genetic properties compared with integer order. Although fractional calculus has not yet emerged, it has recently attracted the attention of researchers, and it remains a great field of study. So far, some excellent results have been presented on the dynamic analysis of FO systems in [7]-[9]. Many important and interesting conclusions have been acquired in fractional-order neural networks (FONNs) systems and all kinds of issues have been investigated by many authors in [10]-[17]. From these results, it can be proved that the FONNs can more correctly depict and imitate the neurons than the traditional integer neural networks in the human brain.

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Because fractional calculus has the advantages of infinite memory and genetic characteristics when representing system models in biological engineering, neural network, fluid dynamics and other fields, it has received more and more concern. It is well known that the long memory properties have been ignored in NNs. Additionally, previous studies have indicated that electroconductibilities of biological cell membranes are FO [32]. Therefore, it is extraordinarily proper and accurate to use FO differential equations to simulate and study the dynamics of real NNs.

It is well known that stability is the most important issue in control systems, and it is the first condition for the system to work properly. For FO systems, numerous interesting conclusions have been obtained in the literature of proving system stability by Lyapunov function, containing asymptotic stability [20], consistent stability [21], stability[22], Hopf-Bifurcation research [23, 24], Mittage-Leffler stability [18, 19]. At present, the research has attracted more and more researchers' attention on the robustness and performance for FO systems, specially for FONN systems with uncertainties. For instance, the delay-dependent stability and stabilization for a kind of FONNs with uncertainty and time-varying delays are discussed in [10]. The asymptotic stability condition for the FONNs is derived by the Lyapunov method, a neural network property and the FO Razumikhin theorem. In [12], sufficient conditions that ensure the stability of a estimated fractional uncertain neural network error system are established by the fractional Lyapunov method.

From the engineering point of view, it is necessary to scheme a controller to make the closed-loop system stable. The controller is used to stabilize the system in [7]-[10] and [12]-[17], and the impulsive control is used to stabilize the system in [11]. However, although the above controllers effectively make the system stable, the system receives data in real time, which may increase the burden of the network channel. As high-speed computing technology develops, the sampled data controller [25] has been widely used in industry due to its low price, high reliability, convenient preservation. Compared with the state feedback control, this method greatly improves the bandwidth utilization and reduces the pressure of receiving information for system. But because the control input is invariant in the following sampling time interval, it is not easy to acquire the stability criteria for the control system. So researcher have put forward numerous ways to research it, and the input delay method [26] has been a well-prevalent way for the past few years. Through the input delay method, the system can be considered as a sequential system with time-varying delay produced by a zero-order holder (ZOH). Afterwards the stability conditions are established by Lyapunov method in LMIs.

Determining the sampling period is an important issue in analyzing the stability of FONNs system using sampled data controller. We note that a larger sampling period brings advantages, for instance, fewer informational communication, less controller drivers, and fewer channel occupancy. Then, a large sampling period is obtained by using some methods to make the FONNs system stable. In most cases, because the characteristics of the plant are hard to judge precisely, the internal parameters of the system are unpredictable, which can change with the variations of the outside situation. Consequently, FONNs system has parameter uncertainty and interference. And the stability is easily destroyed for FONNs system with parameter uncertainty and interference. Although much effort has been made in the existing literature, The reaserch is rarely studied at present about sampled data controller for FONNs systems with uncertainties , which is also the purpose of this paper

In this paper, some new stability criteria are proposed for FONNs system with uncertainties under the sampled data controller. The sampling periods are changeable and smaller than a known maximal admissible upper bound. The order-dependent criteria with lower conservativeness are gained by a set of LMIs. Compared with the now available studies, the main contributions of this study are as follows:

i) In practical application, the influence of parameter uncertainties always exist. The sampled-data control with variable period is considered in the FONNs. Different from the previous methods, by using the input delay method, the sampled-data controller can be used to assure that the uncertain FONNs is asymptotically stable.

ii) Order-dependent and delay-dependent stability criteria are developed for uncertain FONNs by Razumikhin theorem and LMIs. Moreover, in order to stabilize the FONNs system, a design method of the sampled-data control is presented.

iii) The sampling intervals generated by ZOH are time-varying stochastically for FONNs. The alterable sampling peculiarity can be displayed in the simulation example.

