Quantum erasure of decoherence

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Abstract. We consider the classical algebra of observables that are diagonal in a given orthonormal basis, and define a complete decoherence process as a completely positive map that asymptotically converts any quantum observable into a diagonal one, while preserving the elements of the classical algebra. For quantum systems in dimension two and three any decoherence process can be undone by collecting classical information from the environment and using such an information to restore the initial system state. As a relevant example, we illustrate the quantum eraser of Scully *et al.* [Nature **351**, 111 (1991)] as an example of environment-assisted correction. Moreover, we present the generalization of the eraser setup for $d-$dimensional systems, showing that any von Neumann measurement on a system can be undone by a complementary measurement on the environment.

1. Introduction

One of the fundamental postulates in Quantum Mechanics states that a closed system naturally evolves according to a suitable unitary transformation. It is then understood that every open system can in principle be *closed*, in the sense that, by extending the boundaries of the subsystem of interest, it is in principle possible to reach a situation in which everything inside the boundaries obeys a global unitary evolution. In this case, *information* is conserved, that is, there is no net flow of information from the global system. The global evolution preserves indeed the amount of information that can be extracted from an arbitrary set of signal states in which a classical alphabet is encoded, allowing only transfers of information from a subsystem to another.

Here we are interested in a much more particular situation, that is, when the quantum system of interest—the *input* system—unitarily interacts with an environment on which we can perform measurements. In other words, even if the system itself evolves as an open quantum system, according to the dynamics described by a *quantum channel* [1], the complementary subsystem closing the main system is bounded and can be monitored by suitable measurements. We can then exploit a kind of feedback control on the main system, in which we apply some opportune corrections to the system, conditional on the outcomes of the measurement that was performed on the environment. This procedure is called *environment-assisted channel correction* [2] and recently attracted a lot of interest [3], also in connection with the recently discovered *state merging* protocol [4].

In the present paper, we focus on a particular type of open system dynamics, which are usually believed to play a fundamental role in understanding the *quantum-to-classical* transition, namely, decohering evolutions [5]. This kind of channels causes loss of coherence in quantum systems, and this phenomenon usually constitutes the major practical limitation in quantum information processing. A large part of the literature concerning quantum error correction is devoted to engineering methods to combat the effects of decoherence [6]. Here we propose a decoherence correction method based on an environment-assisted control, providing necessary and sufficient conditions for such a method to be effective. Moreover, our analysis will be able to shed some light on the information exchange dynamics between a quantum system and the environment during a decohering evolution. From this point of view, we will also review the quantum eraser arrangement [7] as a particular example of decohering evolution with a controllable environment, in which a *re-coherence* is possible conditional on the outcomes of a suitable environment observable.

2. Completely decohering evolutions

Let's denote by $\mathscr{A}_q$ the "quantum algebra" of all bounded operators on the Hilbert space $\mathcal{H}$, with $\dim \mathcal{H} = d < \infty$, and by $\mathscr{A}_c$ the "classical algebra", namely any maximal Abelian subalgebra $\mathscr{A}_c \subset \mathscr{A}_q$. Clearly, all operators in $\mathscr{A}_c$ can be jointly diagonalized on a common orthonormal basis, which in the following will be denoted as $\mathsf{B} = \{|k\rangle \mid k = 1, \dots, d\}$. Then, the classical algebra $\mathscr{A}_c$ is also the linear span of the one-dimensional projectors $|k\rangle\langle k|$, whence $\mathscr{A}_c$ is a d-dimensional vector space. According to the above general framework, we call *(complete) decoherence map* a completely positive identity-preserving (i. e. trace-preserving in the Schrödinger picture) map $\mathcal{E}$ which asymptotically maps any observable $O \in \mathscr{A}_q$ into a corresponding classical observable $O_c \in \mathscr{A}_c$, while preserving any element of the classical algebra $\mathscr{A}_c$. The defining properties of a decoherence map are then written explicitely as:

$$\forall O \in \mathscr{A}_q : \qquad \exists \lim_{n\to\infty} \mathcal{E}^n(O) \in \mathscr{A}_c \tag{1}$$

and

$$\forall O_c \in \mathscr{A}_c : \qquad \mathcal{E}(O_c) = O_c \ . \tag{2}$$

An important requirement in the above definition of decoherence processes is that any classical observable is preserved. Notice that, for example, the case of amplitude damping channels is not covered by the definition, since in this case any state is driven to a fixed state, namely not all classical observables are preserved.

