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Research Article

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Posted Date: February 28th, 2023

DOI: <https://doi.org/10.21203/rs.3.rs-2622815/v1>

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Additional Declarations: No competing interests reported.

Version of Record: A version of this preprint was published at Neural Processing Letters on July 22nd, 2023. See the published version at <https://doi.org/10.1007/s11063-023-11369-z>.

Event-triggered controller on practically exponential input-to-state stabilization of stochastic reaction-diffusion neural networks and its application to image encryption

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Abstract

The stabilization problem for a class of stochastic reaction-diffusion delayed Cohen-Grossberg neural networks (SRDDCGNNs) with event-triggered controller is addressed in this paper. To address such a problem, Neumann boundary condition, distributed and boundary external disturbances are introduced. New sufficient criteria are derived by using the Lyapunov method, event-triggered mechanism, and the linear matrix inequality (LMI) approach to ensure the proposed controlled systems achieve practically exponential input-to-state stabilization. In light of these criteria, the impact of an event-triggered controller on practically exponential input-to-state stability (PEISS) is examined. Furthermore, the obtained results are successfully applied to stochastic reaction-diffusion delayed cellular neural networks (SRDDCNNs) and stochastic reaction-diffusion delayed Hopfield neural networks (SRDDHNNs). At last, simulation results are given to illustrate the main results, and the SRDDHNNs are applied to the image encryption.

Keywords: Neural networks, Stochastic inputs, Reaction-diffusion terms, External disturbances, Event-triggered control, Image encryption.

1 Introduction

Neural networks (NNs) plays an significant role in many real-world applications such as image encryption [1–7], signal processing [8], pattern recognition [9], optimization problem [10], and secure communication [11]. Cohen-Grossberg NNs (CGNNs) was initially proposed and studied by Cohen and Grossberg in 1983 [12]. Some other models, cellular NNs [13], Hopfield NNs [14], and recurrent NNs [15] are special cases of CGNNs. As a dynamical behavior in CGNNs, stabilization plays an unique role in various fields such as engineering and science. For different aims, various types of stabilization were studied, such as asymptotic stabilization [16], exponential stabilization [17], finite-time stabilization [18], fixed-time stabilization [19], input-to-state stabilization [20], and H_∞ -stabilization [21]. Most of the authors previous focused on stabilization of CGNNs modeled by ordinary differential equations. In the wide range of applications, COVID -19 [22], dengue fever [23], HCV infections epidemic model [24], Alzheimer’s disease [25], chemical reaction [26], and image encryption [27] are depend on both space and time variables. These behaviors are modeled by partial differential equations (PDE). However, the diffusion effects inevitable in NNs while electrons move through unsymmetrical electromagnetic fields [28–31].

Delays are inevitable in NNs because of the limited switching speeds of the neurons and amplifiers. It has been shown that delays can cause oscillation and instability in NNs. Consequently, stability analysis of stochastic CGNNs with time delays has received extensive attention in the literatures [32–35]. However, input-to-state stability (ISS) is required in several engineering fields such as share market and simple pendulum. ISS only requires the state of the system to be within a bounded interval, as compared to asymptotic stability which needs that the states of the system tends to zero equilibrium. In the recent decades, a lot of results has been published on exponential input-to-state stability (EISS) of NNs with respect to distributed external disturbances [36–41]. Especially, in [42], the author studied the practical exponential stability (PES) of impulsive CGNNs with respect to h -manifolds. In [43], the author studied the PEISS of stochastic delayed nonlinear systems with respect to distributed external disturbances. In [44], the author studied the G-Lyapunov function on PES of stochastic delayed systems via G-Brownian motion. In [45], the author studied the PES of stochastic impulsive systems. In [46], the author studied the PES of stochastic impulsive delayed reaction-diffusion systems. In [47], the author studied the PES of stochastic impulsive functional differential equations.

On the other hand, modern technology and microelectronics have increased interest in event-triggered control system analysis and synthesis. Event-triggered control usually comprises of a feedback control rule that stabilize the system and an event-triggered mechanism with a triggering condition that decides when the control has to be updated. In event-triggered control, emulation and co-design are used to design unknown control and event-triggered parameters. In recent decades, the event-triggered control mechanism has been

CGNNs	[20]	[32, 33]	[37–39]	[40, 41]	[43]	[51, 52]	[53]	This paper
Stochastic inputs	✓	✓	×	✓	×	×	✓	✓
Reaction-diffusion	×	✓	×	×	×	×	×	✓
DEIs	✓	×	✓	✓	×	×	✓	✓
BEIs	×	×	×	×	×	×	×	✓
ETC	×	×	×	×	×	✓	✓	✓
Stabilization	×	✓	×	×	×	×	✓	✓
Image encryption	×	×	×	×	×	×	×	✓

Table 1 Comparison for CGNNs with other works.

introduced to NNs [48–50]. Especially, in [51], the author studied the fixed-time synchronization of inertial CGNNs via event-triggered control. In [52], the author studied the asymptotic synchronization of memristive CGNNs via event-triggered control. In [53], the author studied the ISS of stochastic fuzzy CGNNs via event-triggered control.

To best of our knowledge, there are few works concerning PES of stochastic neural networks based on PDE systems and the practically exponential input-to-state stabilization problem of how to act external disturbances on spatial boundary point of stochastic reaction-diffusion NNs has not been studied. Inspired by previous discussions, this paper aims to study the practically exponential input-to-state stabilization problem of SRDDCGNNs with distributed and boundary external disturbances via event-triggered control mechanisms. The main contributions are listed as follows:

- We used an event-triggered controller to investigate the stabilization of SRDDCGNNs with distributed and boundary external disturbances.
- New sufficient criteria are derived to guarantee the PEISS of SRDDCGNNs using appropriate Lyapunov-Krasovskii functional (LKF), Neumann boundary condition, Wirtinger’s inequality, event-triggered mechanism, and LMI method.
- The effects of the control gain matrix on PEISS is reflected in the theoretical results.
- The proposed SRDDHNNs are applied to image encryption, and the efficiency of the encryption system is demonstrated by some security analyses and tests.

To better illustrate the main contributions and innovations of this paper, we provide Table 1 for comparison with other works on CGNNs, where stochastic inputs, reaction-diffusion, distributed external inputs (DEIs), boundary external inputs (BEIs), event-triggered control (ETC), stabilization, and image encryption. Furthermore, ✓ - this term is included in that paper and × - this term is not included in that paper.

Notations: $\mathbb{W}$ – set of all whole numbers; $\mathbb{R}$ – set of all real numbers; $\mathbb{R}_+$ – set of all positive real numbers; $\mathbb{R}^n$ – (n) -dimensional Euclidean space; $\mathbb{R}^{m \times n}$ – $(m \times n)$ -dimensional Euclidean space; $\mathcal{Z}^T = \mathcal{Z} > 0$ (respectively, $\mathcal{Z}^T = \mathcal{Z} < 0$) – positive definite matrix (negative definite matrix); $\mathcal{Z}^T$ – transpose of $\mathcal{Z}$;

$\Pi_{\min}(\mathcal{Z})$ – minimum eigen value of $\mathcal{Z}$; $\Pi_{\max}(\mathcal{Z})$ – maximum eigen value of $\mathcal{Z}$; $*$ – the entry is symmetric; $\text{sym}(\mathcal{Z}) = (\mathcal{Z} + \mathcal{Z}^T)$; $\|\cdot\|$ – Euclidean norm; $\mathbb{E}(\mathcal{Z})$ – mathematical expectation of $\mathcal{Z}$; $W^{1,2}([0, \mathcal{M}]; \mathbb{R}^n)$ – Soblev space of absolutely continuous function; $\int_0^1 \mathfrak{R}^T(t, \varrho) \mathfrak{R}(t, \varrho) d\varrho = \|\mathfrak{R}(t, \varrho)\|^2$.

The rest of this paper is structured as follows. In Section 2, system model, event-triggered control problem, and preliminaries are introduced. In Section 3, we investigate our main results: (i) To obtain the PEISS of proposed controlled system; (ii) To prove the practically exponential input-to-state stabilizable result by designing control gain matrix for proposed controller. In Section 4, numerical simulations are shown that the efficiency of event-triggered controller. In Section 5, the SRDDHNNs are applied to the image encryption. Finally, conclusion and our future works are shown in Section 6.

2 System Description and Preliminaries

Consider the following SRDDCGNNs with distributed and boundary external disturbances:

$$\begin{cases} d\mathfrak{R}(t, \varrho) = \left[D \frac{\partial^2 \mathfrak{R}(t, \varrho)}{\partial \varrho^2} - \alpha(\mathfrak{R}(t, \varrho)) \left(\beta(\mathfrak{R}(t, \varrho)) - Af(\mathfrak{R}(t, \varrho)) \right. \right. \\ \quad \left. \left. - Bg(\mathfrak{R}(t - \zeta, \varrho)) - Cu(t, \varrho) - \phi(t, \varrho) \right) \right] dt \\ \quad + \sigma(\mathfrak{R}(t, \varrho), \mathfrak{R}(t - \zeta, \varrho)) d\omega(t), \\ \mathfrak{R}(z, \varrho) = \varpi(z, \varrho), \quad \varrho \in (0, 1), \quad z \in [-\zeta, 0], \\ \mathfrak{R}(t, 0) = 0, \quad \frac{\partial \mathfrak{R}(t, \varrho)}{\partial \varrho} \Big|_{\varrho=1} = \psi(t), \end{cases} \quad (1)$$

where $\mathfrak{R}(t, \varrho) \in \mathbb{R}^n$ denote the state variable; $\varrho \in (0, 1)$ denote the space variable; $t > 0$ denote the time variable. $\varpi(z, \varrho) \in \mathbb{R}^n$ denote the initial function. $u(t, \varrho) \in \mathbb{R}^n$ denote the event-triggered control input vector. $\phi(t, \varrho) \in \mathbb{R}^n$ and $\psi(t) \in \mathbb{R}^n$ are denotes the distributed and boundary external disturbances, respectively. $D = \text{diag}\{D_1, D_2, \dots, D_n\} > 0$ is a diffusion matrix. $\alpha(\mathfrak{R}(t, \varrho))$ is a amplification function. $\beta(\mathfrak{R}(t, \varrho))$ is a behaved function. $f(\cdot)$ and $g(\cdot)$ are denotes the neuron activation functions. $\sigma(\cdot)$ is a stochastic input. $\omega(t) \in \mathbb{R}^m$ denote the Brownian motion. ζ is a constant delay. A, B are the connection weight matrices. C is a constant matrix with suitable dimension.

2.1 Event-triggered controller design of SRDDCGNNs:

We used an event-triggered control mechanism in this study, which differs from the usual time-triggered method. $\{t_q : q \in \mathbb{W}\}$ is a sampling sequence, which satisfies $t_0 = 0$ and

$$t_{q+1} = \inf\{t : t > t_q, \aleph(t, \varrho) > 0\},$$

where $\aleph(t, \varrho)$ denote the event generator function. For the sampled-data control with zero-order hold (ZOH), the event-triggered controller is designed as

follows:

$$u(t, \varrho) = J\mathfrak{R}(t_q, \varrho), \quad t \in [t_q, t_{q+1}), \quad q \in \mathbb{W}, \quad (2)$$

where J is a control gain matrix.

