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Bipartite Synchronization of Fractional Order Multiple Memristor Coupled Delayed Neural Networks with Event Triggered Pinning Control

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Abstract
This paper investigates the leader and leaderless bipartite synchronization with the signed network utilizing the model of multiple memristor and coupled delayed neural network in an event-triggered pinning control. The usage of the descriptor method in fractional-order neural networks in case of a non-differentiable delay can be seen in this paper. Further, Lyapunov functional criteria, including Lur'e Postnikov Lyapunov functional, is established, and bipartite leader and leaderless synchronization are proved. The obtained numerical results can be seen as accurate to the theoretical results.

Keywords Bipartite synchronization · Caputo derivative · Filippov sense · Memristor · Structurally balanced · Descriptor method · Lur'e Postnikov · Event triggered pinning control

Mathematics Subject Classification 34D06 · 93D20 · 93D15 · 37M99

1 Introduction

Fractional differential calculus (FDC) has more properties than ordinary differential calculus (ODC). The main attractive application of FDC is its weak singularity, degrees of freedom, non-locality, and infinite memory [1]. Taking arbitrary order in integrals and derivatives is an attractive usage of FDC and can be applied to various medical applications. There are many models of FDCs which are widely used in the fields of engineering and sciences. FDC plays

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an important role in biological sciences as well. It is used in analysis of breast cancer [2], tumor-immune system [3], diabetes [4], Nipah virus [5], and coronavirus [6], HIV model [7] and many more. Apart from this, applications of FDC can be seen in real-life science and engineering problems [8], fractional fractal system [9], the fractional ecological system [10], heat transfer, mechanics, wide range fields, electric circuits [11] and fractional partial differential equations [12, 13].

In the 17th century, Christiaan Huygens described the phenomenon of synchronization. Finding an oscillating object's rhythm from weak interaction is known as synchronization. It has been applied in pattern recognition, image processing, and object detection [14], references therein. There are many kinds of synchronization like mutual and high-order synchronization, Kuramoto self-synchronization, complete synchronization, weak and robust synchronization, global synchronization, quasi synchronization, sampled data synchronization, global non-fragile synchronization, bipartite synchronization and references therein [15–19]. Bipartite consensus means all the processing elements converge to a value and have the same modulus except for its sign [20, 21]. A network of structurally balanced signed graphs is known as a bipartite consensus [20]. If it is not structurally balanced, the network reaches an interval bipartite consensus [22]. This consensus of synchronization is said to be bipartite synchronization. Over the years, several results have been established in bipartite synchronization, bipartite synchronization of Lur'e network [23], fractional order bipartite fixed time synchronization [24], coupled delayed neural network (CDNN) of bipartite synchronization [19] and bipartite synchronization of memristor-based fractional-order CDNN [25].

Leon O. Chua postulated a new circuit element. The two-terminal device, obtained by applying the initial conditions of the pinched hysteresis loop in the voltage-current (v-i) plane powered by the bipolar periodic voltage, is called a memristor. It was an experimental observation in HP (Hewlett−Packard) laps. The main advantage of a memristor is non-volatile memory. It contains many applications, such as high density, lower power, ease of integration, nano-dimension and good scalability [26, 27]. The neural network is parallel distributed information to a processing structure. There are several types of neural networks, such as the stochastic neural network [28], Chebyshev neural network [10], BAM neural network [29], discontinuous neural network [18] and multiple memristor neural network [30]. Delay is unavoidable in neural networks, synchronization of memristor delayed neural network, synchronization of coupled delayed neural network (CDNN), synchronization of recurrent neural networks with time varying delay [31], memristor based inertial neural network with mixed time varying delay by using new non-reduced order method [27], switching law with time-varying delay [32], further refer [15, 19, 33].

Now, FDC-based neural networks get more attention from many authors; for example, fractional neural network (FNN) [24, 34–36], fractional memristor neural network (FMNN) [37], fractional memristor delayed neural network (FMDNN) [15], memristor-based fractional-order coupled delayed neural network (FCDNN) [25] are widely studied. Different kinds of controllers are used in ODC and FDC. Some of the widely used ones are pinning control [19, 23, 25], event triggered, periodic and self triggered controllers [35, 36, 38], event triggered pinning control [21], adaptive event triggered control [39], impulsive control [40–42], sampled data control [17, 23], terminal and quantized slide mode control [10, 31, 43], delay-dependent feedback controller [27] and distributed control [44]. The main reason for using pinning control is to reduce large network costs. It should be observed that continuous communication at every time may exist redundant and high energy consumption in control systems. It may be stopped when the event-triggered mechanism maintains the designed controller between two trigger instants. Mainly using event-triggered control reduced superfluous utilization [21]. periodic event-triggered of singular system with cyber-attacks refer [38], event triggered fractional-order multi-agent system [39], event triggered of multiple fractional order neural networks with time varying delays [35], event triggered bipartite synchronization of multi-order fractional neural networks [36]. The Laplace transform [2, 3, 6], the Lyapunov function [10, 16, 18, 37], and linear matrix inequality (LMI) [25, 34] are used in the study of FDC. Lyapunov function is classified as Lyapunov Razumikhin [40, 41] and Lyapunov krasovskii function (LKF) [21, 25, 28, 29, 45]. Moreover, a neuron's activation function applied in the Lyapunov function is the Lur'e-Postnikov Lyapunov function (LPLF) [18, 34]. It helps to reduce conservatism.

The essential novelties of this paper are the fractional order multiple memristor coupled delayed neural network (FMMCDNN) and the distributed event-triggered pinning control and descriptor method proposed by developing fractional order Jensen's inequality to achieve the bipartite leader and leaderless synchronization. Secondly, the bipartite leader and leaderless synchronization are proven for an FMMCDNN using LPLF with LKF.

Based on the above discussions, this paper's major contributions are as follows:

(i) The rarely sought descriptor method in FMMCDNN with LKF of delay-dependent stability criteria is discussed here.
(ii) This paper focuses on the bipartite synchronization of signed networks and FMMCDNN using an event-triggered pinning control with delay conditions,
(iii) we prove that the Zeno behavior is avoided in an event triggered pinning control,
(iv) we also prove the above three results in the case of an FMMCDNN with LPLF added to LKF.

We have formulated an FMMCDNN in Sect. 2, along with some necessary definitions. In Sect. 3, the formulation and leader bipartite synchronization of an event-triggered pinning control is discussed, and the leaderless bipartite synchronization can be seen in Sect. 4. Section 5 deals with the Lur'e Postnikov Lyapunov function criteria. And we can see numerical examples and the conclusion in Sects. 6 and 7, respectively.

Notations

sign$(\cdot)$ represent the signum or sign function, $\otimes$ means the Kronecker product, w_p denotes the gauge transformation, l_{pj} represents a Laplacian matrix, L^s and L^u are Laplacian of signed and unsigned matrices respectively, and $\bar{co}$ means convexity closure. $\varpi = \begin{pmatrix} P & Q \\ * & S \end{pmatrix}$, where $*$ represent transpose of Q and $\mathcal{G}^{SGN}$ denotes signed network.

2 Preliminaries

In this section, we introduce a few prerequisites for the results that follow.

Lemma 1 [46] *Let A be the M-matrix then all its eigenvalues will be positive and all the diagonal entries will be non-positive.*

Definition 1 [20] The $\mathcal{G}^{SGN}$ satisfies the following conditions

(i) Let V be the set of vertices of signed network $\mathcal{G}^{SGN}$, and $V_1 \cup V_2 = V$, $V_1 \cap V_2 = \emptyset$ for some $V_1, V_2 \in V$.
(ii) If the adjacency matrix a_{rs} is positive for all $V_r, V_s \in V_t$, $t \in \{1, 2\}$, and a_{rs} is negative for all $V_r \in V_t$, $V_s \in V_q$, $t \neq q$, where $t, q \in \{1, 2\}$, then the network is said to be structurally balanced. Otherwise, the $\mathcal{G}^{SGN}$ is called structurally unbalanced.

Lemma 2 [19, 23] *Let the Laplacian L^s be defined as $L^s = D\text{-}A^s$ of the $\mathcal{G}^{SGN}$ such that $l^u_{ij} = W l^s_{ij} W = -|a^s_{ij}|$ for $i \neq j$ and $l^u_{ii} = \sum_{k=1,k\neq i}^{N} |a^s_{ij}|$, holds the above conditions then the H-matrix is defined as,*

$$H = (h_{ij}) = L^u + D, \tag{1}$$

here $W = diag(w_1, w_2 \ldots w_N)$, D is the pinning feedback matrix, where A^u and A^s are unsigned and signed adjacency matrices.

Lemma 3 [47] *If the following LMI, $\begin{pmatrix} Q & T \\ T^T & Q_1 \end{pmatrix} > 0$ holds, then either one of the following conditions exist,*

(i) $Q > 0,\ Q_1 - T^T Q^{-1} T > 0,$
(ii) $Q_1 > 0,\ Q - T Q_1^{-1} T^T > 0.$

Definition 2 [1] The integration of fractional-order function $\mho(t)$ is defined as follows,

$${}_{t_0}I_t^q \mho(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} \frac{\mho(\tau) d\tau}{(t-\tau)^{1-q}}, q \in (0, 1]. \tag{2}$$

Definition 3 [1] Caputo derivative of $\mho(t)$ is defined as

$${}_{t_0}^{C}D_t^q \mho(t) = \frac{1}{\Gamma(n-q)} \int_{t_0}^{t} \frac{\mho^n(s)}{(t-s)^{q-n+1}} ds,$$

where $q > 0$, $n \in Z^+$, satisfying $(n-1) < q < n$. Particularly, if $q \in (0, 1)$ then,

$${}_{t_0}^{C}D_t^q \mho(t) = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} \frac{\mho'(s)}{(t-s)^q} ds. \tag{3}$$

The Caputo derivative of a constant is zero.