It is easy to see that the set of decoherence maps is convex (i. e. if we mix two decoherence maps we obtain again a decoherence map). According to Eq. (2), the set of decoherence maps is a subset of the convex set of maps that preserve the elements of the classical algebra $\mathscr{A}_c$. The convex structure of decoherence maps has been analysed in Ref. [8] using the following representation theorem

THEOREM 1. *A map $\mathcal{E}$ preserves all elements of the classical algebra $\mathcal{A}_c$ if and only if it has the form*

$$\mathcal{E}(O) = \xi \circ O. \tag{3}$$

$A \circ B$ denoting the Schur product of operators A and B, i. e.

$$A \circ B \doteq \sum_{k,l=1}^{d} A_{kl} B_{kl} |k\rangle\langle l| \ , \tag{4}$$

$\{A_{kl}\}$ and $\{B_{kl}\}$ being the matrix elements of A and B in the basis B, and ξ_{kl} being a correlation matrix, i.e. a positive semidefinite matrix with $\xi_{kk} = 1$ for all $k = 1, \ldots, d$.

Incidentally, notice that the operator ξ in Eq. (3) is isometrically equivalent to the Choi operator [9] $R_C = \sum_{k,l} \ \xi_{kl} \ |k\rangle|k\rangle\langle l|\langle l|$, which in turn is in one-to-one linear correspondence with the Jamiołkowski operator [10] $R_J = \sum_{k,l} \ \xi_{kl} \ |l\rangle|k\rangle\langle k|\langle l|$. Theorem 1 establishes a one-to-one linear correspondence between maps that preserve the classical algebra $\mathscr{A}_c$ and correlation matrices. This means that both sets share exactly the same convex structure, whence a map is extremal if and only if the corresponding correlation matrix is an extreme point. The decohering evolutions, that have the additional property of Eq. (1), are represented by correlation matrices ξ with the property $|\xi_{kl}| < 1, \quad \forall k \neq l$.

The extreme points of the set of correlation matrices have been characterized by Li and Tam in Ref. [11]. They proved that for $d = 2, 3$, a correlation matrix is extremal if and only if it is rank-one. This statement, translated in terms of maps, informs us that, for $d = 2, 3$ extreme points of the convex set of maps that preserve the classical algebra are unitary maps [8]. As a consequence, for qubits and qutrits, every decoherence map can be written as

$$\mathcal{E}(O) = \sum_i p_i U_i^\dagger O U_i, \tag{5}$$

where U_i's are unitary operators and p_i is a probability distribution, namely any decoherence map is *random-unitary*. However, already for $d = 4$ it is possible to explicitly show [8] that there exist extreme correlation matrices with rank greater than one, and hence, decoherence maps that are not random-unitary.

Notice that the action of the map $\mathcal{E}$ in the Schrödinger picture can be simply and *uniquely* derived from the trace-duality formula $\mathrm{Tr}[\mathcal{E}(O)\ \rho] = \mathrm{Tr}[O\ \mathcal{E}'(\rho)]$. From Eq. (3) it follows that, in the case of decohering maps, $\mathcal{E}'(\rho) = \xi^T \circ \rho$, where ξ^T denotes the transposition of the matrix ξ with respect to the fixed basis B diagonalizing the classical algebra. As a consequence, one has exponential decay of the off-diagonal elements of ρ, since $|[\mathcal{E}'^n(\rho)]_{kl}| = |\xi_{lk}|^n \cdot |\rho_{kl}|$ and $|\xi_{kl}| < 1 \quad \forall k \neq l$. In other words, any initial state ρ decays exponentially towards the completely decohered state ρ_∞ defined as

$$\rho_\infty \doteq \sum_k \rho_{kk} |k\rangle\langle k| \ , \tag{6}$$

namely, its diagonal with respect to the fixed basis B. Since a matrix ξ is a correlation matrix if and only if its transposition ξ^T is, in the following, when there is no possibility of confusion, we will use the same symbol $\mathcal{E}$ to denote the action of the map on operators as well as on density matrices, also omitting the transposition over ξ.