By virtue of controller (2), the system (1) can be rewritten as follows:

$$\begin{cases} d\mathfrak{R}(t, \varrho) = \left[D \frac{\partial^2 \mathfrak{R}(t, \varrho)}{\partial \varrho^2} - \alpha(\mathfrak{R}(t, \varrho)) \left(\beta(\mathfrak{R}(t, \varrho)) - Af(\mathfrak{R}(t, \varrho)) \right. \right. \\ \quad \left. \left. - Bg(\mathfrak{R}(t - \zeta, \varrho)) - CJ\mathfrak{R}(t_q, \varrho) - \phi(t, \varrho) \right) \right] dt \\ \quad + \sigma(\mathfrak{R}(t, \varrho), \mathfrak{R}(t - \zeta, \varrho)) d\omega(t), \\ \mathfrak{R}(z, \varrho) = \varpi(z, \varrho), \quad \varrho \in (0, 1), \quad z \in [-\zeta, 0], \\ \mathfrak{R}(t, 0) = 0, \quad \frac{\partial \mathfrak{R}(t, \varrho)}{\partial \varrho} \big|_{\varrho=1} = \psi(t). \end{cases} \quad (3)$$

Let $\mathfrak{Z}(t, \varrho)$ be measurement error between current state $\mathfrak{R}(t, \varrho)$ and sampled state $\mathfrak{R}(t_q, \varrho)$. Then, we get

$$\mathfrak{Z}(t, \varrho) = \mathfrak{R}(t_q, \varrho) - \mathfrak{R}(t, \varrho), \quad t \in [t_q, t_{q+1}), \quad q \in \mathbb{W}. \quad (4)$$

By virtue of (4), the system (3) can be rewritten as follows:

$$\begin{cases} d\mathfrak{R}(t, \varrho) = \left[D \frac{\partial^2 \mathfrak{R}(t, \varrho)}{\partial \varrho^2} - \alpha(\mathfrak{R}(t, \varrho)) \left(\beta(\mathfrak{R}(t, \varrho)) - Af(\mathfrak{R}(t, \varrho)) \right. \right. \\ \quad \left. \left. - Bg(\mathfrak{R}(t - \zeta, \varrho)) - CJ(\mathfrak{Z}(t, \varrho) + \mathfrak{R}(t, \varrho)) - \phi(t, \varrho) \right) \right] dt \\ \quad + \sigma(\mathfrak{R}(t, \varrho), \mathfrak{R}(t - \zeta, \varrho)) d\omega(t), \\ \mathfrak{R}(z, \varrho) = \varpi(z, \varrho), \quad \varrho \in (0, 1), \quad z \in [-\zeta, 0], \\ \mathfrak{R}(t, 0) = 0, \quad \frac{\partial \mathfrak{R}(t, \varrho)}{\partial \varrho} \big|_{\varrho=1} = \psi(t). \end{cases} \quad (5)$$

In this paper, we choose the following event-generator function $\aleph(t, \varrho)$ as:

$$\aleph(t, \varrho) = \|\mathfrak{Z}(t, \varrho)\|^2 - \varepsilon_1 \|\mathfrak{R}(t_q, \varrho)\|^2 - \varepsilon_2, \quad q \in \mathbb{W}, \quad (6)$$

where $\varepsilon_1, \varepsilon_2 \in \mathbb{R}_+$ denotes the event-triggered parameters, and satisfying the condition $\varepsilon_1^2 + \varepsilon_2^2 \neq 0$.

Assumption ($\mathcal{H}_1$): There exist scalars $\underline{\alpha}_i, \bar{\alpha}_i \in \mathbb{R}_+$ such that

$$0 < \underline{\alpha}_i \leq \alpha_i(h) \leq \bar{\alpha}_i, \quad \forall h \in \mathbb{R}^n, \quad i = \{1, 2, \dots, n\}.$$

Assumption ($\mathcal{H}_2$): There exist scalar $\gamma_i > 0$ such that

$$(\bar{h}_1 - \bar{h}_2)(\beta_i(\bar{h}_1) - \beta_i(\bar{h}_2)) \geq \gamma_i(\bar{h}_1 - \bar{h}_2)^T(\bar{h}_1 - \bar{h}_2), \quad \forall \bar{h}_1, \bar{h}_2 \in \mathbb{R}^n.$$

Assumption ($\mathcal{H}_3$): There exist scalars $\chi_1 > 0$ and $\chi_2 > 0$ such that

$$(f(\bar{h}_1) - f(\bar{h}_2))^T(f(\bar{h}_1) - f(\bar{h}_2)) \leq \chi_1(\bar{h}_1 - \bar{h}_2)^T(\bar{h}_1 - \bar{h}_2), \quad \forall \bar{h}_1, \bar{h}_2 \in \mathbb{R}^n,$$

$$(g(k_1) - g(k_2))^T (g(k_1) - g(k_2)) \leq \chi_2(k_1 - k_2)^T (k_1 - k_2), \quad \forall k_1, k_2 \in \mathbb{R}^n.$$

Assumption ($\mathcal{H}_4$): There exist scalars $\delta_1 > 0$ and $\delta_2 > 0$ such that

$$\begin{aligned} \text{trace}[(\sigma(\bar{h}_1, \bar{h}_2) - \sigma(\bar{h}_3, \bar{h}_4))^T (\sigma(\bar{h}_1, \bar{h}_2) - \sigma(\bar{h}_3, \bar{h}_4))] &\leq \delta_1 (\bar{h}_1 - \bar{h}_3)^T (\bar{h}_1 - \bar{h}_3) \\ &\quad + \delta_2 (\bar{h}_2 - \bar{h}_4)^T (\bar{h}_2 - \bar{h}_4), \end{aligned}$$

for any $\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_4 \in \mathbb{R}^n$.

Lemma 1 [31] For a matrix $\mathcal{R} > 0$ and a state vector $\bar{h}(\cdot) \in W^{1,2}([0, \mathcal{M}]; \mathbb{R}^n)$ with $\bar{h}(0) = 0$ or $\bar{h}(\mathcal{M}) = 0$, we have

$$\int_0^{\mathcal{M}} \bar{h}^T(s) \mathcal{R} \bar{h}(s) ds \leq \frac{4\mathcal{M}^2}{\pi^2} \int_0^{\mathcal{M}} \left(\frac{d\bar{h}(s)}{ds} \right)^T \mathcal{R} \left(\frac{d\bar{h}(s)}{ds} \right) ds.$$

Lemma 2 [28] There exist real matrices Υ_1, Υ_2 , and a positive definite matrix $\mathcal{W}$ such that the following inequality is holds:

$$\Upsilon_1^T \Upsilon_2 + \Upsilon_2^T \Upsilon_1 \leq \Upsilon_1^T \mathcal{W}^{-1} \Upsilon_1 + \Upsilon_2^T \mathcal{W} \Upsilon_2.$$

Lemma 3 [35] Let $\Theta_1, \Theta_2, \Theta_3$ be given matrices such that $\Theta_1^T = \Theta_1 > 0$ and $\Theta_2^T = \Theta_2 > 0$. Then

$$\Theta_1 + \Theta_3^T \Theta_2^{-1} \Theta_3 < 0 \Leftrightarrow \begin{bmatrix} \Theta_1 & \Theta_3^T \\ * & -\Theta_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Theta_2 & \Theta_3 \\ * & \Theta_1 \end{bmatrix} < 0.$$

Definition 1 [36] The system (5) is called practically exponentially input-to-state stable (PEISS) with respect to external disturbances $\phi(t, \varrho)$ and $\psi(t)$ if there exist scalars $\lambda > 0, \mu > 0, \eta_1 > 0, \eta_2 > 0$, and $\theta \geq 0$ such that

$$\mathbb{E} \|\mathfrak{R}(t, \varrho)\|^2 \leq \lambda e^{-\mu t} \sup_{-\zeta \leq z \leq 0} \mathbb{E} \|\varpi(z, \varrho)\|^2 + \eta_1 \|\phi(t, \varrho)\|_\infty^2 + \eta_2 \|\psi(t)\|_\infty^2 + \theta,$$

where

$$\|\phi(t, \varrho)\|_\infty^2 = \sup_{t>0} \|\phi(t, \varrho)\|^2 \text{ and } \|\psi(t)\|_\infty^2 = \sup_{t>0} \|\psi(t)\|^2.$$

Especially, when $\theta = 0$, the system (5) is exponentially input-to-state stable (EISS) with respect to external disturbances $\phi(t, \varrho)$ and $\psi(t)$. Furthermore, when $\theta = 0$, $\phi(t, \varrho) = 0$, and $\psi(t) = 0$, the system (5) is exponentially stable.

Definition 2 [43] The system (5) is called practically exponentially input-to-state stabilizable if there exist control gain matrix J and event-triggering parameters $\varepsilon_1, \varepsilon_2$ such that the system (5) is PEISS with respect to external disturbances $\phi(t, \varrho)$ and $\psi(t)$. In particular, when $\theta = 0$, the system (5) is exponentially input-to-state stabilizable with respect to external disturbances $\phi(t, \varrho)$ and $\psi(t)$. Furthermore, when $\theta = 0$, $\phi(t, \varrho) = 0$, and $\psi(t) = 0$, the system (5) is exponentially stabilizable.

3 Main Results

In this section, we investigate the practically exponential input-to-state stabilization of SRDDCGNNS, SRDDCNNs, and SRDDHNNs with distributed and boundary external disturbances via event-triggered control mechanisms and the Lyapunov method.

For our convenience, we let

$$\underline{\mathcal{P}} = \text{diag}\{\underline{\alpha}_1, \underline{\alpha}_2, \dots, \underline{\alpha}_n\}, \quad \overline{\mathcal{P}} = \text{diag}\{\overline{\alpha}_1, \overline{\alpha}_2, \dots, \overline{\alpha}_n\}, \quad \mathcal{Q} = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_n\}, \\ \sigma(t) = \sigma(\mathfrak{R}(t, \varrho), \mathfrak{R}(t - \zeta, \varrho)).$$

3.1 Practically exponential input-to-state stabilization of SRDDCGNNS:

In this section, by using event-triggered control mechanisms to obtain the PEISS of system (5). Furthermore, we obtain the practically exponential input-to-state stabilization of system (5) through the control gain matrix for proposed controller.