2. Problem statement and preliminary

Consider the following n-dimensional FONNS system with parameter uncertainties described by

$$D^{\gamma}x(t) = -\widetilde{A}x(t) + \widetilde{B}f(x(t)) + u(t), \tag{1}$$

where $x(t) = (x_1(t), \dots, x_n(t))^T \in R^n$ represents the state vector; $f(x(t)) = (f_1(x(t)), ..., f_n(x(t)))^T \in R^n$ represents the activation function; $u(t) = (u_1(t), ..., u_n(t))^T$ denotes control input; $\gamma \in (0, 1)$; $\widetilde{A} = A + \Delta A(t)$; $\widetilde{B} = B + \Delta B(t)$; $A = diag\{a_1, a_2, ..., a_n\}$ is the positive definite diagonal matrix; $B \in R^{n\times n}$ is the constant matrix; $\Delta A(t)$, $\Delta B(t)$ are time-varying matrices with compatible dimensions satisfying:

$$[\Delta A(t) \quad \Delta B(t)] = ME(t)[N_1 \quad N_2], \tag{2}$$

where M, N_1 and N_2 are given constant matrices; unidentified matrix function $E(t)$ is fulfilled as follows:

$$E^T(t)E(t) \leq I. \tag{3}$$

In actual applications, some hardware communication is transmitted by digital signals with the popularity of digital signals. In order to make full use of networking technique, a sampled data controller is designed for the stability of FONNs (1). Obviously, the sampled-data controller is given as follows:

$$u(t) = Kx(t_k), \tag{4}$$

where K is controller gain matrix.
In this paper, the control message is generated by ZOH function at the sampling instant $0 = t_0 \leq t_1 \leq \cdots \leq \lim_{k\to+\infty} t_k = +\infty$. In addition, the sampling period is changeable and nonperiodic. In other words, the sampling period is described as follows:

$$0 < t_{k+1} - t_k = d_k \leq d, \quad \forall k \geq 0 \tag{5}$$

where $d > 0$ represents the upper bound of sampling periods. And define $d(t) = t - t_k, t \in [t_k, t_{k+1}]$. Then, the state feedback controller is rewritten as:

$$u(t) = Kx(t - d(t)),\ t_k \leq t < t_{k+1}. \tag{6}$$

Through the controller (6), the FONNs system (1) is written by

$$D^{\gamma}x(t) = -\widetilde{A}x(t) + \widetilde{B}f(x(t)) + Kx(t - d(t)), t > 0, \tag{7}$$

Here, it is supposed that activation function $f_i(.)$ meets the following assumption.
Assumption 1. [31] For any $a, b \in R$, $a \neq b$, we have

$$l_i^- < \frac{f_i(b) - f_i(a)}{b - a} < l_i^+, \tag{8}$$

where l_i^-, l_i^+ are known constants.

Under the sampled-data controller, the stability is guaranteed for FONNs system (7). To this aim, the following definition and lemmas are presented firstly.

Definition 1. [27]. The Caputo derivative of FO α of function $x(t)$ is described as follows:

$$D^{\alpha}x(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(t-\tau)^{n-\alpha-1}x^{(n)}(\tau)d\tau, \tag{9}$$

where $n-1<\alpha<n\in Z^{+}$. $\Gamma(\cdot)$ is the Gamma function $\Gamma(s)=\int_0^{\infty}t^{s-1}e^{-\mathrm{t}}dt$.