Lemma 4 [45] *Let $\eta(t) \in \mathbb{R}^n$ be an integrable function, and U be positive definite matrix, then the fractional order Jensen's inequality is defined as follows,*

$${}_{t_0}I_t^q \big(\eta^T(t) U \eta(t)\big) \geq \frac{\Gamma(q+1)}{\big(t-t_0\big)^q} \big({}_{t_0}I_t^q \eta(t)\big)^T U \big({}_{t_0}I_t^q \eta(t)\big), q \in (0, 1). \tag{4}$$

3 Problem Formulation

Consider FMMCDNN of signed network $\mathcal{G}^{SGN}$ expressed by:

$$\begin{aligned} {}_{t_0}^{C}D_t^q \big(x_i(t)\big) &= -Ax_i(t) + B\big(x_i(t)\big) f\big(x_i(t)\big) + C\big(x_i(t)\big) g\big(x_i\big(t-\tau(t)\big)\big) \\ &\quad -\sigma \sum_{j=1}^{N} \big|a^s_{ij}\big| \big(x_i\big(t_k\big) - sign\big(a^s_{ij}\big) x_j\big(t_k\big)\big) + u_i, \end{aligned} \tag{5}$$

where the state variable $x_i(t)$, $(t \geq 0)$, $x_i(t) = (x_{i1}(t), \ldots x_{in}(t))^T$ (capacitor's voltage) is the ith neuron, $i = \{1, 2, \ldots N\}$, diagonal matrices of neurons denoted by $A = diag(a_1, a_2, \ldots a_n)$. The connective weighted memristor matrices of B & C are defined as follows, $B(x_i(t)) = [b_{gj}(x_{ij}(t))]_{N\times N}$ and $C(x_i(t)) = [c_{gj}(x_{ij}(t))]_{N\times N}$ respectively. Let f & g are bounded feedback functions with and without delay, and $f(x_i(t)) =$

$(f_1(x_{i1}(t)), \dots f_n(x_{in}(t)))^T$ and $g(x_i(t-\tau(t))) = (g_1(x_{i1}(t-\tau(t))), \dots g_n(x_{in}(t-\tau(t))))^T$. The coupling strength of the network is represented by σ, and $\tau(t)$ stands for bounded and non-differentiable node delay $0 \le \tau(t) < \bar{\tau}$. Using a memristor instead of a resister in a network forms a memristor neural network. The memristor neural network with a coupled term forms a multiple memristor neural network [26, 30]. If a coupling is symmetric, the adjacency matrix satisfies $a_{ij} = a_{ji}$. But it is not necessarily symmetric. If signed network $\mathcal{G}^{SGN}$ (5) is strongly connected and a_{ij} is positive then the coupling term $a^s_{ij}(x_i(t) - x_j(t))$, otherwise -$a^s_{ij}(x_i(t) + x_j(t))$. Let the initial condition of $x_i(t)$ be defined as $x_i(t) = \phi_i(t)$, $t \in [-r, 0]$, where $r = \sup_{t \ge 0}\{\tau(t)\}$. The initial condition $\phi_i(t)$, which belongs to the bounded feedback functions on $[-r, 0]$. The leader node of network (5) is defined as

$$ {}^{C}_{t_0}D^q_t(s_i(t)) = -As_i(t) + B(s_i(t))f(s_i(t)) + C(s_i(t))g(s_i(t-\tau(t))), \tag{6} $$

$b_{gj}(s_{ij}(t))$ and $c_{gj}(s_{ij}(t))$ represent the memristor connective weights of matrices, where $s_i(t) = (s_{i1}(t), \dots s_{in}(t))^T$.

Remark 1 Let FMMCDNN (5) of connective weights of memristor matrices $b_{gj}(s_{ij}(t))$, $c_{gj}(s_{ij}(t))$ are discontinuous then the following definitions cannot be applied directly to these theories. Here, we use the method of differential inclusions in the Filippov solution of an ODC with a discontinuous right-hand side.

Definition 4 [48] Let the set-valued map $F : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ be defined as

$$ F(t, \ell) = \cap_{\delta>0} \cap_{\mu(\mathcal{M})=0} \bar{co}[f(t, \mathbb{B}(\ell, \delta))/\mathcal{M}], $$

let $\ell(t)$ be a vector value function defined on $I \subset \mathbb{R}$ and, if absolutely continuous on I, then the vector value function satisfies the solution of Filippov system and differential inclusion $\frac{d\ell}{dt} \in F(t, \ell)$ holds, where an open ball $\mathbb{B}(\ell, \delta)$ with center ℓ and radius δ, $\bar{co}$ represents convex closure, $\mu(\mathcal{M})$ is known as Lebesgue measure, which overall intersection measure is zero.
Memristor connection weights of $B(x_i(t))$ and $C(x_i(t))$ in $\mathcal{G}^{SGN}$ are defined as $b_{gj}(x_{ij}(t)) = \begin{cases} \hat{b}_{gj}, |x_{ij}(t)| > \mathcal{T}_j, \\ \check{b}_{gj}, |x_{ij}(t)| < \mathcal{T}_j, \end{cases}$ $c_{gj}(x_{ij}(t)) = \begin{cases} \hat{c}_{gj}, |x_{ij}(t)| > \mathcal{T}_j, \\ \check{c}_{gj}, |x_{ij}(t)| < \mathcal{T}_j. \end{cases}$ Memristor connection weights of $B(s_i(t))$ and $C(s_i(t))$ in network of leader node are defined as $b_{gj}(s_{ij}(t)) =$

$$ \begin{cases} \hat{b}_{gj}, |s_{ij}(t)| > \mathcal{T}_j, \\ \check{b}_{gj}, |s_{ij}(t)| < \mathcal{T}_j, \end{cases} \quad c_{gj}(s_{ij}(t)) = \begin{cases} \hat{c}_{gj}, |s_{ij}(t)| > \mathcal{T}_j, \\ \check{c}_{gj}, |s_{ij}(t)| < \mathcal{T}_j, \end{cases} $$

$B(\pm\mathcal{T}_j) = \hat{b}_{gj}$ (or) $\check{b}_{gj}$, $C(\pm\mathcal{T}_j) = \hat{c}_{gj}$ (or) $\check{c}_{gj}$, where weights $\hat{b}_{gj}, \check{b}_{gj}, \hat{c}_{gj}, \check{c}_{gj}$ are constant, switching jumps denotes $\mathcal{T}_j > 0$, $g, j \in N$.

Assumption 1 (A_1) The activation of odd functions f_k, g_k are bounded, $f_k(\pm\mathcal{T}_k) = 0$ and satisfies the following Lipschitz condition

$$ \begin{aligned} |f_k(y) - f_k(z)| &\le \mathcal{F}_k|y - z|, \\ |g_k(y) - g_k(z)| &\le \mathcal{G}_k|y - z|, \end{aligned} \tag{7} $$

where $\mathcal{F}_k > 0, \mathcal{G}_k > 0$, $k = \{1, 2, \dots n\}$, $y, z \in \mathbb{R}$.

Lemma 5 [15] *If* $f(\pm\mathcal{T}_i) = g(\pm\mathcal{T}_i) = 0$*, by the assumption, then*

$$|co[b_{gj}x_{ij}(t)]f(x_i(t)) - co[b_{gj}s_{ij}(t)]f(s_i(t))| \le \mathcal{BF}_i|x_i(t) - s_i(t)|,$$
$$|co[c_{gj}x_{ij}(t)]g(x_i(t)) - co[c_{gj}s_{ij}(t)]g(s_i(t))| \le \mathcal{CG}_i|x_i(t) - s_i(t)|,$$

where $\mathcal{B} = max\{|\hat{b}_{gj}|, |\check{b}_{gj}|\}$, $\mathcal{C} = max\{|\hat{c}_{gj}|, |\check{c}_{gj}|\}$, *and* $\mathcal{F}_i > 0, \mathcal{G}_i > 0$, $i \in \{1, 2, \dots N\}$.

Remark 2 The set valued map defined in the leader node of network (6) is described by,

$$co[(b_{gj}(s_{ij}(t)))] = \begin{cases} \hat{b}_{gj}, |s_{ij}(t)| > \mathcal{T}_j, \\ \bar{co}\{\hat{b}_{gj}, \check{b}_{ij}\}, |s_{ij}(t)| = \mathcal{T}_j, \\ \check{b}_{gj}|s_{ij}(t)| < \mathcal{T}_j. \end{cases}$$

Applying Filippov sense into the leader node $s_i(t)$, which is absolutely continuous on $[0, \mathcal{T})$, the Filippov solution of (6) will be defined as,

$${}_{t_0}^{C}D_t^q(s_i(t)) \in -As_i(t) + co[b_{gj}(s_{ij}(t))]f(s_i(t)) + co[c_{gj}(s_{ij}(t))]g(s_i(t - \tau(t))),$$

for $t \ge 0$, where $0 < q < 1$, or there exist $\mathcal{B} \in co[b_{gj}(s_{ij}(t))]$, $\mathcal{C} \in co[c_{gj}(s_{ij}(t))]$ such that

$${}_{t_0}^{C}D_t^q(s_i(t)) = -As_i(t) + \mathcal{B}f(s_i(t)) + \mathcal{C}g(s_i(t - \tau(t))). \tag{8}$$

Differential inclusion of fractional order with Filippov sense based FMMCDNN (5) is written as,

$${}_{t_0}^{C}D_t^q(x_i(t)) = -Ax_i(t) + \mathcal{B}f(x_i(t)) + \mathcal{C}g(x_i(t - \tau(t))) - \sigma\sum_{j=1}^{N} l_{ij}^u x_j(t_k) + u_i, \tag{9}$$

where $\mathcal{B} \in co[b_{gj}(x_{ij}(t))]$, $\mathcal{C} \in co[c_{gj}(x_{ij}(t))]$.