Theorem 1 *Suppose that Assumptions $(\mathcal{H}_1) - (\mathcal{H}_4)$ holds, and the event-triggered parameters $\varepsilon_1, \varepsilon_2 \in \mathbb{R}_+$ satisfy $0 \leq \varepsilon_1 < 1 - \tau$, $\varepsilon_2 \geq 0$, and $0 < \tau < 1$, the system (5) is said to be PEISS if there exist positive definite matrices $\mathcal{U}, \mathcal{V}, \mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3$, and $\mathcal{W}_4$ such that the following LMI feasible for*

$$(i) \quad \Omega = \begin{bmatrix} \Omega_{11} & -\mathcal{U}D & 0 \\ * & \left(1 - \frac{\pi^2}{4}\right)\mathcal{U}D & 0 \\ * & * & -\mathcal{V} + \chi_2\mathcal{W}_2 + \Pi_{\max}(\mathcal{U})\delta_2. \end{bmatrix} < 0, \quad (7)$$

where

$$\Omega_{11} = \text{sym}(\overline{\mathcal{P}}\mathcal{C}\mathcal{U}J - \underline{\mathcal{P}}\mathcal{U}\mathcal{Q}) + \left(1 - \frac{\pi^2}{4}\right)\mathcal{U}D + \mathcal{V} + \Pi_{\max}(\mathcal{U})\delta_1 + \chi_1\mathcal{W}_1 \\ + \overline{\mathcal{P}}\mathcal{U}A\mathcal{W}_1^{-1}A^T\mathcal{U}\overline{\mathcal{P}} + \overline{\mathcal{P}}\mathcal{U}B\mathcal{W}_2^{-1}B^T\mathcal{U}\overline{\mathcal{P}} + \overline{\mathcal{P}}\mathcal{C}\mathcal{U}J\mathcal{W}_3^{-1}J^T\mathcal{U}\mathcal{C}^T\overline{\mathcal{P}} + \overline{\mathcal{P}}\mathcal{U}\mathcal{W}_4^{-1}\mathcal{U}\overline{\mathcal{P}} \\ + \Pi_{\max}(\mathcal{W}_3)\frac{(1 - \tau)\varepsilon_1}{\tau(1 - \tau - \varepsilon_1)}.$$

Proof: Consider the following Lyapunov-Krasovskii functional (LKF) candidate as:

$$V(\mathfrak{R}(t, \varrho)) = \int_0^1 \mathfrak{R}^T(t, \varrho)\mathcal{U}\mathfrak{R}(t, \varrho)d\varrho + \int_0^1 \int_{t-\zeta}^t \mathfrak{R}^T(\ell, \varrho)\mathcal{V}\mathfrak{R}(\ell, \varrho)d\ell d\varrho. \quad (8)$$

Calculating the derivative of $V(\mathfrak{R}(t, \varrho))$ along the trajectories of system (5) by using Ito's formula, we obtain that

$$\mathcal{L}V(\mathfrak{R}(t, \varrho)) = 2 \int_0^1 \mathfrak{R}^T(t, \varrho)\mathcal{U} \left[D \frac{\partial^2 \mathfrak{R}(t, \varrho)}{\partial \varrho^2} - \alpha(\mathfrak{R}(t, \varrho)) \left(\beta(\mathfrak{R}(t, \varrho)) - Af(\mathfrak{R}(t, \varrho)) \right) \right]$$

$$\begin{aligned}
& -Bg(\Re(t-\zeta, \varrho)) - CJ(\Im(t, \varrho) + \Re(t, \varrho)) - \phi(t, \varrho) \Big) d\varrho \\
& + \int_0^1 \text{trace}[\sigma^T(t)\mathcal{U}\sigma(t)]d\varrho + \int_0^1 \Re^T(t, \varrho)\mathcal{V}\Re(t, \varrho)d\varrho \\
& - \int_0^1 \Re^T(t-\zeta, \varrho)\mathcal{V}\Re(t-\zeta, \varrho)d\varrho.
\end{aligned} \tag{9}$$

By virtue of boundary condition and integration by parts, we get

$$\begin{aligned}
\int_0^1 \Re^T(t, \varrho)\mathcal{U}D\frac{\partial^2 \Re(t, \varrho)}{\partial \varrho^2}d\varrho & = \left[\Re^T(t, \varrho)\mathcal{U}D\frac{\partial \Re(t, \varrho)}{\partial \varrho} \right]_{\varrho=0}^{\varrho=1} \\
& - \int_0^1 \frac{\partial \Re^T(t, \varrho)}{\partial \varrho}\mathcal{U}D\frac{\partial \Re(t, \varrho)}{\partial \varrho}d\varrho \\
& = \Re^T(t, 1)\mathcal{U}D \int_0^1 \psi(t)d\varrho \\
& - \int_0^1 \frac{\partial \Re^T(t, \varrho)}{\partial \varrho}\mathcal{U}D\frac{\partial \Re(t, \varrho)}{\partial \varrho}d\varrho.
\end{aligned} \tag{10}$$

To get $\bar{\Re}(t, \varrho) = 0$, we introduce a new state variable $\bar{\Re}(t, \varrho) = \Re(t, \varrho) - \Re(t, 1)$ and satisfy the following condition:

$$\frac{\partial \Re^T(t, \varrho)}{\partial \varrho}\mathcal{U}D\frac{\partial \Re(t, \varrho)}{\partial \varrho} = \frac{\partial \bar{\Re}^T(t, \varrho)}{\partial \varrho}\mathcal{U}D\frac{\partial \bar{\Re}(t, \varrho)}{\partial \varrho}. \tag{11}$$

Applying Lemma 1, we have

$$\begin{aligned}
\int_0^1 \Re^T(t, \varrho)\mathcal{U}D\frac{\partial^2 \Re(t, \varrho)}{\partial \varrho^2}d\varrho & = \Re^T(t, 1)\mathcal{U}D \int_0^1 \psi(t)d\varrho \\
& - \frac{1}{2} \int_0^1 \frac{\partial \Re^T(t, \varrho)}{\partial \varrho}\mathcal{U}D\frac{\partial \Re(t, \varrho)}{\partial \varrho}d\varrho \\
& - \frac{1}{2} \int_0^1 \frac{\partial \bar{\Re}^T(t, \varrho)}{\partial \varrho}\mathcal{U}D\frac{\partial \bar{\Re}(t, \varrho)}{\partial \varrho}d\varrho \\
& \leq \int_0^1 (\Re^T(t, \varrho) - \bar{\Re}^T(t, \varrho))\mathcal{U}D\psi(t)d\varrho \\
& - \frac{\pi^2}{8} \int_0^1 \Re^T(t, \varrho)\mathcal{U}D\Re(t, \varrho)d\varrho \\
& - \frac{\pi^2}{8} \int_0^1 \bar{\Re}^T(t, \varrho)\mathcal{U}D\bar{\Re}(t, \varrho)d\varrho.
\end{aligned} \tag{12}$$

Based on Assumptions $(\mathcal{H}_1)$ and $(\mathcal{H}_2)$, we have

$$\begin{aligned}
 -2\mathfrak{R}^T(t, \varrho)\mathcal{U}\alpha(\mathfrak{R}(t, \varrho))\beta(\mathfrak{R}(t, \varrho)) &= -2\mathcal{U} \sum_{i=1}^n \mathfrak{R}_i(t, \varrho)\alpha_i(\mathfrak{R}_i(t, \varrho))\beta_i(\mathfrak{R}_i(t, \varrho)) \\
 &= -2\mathcal{U} \sum_{i=1}^n \alpha_i(\mathfrak{R}_i(t, \varrho)) [\mathfrak{R}_i(t, \varrho)\beta_i(\mathfrak{R}_i(t, \varrho))] \\
 &\leq -2\mathcal{U} \sum_{i=1}^n \underline{\alpha}_i \gamma_i \mathfrak{R}_i^2(t, \varrho) \\
 &\leq -2\mathfrak{R}^T(t, \varrho)\underline{\mathcal{P}}\mathcal{U}\mathcal{Q}\mathfrak{R}(t, \varrho).
 \end{aligned} \tag{13}$$

By virtue of Assumption $(\mathcal{H}_3)$ and Lemma 2, we get

$$\begin{aligned}
 2\mathfrak{R}^T(t, \varrho)\alpha(\mathfrak{R}(t, \varrho))\mathcal{U}Af(\mathfrak{R}(t, \varrho)) &= \mathfrak{R}^T(t, \varrho)\alpha(\mathfrak{R}(t, \varrho))\mathcal{U}Af(\mathfrak{R}(t, \varrho)) \\
 &\quad + f^T(\mathfrak{R}(t, \varrho))A^T\mathcal{U}\alpha^T(\mathfrak{R}(t, \varrho))\mathfrak{R}(t, \varrho) \\
 &\leq \mathfrak{R}^T(t, \varrho)\alpha(\mathfrak{R}(t, \varrho))\mathcal{U}A\mathcal{W}_1^{-1}A^T\mathcal{U}\alpha^T(\mathfrak{R}(t, \varrho))\mathfrak{R}(t, \varrho) \\
 &\quad + f^T(\mathfrak{R}(t, \varrho))\mathcal{W}_1f(\mathfrak{R}(t, \varrho)) \\
 &\leq \mathfrak{R}^T(t, \varrho)\overline{\mathcal{P}}\mathcal{U}A\mathcal{W}_1^{-1}A^T\mathcal{U}\overline{\mathcal{P}}\mathfrak{R}(t, \varrho) \\
 &\quad + \mathfrak{R}^T(t, \varrho)\chi_1\mathcal{W}_1\mathfrak{R}(t, \varrho),
 \end{aligned} \tag{14}$$

similarly

$$\begin{aligned}
 2\mathfrak{R}^T(t, \varrho)\alpha(\mathfrak{R}(t, \varrho))\mathcal{U}Bg(\mathfrak{R}(t - \zeta, \varrho)) &\leq \mathfrak{R}^T(t, \varrho)\overline{\mathcal{P}}\mathcal{U}B\mathcal{W}_2^{-1}B^T\mathcal{U}\overline{\mathcal{P}}\mathfrak{R}(t, \varrho) \\
 &\quad + \mathfrak{R}^T(t - \zeta, \varrho)\chi_2\mathcal{W}_2\mathfrak{R}(t - \zeta, \varrho),
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 2\mathfrak{R}^T(t, \varrho)\alpha(\mathfrak{R}(t, \varrho))\mathcal{C}\mathcal{U}J\mathfrak{Z}(t, \varrho) &\leq \mathfrak{R}^T(t, \varrho)\overline{\mathcal{P}}\mathcal{C}\mathcal{U}J\mathcal{W}_3^{-1}J^T\mathcal{U}\mathcal{C}^T\overline{\mathcal{P}}\mathfrak{R}(t, \varrho) \\
 &\quad + \mathfrak{Z}^T(t, \varrho)\mathcal{W}_3\mathfrak{Z}(t, \varrho),
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 2\mathfrak{R}^T(t, \varrho)\alpha(\mathfrak{R}(t, \varrho))\mathcal{U}\phi(t, \varrho) &\leq \mathfrak{R}^T(t, \varrho)\overline{\mathcal{P}}\mathcal{U}\mathcal{W}_4^{-1}\mathcal{U}\overline{\mathcal{P}}\mathfrak{R}(t, \varrho) \\
 &\quad + \phi^T(t, \varrho)\mathcal{W}_4\phi(t, \varrho).
 \end{aligned} \tag{17}$$

Noting that $\mathcal{U}$, D , and $\mathcal{U}D$ are positive definite matrices, we get

$$\begin{aligned}
 2(\mathfrak{R}^T(t, \varrho) - \bar{\mathfrak{R}}^T(t, \varrho))\mathcal{U}D\psi(t) &\leq (\mathfrak{R}^T(t, \varrho) - \bar{\mathfrak{R}}^T(t, \varrho))\mathcal{U}D(\mathfrak{R}(t, \varrho) - \bar{\mathfrak{R}}(t, \varrho)) \\
 &\quad + \psi^T(t)\mathcal{U}D\psi(t).
 \end{aligned} \tag{18}$$