Lemma 1. [28]. Let $x(t)\in R^{n}$ be a differentiable vector value function. Then, for any time instant $t\geq t_0$, we have

$$D^{\alpha}(x^{T}(t)Px(t))\leq(x^{T}(t)P)D^{\alpha}x(t)+(D^{\alpha}x(t))^{T}Px(t),$$

Lemma 2. [29]. For given matrices $Q=Q^{T},E,H$ and $F(t)$ with compatible dimensions, $R=R^{T}>0$ of appropriate dimensions, the following inequality holds:

$$Q+HF(t)E+E^{T}F^{T}(t)H^{T}<0,$$

Then, for all $F(t)$ satisfying $F^{T}(t)F(t)\leq I$, if and only if there exist a scalar $\lambda>0$ such that

$$Q+\lambda^{-1}HH^{T}+\lambda E^{T}E<0,$$

or, equivalentiy,

$$\begin{bmatrix} Q & \lambda H & E^{T} \\ * & -\lambda I & 0 \\ * & * & -\lambda I \end{bmatrix}<0.$$

Lemma 3. [30]. Assume that there exist three positive constants $\alpha_1,\alpha_2,\alpha_3$ and a quadratic Lyapunov function $V(.)$: $R^{+}\times R^{n}\rightarrow R^{+}$ such that
(i) $\alpha_1\parallel x(t)\|^2\leq V(t,x(t))\leq\alpha_2\parallel x(t)\|^2$,
(ii) $D_t^{\alpha}V(t,x(t))\leq-\alpha_3\parallel x(t)\|^2$,

whenever $V(t+\theta,x(t+\theta))<\rho V(t,x(t)),\forall\theta\in[-h,0],t\geq 0$, for some $\rho>1$, then under the zero initial condition, FO system $D_t^{\alpha}x(t)=f(t,x_t),\alpha\in(0,1)$ is asymptotically stable.

3. Main results

In this section, the stability is discussed for the FONNs system (7) by the sampled-date controller (6). The delay-dependent and order-dependent stability criteria in the form of LMIs for system (7) are proposed, which are given in the next theorem.

Theorem 1. Given scalars d, $\gamma\in(0,1]$ and matrix K, if there exist matrices $P>0$, $X>0$, $Z>0$, $W_1>0$ and any suitable matrix Y, a scalar $\lambda_1>0$ such that the next LMIs hold:

$$\psi=\begin{bmatrix} \psi_{11}+P & \psi_{12} & \psi_{13} & \psi_{14} & \psi_{15} & \lambda_1 N_2^{T} \\ * & -W_1 & 0 & \psi_{24} & 0 & -\lambda_1 N_1^{T} \\ * & * & -P & \psi_{34} & 0 & 0 \\ * & * & * & \psi_{44} & \psi_{45} & 0 \\ * & * & * & * & -\lambda_1 I & 0 \\ * & * & * & * & * & -\lambda_1 I \end{bmatrix}<0, \tag{10}$$

$$\Pi=\begin{bmatrix} X & Y \\ Y^{T} & Z \end{bmatrix}\geq 0, \tag{11}$$

where
$\psi_{11} = -PA - AP + LW_1L + d^\gamma\gamma^{-1}(X - YA - AY^T)$,
$\psi_{12} = PB + d^\gamma\gamma^{-1}YB$,
$\psi_{13} = PK + d^\gamma\gamma^{-1}YK$,
$\psi_{14} = -d^\gamma\gamma^{-1}A^TZ$,
$\psi_{15} = -(P + d^\gamma\gamma^{-1}Y)M$,
$\psi_{24} = d^\gamma\gamma^{-1}B^TZ$,
$\psi_{34} = d^\gamma\gamma^{-1}K^TZ$,
$\psi_{44} = -d^\gamma\gamma^{-1}Z$,
$\psi_{45} = d^\gamma\gamma^{-1}ZM$.
Then, the closed-loop system (7) is asymptotically stable.
Proof: Building the Lyapunov function candidate $V(x(t)) = x^T(t)Px(t)$ and computing the derivative of $V(x(t))$ for system (7) yield in light of Lemma 1

$$\begin{aligned} D^\gamma V(x(t)) &\leq x^T(t)PD^\gamma x(t) + (D^\gamma x(t))^T Px(t) \\ &= x^T(t)(-P\widetilde{A} - \widetilde{A}P)x(t) + 2x^T(t)P\widetilde{B}f(x(t)) + 2x^T(t)PKx(t - d(t)). \end{aligned} \tag{12}$$