Lemma 6 [39] *Let* $\hat{\varpi}, \psi, \eta > 0$, $\exp(t)$ *and* $E_q(t)$ *are bounded on* $[t_k, t_{k+1})$, *and also satisfies* $E_q(-\eta(t - t_0)^q) \le \hat{\varpi}\exp(-\psi(t - t_0))$ *then* $\prod_{i=1}^{k} E_q[-\eta(t_i - t_{i-1})^q] \le \hat{\varpi}^k \exp(-\psi(t_k - t_0))$, *where* $E_q(t) = \sum_{h=0}^{\infty}\frac{t^h}{\Gamma(hq+1)}$ *is called Mittag-Leffler function,* $q \in (0, 1)$.

3.1 Scheme of Event Triggered Pinning Control Formulation

The impulsive instant t_k is defined by

$$t_{k+1} = \inf\{t > t_k \mid g_i(t) \ge 0\}, \quad k \in \mathbb{N}.$$

If $\Gamma = \{\Gamma_1, \Gamma_2, \Gamma_3\}$ be positive definite matrix and $0 \le \zeta_1, \zeta_2 < 1$, define $\tau(t) = t - t_k \in [t_k, t_{k+1})$, then triggering function is defined as follows,

$$\begin{aligned} g_i(t) = {} & \varpi_i^T(t)\Gamma_1\varpi_i(t) - \zeta_1\big[\varpi_i^T(t) + e_i^T(t - \tau(t))\big]\Gamma_2\big[\varpi_i(t) + e_i(t - \tau(t))\big] \\ & -\zeta_2 e_i^T(t - \tau(t))\Gamma_3 e_i(t - \tau(t)) \ge 0 \end{aligned} \tag{10}$$

where triggering error $\varpi_i(t) = e_i(t_k) - e_i(t)$.

Definition 5 [19, 23] If the FMMCDNN is said to be bipartite synchronization with leader node of network then the following condition exist, $\lim_{t\to\infty}(x_i(t) - w_i s_i(t)) = 0$. If FMMCDNN is said to be bipartite synchronization with leaderless node of network then $\lim_{t\to\infty}(x_i(t) - w_i x_j(t)) = 0$.

Now, we define the event triggered pinning controller $u_i(t) = -\sigma d_i(x_i(t_k) - w_i s_i(t_k))$, $t \in [t_k, t_{k+1})$, where d_i is the pinning feedback gain. If d_i is positive then the vertices are pinned, otherwise the value is zero. If $w_i = 1$ then $i \in V_1$, and if $w_i = -1$ then $i \in V_2$. Suppose, if $w_i = I_N$, bipartite synchronization changes into the traditional leader-follower synchronization.

Remark 3 The augmented network contains the leader node (8) and the signed network (9), which is the spanning tree when the leader node is the network's root, i.e., the network of the leader node is a direct path to every node of a $\mathcal{G}^{SGN}$ [19, 20, 22, 23]. We apply this theory to directed networks [19, 21, 23–25, 44] and undirected networks [23, 30], based on their properties.

4 Leader Bipartite Synchronization of FMMCDNN

Applying gauge transformation, the error system $e_i(t) = \bar{x}_i(t) - s_i(t)$, which contains FMMCDNN of signed network (9) and leader node of network (8), where $\bar{x}_i(t) = w_i x_i(t)$, $w_i^2 = 1$. The error system is described as

$$ {}_{t_0}^{C}D_t^q(e_i(t)) = -Ae_i(t) + \mathcal{B}\mathcal{F}_i e_i(t) + \mathcal{C}\mathcal{G}_i e_i(t - \tau(t)) - \sigma \sum_{j=1}^{N} h_{ij} e_j(t_k). \tag{11} $$

In this case, let $a_{ik}^s < 0$ and $\sum_{k=1,k\neq j}^{N} l_{ik}^s \neq 0$, then row sum matrix of L^s is not zero. Its different from unsigned Laplacian matrix.

Lemma 7 [34] *Let f and $\hat{f}$ be continuous functions, if $f(0) = 0$, $tf(t) \geq 0$, $\hat{f}(t) \geq 0$, and $s \in \mathbb{R}$, then*

$$ sign\left(\int_0^s f(t)\hat{f}(t)dt\right) = sign\left(\int_0^s f(t)dt\right). \tag{12} $$

Lemma 8 [18] *If $q \in (0, 1)$ and $\gamma_i(t) \in co\big[b_{gj}x_{ij}(t)\big] f(x_i(t)) - co\big[b_{gj}s_{ij}(t)\big] f(s_i(t))$, then*

$$ {}_{t_0}^{C}D_t^q\left[\int_0^{e_i(t)} f_i(s)ds\right] \leq \gamma_i(t) {}_{t_0}^{C}D_t^q e_i(t). \tag{13} $$

4.1 Descriptor Method (DM)

The advantage of DM is having less conservative properties for uncertain systems. Let us consider the LMI's for the continuous time systems, discrete time systems and singularly perturbed systems from [49]. And if we take delay as non-differentiable then the LMIs are found to be more efficient when compared to the differentiable delay. Then the descriptor method of ODC can be seen in [19, 49]. The above conditions are applied to the FDC,

$$ {}_{t_0}^{C}D_t^q(e_i(t)) = y_i(t), \tag{14} $$

where ${}_{t_0}^{C}D_t^q(e_i(t)) = A_0 e_i(t) + A_1 e_i(t - \tau(t)) + A_2 \varpi_i(t)$, $A_0 = -I_N \otimes A + I_N \otimes \mathcal{B} - \sigma(H \otimes I_N)$, $A_1 = I \otimes \mathcal{C}$, $A_2 = -\sigma(H \otimes I_N)$.

Theorem 1 *Assume the hypothesis* (A_1)*, let* $P, P_1, P_2, \Gamma_1, \Gamma_2, \Gamma_3 \in \mathbb{R}^{n\times n}$ *be positive definite matrices,* $\zeta_1, \zeta_2 \in [0, 1)$*, there exist a matrix* Q *and the following LMI condition holds,*

$$\Pi = \begin{pmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & 0 & 0 & \Pi_{17} \\ * & -2Q^T & \Pi_{23} & 0 & 0 & 0 & \Pi_{27} \\ * & * & \Pi_{33} & 0 & 0 & 0 & \Pi_{37} \\ * & * & * & -P_1 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 \\ * & * & * & * & * & \Pi_{66} & 0 \\ * & * & * & * & * & * & \Pi_{77} \end{pmatrix} < 0; \tag{15}$$

where $\Pi_{11} = P_1 + 2Q^T A_0$, $\Pi_{12} = P - Q^T + Q^T A_0$, $\Pi_{13} = Q^T A_1 G$, $\Pi_{17} = Q^T A_2$, $\Pi_{23} = Q^T A_1$, $\Pi_{27} = Q^T A_2$, $\Pi_{33} = \zeta_1\Gamma_2 + \zeta_2\Gamma_3$, $\Pi_{37} = \zeta_1\Gamma_2$, $\Pi_{55} = \frac{\bar{\tau}^q}{\Gamma(q+1)} P_2$, $\Pi_{66} = -\frac{\Gamma(q+1)}{\bar{\tau}^q} P_2$, $\Pi_{77} = \zeta_1\Gamma_2 - \Gamma_1$, $A_0 = -I_N \otimes A + I_N \otimes \mathcal{B} - \sigma(H \otimes I_n)$, $A_1 = I_N \otimes \mathcal{C}$, $A_2 = -\sigma(H \otimes I_N)$*, then the FMMCDNN of the error system (11) is bipartite synchronized with leader node of the network.*

Proof Take the candidate of Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t) &= \sum_{k=1}^{3} V_k(t), \\ V_1(t) &= e_i^T(t) P e_i(t), \\ V_2(t) &= \int_{t-\bar{\tau}}^{t} e_i^T(s) P_1 e_i(s) ds, \\ V_3(t) &= \int_{-\bar{\tau}}^{0} (-\theta)^{q-1} \int_{t+\theta}^{t} ({}_{t-\bar{\tau}}^{C}D_t^q e_i(s))^T P_2 ({}_{t-\bar{\tau}}^{C}D_t^q e_i(s)) ds d\theta. \end{aligned}$$