By virtue of Assumption $(\mathcal{H}_4)$, we obtain that

$$\begin{aligned}
 \text{trace}[\sigma^T(t)\mathcal{U}\sigma(t)] &\leq \Pi_{\max}(\mathcal{U}) \left[\delta_1 \mathfrak{R}^T(t, \varrho)\mathfrak{R}(t, \varrho) \right. \\
 &\quad \left. + \delta_2 \mathfrak{R}^T(t - \zeta, \varrho)\mathfrak{R}(t - \zeta, \varrho) \right].
 \end{aligned} \tag{19}$$

Combining the inequalities (9)–(19), we have

$$\begin{aligned}
\mathcal{LV}(\mathfrak{R}(t, \varrho)) &\leq \int_0^1 \left\{ \mathfrak{R}^T(t, \varrho) \left[-2\underline{P}\mathcal{U}\mathcal{Q} + 2\overline{P}C\mathcal{U}J + \left(1 - \frac{\pi^2}{4}\right)\mathcal{U}D + \mathcal{V} \right. \right. \\
&\quad + \Pi_{\max}(\mathcal{U})\delta_1 + \chi_1\mathcal{W}_1 + \overline{P}\mathcal{U}A\mathcal{W}_1^{-1}A^T\mathcal{U}\overline{P} + \overline{P}\mathcal{U}B\mathcal{W}_2^{-1}B^T\mathcal{U}\overline{P} \\
&\quad + \overline{P}C\mathcal{U}J\mathcal{W}_3^{-1}J^T\mathcal{U}C^T\overline{P} + \overline{P}\mathcal{U}\mathcal{W}_4^{-1}\mathcal{U}\overline{P} \left. \right] \mathfrak{R}(t, \varrho) - \mathfrak{R}^T(t, \varrho)\mathcal{U}D\mathfrak{R}(t, \varrho) \\
&\quad + \mathfrak{R}^T(t, \varrho) \left(1 - \frac{\pi^2}{4}\right)\mathcal{U}D\mathfrak{R}(t, \varrho) - \mathfrak{R}^T(t, \varrho)\mathcal{U}D\mathfrak{R}(t, \varrho) + \mathfrak{R}^T(t - \zeta, \varrho) \\
&\quad \times \left[-\mathcal{V} + \chi_2\mathcal{W}_2 + \Pi_{\max}(\mathcal{U})\delta_2 \right] \mathfrak{R}(t - \zeta, \varrho) \Big\} d\varrho + \Pi_{\max}(\mathcal{W}_3)\|\mathfrak{Z}(t, \varrho)\|^2 \\
&\quad + \Pi_{\max}(\mathcal{W}_4)\|\phi(t, \varrho)\|^2 + \Pi_{\max}(\mathcal{U}D)\|\psi(t)\|^2. \tag{20}
\end{aligned}$$

By the definition of t_{q+1} , since $\mathfrak{R}(t, \varrho) \leq 0$ for $t \in [t_q, t_{q+1})$. That is,

$$\mathfrak{R}(t, \varrho) = \|\mathfrak{Z}(t, \varrho)\|^2 - \varepsilon_1\|\mathfrak{R}(t_q, \varrho)\|^2 - \varepsilon_2 \leq 0,$$

and so

$$\begin{aligned}
\|\mathfrak{Z}(t, \varrho)\|^2 &\leq \varepsilon_1\|\mathfrak{R}(t_q, \varrho)\|^2 + \varepsilon_2 \\
&\leq \varepsilon_1\|\mathfrak{Z}(t, \varrho) + \mathfrak{R}(t, \varrho)\|^2 + \varepsilon_2 \\
&\leq \varepsilon_1 \left(\frac{\|\mathfrak{Z}(t, \varrho)\|^2}{1 - \tau} + \frac{\|\mathfrak{R}(t, \varrho)\|^2}{\tau} \right) + \varepsilon_2 \\
\|\mathfrak{Z}(t, \varrho)\|^2 &\leq \frac{(1 - \tau)\varepsilon_1}{\tau(1 - \tau - \varepsilon_1)}\|\mathfrak{R}(t, \varrho)\|^2 + \frac{(1 - \tau)\varepsilon_2}{1 - \tau - \varepsilon_1}. \tag{21}
\end{aligned}$$

From the inequalities (20) and (21), we get

$$\begin{aligned}
\mathcal{LV}(\mathfrak{R}(t, \varrho)) &\leq \int_0^1 \xi^T(t, \varrho)\Omega\xi(t, \varrho)d\varrho + \Pi_{\max}(\mathcal{W}_3)\frac{(1 - \tau)\varepsilon_2}{1 - \tau - \varepsilon_1} \\
&\quad + \Pi_{\max}(\mathcal{W}_4)\|\phi(t, \varrho)\|^2 + \Pi_{\max}(\mathcal{U}D)\|\psi(t)\|^2, \tag{22}
\end{aligned}$$

where

$$\xi^T(t, \varrho) = \begin{bmatrix} \mathfrak{R}^T(t, \varrho) & \mathfrak{R}^T(t, \varrho) & \mathfrak{R}^T(t - \zeta, \varrho) \end{bmatrix}.$$

Let $b = \Pi_{\min}(-\Omega)$. Since $\Omega < 0$, we have $b > 0$. Thus, we obtain that

$$\begin{aligned}
\mathcal{LV}(\mathfrak{R}(t, \varrho)) &\leq -b\|\mathfrak{R}(t, \varrho)\|^2 + \Pi_{\max}(\mathcal{W}_3)\frac{(1 - \tau)\varepsilon_2}{1 - \tau - \varepsilon_1} \\
&\quad + \Pi_{\max}(\mathcal{W}_4)\|\phi(t, \varrho)\|^2 + \Pi_{\max}(\mathcal{U}D)\|\psi(t)\|^2. \tag{23}
\end{aligned}$$

From the LKF (8), we get

$$\Pi_{\min}(\mathcal{U})\|\mathfrak{R}(t, \varrho)\|^2 \leq V(\mathfrak{R}(t, \varrho)) \leq \Pi_{\max}(\mathcal{U})\|\mathfrak{R}(t, \varrho)\|^2$$

$$+ \Pi_{\max}(\mathcal{V}) \int_{t-\zeta}^t \|\mathfrak{R}(\ell, \varrho)\|^2 d\ell. \quad (24)$$

By using Theorem 1 in [43], there exist a unique constant $\mu > 0$ such that

$$\mu \Pi_{\max}(\mathcal{U}) + \mu \Pi_{\max}(\mathcal{V}) \zeta e^{\mu\zeta} = b. \quad (25)$$

By virtue of Dynkin formula and inequality (23), we obtain

$$\begin{aligned} \mathbb{E} e^{\mu t} V(\mathfrak{R}(t, \varrho)) - \mathbb{E} V(\mathfrak{R}(0, \varrho)) &= \mathbb{E} \int_0^t \mathcal{L}(e^{\mu\ell} V(\mathfrak{R}(\ell, \varrho))) d\ell \\ &= \mathbb{E} \int_0^t e^{\mu\ell} (\mu V(\mathfrak{R}(\ell, \varrho)) + \mathcal{L}V(\mathfrak{R}(\ell, \varrho))) d\ell \\ &\leq \mathbb{E} \int_0^t e^{\mu\ell} [\mu \Pi_{\max}(\mathcal{U}) \|\mathfrak{R}(\ell, \varrho)\|^2 \\ &\quad + \mu \Pi_{\max}(\mathcal{V}) \int_{\ell-\zeta}^{\ell} \|\mathfrak{R}(z, \varrho)\|^2 dz \\ &\quad - b \|\mathfrak{R}(\ell, \varrho)\|^2 + \Pi_{\max}(\mathcal{W}_3) \frac{(1-\tau)\varepsilon_2}{1-\tau-\varepsilon_1} \\ &\quad + \Pi_{\max}(\mathcal{W}_4) \|\phi(t, \varrho)\|^2 + \Pi_{\max}(\mathcal{UD}) \|\psi(t)\|^2] d\ell \\ &\leq \mathbb{E} \int_0^t e^{\mu\ell} [\mu \Pi_{\max}(\mathcal{U}) + \mu \Pi_{\max}(\mathcal{V}) \zeta e^{\mu\zeta} - b] \\ &\quad \times \|\mathfrak{R}(\ell, \varrho)\|^2 d\ell + \Pi_{\max}(\mathcal{V}) \mu \zeta e^{\mu\zeta} \int_{-\zeta}^0 \mathbb{E} e^{\mu\ell} \\ &\quad \times \|\mathfrak{R}(\ell, \varrho)\|^2 d\ell + \int_0^t e^{\mu\ell} \Pi_{\max}(\mathcal{W}_3) \frac{(1-\tau)\varepsilon_2}{1-\tau-\varepsilon_1} d\ell \\ &\quad + \int_0^t e^{\mu\ell} \Pi_{\max}(\mathcal{W}_4) \|\phi(t, \varrho)\|^2 d\ell \\ &\quad + \int_0^t e^{\mu\ell} \Pi_{\max}(\mathcal{UD}) \|\psi(t)\|^2 d\ell \\ &\leq \Pi_{\max}(\mathcal{V}) \mu \zeta^2 e^{\mu\zeta} \sup_{-\zeta \leq z \leq 0} \mathbb{E} \|\varpi(z, \varrho)\|^2 \\ &\quad + \frac{(1-\tau)\varepsilon_2}{1-\tau-\varepsilon_1} \Pi_{\max}(\mathcal{W}_3) \frac{1}{\mu} (e^{\mu t} - 1) \\ &\quad + \Pi_{\max}(\mathcal{W}_4) \|\phi(t, \varrho)\|_{\infty}^2 \frac{1}{\mu} (e^{\mu t} - 1) \\ &\quad + \Pi_{\max}(\mathcal{UD}) \|\psi(t)\|_{\infty}^2 \frac{1}{\mu} (e^{\mu t} - 1). \end{aligned} \quad (26)$$

From the inequalities (24)–(26), we have

$$\begin{aligned}
\Pi_{\min}(\mathcal{U})e^{\mu t}\mathbb{E}\|\mathfrak{R}(t, \varrho)\|^2 &\leq \Pi_{\max}(\mathcal{U})\mathbb{E}\|\mathfrak{R}(0, \varrho)\|^2 \\
&\quad + \Pi_{\max}(\mathcal{V})\mu\zeta^2e^{\mu\zeta}\sup_{-\zeta\leq z\leq 0}\mathbb{E}\|\varpi(z, \varrho)\|^2 \\
&\quad + \frac{(1-\tau)\varepsilon_2}{1-\tau-\varepsilon_1}\Pi_{\max}(\mathcal{W}_3)\frac{1}{\mu}(e^{\mu t}-1) \\
&\quad + \Pi_{\max}(\mathcal{W}_4)\|\phi(t, \varrho)\|_\infty^2\frac{1}{\mu}(e^{\mu t}-1) \\
&\quad + \Pi_{\max}(\mathcal{UD})\|\psi(t)\|_\infty^2\frac{1}{\mu}(e^{\mu t}-1) \\
&\leq \left[\Pi_{\max}(\mathcal{U}) + \Pi_{\max}(\mathcal{V})\mu\zeta^2e^{\mu\zeta} \right] \sup_{-\zeta\leq z\leq 0}\mathbb{E}\|\varpi(z, \varrho)\|^2 \\
&\quad + \frac{(1-\tau)\varepsilon_2}{1-\tau-\varepsilon_1}\Pi_{\max}(\mathcal{W}_3)\frac{1}{\mu}(e^{\mu t}-1) \\
&\quad + \Pi_{\max}(\mathcal{W}_4)\|\phi(t, \varrho)\|_\infty^2\frac{1}{\mu}(e^{\mu t}-1) \\
&\quad + \Pi_{\max}(\mathcal{UD})\|\psi(t)\|_\infty^2\frac{1}{\mu}(e^{\mu t}-1). \tag{27}
\end{aligned}$$