There exist any real matrices $X = X^T > 0, Y$ and $Z = Z^T > 0$, satisfying (11). Then, the following result holds,

$$d^\gamma\gamma^{-1}\upsilon^T(t)\Pi\upsilon(t) - \int_{t-d(t)}^{t}(t-s)^{\gamma-1}\upsilon^T(t)\Pi\upsilon(t)ds \geq 0, \tag{13}$$

where $\upsilon(t) = \left[x^T(t), (D^\gamma x(t))^T\right]^T$.
Under the assumption 1, the following inequality holds for any diagonal matrix $W_1 > 0$,

$$x^T(t)LW_1Lx(t) - f^T(x(t))W_1f(x(t)) \geq 0, \tag{14}$$

where $L = diag\{l_1, l_2, ..., l_n\}$.
By Lemma 2, for any real number $\rho > 1$, we suppose that

$$V(t + \theta, x(t + \theta)) < \rho V(t, x(t)).$$

Then, we can obtain

$$\rho x^T(t)Px(t) - x^T(t - d(t))Px(t - d(t)) \geq 0. \tag{15}$$

Combining (12)-(15), we have

$$\begin{aligned} D^\gamma V(x(t)) \leq\ & x^T(t)(-P\widetilde{A} - \widetilde{A}P + LW_1L + \rho P + d^\gamma\gamma^{-1}(X - Y\widetilde{A} - \widetilde{A}Y^T + \widetilde{A}Z\widetilde{A}))x(t) \\ & + 2x^T(t)(P\widetilde{B} + d^\gamma\gamma^{-1}(Y\widetilde{B} - \widetilde{A}Z\widetilde{B}))f(x(t)) \\ & + 2x^T(t)(PK + d^\gamma\gamma^{-1}(YK - \widetilde{A}ZK))x(t - d(t)) \\ & + 2f^T(x(t))d^\gamma\gamma^{-1}\widetilde{B}^TZKx(t - d(t)) \\ & + f^T(x(t))(-W_1 + d^\gamma\gamma^{-1}\widetilde{B}^TZ\widetilde{B})f(x(t)) \\ & + x^T(t - d(t))(-P + d^\gamma\gamma^{-1}K^TZK)x(t - d(t)) - \int_{t-d(t)}^{t}(t-s)^{\gamma-1}\upsilon(t)\Pi\upsilon(t)^Tds \\ =:\ & \vartheta^T(t)\phi\vartheta(t) - \int_{t-d(t)}^{t}(t-s)^{\gamma-1}\upsilon^T(t)\Pi\upsilon(t)ds, \end{aligned} \tag{16}$$

where $\vartheta(t) = [x^T(t), f^T(x(t)), x^T(t-d(t))]^T$,

$$\phi = \begin{bmatrix} \phi_{11} + \rho P + d^\gamma \gamma^{-1} \widetilde{A} Z \widetilde{A} & \phi_{12} - d^\gamma \gamma^{-1} \widetilde{A} Z \widetilde{B} & \phi_{13} - d^\gamma \gamma^{-1} \widetilde{A} Z K \\ * & -W_1 + d^\gamma \gamma^{-1} \widetilde{B}^T Z \widetilde{B} & d^\gamma \gamma^{-1} \widetilde{B}^T Z K \\ * & * & -P + d^\gamma \gamma^{-1} K^T Z K \end{bmatrix},$$