Now, using Caputo fractional order derivative and the descriptor method (14), we get

$$\begin{aligned} {}_{t_0}^{C}D_t^q V_1(t) &\le 2e_i^T(t) P y_i(t), \\ {}_{t_0}^{C}D_t^q V_2(t) &= e_i^T(t) P_1 e_i(t) - e_i^T\big(t-\bar{\tau}\big) P_1 e_i\big(t-\bar{\tau}\big), \\ {}_{t_0}^{C}D_t^q V_3(t) &\le \frac{\bar{\tau}^q}{\Gamma(q+1)} \big({}_{t-\bar{\tau}}^{C}D_t^q e_i(t)\big)^T P_2 \big({}_{t-\bar{\tau}}^{C}D_t^q e_i(t)\big) \\ &\quad - \frac{1}{\Gamma(q)} \int_{\bar{\tau}}^{0} (-\theta)^{q-1} \big({}_{t+\theta-\bar{\tau}}^{C}D_{t+\theta}^q e_i(t+\theta)\big)^T P_2 \big({}_{t+\theta-\bar{\tau}}^{C}D_{t+\theta}^q e_i(t+\theta)\big) \\ &= \Big\{ \frac{\bar{\tau}^q}{\Gamma(q+1)} \big({}_{t-\bar{\tau}}^{C}D_t^q e_i(t)\big)^T P_2 \big({}_{t-\bar{\tau}}^{C}D_t^q e_i(t)\big) \Big\} \\ &\quad - \int_{t-\bar{\tau}}^{t} \frac{1}{(t-\theta)^{1-q}\Gamma(q)} \big({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_i(\theta)\big)^T P_2 \big({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_i(\theta)\big) d\theta. \end{aligned}$$

Now, consider

$$\begin{aligned} &\int_{t-\bar{\tau}}^{t} \frac{1}{(t-\theta)^{1-q}\Gamma(q)} \big({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_i(\theta)\big)^T P_2 \big({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_i(\theta)\big) d\theta \\ &\ge \frac{\Gamma(q+1)}{\bar{\tau}^q} \left(\frac{1}{\Gamma(q)} \int_{t-\bar{\tau}}^{t} (t-\theta)^{q-1} \big({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_i(\theta) d\theta\big) \right)^T P_2 \\ &\quad \left(\frac{1}{\Gamma(q)} \int_{t-\bar{\tau}}^{t} (t-\theta)^{q-1} \left({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_i(\theta) d\theta\right) \right) \end{aligned}$$

$$
\begin{aligned}
&= \frac{\Gamma(q+1)}{\bar{\tau}^q}\left(\frac{1}{\Gamma(q)}\int_{t-\bar{\tau}}^{t}(t-\theta)^{q-1}\left({}^{C}_{\theta-\bar{\tau}}D^q_\theta e_i(\theta)-\hat{y}_i(\theta)d\theta\right)^T P_2\right.\\
&\quad\left(\frac{1}{\Gamma(q)}\int_{t-\bar{\tau}}^{t}(t-\theta)^{q-1}\left({}^{C}_{\theta-\bar{\tau}}D^q_\theta e_i(\theta)-\hat{y}_i(\theta)\right)d\theta\right)\\
&= \frac{\Gamma(q+1)}{\bar{\tau}^q}\left(e_i(t)-e_i(t-\bar{\tau})-{}_{t-\bar{\tau}}I^q_t\hat{y}_i(t)\right)^T P_2\\
&\quad\left(e_i(t)-e_i(t-\bar{\tau})-{}_{t-\bar{\tau}}I^q_t\hat{y}_i(t)\right)
\end{aligned}
$$

where $\hat{y}_i(\theta) = \frac{1}{\Gamma(1-q)}\int_{t-\bar{\tau}}^{\theta-\bar{\tau}}(u-\theta)^{-q}\dot{e}_i(u)du$, $e(t) = (e_1(t), e_2(t), \dots e_N(t))^T$, $e(t-\tau(t)) = (e_1(t-\tau(t)), e_2(t-\tau(t)), \dots e_N(t-\tau(t)))^T$, $e(t-\bar{\tau}) = (e_1(t-\bar{\tau}), e_2(t-\bar{\tau}), \dots e_N(t-\bar{\tau}))^T$, $y(t) = (y_1(t), y_2(t)\dots y_N(t))^T$, $\varpi(t) = (\varpi_1(t), \varpi_2(t), \dots \varpi_N(t))^T$ and satisfying, $0 = -2(e_i^T(t) + {}^{C}_{t_0}(D^q_t e_i(t))^T)Q^T({}^{C}_{t_0}D^q_t e_i(t) - A_0 e_i(t) - A_1 e_i(t-\tau(t)) - A_2\varpi_i(t))$,

$$
\begin{aligned}
{}^{C}_{t_0}D^q_t V(t) \le\ & 2e_i^T(t)P\{y_i(t)\} + e_i^T(t)P_1 e_i(t) - e_i^T(t-\bar{\tau})P_1 e_i(t-\bar{\tau})\\
&+\frac{\bar{\tau}^q}{\Gamma(q+1)}({}^{C}_{t-\bar{\tau}}D^q_t e_i(t))^T P_2({}^{C}_{t-\bar{\tau}}D^q_t e_i(t))\\
&-\frac{\Gamma(q+1)}{\bar{\tau}^q}(e_i(t)-e_i(t-\bar{\tau})-{}_{t-\bar{\tau}}I^q_t\hat{y}_i(t))^T P_2(e_i(t)-e_i(t-\bar{\tau})-{}_{t-\bar{\tau}}I^q_t\hat{y}_i(t)).
\end{aligned}
$$

By the triggering condition (10),

$$
\begin{aligned}
&\le 2e^T(t)Py(t) + e^T(t)P_1 e(t) - e^T(t-\bar{\tau})P_1 e(t-\bar{\tau})\\
&\quad+\frac{\bar{\tau}^q}{\Gamma(q+1)}\Omega_1^T(t)P_2\Omega_1(t) - \frac{\Gamma(q+1)}{\bar{\tau}^q}\Omega_2^T(t)P_2\Omega_2(t)\\
&\quad+\varpi^T(t)\{\zeta_1\Gamma_2-\Gamma_1\}\varpi(t) + e^T(t-\tau(t))\{\zeta_1\Gamma_2+\zeta_2\Gamma_3\}e(t-\tau(t))\\
&\quad+\varpi^T(t)\{\zeta_1\Gamma_2\}e(t-\tau(t)) + e^T(t-\tau(t))\{\zeta_1\Gamma_2\}\varpi(t)\\
&\quad+e^T(t)\{-2Q^T\}\Omega_1 + e^T(t)\{2Q^T A_0\}e(t)\\
&\quad+e^T(t)\{2Q^T A_1 G\}e(t-\tau(t)) + e^T(t)\{2Q^T A_2\}\varpi(t) + y^T(t)\{-2Q^T\}y(t)\\
&\quad+y^T(t)\{2Q^T A_0\}e(t) + y^T(t)\{2Q^T A_1\}e(t-\tau(t))\\
&\quad+y^T(t)\{2Q^T A_2\}\varpi(t)\\
&\le z_i^T(t)\Pi z_i(t),
\end{aligned}
$$

here $z_i(t)$=col $(e_i(t), y_i(t), e_i(t-\tau(t)), e_i(t-\bar{\tau}), \Omega_1(t), \Omega_2(t), \varpi_i(t))$, $\hat{y}(\theta) = (\hat{y}_1(\theta), \hat{y}_2(\theta), \dots \hat{y}_N(\theta))^T$, $\Omega_1(t) = {}^{C}_{t-\bar{\tau}}D^q_t e_i(t)$, $\Omega_2(t) = e_i(t) - e_i(t-\bar{\tau}) - {}_{t-\bar{\tau}}I^q_t\hat{y}_i(t)$. Hence, by using the LMI (15) and lemma 3, the error system (11) is globally asymptotically stable. Therefore, we attained the desired result. □

Theorem 2 *In Theorem 1, the concerned FMMCDNN of $\mathcal{G}^{SGN}$ and leader node of network avoid Zeno behaviour.*

Proof Assume the hypothesis (A_1), by the definition of $w_i(t)$

$$
\begin{aligned}
\parallel \varpi_i(t) \parallel &= \parallel e_i(t_k) - e_i(t) \parallel\\
&= \left\| {}^{C}_{t_0}D^{-q}_t\left({}^{C}_{t_0}D^q_t(e_i(t_k))\right) - {}^{C}_{t_0}D^{-q}_t\left({}^{C}_{t_0}D^q_t(e_i(t))\right)\right\|
\end{aligned}
$$

$$= \left\| \frac{1}{\Gamma(q)} \left(\int_{t_0}^{t_k} \frac{{}_{t_0}^{C}D_t^q(e_i(s))}{(t_k - s)^{1-q}} \right) ds - \left(\int_{t_0}^{t} \frac{{}_{t_0}^{C}D_t^q(e_i(s))}{(t - s)^{1-q}} \right) ds \right\|$$

$$\leq \frac{1}{\Gamma(q)} \left\| \int_{t_0}^{t_k} \left(\frac{{}_{t_0}^{C}D_t^q(e_i(s))}{(t_k - s)^{1-q}} - \frac{{}_{t_0}^{C}D_t^q(e_i(s))}{(t - s)^{1-q}} \right) ds \right\| + \frac{1}{\Gamma(q)} \left\| \int_{t_k}^{t} \frac{{}_{t_0}^{C}D_t^q(e_i(s))}{(t - s)^{1-q}} ds \right\|$$