By virtue of (27), we obtain

$$\begin{aligned}
\mathbb{E}\|\mathfrak{R}(t, \varrho)\|^2 &\leq \frac{1}{\Pi_{\min}(\mathcal{U})} \left[\Pi_{\max}(\mathcal{U}) + \Pi_{\max}(\mathcal{V})\mu\zeta^2e^{\mu\zeta} \right] e^{-\mu t} \sup_{-\zeta\leq z\leq 0}\mathbb{E}\|\varpi(z, \varrho)\|^2 \\
&\quad + \frac{\Pi_{\max}(\mathcal{W}_4)}{\Pi_{\min}(\mathcal{U})\mu}\|\phi(t, \varrho)\|_\infty^2 + \frac{\Pi_{\max}(\mathcal{UD})}{\Pi_{\min}(\mathcal{U})\mu}\|\psi(t)\|^2 \\
&\quad + \frac{(1-\tau)\varepsilon_2\Pi_{\max}(\mathcal{W}_3)}{(1-\tau-\varepsilon_1)\Pi_{\min}(\mathcal{U})\mu} \\
&= \lambda e^{-\mu t} \sup_{-\zeta\leq z\leq 0}\mathbb{E}\|\varpi(z, \varrho)\|^2 + \eta_1\|\phi(t, \varrho)\|_\infty^2 + \eta_2\|\psi(t)\|_\infty^2 + \theta, \tag{28}
\end{aligned}$$

where

$$\begin{aligned}
\lambda &= \frac{1}{\Pi_{\min}(\mathcal{U})} [\Pi_{\max}(\mathcal{U}) + \Pi_{\max}(\mathcal{V})\mu\zeta^2e^{\mu\zeta}], \quad \eta_1 = \frac{\Pi_{\max}(\mathcal{W}_4)}{\Pi_{\min}(\mathcal{U})\mu}, \\
\eta_2 &= \frac{\Pi_{\max}(\mathcal{UD})}{\Pi_{\min}(\mathcal{U})\mu}, \quad \theta = \frac{(1-\tau)\varepsilon_2\Pi_{\max}(\mathcal{W}_3)}{(1-\tau-\varepsilon_1)\Pi_{\min}(\mathcal{U})\mu}.
\end{aligned}$$

The proof is completed.

The next theorem is control gain matrix designed to obtain the practically exponential input-to-state stabilization of system (5).

Theorem 2 Suppose that Assumptions $(\mathcal{H}_1) - (\mathcal{H}_4)$ holds, and the event-triggered parameters $\varepsilon_1, \varepsilon_2 \in \mathbb{R}_+$ satisfy $0 \leq \varepsilon_1 < 1 - \tau$, $\varepsilon_2 \geq 0$, and $0 < \tau < 1$, the system (5) is practically exponentially input-to-state stabilizable if there exist positive definite matrices $\mathcal{U}, \mathcal{V}, \mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \mathcal{W}_4$, and a constant matrix K such that the following LMI feasible for

$$(ii) \quad \mathcal{U} = \begin{bmatrix} \mathcal{U}_{11} & -\mathcal{U}D & 0 & \overline{\mathcal{P}}\mathcal{U}A & \overline{\mathcal{P}}\mathcal{U}B & \overline{\mathcal{P}}\mathcal{C}K & \overline{\mathcal{P}}\mathcal{U} \\ * & \mathcal{U}_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & \mathcal{U}_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -\mathcal{W}_1 & 0 & 0 & 0 \\ * & * & * & * & -\mathcal{W}_2 & 0 & 0 \\ * & * & * & * & * & -\mathcal{W}_3 & 0 \\ * & * & * & * & * & * & -\mathcal{W}_4 \end{bmatrix} < 0, \quad (29)$$

where

$$\begin{aligned} \mathcal{U}_{11} &= \text{sym}(\overline{\mathcal{P}}\mathcal{C}K - \mathcal{P}\mathcal{U}\mathcal{Q}) + \left(1 - \frac{\pi^2}{4}\right)\mathcal{U}D + \mathcal{V} + \Pi_{\max}(\mathcal{U})\delta_1 + \chi_1\mathcal{W}_1 \\ &+ \Pi_{\max}(\mathcal{W}_3)\frac{(1-\tau)\varepsilon_1}{\tau(1-\tau-\varepsilon_1)}, \quad \mathcal{U}_{22} = \left(1 - \frac{\pi^2}{4}\right)\mathcal{U}D, \quad \mathcal{U}_{33} = -\mathcal{V} + \chi_2\mathcal{W}_2 + \Pi_{\max}(\mathcal{U})\delta_2. \end{aligned}$$

Moreover, the control gain matrix J is defined by

$$(iii) \quad J = K\mathcal{U}^{-1}. \quad (30)$$

Proof: Clearly, the proof of this theorem follows from Theorem 1 and Lemma 3.

Remark 1 Compared with the existing results [43, 51–53], the following are the key elements and advantages of this paper:

- We introduce the reaction-diffusion terms to analyze its dynamic behaviors. This gives our research findings more practical significance.
- In this paper, the designed controller are simpler and more powerful than.
- In Theorem 1 and Theorem 2, new sufficient criterions of SRDDCGNNs are obtained by construct a suitable LKF, and using the Neumann boundary condition, Wirtinger's inequality, event-triggered control mechanisms, and LMI approach to guarantee PEISS and practically exponential input-to-state stabilization, respectively. These stability criterions are formulated without algebraic conditions, which can be lead to less conservative results.

Remark 2 In [17–19], the authors studied the stabilization of CGNNs without stochastic disturbance and diffusion effects. In fact, noise was a major problem in the way information was sent, and it affected every part of how the neuron systems worked. In NNs, diffusion effects are essentially inevitable because electrons move through nonuniform electromagnetic fields. Thus, the introduction of stochastic disturbance and diffusion effects into the CGNNs. It is more suitable for practical applications such as image encryption, Alzheimer's disease, and COVID-19.

Remark 3 The main results in this paper are more general than the results obtained in [37–42]. In [37–41], the authors investigated the EISS of CGNNs. In [42], the

author investigated the PES of CGNNs. The preceding results only addressed system stability, not system stabilization. In this paper, we investigate the practically exponential input-to-state stabilization of SRDDCGNNs with distributed and boundary external disturbances via event-triggered control.

Remark 4 If the distributed external disturbance $\phi(t, \varrho) = 0$, and boundary external disturbance $\psi(t) = 0$ in the system (5), then the corresponding system can be rewritten as follows:

$$\begin{cases} d\mathfrak{R}(t, \varrho) = \left[D \frac{\partial^2 \mathfrak{R}(t, \varrho)}{\partial \varrho^2} - \alpha(\mathfrak{R}(t, \varrho)) \left(\beta(\mathfrak{R}(t, \varrho)) - Af(\mathfrak{R}(t, \varrho)) \right. \right. \\ \quad \left. \left. - Bg(\mathfrak{R}(t - \zeta, \varrho)) - CJ(\mathfrak{I}(t, \varrho) + \mathfrak{R}(t, \varrho)) \right) \right] dt \\ \quad + \sigma(\mathfrak{R}(t, \varrho), \mathfrak{R}(t - \zeta, \varrho)) d\omega(t), \\ \mathfrak{R}(z, \varrho) = \varpi(z, \varrho), \quad \varrho \in (0, 1), \quad z \in [-\zeta, 0], \\ \mathfrak{R}(t, 0) = 0, \quad \frac{\partial \mathfrak{R}(t, \varrho)}{\partial \varrho} \big|_{\varrho=1} = 0. \end{cases} \quad (31)$$

In Theorem 2, we have following corollary.

Corollary 1 *Suppose that Assumptions $(\mathcal{H}_1) - (\mathcal{H}_4)$ holds, and the event-triggered parameters $\varepsilon_1, \varepsilon_2 \in \mathbb{R}_+$ satisfy $0 \leq \varepsilon_1 < 1 - \tau$, $\varepsilon_2 \geq 0$, and $0 < \tau < 1$, the system (31) is said to be practically exponentially stabilizable if there exist positive definite matrices $\mathcal{U}, \mathcal{V}, \mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3$, and a constant matrix K such that the following LMI feasible for*

$$(iv) \quad \Phi = \begin{bmatrix} \Phi_{11} & 0 & 0 & \overline{\mathcal{P}}\mathcal{U}\mathcal{A} & \overline{\mathcal{P}}\mathcal{U}\mathcal{B} & \overline{\mathcal{P}}\mathcal{C}K \\ * & -\frac{\pi^2}{4}\mathcal{U}D & 0 & 0 & 0 & 0 \\ * & * & -\mathcal{V} + \chi_2\mathcal{W}_2 + \Pi_{\max}(\mathcal{U})\delta_2 & 0 & 0 & 0 \\ * & * & * & -\mathcal{W}_1 & 0 & 0 \\ * & * & * & * & -\mathcal{W}_2 & 0 \\ * & * & * & * & * & -\mathcal{W}_3 \end{bmatrix} < 0, \quad (32)$$

where

$$\Phi_{11} = \text{sym}(\overline{\mathcal{P}}\mathcal{C}K - \underline{\mathcal{P}}\mathcal{U}\mathcal{Q}) - \frac{\pi^2}{4}\mathcal{U}D + \mathcal{V} + \Pi_{\max}(\mathcal{U})\delta_1 + \chi_1\mathcal{W}_1 + \Pi_{\max}(\mathcal{W}_3) \frac{(1 - \tau)\varepsilon_1}{\tau(1 - \tau - \varepsilon_1)}.$$

Moreover, the control gain matrix J is defined by (30).