$\phi_{11} = -P\widetilde{A} - \widetilde{A}P + LW_1L + d^\gamma \gamma^{-1}(X - Y\widetilde{A} - \widetilde{A}Y^T)$,
$\phi_{12} = P\widetilde{B} + d^\gamma \gamma^{-1} Y \widetilde{B}$,
$\phi_{13} = PK + d^\gamma \gamma^{-1} YK$.
By Schur complement, $\phi < 0$ is equivalent to

$$\begin{bmatrix} \phi_{11} + \rho P & \phi_{12} & \phi_{13} & \widetilde{\psi}_{14} \\ * & -W_1 & 0 & \widetilde{\psi}_{24} \\ * & * & -P & \widetilde{\psi}_{34} \\ * & * & * & \widetilde{\psi}_{44} \end{bmatrix} < 0, \tag{17}$$

where
$\widetilde{\psi}_{14} = -d^\gamma \gamma^{-1} \widetilde{A} Z, \widetilde{\psi}_{24} = d^\gamma \gamma^{-1} \widetilde{B}^T Z, \widetilde{\psi}_{34} = d^\gamma \gamma^{-1} K^T Z, \widetilde{\psi}_{44} = -d^\gamma \gamma^{-1} Z$,
Replacing $\Delta A(t)$ and $\Delta B(t)$ in (17) with $ME(t)N_1$ and $ME(t)N_2$, respectively, one has

$$\begin{aligned} &\begin{bmatrix} \psi_{11} + \rho P & \psi_{12} & \psi_{13} & \psi_{14} \\ * & -W_1 & 0 & \psi_{24} \\ * & * & -P & \psi_{34} \\ * & * & * & \psi_{44} \end{bmatrix} + \begin{bmatrix} \psi_{15} \\ 0 \\ 0 \\ \psi_{45} \end{bmatrix} E(t) \begin{bmatrix} N_1 & -N_2 & 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} N_1^T \\ -N_2^T \\ 0 \\ 0 \end{bmatrix} E^T(t) \begin{bmatrix} {\psi_{15}}^T & 0 & 0 & {\psi_{45}}^T \end{bmatrix} < 0. \end{aligned} \tag{18}$$

Applying Lemma 2 to (18), there exists a positive scalar λ_1, such that

$$\begin{aligned} &\begin{bmatrix} \psi_{11} + \rho P & \psi_{12} & \psi_{13} & \psi_{14} \\ * & -W_1 & 0 & \psi_{24} \\ * & * & -P & \psi_{34} \\ * & * & * & \psi_{44} \end{bmatrix} + {\lambda_1}^{-1} \begin{bmatrix} \psi_{15} \\ 0 \\ 0 \\ \psi_{45} \end{bmatrix} \begin{bmatrix} {\psi_{15}}^T & 0 & 0 & {\psi_{45}}^T \end{bmatrix} \\ &+ \lambda_1 \begin{bmatrix} N_1^T \\ -N_2^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} N_1 & -N_2 & 0 & 0 \end{bmatrix} < 0. \end{aligned}$$

Taking $\rho \to 1^+$, ψ is equivalent to (10) by Schur complement. Since $\int_{t-d(t)}^{t} (t-s)^{\gamma-1} \upsilon(t) \Pi \upsilon(t)^T ds > 0$, we have $D^\gamma V(x(t)) < \vartheta^T(t) \psi \vartheta(t)$. From (10), we have $D^\gamma V(x(t)) < 0$, then, condition (ii) in Lemma 3 is also satisfied. Thus, system (7) with sampled-data controler K is asymptotically stable.