3. Environment-assisted control

In Ref. [2], the following general situation is considered. A channel $\mathcal{E}$, acting on density matrices ρ on the input Hilbert space $\mathcal{H}$, is given. As a consequence of the Stinespring theorem [12], we can always write it as follows [13]

$$\mathcal{E}(\rho) = \mathrm{Tr}_e[U(\rho \otimes |0\rangle\langle 0|_e) U^\dagger], \tag{7}$$

namely, as a unitary interaction between the system and an *environment*, described by the Hilbert space $\mathcal{H}_e$, followed by a trace over the environment degrees of freedom. If the environment input state is a pure one—like in Eq. (7)—Gregoratti and Werner [2] proved that, assuming a somehow "controllable" environment, for all possible unitary interactions U in Eq. (7), and for all possible decompositions of the channel $\mathcal{E}$ into pure Kraus representations $\mathcal{E}(\rho) = \sum_i E_i \rho E_i^\dagger$ [1], there exists a suitable rank-one POVM on the environment, let us call it $\{|v_i\rangle\langle v_i|_e\}$, $\sum_i |v_i\rangle\langle v_i|_e = I_e$, such that

$$E_i \rho E_i^\dagger = \mathrm{Tr}_e[U(\rho \otimes |0\rangle\langle 0|_e) U^\dagger \ (I \otimes |v_i\rangle\langle v_i|_e)]. \tag{8}$$

Within this setting, one can then think of performing a correction $\mathcal{C}_i$ on the system conditional on the i-th outcome of the environment measurement, thus obtaining the following overall corrected channel

$$\mathcal{E}_{\mathrm{corr}}(\rho) = \sum_i \mathcal{C}_i(E_i \rho E_i^\dagger). \tag{9}$$

In Ref. [2] it is shown that the only channels that can be perfectly inverted by monitoring the environment—i. e. such that it is possible to have $\mathcal{E}_{\rm corr}(\rho) = \rho$, for all ρ—are the random unitary ones. Therefore, it follows that one can perfectly correct any decoherence map for qubits and qutrits by monitoring the environment. The correction is achieved by retrieving the index i in Eq. (5) via the measurement on the environment represented by the rank-one POVM $\{|v_i\rangle\langle v_i|_e\}$, and then by applying the inverse of the unitary transformation U_i on the system. Therefore, the random-unitary map simply leaks $H(p_i)$ bits of classical information into the environment, where H denotes the Shannon entropy and p_i is the probability of the outcome "i". The effects of decoherence can be completely eliminated by recovering such classical information, without any prior knowledge about the input state.

4. Bounds on the information flow

It is now interesting to address the problem of estimating the amount of classical information needed in order to invert a random-unitary decoherence map. If the environment is initially in a pure state, say $|0\rangle_e$, a useful quantity to deal with is the so-called entropy exchange [14] $S_{\rm ex}$ defined as

$$S_{\rm ex}(\rho) = S(\sigma_e^\rho), \tag{10}$$

where σ_e^ρ is the reduced environment state after the interaction with the system in the state ρ, and $S(\rho) = -\operatorname{Tr}[\rho \log \rho]$ is the von Neumann entropy. The entropy exchange quantifies the information flow from the system to the environment and, for all input states ρ, one has the bound [14] $|S(\mathcal{E}(\rho)) - S(\rho)| \leq S_{\rm ex}(\rho)$, namely the entropy exchange $S_{\rm ex}$ bounds the entropy production at each step of the decoherence process.

In the case of initially pure environment, the entropy exchange depends only on the map $\mathcal{E}$ and on the input state of the system ρ, regardless of the particular system-environment interaction U that is chosen to model $\mathcal{E}$ via Eq. (7). In particular, by the Kolmogorov decomposition for nonnegative definite matrices it is always possible to write $\xi_{kl} = \langle e_l|e_k\rangle$ for a suitable set of normalized vectors $\{|e_k\rangle\}$, and the map $\mathcal{E}(\rho) = \xi \circ \rho$ can be realized as $\mathcal{E}(\rho) = \operatorname{Tr}_e[U(\rho \otimes |0\rangle\langle 0|_e)U^\dagger]$, where the unitary interaction U gives the transformation