3.2 Practically exponential input-to-state stabilization of SRDDCNNs

Let $\alpha(\mathfrak{R}(t, \varrho)) = 1$ and $\beta(\mathfrak{R}(t, \varrho)) = F\mathfrak{R}(t, \varrho)$ in system (5). Then, the system (5) turns out to be the following SRDDCNNs with distributed and boundary external disturbances:

$$\begin{cases} d\mathfrak{R}(t, \varrho) = \left[D \frac{\partial^2 \mathfrak{R}(t, \varrho)}{\partial \varrho^2} - F\mathfrak{R}(t, \varrho) + Af(\mathfrak{R}(t, \varrho)) + Bg(\mathfrak{R}(t - \zeta, \varrho)) \right. \\ \quad \left. + CJ(\mathfrak{I}(t, \varrho) + \mathfrak{R}(t, \varrho)) + \phi(t, \varrho) \right] dt \\ \quad + \sigma(\mathfrak{R}(t, \varrho), \mathfrak{R}(t - \zeta, \varrho)) d\omega(t), \\ \mathfrak{R}(z, \varrho) = \varpi(z, \varrho), \quad \varrho \in (0, 1), \quad z \in [-\zeta, 0], \\ \mathfrak{R}(t, 0) = 0, \quad \frac{\partial \mathfrak{R}(t, \varrho)}{\partial \varrho} \big|_{\varrho=1} = \psi(t). \end{cases} \quad (33)$$

Corollary 2 Suppose that Assumptions $(\mathcal{H}_3) - (\mathcal{H}_4)$ holds, and the event-triggered parameters $\varepsilon_1, \varepsilon_2 \in \mathbb{R}_+$ satisfy $0 \leq \varepsilon_1 < 1 - \tau$, $\varepsilon_2 \geq 0$, and $0 < \tau < 1$, the system (33) is practically exponentially input-to-state stabilizable if there exist positive definite matrices $\mathcal{U}, \mathcal{V}, \mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \mathcal{W}_4$, and a constant matrix K such that the following LMI feasible for

$$(v) \quad \Xi = \begin{bmatrix} \Xi_{11} & -\mathcal{U}D & 0 & \mathcal{U}A & \mathcal{U}B & CK & \mathcal{U} \\ * & \Xi_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -\mathcal{W}_1 & 0 & 0 & 0 \\ * & * & * & * & -\mathcal{W}_2 & 0 & 0 \\ * & * & * & * & * & -\mathcal{W}_3 & 0 \\ * & * & * & * & * & * & -\mathcal{W}_4 \end{bmatrix} < 0, \quad (34)$$

where

$$\begin{aligned} \Xi_{11} &= \text{sym}(CK - \mathcal{U}F) + \left(1 - \frac{\pi^2}{4}\right)\mathcal{U}D + \mathcal{V} + \Pi_{\max}(\mathcal{U})\delta_1 + \chi_1\mathcal{W}_1 \\ &+ \Pi_{\max}(\mathcal{W}_3)\frac{(1-\tau)\varepsilon_1}{\tau(1-\tau-\varepsilon_1)}, \quad \Xi_{22} = \left(1 - \frac{\pi^2}{4}\right)\mathcal{U}D, \quad \Xi_{33} = -\mathcal{V} + \chi_2\mathcal{W}_2 + \Pi_{\max}(\mathcal{U})\delta_2. \end{aligned}$$

Moreover, the control gain matrix J is defined by (30).

3.3 Practically exponential input-to-state stabilization of SRDDHNNs

Let $A = 0$ in system (33). Then, the system (33) turns out to be the following SRDDHNNs with distributed and boundary external disturbances:

$$\begin{cases} d\mathfrak{R}(t, \varrho) = \left[D \frac{\partial^2 \mathfrak{R}(t, \varrho)}{\partial \varrho^2} - F\mathfrak{R}(t, \varrho) + Bg(\mathfrak{R}(t - \zeta, \varrho)) + CJ(\mathfrak{Z}(t, \varrho) \right. \\ \quad \left. + \mathfrak{R}(t, \varrho) + \phi(t, \varrho) \right] dt + \sigma(\mathfrak{R}(t, \varrho), \mathfrak{R}(t - \zeta, \varrho)) d\omega(t), \\ \mathfrak{R}(z, \varrho) = \varpi(z, \varrho), \quad \varrho \in (0, 1), \quad z \in [-\zeta, 0], \\ \mathfrak{R}(t, 0) = 0, \quad \frac{\partial \mathfrak{R}(t, \varrho)}{\partial \varrho} \big|_{\varrho=1} = \psi(t). \end{cases} \quad (35)$$

In Corollary 2, we have following corollary.

Corollary 3 Suppose that Assumptions $(\mathcal{H}_3) - (\mathcal{H}_4)$ holds, and the event-triggered parameters $\varepsilon_1, \varepsilon_2 \in \mathbb{R}_+$ satisfy $0 \leq \varepsilon_1 < 1 - \tau$, $\varepsilon_2 \geq 0$, and $0 < \tau < 1$, the system (35) is practically exponentially input-to-state stabilizable if there exist positive definite matrices $\mathcal{U}, \mathcal{V}, \mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \mathcal{W}_4$, and a constant matrix K such that the following LMI feasible for

$$(vi) \quad \Delta = \begin{bmatrix} \Delta_{11} & -\mathcal{U}D & 0 & \mathcal{U}B & CK & \mathcal{U} \\ * & \Delta_{22} & 0 & 0 & 0 & 0 \\ * & * & \Delta_{33} & 0 & 0 & 0 \\ * & * & * & -\mathcal{W}_2 & 0 & 0 \\ * & * & * & * & -\mathcal{W}_3 & 0 \\ * & * & * & * & * & -\mathcal{W}_4 \end{bmatrix} < 0, \quad (36)$$

where

$$\Delta_{11} = \text{sym}(CK - \mathcal{U}F) + \left(1 - \frac{\pi^2}{4}\right)\mathcal{U}D + \mathcal{V} + \Pi_{\max}(\mathcal{U})\delta_1 + \Pi_{\max}(\mathcal{W}_3)\frac{(1-\tau)\varepsilon_1}{\tau(1-\tau-\varepsilon_1)},$$

$$\Delta_{22} = \left(1 - \frac{\pi^2}{4}\right)\mathcal{U}D, \quad \Delta_{33} = -\mathcal{V} + \chi_2\mathcal{W}_2 + \Pi_{\max}(\mathcal{U})\delta_2.$$

Moreover, the control gain matrix J is defined by (30).

4 Numerical Simulation

In this section, two numerical examples are presented to show how well the event-triggered controller works.

Example 1 Consider the following 3-dimensional SRDDCGNNs with distributed and boundary external disturbances:

$$\begin{cases} d\mathfrak{R}(t, \varrho) = \left[D \frac{\partial^2 \mathfrak{R}(t, \varrho)}{\partial \varrho^2} - \mathfrak{R}(t, \varrho) \left(3\mathfrak{R}(t, \varrho) - 2A \tanh(\mathfrak{R}(t, \varrho)) \right. \right. \\ \quad \left. \left. - B \tanh(\mathfrak{R}(t - 0.5, \varrho)) - Cu(t, \varrho) - 0.2 \sin(t) \cos(\varrho) \right) \right] dt \\ \quad + [0.2\mathfrak{R}(t, \varrho) + 0.3\mathfrak{R}(t - 0.5, \varrho)] d\omega(t), \\ \mathfrak{R}(z, \varrho) = \varpi(z, \varrho), \quad \varrho \in (0, 1), \quad z \in [-0.5, 0], \\ \mathfrak{R}(t, 0) = 0, \quad \frac{\partial \mathfrak{R}(t, \varrho)}{\partial \varrho} \big|_{\varrho=1} = 0.2 \cos(t), \end{cases} \quad (37)$$

where

$$D = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad A = \begin{bmatrix} 3.5 & 5.5 & -2.5 \\ 2.5 & 3.2 & -4.2 \\ 2.1 & 1.1 & 3.3 \end{bmatrix},$$

$$B = \begin{bmatrix} 4.5 & 2.6 & -3.5 \\ 2.8 & 1.5 & -4.5 \\ 5.1 & 2.1 & 4.2 \end{bmatrix}, \quad C = \begin{bmatrix} 2.5 & 1.6 & 2.5 \\ 1.4 & 2.5 & 3.5 \\ 1.1 & 2.5 & 3.5 \end{bmatrix}.$$

The initial values of (37) are

$$\begin{cases} \varpi_1(z, \varrho) = 0.4 \sin(z) + 0.4 \sin(0.4\pi\varrho), \quad z \in [-0.5, 0], \\ \varpi_2(z, \varrho) = 0.2 \sin(z) + 0.3 \sin(0.9\pi\varrho), \quad z \in [-0.5, 0], \\ \varpi_3(z, \varrho) = 0.3 \sin(z) + 0.4 \sin(0.6\pi\varrho), \quad z \in [-0.5, 0]. \end{cases}$$

By virtue of Theorem 2, solve LMI (29) with the MATLAB LMI toolbox, and the feasible solutions are obtained as follows:

$$\mathcal{U} = \begin{bmatrix} 1.0971 & -1.4402 & -0.6595 \\ -1.4402 & 1.3108 & -0.4440 \\ -0.6595 & -0.4440 & 0.3370 \end{bmatrix}, \quad \mathcal{V} = \begin{bmatrix} 15.3418 & 0.5919 & -1.0466 \\ 0.5919 & 15.9819 & 0.7796 \\ -1.0466 & 0.7796 & 16.2925 \end{bmatrix},$$

$$\mathcal{W}_1 = \begin{bmatrix} 5.0941 & -1.3161 & -0.0415 \\ -1.3161 & 5.6957 & -0.6536 \\ -0.0415 & -0.6536 & 6.0423 \end{bmatrix}, \quad \mathcal{W}_2 = \begin{bmatrix} 8.4699 & 0.8105 & -1.1363 \\ 0.8105 & 9.2787 & 0.9399 \\ -1.1363 & 0.9399 & 9.7374 \end{bmatrix},$$

$$\mathcal{W}_3 = \begin{bmatrix} 13.0513 & 3.2221 & 2.4566 \\ 3.2221 & 11.2726 & 2.8923 \\ 2.4566 & 2.8923 & 11.7852 \end{bmatrix}, \quad \mathcal{W}_4 = \begin{bmatrix} 14.7244 & 3.6218 & 1.7195 \\ 3.6218 & 15.0787 & -1.5598 \\ 1.7195 & -1.5598 & 16.3115 \end{bmatrix},$$

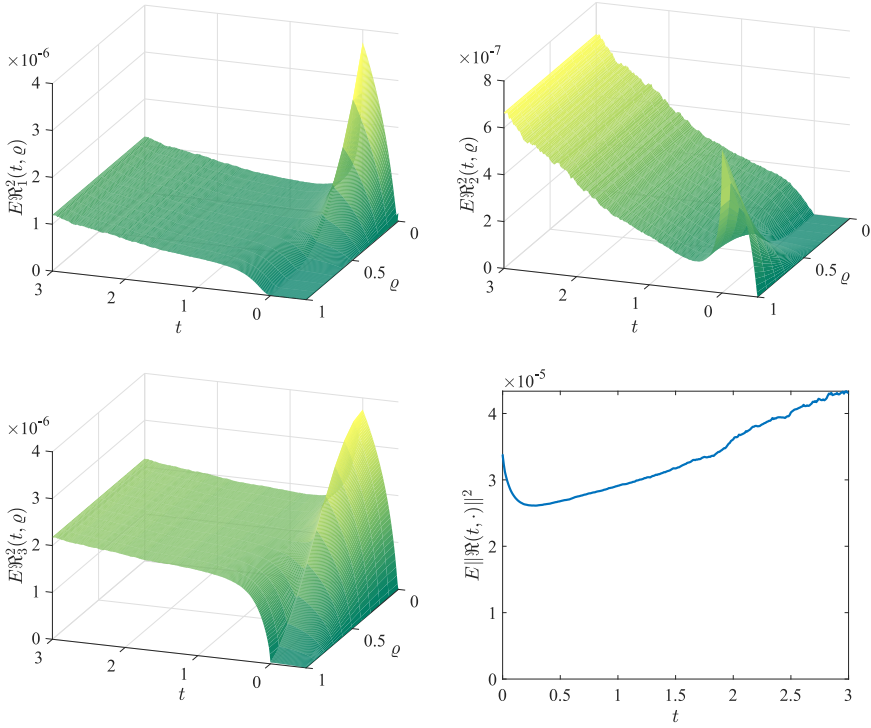


Fig. 1 State responses $\mathbb{E}\mathfrak{R}^2(t, \varrho)$ and state norm $\mathbb{E}||\mathfrak{R}(t, \cdot)||^2$ of system (37) without control

$$K = \begin{bmatrix} -4.2015 & -1.2587 & 1.3441 \\ -1.2587 & 0.0082 & -0.8091 \\ 1.3441 & -0.8091 & -1.7524 \end{bmatrix}.$$