When the controller gain matrix K is unknown, it is evident that the (10) is not an LMI because some crosses of these determined parameters are described in (10) in a nonlinear fashion PK. However, it can be transformed into an LMI by the following Theorem 2.
Theorem 2. Given scalars d, $\gamma \in (0, 1]$. If there exist $P > 0$, $X > 0$, $Z > 0$, $W_1 > 0$ and any suitable matrix

Y, a scalar $\lambda_1 > 0$ such that the next LMIs hold

$$\begin{bmatrix} \widehat{\psi}_{11}+P & PB & \overline{X} & -d^{\gamma}\gamma^{-1}AP & -PM & \lambda_1 {N_1}^T \\ * & -W_1 & 0 & d^{\gamma}\gamma^{-1}B^TP & 0 & -\lambda_1 {N_2}^T \\ * & * & -P & d^{\gamma}\gamma^{-1}\overline{X}^T & 0 & 0 \\ * & * & * & -d^{\gamma}\gamma^{-1}P & d^{\gamma}\gamma^{-1}PM & 0 \\ * & * & * & * & -\lambda_1 I & 0 \\ * & * & * & * & * & -\lambda_1 I \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} X & 0 \\ 0 & P \end{bmatrix} \geq 0, \tag{20}$$

where $\widehat{\psi}_{11} = -PA - AP + LW_1L + d^{\gamma}\gamma^{-1}X$.
Moreover, a controller gain matrix is given by

$$K = P^{-1}\overline{X} \tag{21}$$

Proof: To simplify the structure of the LMI, let $Y = 0, Z = P$ in (10). The proof is completed.
In order to show the advantages of the controller (6), the following result without uncertainties is given for comparison. Excepting for the uncertainties, the system (7) is describe by

$$D^{\alpha}x(t) = -Ax(t) + Bf(x(t)) + Kx(t-d(t)), t > 0, \tag{22}$$

Then, we can obtain the following criterion. From Theorem 1, we can get the following result for FONNs.
Corollary 1. : Given scalars d, $\gamma \in (0, 1]$ and matrix K. If there exist $P > 0$, $X > 0$, $W_1 > 0$, $Z > 0$, and any suitable matrix Y, such that the next LMIs hold

$$\begin{bmatrix} \psi_{11}+P & \psi_{12} & \psi_{13} & \psi_{14} \\ * & -W_1 & 0 & \psi_{24} \\ * & * & -P & \psi_{34} \\ * & * & * & \psi_{44} \end{bmatrix} < 0, \tag{23}$$

$$\Pi = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0, \tag{24}$$

Similarly, the system (22) remains stable when the uncertainty disappears.
When the controller gain matrix K is unknown, it is evident that the (23) isn't an LMI because some crosses of these determined parameters are described in (23) in a nonlinear fashion PK. However, it can be transformed into an LMI by the following Corollary 2.
Corollary 2. Given scalars d, $\gamma \in (0, 1]$. If there exist $P > 0$, $W_1 > 0$, $X > 0$, $Z > 0$, and any suitable matrix Y, such that the next LMIs hold

$$\begin{bmatrix} \widehat{\psi}_{11}+P & PB & \overline{X} & -d^{\gamma}\gamma^{-1}AP \\ * & -W_1 & 0 & d^{\gamma}\gamma^{-1}B^TP \\ * & * & -P & d^{\gamma}\gamma^{-1}\overline{X}^T \\ * & * & * & -d^{\gamma}\gamma^{-1}P \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} X & 0 \\ 0 & P \end{bmatrix} \geq 0, \tag{26}$$

Similarly, a controller gain matrix is given by

$$K = P^{-1}\overline{X}. \tag{27}$$

Proof: To simplify the structure of the LMI, let $Y = 0, Z = P$ in (23). The proof is completed.

Remark 1. The conditions in criteria can guarantee the stabilization of the FONNs under a sampled-data controller. In this paper, by using the FO Razumikin theorem, appropriate inequalities are established. Then, the order-dependent stability conclusions are obtained for the FONNs with sampled-data control. Finally, the LMI toolbox can be used to overcome the issue quickly. In addition, the conclusions are easy to be applied to engineering applications. In many papers, the authors have reflected on the issue of stabilization for regions or conditions without sampled-data control in [11]-[17]. However, the results are graphic or numerical, so they are difficult to verify. Hence, we take account of this truth.