${}_{t_0}^{C}D_t^q(e_i(t))$ is bounded, since $e_i(t)$ is bounded. There exist $Z > 0$, we get $({}_{t_0}^{C}D_t^q(e_i(t))) \leq Z$, which yields that

$$\begin{aligned}
\| \varpi_i(t) \| &\leq \frac{Z}{\Gamma(q)} \left| \int_{t_0}^{t_k} \left(\frac{1}{(t_k - s)^{1-q}} - \frac{1}{(t - s)^{1-q}} \right) ds \right| + \frac{Z}{\Gamma(q)} \left| \int_{t_k}^{t} \frac{1}{(t - s)^{1-q}} ds \right| \\
&= \frac{Z}{\Gamma(q+1)} (|(t - t_k)^q - (t - t_0)^q + (t_k - t_0)^q| + (t - t_k)^q) \\
&\leq \frac{2Z(t - t_k)^q}{\Gamma(q+1)}.
\end{aligned}$$

We obtain

$$\| \varpi_i(t_{k+1}) \|^2 = \frac{4Z^2 \mathcal{T}_k^{2q}}{\Gamma^2(q+1)}, \tag{15}$$

where $\mathcal{T}_k = t_{k+1} - t_k$. It is clear that $\mathcal{T}_k > 0$ then zeno behaviour does not hold. Suppose $\mathcal{T}_k = 0$, by the triggering condition (10), we get

$$\begin{aligned}
&\varpi_i^T(t)\Gamma_1\varpi_i(t) - \varpi_i^T(t)\zeta_1\Gamma_2\varpi_i(t) - \varpi_i^T(t)\zeta_1\Gamma_2 e_i(t - \tau(t)) \\
&\quad - e^T(t - \tau(t))\zeta_1\Gamma_2 e_i(t - \tau(t)) \\
&\quad e^T(t - \tau(t))\zeta_1\Gamma_2\varpi_i(t) - e^T(t - \tau(t))\zeta_2\Gamma_3 e_i(t - \tau(t)) \geq 0 \\
&\quad \mathcal{W} \| \varpi_i(t) \|^2 \geq \| e_i(t - \tau(t)) \|^2 \, \overline{\mathcal{W}}
\end{aligned} \tag{16}$$

by 15, we get $\| e_i(t_{k+1} - \tau(t_{k+1})) \|^2 \leq 0$, where $\mathcal{W} = \Gamma_1 - \zeta_1\Gamma_2 - \epsilon\zeta_1\Gamma_2 - \frac{1}{\hat{\epsilon}}\zeta_1\Gamma_2$ and $\overline{\mathcal{W}} = \zeta_1\Gamma_2 + \zeta_2\Gamma_3 + \frac{1}{\epsilon} + \hat{\epsilon}\zeta_1\Gamma_2$, $\epsilon, \hat{\epsilon} \in \mathbb{R} - \{0\}$, which is contradiction, then we concluded Zeno behaviour does not exist. □

5 Leaderless Bipartite Synchronization of FMMCDNN

A synchronization without an external force is called self-synchronization or leaderless synchronization of FMMCDNN. In case of such self synchronized signed networks (9), the pinned value is zero. Let $\bar{x}_i(t) = w_i x_i(t)$, which also gives $x_i = w_i \bar{x}_i(t)$. Then from (9),

$${}_{t_0}^{C}D_t^q(\bar{x}_i(t)) = -A\bar{x}_i(t) + \mathcal{B}w_i f(w_i\bar{x}_j(t)) + \mathcal{C}w_i g(w_i\bar{x}_i(t - \tau(t))) - \sigma \sum_{j=1}^{N} l_{ij}^u \bar{x}_j(t_k),$$

$${}_{t_0}^{C}D_t^q(\bar{x}_i(t)) = -A\bar{x}_i(t) + \mathcal{B}f(\bar{x}_i(t)) + \mathcal{C}g(\bar{x}_i(t - \tau(t))) - \sigma \sum_{j=1}^{N} l_{ij}^u \bar{x}_j(t_k).$$

The error system $e_i(t) = \bar{x}_i(t) - \bar{x}_r(t)$ is written as,

$${}_{t_0}^{C}D_t^q(e_i(t)) = -Ae_i(t) + \mathcal{B}F_i e_i(t) + \mathcal{C}G_i e_i(t - \tau(t)) - \sigma \sum_{j=1}^{N} L_{ij}^u e_j(t_k), \tag{17}$$

where, $L^u_{ij} = l^u_{ij} - l^u_{rj}$, and $\sum_{j=1}^{N} l^u_{ij} = 0$.

Theorem 3 *Assume the hypothesis* (A_1), $\zeta_1, \zeta_2 \in [0, 1)$, *let* $P, P_1, P_2, \Gamma_1, \Gamma_2, \Gamma_3 \in \mathbb{R}^{n\times n}$ *be positive definite matrices, there exist a matrix* Q *and the following LMI condition holds,*

$$\bar{\Pi} = \begin{pmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & 0 & 0 & 0 & \bar{\Pi}_{17} \\ * & -2Q^T & \bar{\Pi}_{23} & 0 & 0 & 0 & \bar{\Pi}_{27} \\ * & * & \bar{\Pi}_{33} & 0 & 0 & 0 & \bar{\Pi}_{37} \\ * & * & * & -P_1 & 0 & 0 & 0 \\ * & * & * & * & \bar{\Pi}_{55} & 0 & 0 \\ * & * & * & * & * & \bar{\Pi}_{66} & 0 \\ * & * & * & * & * & * & \bar{\Pi}_{77} \end{pmatrix} < 0; \tag{19}$$

$\bar{\Pi}_{11} = P_1 + 2Q^T A_0$, $\bar{\Pi}_{12} = P - Q^T + Q^T A_0$, $\bar{\Pi}_{13} = Q^T A_1 G$, $\bar{\Pi}_{17} = Q^T A_2$, $\bar{\Pi}_{23} = Q^T A_1$, $\bar{\Pi}_{27} = Q^T A_2$, $\bar{\Pi}_{33} = \zeta_1\Gamma_2 + \zeta_2\Gamma_3$, $\bar{\Pi}_{37} = \zeta_1\Gamma_2$, $\bar{\Pi}_{55} = \frac{\bar{\tau}^q}{\Gamma(q+1)} P_2$, $\bar{\Pi}_{66} = -\frac{\Gamma(q+1)}{\bar{\tau}^q} P_2$, $\bar{\Pi}_{77} = \zeta_1\Gamma_2 - \Gamma_1$, $A_0 = -I_N \otimes A + I_N \otimes B - \sigma(L \otimes I_n)$, $A_1 = I_N \otimes C$, $A_2 = -\sigma(L \otimes I_N)$ *then error system (17) is leaderless bipartite synchronized.*

Proof Proof is similar to Theorem 1. □

6 Lur'e Postnikov Lyapunov Function

In neural networks, we use LPLF, which is Lyapunov function along with activation function. Quadratic Lyapunov function with LPLF is discussed in [18, 34]. LPLF with LKF will be discussed in this theorem.

Theorem 4 *Consider the hypothesis* (A_1), $\zeta_1, \zeta_2 \in [0, 1)$, $\beta \in \mathbb{R}$, *positive definite matrices* $P, P_1, P_2, \Gamma_1, \Gamma_2, \Gamma_3, R \in \mathbb{R}^{n\times n}$, *there exist a matrix* Q *and the following LMI condition holds,*

$$\hat{\Pi} = \begin{pmatrix} \hat{\Pi}_{11} & \hat{\Pi}_{12} & \hat{\Pi}_{13} & 0 & 0 & 0 & \hat{\Pi}_{17} & 0 \\ * & -2Q^T & \hat{\Pi}_{23} & 0 & 0 & 0 & \hat{\Pi}_{27} & \hat{\Pi}_{28} \\ * & * & \hat{\Pi}_{33} & 0 & 0 & 0 & \hat{\Pi}_{37} & 0 \\ * & * & * & -P_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & \hat{\Pi}_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66} & 0 & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{77} & 0 \\ * & * & * & * & * & * & 0 & 0 \end{pmatrix} < 0; \tag{20}$$

$\hat{\Pi}_{11} = P_1 + 2Q^T A_0$, $\hat{\Pi}_{12} = P - Q^T + Q^T A_0$, $\hat{\Pi}_{13} = Q^T A_1 G$, $\hat{\Pi}_{17} = Q^T A_2$, $\hat{\Pi}_{23} = Q^T A_1$, $\hat{\Pi}_{27} = Q^T A_2$, $\hat{\Pi}_{28} = \frac{\beta R}{2}$, $\hat{\Pi}_{33} = \zeta_1\Gamma_2 + \zeta_2\Gamma_3$, $\hat{\Pi}_{37} = \zeta_1\Gamma_2$, $\hat{\Pi}_{55} = \frac{\bar{\tau}^q}{\Gamma(q+1)} P_2$, $\hat{\Pi}_{66} = -\frac{\Gamma(q+1)}{\bar{\tau}^q} P_2$, $\hat{\Pi}_{77} = \zeta_1\Gamma_2 - \Gamma_1$, $A_0 = -I_N \otimes A + I_N \otimes \mathcal{B} - \sigma(H \otimes I_n)$, $A_1 = I_N \otimes \mathcal{C}$, *then the FMMCDNN of the error system* (11) *is bipartite leader synchronized.*

Proof Consider the LKF and LPLF:

$$V(t) = \sum_{k=1}^{4} V_k(t),$$

$$V_1(t) = e_i^T(t) P e_i(t),$$

$$
\begin{aligned}
V_2(t) &= \int_{t-\bar{\tau}}^{t} e_i^T(s) P_1 e_i(s) ds, \\
V_3(t) &= \int_{-\bar{\tau}}^{0} (-\theta)^{q-1} \int_{t+\theta}^{t} \left({}_{t-\bar{\tau}}^{C} D_t^q e_i(s) \right)^T P_2 \left({}_{t-\bar{\tau}}^{C} D_t^q e_i(s) \right) ds d\theta, \\
V_4(t) &= \beta \sum_{i=1}^{n} r_i \int_0^{e_i(t)} f_i(s) ds.
\end{aligned}
$$