Furthermore, the control gain matrix J is obtained as follows:

$$J = \begin{bmatrix} -0.0076 & 0.6812 & 4.8708 \\ 0.8233 & 1.1618 & 0.7411 \\ 1.5922 & 0.7696 & -1.0701 \end{bmatrix}. \quad (38)$$

The state responses $\mathbb{E}\mathfrak{R}^2(t, \varrho)$ and state norm $\mathbb{E}||\mathfrak{R}(t, \cdot)||^2$ of system (37) without control are depicted in Fig. 1. It is clearly shown that the system (37) does not realize stabilization without control. Under the event-triggered controller (2) and control gain matrix (38), Fig. 2 shows that the event-triggered controller (2) can guarantee practically exponential input-to-state stabilization of the system (37). Thus, our proposed event-triggered controller is effective.

Example 2 Consider the following 3-dimensional SRDDCNNs with distributed and boundary external disturbances:

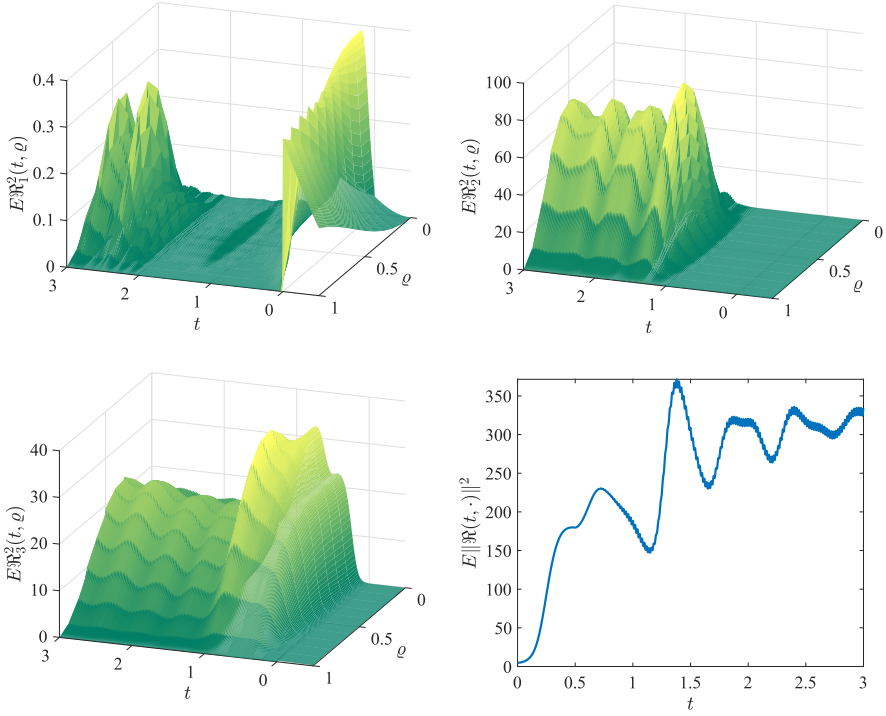


Fig. 2 State responses $\mathbb{E}\mathbb{R}^2(t, \varrho)$ and state norm $\mathbb{E}\|\mathbb{R}(t, \cdot)\|^2$ of system (37) with event-triggered controller (2)

$$\begin{cases} d\mathbb{R}(t, \varrho) = \left[D \frac{\partial^2 \mathbb{R}(t, \varrho)}{\partial \varrho^2} - F\mathbb{R}(t, \varrho) + 2A \tanh(\mathbb{R}(t, \varrho)) \right. \\ \quad \left. + B \tanh(\mathbb{R}(t - 1.5, \varrho)) + Cu(t, \varrho) + 0.15 \sin(t) \cos(\varrho) \right] dt \\ \quad + [0.2\mathbb{R}(t, \varrho) + 0.3\mathbb{R}(t - 1.5, \varrho)] d\omega(t), \\ \mathbb{R}(z, \varrho) = \varpi(z, \varrho), \quad \varrho \in (0, 1), \quad z \in [-1.5, 0], \\ \mathbb{R}(t, 0) = 0, \quad \frac{\partial \mathbb{R}(t, \varrho)}{\partial \varrho} \Big|_{\varrho=1} = 0.2 \cos(t), \end{cases} \quad (39)$$

where

$$D = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad F = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}, \quad A = \begin{bmatrix} 2.5 & 4.5 & -1.5 \\ 1.5 & 2.2 & -3.2 \\ 1.5 & 0.8 & 2.5 \end{bmatrix},$$

$$B = \begin{bmatrix} 4.5 & 2.6 & -3.5 \\ 2.8 & 1.5 & -4.5 \\ 2.1 & 1.1 & 3.5 \end{bmatrix}, \quad C = \begin{bmatrix} 1.1 & 1.3 & 1.5 \\ 0.4 & 0.5 & 1.1 \\ 0.9 & 0.5 & 0.7 \end{bmatrix}.$$

The initial values of (39) are

$$\begin{cases} \varpi_1(z, \varrho) = 0.5 \sin(z) + 0.5 \sin(0.5\pi\varrho), \quad z \in [-1.5, 0], \\ \varpi_2(z, \varrho) = 0.1 \sin(z) + 0.4 \sin(0.8\pi\varrho), \quad z \in [-1.5, 0], \\ \varpi_3(z, \varrho) = 0.5 \sin(z) + 0.6 \sin(0.9\pi\varrho), \quad z \in [-1.5, 0]. \end{cases}$$

By virtue of Corollary 2, solve LMI (34) with the MATLAB LMI toolbox, and the

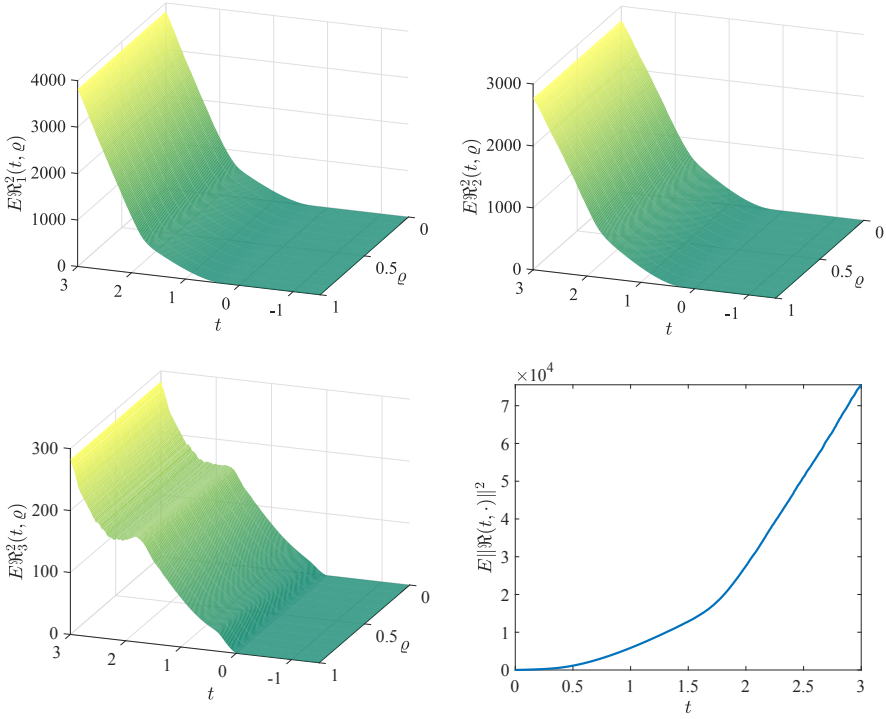


Fig. 3 State responses $\mathbb{E}\mathbb{R}^2(t, \varrho)$ and state norm $\mathbb{E}\|\mathbb{R}(t, \cdot)\|^2$ of system (39) without control

feasible solutions are obtained as follows:

$$\begin{aligned}
 \mathcal{U} &= \begin{bmatrix} 1.1153 & -1.4179 & -0.7106 \\ -1.4179 & 1.0248 & -0.6354 \\ -0.7106 & -0.6354 & 0.2996 \end{bmatrix}, \quad \mathcal{V} = \begin{bmatrix} 16.4835 & 0.6872 & -1.5768 \\ 0.6872 & 15.9922 & 0.2981 \\ -1.5768 & 0.2981 & 17.1515 \end{bmatrix}, \\
 \mathcal{W}_1 &= \begin{bmatrix} 4.2841 & -0.2275 & 0.6700 \\ -0.2275 & 5.0450 & -0.7077 \\ 0.6700 & -0.7077 & 5.8816 \end{bmatrix}, \quad \mathcal{W}_2 = \begin{bmatrix} 9.6055 & 0.8889 & -1.6958 \\ 0.8889 & 8.9933 & 0.4047 \\ -1.6958 & 0.4047 & 10.4564 \end{bmatrix}, \\
 \mathcal{W}_3 &= \begin{bmatrix} 11.6451 & 1.5904 & 1.4176 \\ 1.5904 & 9.8221 & 0.9108 \\ 1.4176 & 0.9108 & 10.3236 \end{bmatrix}, \quad \mathcal{W}_4 = \begin{bmatrix} 14.8356 & 1.9517 & 1.1434 \\ 1.9517 & 14.9079 & -1.6159 \\ 1.1434 & -1.6159 & 15.7424 \end{bmatrix}, \\
 K &= \begin{bmatrix} -0.9387 & 0.9700 & -3.5086 \\ 0.9700 & 0.5706 & -3.1165 \\ -3.5086 & -3.1165 & 2.7482 \end{bmatrix}.
 \end{aligned}$$

Furthermore, the control gain matrix J is obtained as follows:

$$J = \begin{bmatrix} 1.9039 & 2.7948 & -1.2675 \\ 1.8624 & 1.8325 & -2.0984 \\ -0.8317 & -0.8670 & 5.3620 \end{bmatrix}. \quad (40)$$

The state responses $\mathbb{E}\mathbb{R}^2(t, \varrho)$ and state norm $\mathbb{E}\|\mathbb{R}(t, \cdot)\|^2$ of system (39) without control are depicted in Fig. 3. It is clearly shown that the system (39) does not realize stabilization without control. Under the event-triggered controller (2) and control