Remark 2. According to [28], it is possible to analyze the stabilization of FO complex dynamical networks with sampled-data control in the future.

Remark 3. If we consider $\gamma = 1$ in system (1), then the FONNS system with sampled-data control will degenerate to integer order. Accordingly, the asymptotic stability and stability results are still effective for integer order neural network models in [10], [12] and [13].

Remark 4. In [17], the authors used Lyapunov direct method to derive an order-independent analysis criterion for FONNs. The Caputo FO correlation analysis criterion needs to be proved by the FO Razumikhin theorem and by combining some properties of FONNs. This paper is less conservative compared with the results of [17].

4. Numerical Example

In this section, an example is displayed to testify the availability of the proposed method.
Consider system (22) with controller (6), the parameters given as follows:

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}, B = \begin{bmatrix} 2 & -1.2 & 0 \\ 1.8 & 1.71 & 1.15 \\ -4.75 & 0 & 1.1 \end{bmatrix},$$

Setting the activation functions $f_1 = f_2 = f_3 = tanh(s)$ with $l_1^+ = l_2^+ = l_3^+ = 1$, the following matrice is given:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

The initial conditions are chosen as $x(t) = [0.9, 0.8, 0.9]^T$. With the criteria of Corollary 2, it can be obtained the maximum d under different γ in Table 1. The stability of the FONNs is verified.
From Table 1, we can get that d increases as γ increases, which means that the consequences of our method are less conservative and more generalized. Letting $\gamma = 0.98$, the corresponding maximum d is 0.17. Using the Matlab LMI Toolbox, by the solution of LMIS (25)-(27), the K is given as follow:

$$K = \begin{bmatrix} 0.7533 & 0.1617 & 0.0343 \\ 0.6972 & 0.9290 & -0.0694 \\ 0.3172 & -0.0661 & 0.2385 \end{bmatrix},$$

Table 1: The simulation results of the sampling interval with different γ.

γ	0.90	0.92	0.95	0.98
d	0.13	0.14	0.15	0.17

With the above K, the state response curves of state signal $x(t)$ and control input $u(t)$ are given in Fig. 1 and 2, separately. From Fig. 1, it is clear to see that the state $x(t)$ of the system is tending to zero, which means the system (22) in the example is asymptotically stable successfully by the designed sampled-data controller.