Now, using Caputo fractional order derivative and the descriptor method (14),

$$
\begin{aligned}
{}_{t_0}^{C} D_t^q V_1(t) &\leq 2e_i^T(t) P y_i(t), \\
{}_{t_0}^{C} D_t^q V_2(t) &= \Big\{ e_i^T(t) P_1 e_i(t) - e_i^T(t-\bar{\tau}) P_1 e_i(t-\bar{\tau}), \\
{}_{t_0}^{C} D_t^q V_3(t) &\leq \frac{\bar{\tau}^q}{\Gamma(q+1)} \big({}_{t-\bar{\tau}}^{C} D_t^q e_i(t) \big)^T P_2 \big({}_{t-\bar{\tau}}^{C} D_t^q e_i(t) \big) \\
&\quad - \frac{1}{\Gamma(q)} \int_{\bar{\tau}}^{0} (-\theta)^{q-1} \big({}_{t+\theta-\bar{\tau}}^{C} D_{t+\theta}^q e_i(t+\theta) \big)^T P_2 \big({}_{t+\theta-\bar{\tau}}^{C} D_{t+\theta}^q e_i(t+\theta) \big) \\
&= \left\{ \frac{\bar{\tau}^q}{\Gamma(q+1)} \big({}_{t-\bar{\tau}}^{C} D_t^q e_i(t) \big)^T P_2 \big({}_{t-\bar{\tau}}^{C} D_t^q e_i(t) \big) \right\} \\
&\quad - \int_{t-\bar{\tau}}^{t} \frac{1}{(t-\theta)^{1-q} \Gamma(q)} \big({}_{\theta-\bar{\tau}}^{C} D_\theta^q e_i(\theta) \big)^T P_2 \big({}_{\theta-\bar{\tau}}^{C} D_\theta^q e_i(\theta) \big) d\theta. \\
{}_{t_0}^{C} D_t^q V_4(t) &\leq \beta \sum_{i=1}^{n} r_i \, {}_{t_0}^{C} D_t^q \left[\int_0^{e_i(t)} f_i(s) ds \right]
\end{aligned}
$$

Now, consider

$$
\begin{aligned}
&\int_{t-\bar{\tau}}^{t} \frac{1}{(t-\theta)^{1-q} \Gamma(q)} \big({}_{\theta-\bar{\tau}}^{C} D_\theta^q e_i(\theta) \big)^T P_2 \big({}_{\theta-\bar{\tau}}^{C} D_\theta^q e_i(\theta) \big) d\theta \\
&\quad \geq \frac{\Gamma(q+1)}{\bar{\tau}^q} \Big(\frac{1}{\Gamma(q)} \int_{t-\bar{\tau}}^{t} (t-\theta)^{q-1} \big({}_{\theta-\bar{\tau}}^{C} D_\theta^q e_i(\theta) d\theta \big) \Big)^T P_2 \\
&\quad \left(\frac{1}{\Gamma(q)} \int_{t-\bar{\tau}}^{t} (t-\theta)^{q-1} \big({}_{\theta-\bar{\tau}}^{C} D_\theta^q e_i(\theta) d\theta \big) \right) \\
&\quad = \frac{\Gamma(q+1)}{\bar{\tau}^q} \Big(\frac{1}{\Gamma(q)} \int_{t-\bar{\tau}}^{t} (t-\theta)^{q-1} \big({}_{\theta-\bar{\tau}}^{C} D_\theta^q e_i(\theta) - \hat{y}_i(\theta) d\theta \big)^T P_2 \\
&\quad \left(\frac{1}{\Gamma(q)} \int_{t-\bar{\tau}}^{t} (t-\theta)^{q-1} \big({}_{\theta-\bar{\tau}}^{C} D_\theta^q e_i(\theta) - \hat{y}_i(\theta) \big) d\theta \right) \\
&\quad = \frac{\Gamma(q+1)}{\bar{\tau}^q} \big(e_i(t) - e_i(t-\bar{\tau}) - {}_{t-\bar{\tau}} I_t^q \hat{y}_i(t) \big)^T P_2 \\
&\quad \big(e_i(t) - e_i(t-\bar{\tau}) - {}_{t-\bar{\tau}} I_t^q \hat{y}_i(t) \big)
\end{aligned}
$$

where $\hat{y}_i(\theta) = \frac{1}{\Gamma(1-q)} \int_{t-\bar{\tau}}^{\theta-\bar{\tau}} (u-\theta)^{-q} \dot{e}_i(u) du$, $e(t) = (e_1(t), e_2(t), \ldots e_N(t))^T$, $e(t-\tau(t)) = (e_1(t-\tau(t)), e_2(t-\tau(t)), \ldots e_N(t-\tau(t)))^T$, $\varpi(t) = (\varpi_1(t), \varpi_2(t), \ldots \varpi_N(t))^T$, $\gamma(t) = (\gamma_1(t), \gamma_2(t), \ldots \gamma_N(t))^T$,

$$
\begin{aligned}
{}_{t_0}^{C}D_t^q V(t) \le\ & 2\sum_{i=1}^{N} \xi_i e_i^T(t) P\{y_i(t)\} \\
& + \left\{ e_i^T(t) P_1 e_i(t) - e_i^T(t-\bar{\tau}) P_1 e_i(t-\bar{\tau}) \right\} \\
& + \frac{\bar{\tau}^q}{\Gamma(q+1)} \left({}_{t-\bar{\tau}}^{C}D_t^q e_i(t) \right)^T P_2 \big({}_{t-\bar{\tau}}^{C}D_t^q e_i(t) \big) \\
& - \frac{\Gamma(q+1)}{\bar{\tau}^q} \big(e_i(t) - e_i(t-\bar{\tau}) - {}_{t-\bar{\tau}}I_t^q \hat{y}_i(t) \big)^T P_2 \, (e_i(t) \\
& - e_i(t-\bar{\tau}) - {}_{t-\bar{\tau}}I_t^q \hat{y}_i(t) \big) + \beta \sum_{i=1}^{n} q_i \, {}_{t_0}^{C}D_t^q \left[\int_0^{e_i(t)} h_i(s) ds \right] \\
\le\ & 2e^T(t) P y(t) \\
& + e^T(t) P_1 e(t) - e^T(t-\bar{\tau}) P_1 e(t-\bar{\tau}) \\
& + \frac{\bar{\tau}^q}{\Gamma(q+1)} \Omega_1^T(t) P_2 \Omega_1(t) - \frac{\Gamma(q+1)}{\bar{\tau}^q} \Omega_2^T(t) P_2 \Omega_2(t) \\
& + \varpi^T(t) \{\zeta_1 \Gamma_2 - \Gamma_1\} \varpi(t) + e^T(t-\tau(t)) \{\zeta_1 \Gamma_2 + \zeta_2 \Gamma_3\} e(t-\tau(t)) \\
& + \varpi^T(t) \{\zeta_1 \Gamma_2\} e(t-\tau(t)) + e^T(t-\tau(t)) \{\zeta_1 \Gamma_2\} \varpi(t) \\
& + e^T(t) \{-2Q^T\} \Omega_1 + e^T(t) \left\{ 2Q^T A_0 \right\} e(t) \\
& + e^T(t) \left\{ 2Q^T A_1 G \right\} e(t-\tau(t)) + e^T(t) \left\{ 2Q^T A_2 \right\} \varpi(t) \\
& + y^T(t) \{-2Q^T\} y(t) + y^T(t) \left\{ 2Q^T A_0 \right\} e(t) + y^T(t) \left\{ 2Q^T A_1 \right\} e(t-\tau(t)) \\
& + y^T(t) \left\{ 2Q^T A_2 \right\} \varpi(t) + \gamma^T(t) \beta R y(t) \\
\le\ & z_i^T(t) \hat{\Pi} z_i(t)
\end{aligned} \tag{18}
$$

where $z_i(t)=\text{col}\ (e_i(t), y_i(t), e_i(t-\tau(t)), e_i(t-\hat{\tau}), \Omega_1(t), \Omega_2(t), \varpi_i(t), \gamma_i(t))$ and $\hat{y}(\theta) = (\hat{y}_1(\theta), \hat{y}_2(\theta), \ldots \hat{y}_N(\theta))^T$, $\Omega_1(t) = {}_{t-\bar{\tau}}^{C}D_t^q e_i(t)$, $\Omega_2(t) = e_i(t) - e_i(t-\bar{\tau}) - {}_{t-\bar{\tau}}I_t^q \hat{y}_i(t)$. Hence, by using the LMI (20) and Lemma 8, the error system (11) is bipartite leader synchronized. □