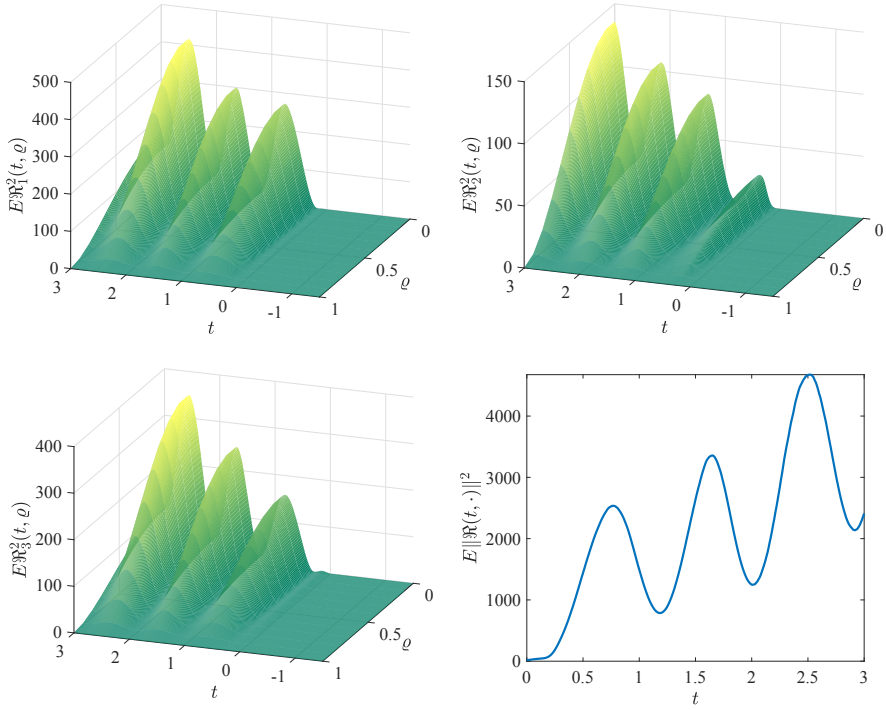


Fig. 4 State responses $\mathbb{E}\mathbb{R}^2(t, \varrho)$ and state norm $\mathbb{E}\|\mathbb{R}(t, \cdot)\|^2$ of system (39) with event-triggered controller (2)

gain matrix (40), Fig. 4 shows that the event-triggered controller (2) can guarantee practically exponential input-to-state stabilization of the system (39). Thus, our proposed event-triggered controller is effective.

5 Application

The application of chaotic systems to image encryption has become a popular research area in recent years [3–7]. This section will explore the implementation of chaotic SRDDHNNs through the secure transmission of cameraman image. The structure of the image encryption and decryption algorithm is shown in Fig. 5. Some security analysis are given to illustrate the efficiency of encryption system.

5.1 Key space analysis:

The encryption system is comprised of the following keys. (i) Initial and Neumann boundary conditions of SRDDHNNs; (ii) The parameters of event-triggered controller that changes in SRDDHNNs; (iii) The time delay that causes chaotic signals in the system; (iv) The number of chaotic SRDDHNNs iterations. An effective image encryption technique has a huge key space. In

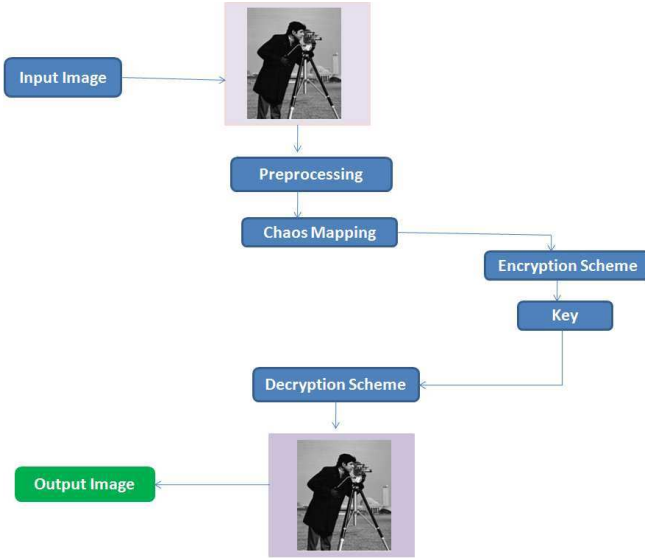


Fig. 5 Encryption and decryption algorithm

order to prevent brute-force attacks, the space of key should be at least 2^{128} [4]. Assume the precision of non integer key is considered as 10^{-14} . Thus, key space of this approach may be large as $10^{84} > 2^{128}$, which is enough to resist brute force attacks.

5.2 Histogram analysis:

The distribution characteristics of image pixel values are examined using histograms. In order to withstand various statistical attacks, the histogram of a well encrypted image must be flat and have an even distribution. Histograms of plain, encrypted, and decrypted images are depicted in Fig. 6. It shows that histogram of encrypted images are more evenly distributed than those of plain images, which is a good sign that they can resist statistical attacks.

5.3 Correlation analysis:

The plain image exhibits a high correlation between pixel location in three directions (horizontal, vertical, and diagonal directions). To make encryption technologies operate, the correlation between adjacent pixels must be destroyed. Fig. 7 depicts the distribution correlation of adjacent pixels in three directions of plain and encrypted images. To check the correlation of two adjacent pixels in an encrypted image, apply the following mathematical

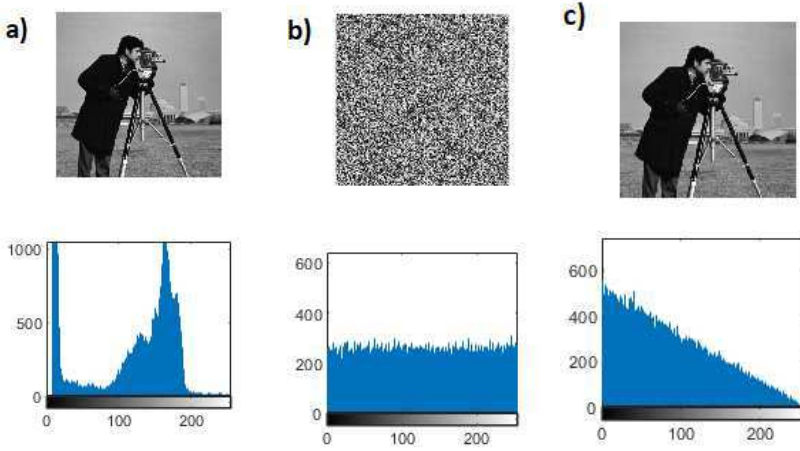


Fig. 6 Cameraman image: (a) plain image and its histogram, (b) encryption image and its histogram, (c) decryption image and its histogram.

Cameraman image	Horizontal	Vertical	Diagonal
Plain	0.9333	0.9569	0.9052
Encrypted (our scheme)	-0.0003	-0.0001	0.0010
Encrypted [3]	-0.0004	-0.0003	0.0030
Encrypted [4]	-0.0031	-0.0006	0.0011
Encrypted [5]	-0.0025	-0.0004	0.0035
Encrypted [6]	-0.0208	-0.0011	0.0323
Encrypted [7]	-0.0186	-0.0053	0.0095

Table 2 Correlation coefficients of pixels location of plain and encrypted image in three direction

expressions:

$$\left\{ \begin{array}{l} \bar{x} = \frac{1}{N} \sum_{r=1}^N x_r, \\ \mathcal{SD}(x) = \frac{1}{N} \sum_{r=1}^N (x_r - \bar{x})^2, \\ cov(x, y) = \frac{1}{N} \sum_{r=1}^N (x_r - \bar{x})(y_r - \bar{y}), \\ \varrho_{xy} = \frac{cov(x, y)}{\sqrt{\mathcal{SD}(x)}\sqrt{\mathcal{SD}(y)}}. \end{array} \right. \quad (41)$$

Thus, the results of correlation coefficients of plain and encrypted images in three direction are shown in Table 2. By virtue of Table 2 and Fig. 7, the correlation between the pixels location of the plain image are 0.9333, 0.9569 and 0.9052, and the correlation of pixels location of the encrypted image are -0.0003, -0.0001 and 0.0010, they are nearest to zero.

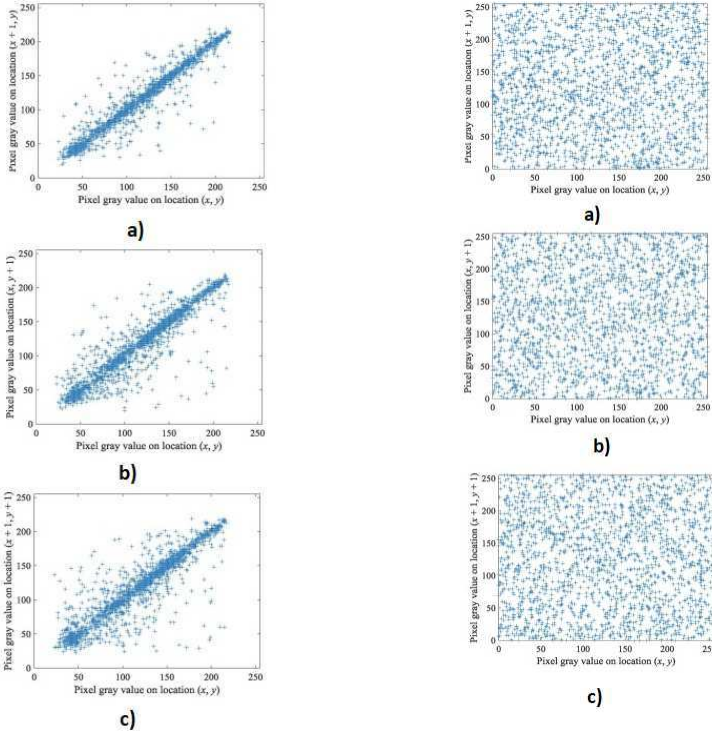


Fig. 7 Correlations of pixels location in three direction of plain and encryption image: (a) horizontal direction, (b) vertical direction, (c) diagonal direction.

6 Conclusion

This paper investigated the practically exponential input-to-state stabilization of stochastic Cohen-Grossberg neural networks, where reaction-diffusion, event-triggered controller, time delays, Neumann boundary condition, distributed and boundary external disturbances are considered. Then, the input-to-state stability criteria is obtained by virtue of suitable Lyapunov-Krasovskii functional, event-triggered mechanism, and some famous inequality techniques. It should be point out that the practically exponential input-to-state stabilization performance investigated through the control gain matrix for event-triggered controller. Furthermore, the obtained results are successfully applied to stochastic reaction-diffusion cellular neural networks and stochastic reaction-diffusion Hopfield neural networks. Finally, the numerical simulations are presented to show that the efficiency of the proposed event-triggered controller matches the theoretical results, and the proposed Hopfield neural networks are applied to image encryption. In the future, we will focus on the event-triggered boundary synchronization and stabilization problems of fractional-order stochastic Cohen-Grossberg neural networks with reaction-diffusion terms.

Acknowledgments. The authors have no conflicts of interest to declare that are relevant to the content of this paper.

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