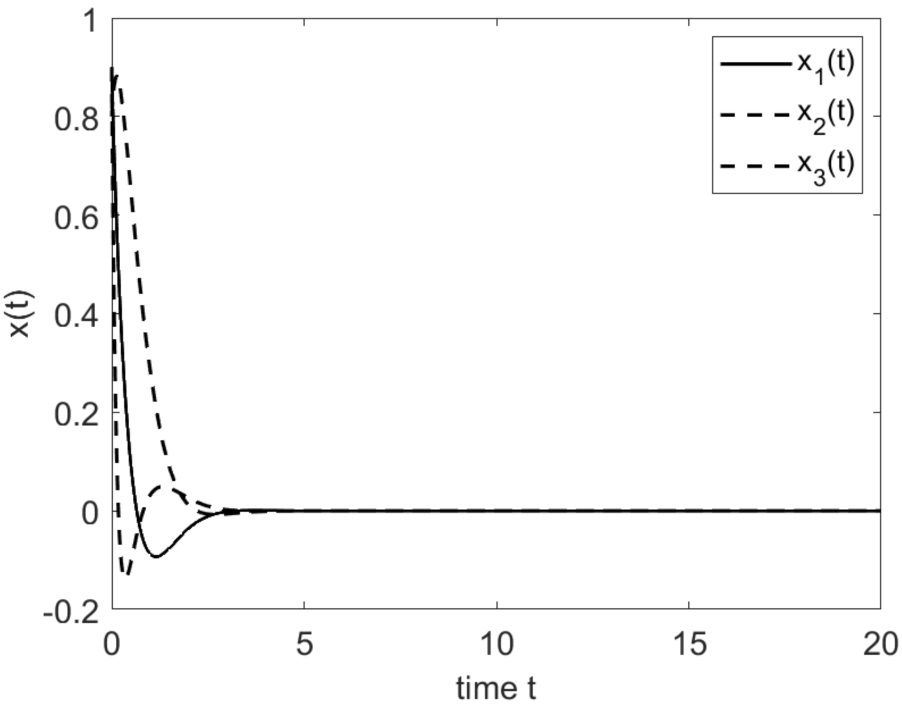

Figure 1: State $x(t)$ in the Example

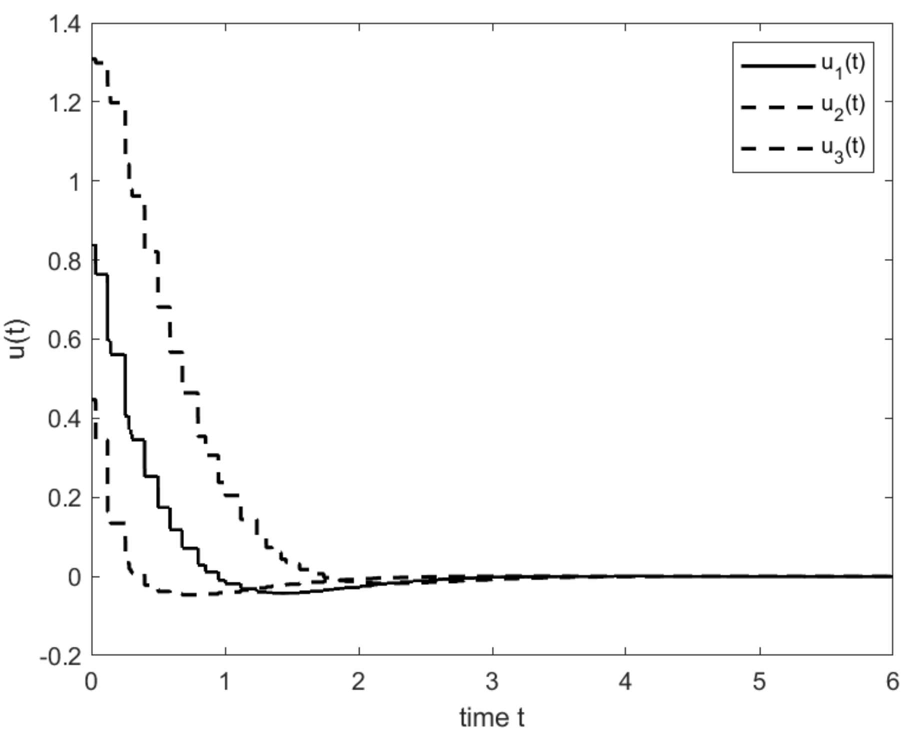

Figure 2: Sampled-data control input $u(t)$ in the Example

5. Conclusion

The order-dependent stability of FMNN with sampled data control is studied in this paper. Through the FO Razumikhin theorem and LMIs, the order-dependent stability criteria are formulated. In view of the stability conditions, the sampled data controller is also proposed. The results are less conservative and restrictive than those reported previously in the literature. In addition, this method is feasible on calculation and can be directly implemented by Matlab toolbox. This method can be applied to the synchronization problem of FNNs systems with uncertainty. The effectiveness of the new criteria is demonstrated by a numerical example. Parameter uncertainty is often encountered in real systems and neural networks. Because of inaccurate modeling of the model or changes in the environment, robust stability and stabilization analysis of delayed FNNs systems will be considered in the future.

Acknowledgments

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Funding

This paper isn't funded by the fund.

Author Contribution

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Ruonan Zhang: Visualization, Investigation, Supervision.
Li Yang: Validation, Project administration.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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