Theorem 5 *In Theorem 4, the concerned FMMCDNN is precluded the Zeno behaviour.*

Proof Proof is similar to Theorem 2. □

Theorem 6 *Consider the hypothesis* (A_1)*,* $\zeta_1, \zeta_2 \in [0, 1)$*,* $\beta \in \mathbb{R}$*, positive definite matrices* $P, P_1, P_2, \Gamma_1, \Gamma_2, \Gamma_3, R \in \mathbb{R}^{n\times n}$*, there exist a matrix* Q *and the following LMI condition holds,*

$$
\mathring{\Pi} = \begin{pmatrix}
\mathring{\Pi}_{11} & \mathring{\Pi}_{12} & \mathring{\Pi}_{13} & 0 & 0 & 0 & \mathring{\Pi}_{17} & 0 \\
* & -2Q^T & \mathring{\Pi}_{23} & 0 & 0 & 0 & \mathring{\Pi}_{27} & \mathring{\Pi}_{28} \\
* & * & \mathring{\Pi}_{33} & 0 & 0 & 0 & \mathring{\Pi}_{37} & 0 \\
* & * & * & -P_1 & 0 & 0 & 0 & 0 \\
* & * & * & * & \mathring{\Pi}_{55} & 0 & 0 & 0 \\
* & * & * & * & * & \mathring{\Pi}_{66} & 0 & 0 \\
* & * & * & * & * & * & \mathring{\Pi}_{77} & 0 \\
* & * & * & * & * & * & 0 & 0
\end{pmatrix} < 0; \tag{22}
$$

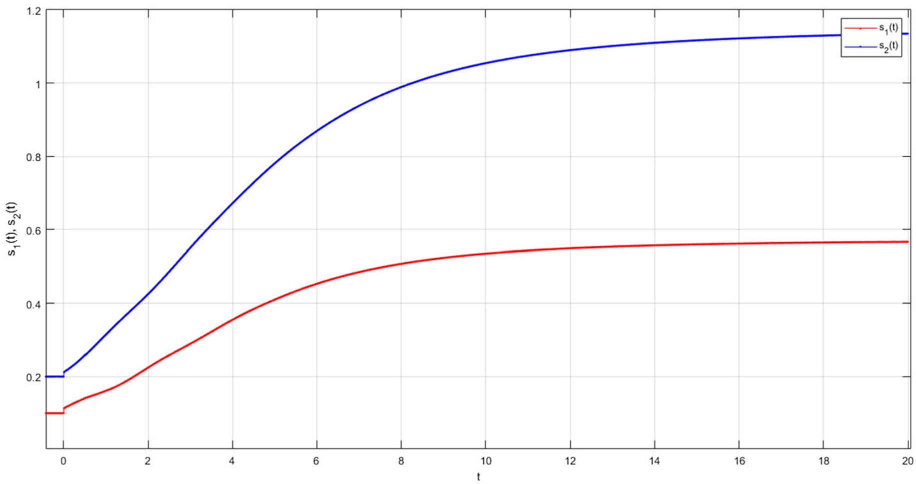

Fig. 1 Leader node of network trajectory

$\mathring{\Pi}_{11} = P_1 + 2Q^T A_0$, $\mathring{\Pi}_{12} = P - Q^T + Q^T A_0$, $\mathring{\Pi}_{13} = Q^T A_1 G$, $\mathring{\Pi}_{17} = Q^T A_2$, $\mathring{\Pi}_{23} = Q^T A_1$, $\mathring{\Pi}_{27} = Q^T A_2$, $\mathring{\Pi}_{28} = \frac{\beta R}{2}$, $\mathring{\Pi}_{33} = \zeta_1\Gamma_2 + \zeta_2\Gamma_3$, $\mathring{\Pi}_{37} = \zeta_1\Gamma_2$, $\mathring{\Pi}_{55} = \frac{\bar{\tau}^q}{\Gamma(q+1)} P_2$, $\mathring{\Pi}_{66} = -\frac{\Gamma(q+1)}{\bar{\tau}^q} P_2$, $\mathring{\Pi}_{77} = \zeta_1\Gamma_2 - \Gamma_1$, $A_0 = -I_N \otimes A + I_N \otimes \mathcal{B} - \sigma(L \otimes I_n)$, $A_1 = I_N \otimes \mathcal{C}$, $A_2 = -\sigma(L \otimes I_N)$, *then the memristor based FMMCDNN of the error system (17) is bipartite leaderless synchronized.*

Proof Proof is similar to Theorem 4. □

Remark 4 Assume by the descriptor method, the LMIs (15), (19), (20) and (22) are higher dimension when compared to the LMI (11) and (15) in [25], since $\tau(t)$ is bounded but not differentiable. If $\beta = 0$ in Theorems 4 and 5, the Lyapunov Postnikov candidates change into the Lyapunov Krasovskii functional, as we have proved in Theorems 1 and 3.

Remark 5 Bipartite synchronization of neural networks without memristor discussed in [19, 36], bipartite synchronization with memristor studied [25, 30], event-triggered without delay [36, 39], event- triggered with delay [35, 38]. Event-triggered bipartite leader synchronization as shown in figure 3, leaderless bipartite synchronization revels in figure 5. We implemented LPLF with bipartite synchronization memristor based event-triggered pinning control.

7 Numerical Examples

Example 1 Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be the diagonal matrix and consider memristor in leader node of the network (8), $\mathcal{B} = \begin{pmatrix} 0.02 & 0.04 \\ 0.01 & 0.03 \end{pmatrix}$, $\mathcal{C} = \begin{pmatrix} 0.05 & 0.01 \\ 0.02 & 0.06 \end{pmatrix}$, where the memristor values are taken from [50]. Figure 1 shows the leader node of the network trajectory (8). Let $A = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{pmatrix}$

be a diagonal matrix, consider FMMCDNN with the following parameters. The memristor and multiple memristor weights are from [30, 43] as

$$b_{11}(x_1) = \begin{cases} 1.2, |x_1(t)| < 1, \\ 1.1, |x_1(t)| > 1, \end{cases} \quad b_{12}(x_1) = \begin{cases} -3, |x_1(t)| < 1, \\ 3, |x_1(t)| > 1, \end{cases}$$

$$b_{13}(x_1) = \begin{cases} -3, |x_1(t)| < 1, \\ 3, |x_1(t)| > 1, \end{cases} \quad b_{21}(x_2) = \begin{cases} -3, |x_2(t)| < 1, \\ 3, |x_2(t)| > 1, \end{cases}$$

$$b_{22}(x_2) = \begin{cases} 1, |x_2(t)| < 1, \\ -1, |x_2(t)| > 1, \end{cases} \quad b_{23}(x_2) = \begin{cases} -4, |x_2(t)| < 1, \\ 4, |x_2(t)| > 1, \end{cases}$$

$$b_{31}(x_3) = \begin{cases} -3, |x_3(t)| < 1, \\ 3, |x_3(t)| > 1, \end{cases} \quad b_{32}(x_3) = \begin{cases} -4, |x_3(t)| < 1, \\ 4, |x_3(t)| > 1, \end{cases}$$

$$b_{33}(x_3) = \begin{cases} 1.2, |x_3(t)| < 1, \\ -1.2, |x_3(t)| > 1, \end{cases} \quad c_{11}(x_1) = \begin{cases} 1.25, |x_1(t)| < 1, \\ -1.25, |x_1(t)| > 1, \end{cases}$$

$$c_{12}(x_1) = \begin{cases} -3.2, |x_1(t)| < 1, \\ 3.2, |x_1(t)| > 1, \end{cases} \quad c_{13}(x_1) = \begin{cases} -3.2, |x_1(t)| < 1, \\ 3.2, |x_1(t)| > 1, \end{cases}$$

$$c_{21}(x_2) = \begin{cases} -3.2, |x_2(t)| < 1, \\ 3.2, |x_2(t)| > 1, \end{cases} \quad c_{22}(x_2) = \begin{cases} 1.1, |x_2(t)| < 1, \\ -1.1, |x_2(t)| > 1, \end{cases}$$

$$c_{23}(x_2) = \begin{cases} -4.4, |x_2(t)| < 1, \\ 4.4, |x_2(t)| > 1, \end{cases} \quad c_{31}(x_3) = \begin{cases} -3.2, |x_3(t)| < 1, \\ 3.2, |x_3(t)| > 1, \end{cases}$$

$$c_{32}(x_3) = \begin{cases} -4.4, |x_3(t)| < 1, \\ 4.4, |x_3(t)| > 1, \end{cases} \quad c_{33}(x_3) = \begin{cases} 1, |x_3(t)| < 1, \\ 1, |x_3(t)| > 1. \end{cases}$$

Using the bipartite consensus $V_1 = \{1, 2, 3, 5, 10\}$, $V_2 = \{4, 6, 7, 8, 9\}$, $W = \{1, 1, 1, -1, 1, -1, -1, -1, -1, 1\}$, $d_4 = 30$,
$\sigma = 25$ and $q = 0.85$, the activation function $f(x_i(t)) = g(x_i(t - \tau(t))) = tanh(x_{ij}(t))$ which satisfies $F = G = 1$. Hence from the lemma 2, we get $\lambda_i = 0.4280$. Using MATLAB LMI toolbox on the hypothesis of Theorem 1, we can see that the inequality (15) is feasible with positive definite matrices.

$$P = \begin{pmatrix} 0.3669 & 0.3971 & 0.1965 \\ 0.3971 & 0.7657 & 0.3495 \\ 0.1965 & 0.3495 & 0.3012 \end{pmatrix}; \quad P_1 = \begin{pmatrix} 0.5171 & 0.4022 & 0.3270 \\ 0.4022 & 0.6312 & 0.3801 \\ 0.3270 & 0.3801 & 0.4862 \end{pmatrix};$$

$$P_2 = \begin{pmatrix} 0.3457 & 0.1945 & 0.1945 \\ 0.1945 & 0.3457 & 0.1945 \\ 0.1945 & 0.1945 & 0.3457 \end{pmatrix}.$$

Figures 2, 3 and 4 represents directed signed graph with ten vertices of cooperative and competitive relationship and the bipartite leader synchronization and transmit interval of leader bipartite synchronization respectively.

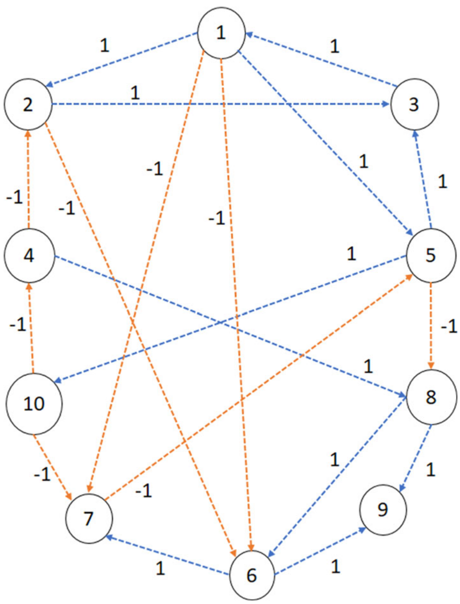

Fig. 2 Signed network with ten vertices. Blue dot line represents cooperative and red line represents competitive

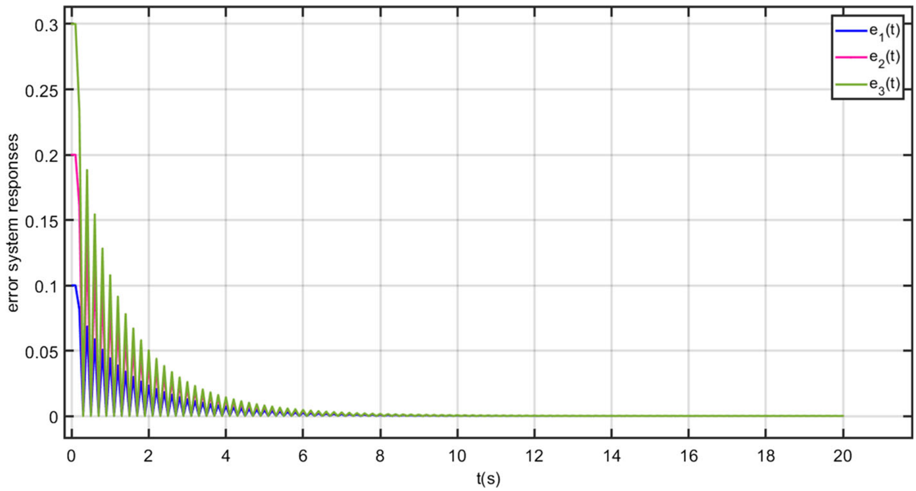

Fig. 3 Leader bipartite synchronization

Example 2 Consider Eq. (17) with $A = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 20 \end{pmatrix}$, the memristor values same as in example 1. By the bipartite consensus $V_1 = \{1, 2, 3, 5, 10\}$, $V_2 = \{4, 6, 7, 8, 9\}$, $W = \{1, 1, 1, -1, 1, -1, -1, -1, -1, 1\}$, $q = 0.9$. Hence all the hypotheses of Theorem 3 are satisfied and with LMI toolbox of MATLAB, inequality (19) is feasible with following positive definite matrices,

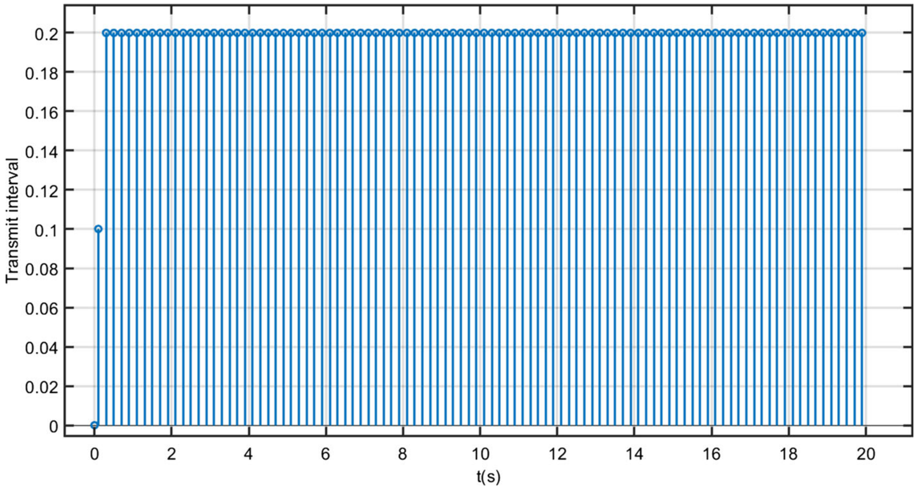

Fig. 4 Transmit interval of leader bipartite synchronization with event triggered control

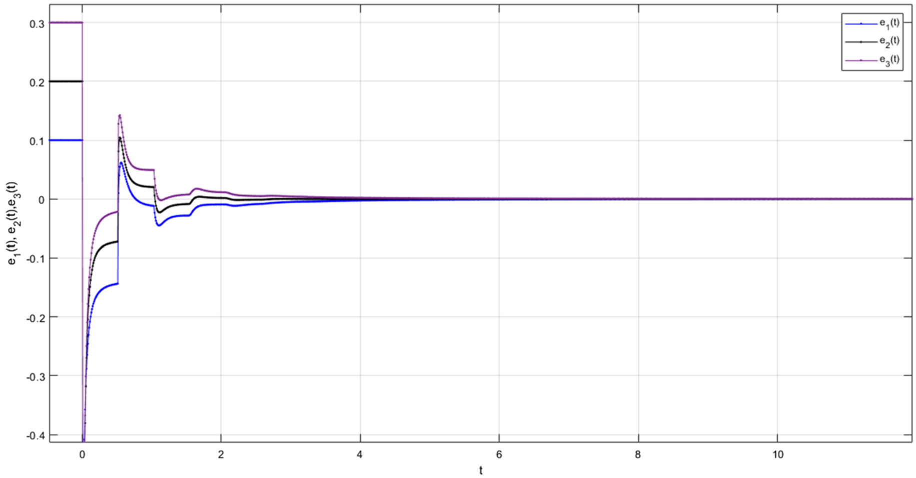

Fig. 5 Leaderless bipartite synchronization

$$P = \begin{pmatrix} 1.4939 & 1.1273 & 0.0979 \\ 1.1273 & 3.3104 & 0.7153 \\ 0.0979 & 0.7153 & 1.0138 \end{pmatrix}; \quad P_1 = \begin{pmatrix} 2.4789 & 1.8289 & 1.1824 \\ 1.8289 & 3.3185 & 1.6717 \\ 1.1824 & 1.6717 & 2.1489 \end{pmatrix};$$
$$P_2 = \begin{pmatrix} 2.1220 & 1.2110 & 1.2110 \\ 1.2110 & 2.1220 & 1.2110 \\ 1.2110 & 1.2110 & 2.1220 \end{pmatrix}.$$

And the error system (17) is the bipartite leaderless synchronization as shown in Fig. 5.

Example 3 With the help of MATLAB LMI toolbox, applied on the hypotheses of Theorem 4 and inequality (20) we can find the positive definite matrices, $P = \begin{pmatrix} 7.7282 & 9.0471 & 4.0231 \\ 9.0471 & 18.5602 & 7.4776 \\ 4.0231 & 7.4776 & 6.0243 \end{pmatrix}$;
$P_1 = \begin{pmatrix} 10.0111 & 7.6591 & 6.2106 \\ 7.6591 & 12.5054 & 7.3478 \\ 6.2106 & 7.3478 & 9.2677 \end{pmatrix}$; $P_2 = \begin{pmatrix} 6.4911 & 3.6451 & 3.6451 \\ 3.6451 & 6.4911 & 3.6451 \\ 3.6451 & 3.6451 & 6.4911 \end{pmatrix}$;
$R = \begin{pmatrix} 7.5565 & 3.2998 & 3.2998 \\ 3.2998 & 7.5565 & 3.2998 \\ 3.2998 & 3.2998 & 7.5565 \end{pmatrix}$.

Example 4 With the help of MATLAB LMI toolbox, applied on the hypotheses of Theorem 6 and inequality (22) we can find the positive definite matrices, $P = \begin{pmatrix} 2.9124 & 2.9066 & 0.3161 \\ 2.9066 & 8.5683 & 2.1383 \\ 0.3161 & 2.1383 & 1.8351 \end{pmatrix}$;
$P_1 = \begin{pmatrix} 4.5388 & 3.6097 & 2.0413 \\ 3.6097 & 6.9409 & 3.2591 \\ 2.0413 & 3.2591 & 3.7073 \end{pmatrix}$; $P_2 = \begin{pmatrix} 3.7011 & 2.0787 & 2.0787 \\ 2.0787 & 3.7011 & 2.0787 \\ 2.0787 & 2.0787 & 3.7011 \end{pmatrix}$;
$R = \begin{pmatrix} 2.5916 & 0.1197 & 0.1197 \\ 0.1197 & 2.5916 & 0.1197 \\ 0.1197 & 0.1197 & 2.5916 \end{pmatrix}$.

8 Conclusion

This paper analyses FMMCDNN and has given proof for the bipartite leader and leaderless synchronization of a structurally balanced signed network with event-triggered pinning control. But it is difficult to do the same in a structurally unbalanced signed network. It can be considered a future problem. And in this paper, we have used LPLF with LKF and successfully proved the bipartite leader and leaderless synchronization.

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