$$U|k\rangle \otimes |0\rangle_e = |k\rangle \otimes |e_k\rangle. \tag{11}$$

With this choice of the interaction U, the final reduced state of the environment is $\sigma_e^\rho = \sum_k \rho_{kk} |e_k\rangle\langle e_k|$. In order to evaluate the entropy exchange $S_{\rm ex}$ for a decoherence map $\mathcal{E}(\rho) = \xi \circ \rho$, one can then use the formula

$$S_{\rm ex}(\rho) = S(\sqrt{\rho_\infty}\xi\sqrt{\rho_\infty}), \tag{12}$$

which follows immediately from the fact that $\sqrt{\rho_\infty}\xi\sqrt{\rho_\infty}$, and σ_e^ρ are both reduced states of the same bipartite state $\sum_i \sqrt{\rho_{ii}}|i\rangle|e_i\rangle$.

When a map can be inverted by monitoring the environment—i. e. in the random-unitary case—the entropy exchange $S_{\rm ex}(I/d)$ provides a lower bound to the amount of classical information $H(p_i)$ that must be collected from the environment in order to perform the correction scheme of Ref. [2]. In fact, assuming a random-unitary decomposition (5) and using the formula [14] $S_{\rm ex}(\rho) = S\left(\sum_{i,j}\sqrt{p_i p_j}\,{\rm Tr}[U_i\rho U_j^\dagger]|i\rangle\langle j|\right)$, we obtain

$$S_{\rm ex}(I/d) \le H(p_i). \tag{13}$$

The inequality comes from the fact that the diagonal entries of a density matrix are always majorized by its eigenvalues, and it becomes equality if and only if ${\rm Tr}[U_iU_j^\dagger]/d = \delta_{ij}$, i. e. the map admits a random-unitary decomposition with *orthogonal* unitary operators. Moreover, from Eq. (12) we have $S_{\rm ex}(I/d) = S(\xi/d)$, whence the relation

$$H(p_i) \ge S(\xi/d)\ , \tag{14}$$

which gives a lower bound on the amount of information needed from the environment in order to invert the decohering evolution.

On the other hand, the random-unitary representation (5), when possible, is highly non unique. This means that, depending on the particular unitary operators chosen, the entropy $H(p_i)$ can be made as large as desired. However, it is still possible to provide a (generally non tight) upper bound to the *minimum value* of the amount of classical information $H(p_i)$. Such a bound is derived in Ref. [15] as $H(p_i) \le 2\log{\rm rank}\,\xi$, and hence it generally holds that

$$S(\xi/d) \le H(p_i) \le 2\log{\rm rank}\,\xi. \tag{15}$$

Eq. (15) is true for all dimensions d. It is then reasonable that it does not accurately describe the peculiar geometry enjoyed by two-dimensional systems. In fact, in Ref. [8] it is proved that for $d = 2$, it always holds that

$$H(p_i) = S(\xi/2), \qquad d = 2. \tag{16}$$

However, already for $d = 3$, there exist random-unitary decoherence maps for which $S(\xi/d) < H(p_i)$ *strictly*, and at the moment we are not able to provide a better upper bound than the one given above.

5. Example: the quantum eraser

Our results about the possibility of inverting decohering evolutions by collecting classical information from the environment can boast a celebrated *ante litteram* example, namely the quantum eraser of Ref. [16]. In this Section we briefly review this example using a compact notation that will turn out to be useful for its generalization to similar cases in higher dimension.

Let an excited atom pass through a double-slit, as depicted in Fig 1. Its state

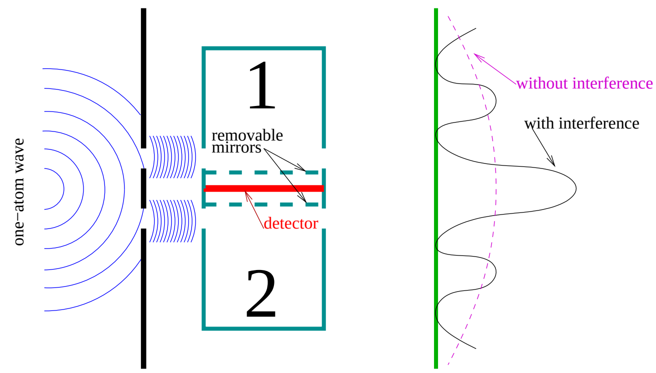

Fig. 1: The Quantum Eraser arrangement.

can be described in full generality by a density matrix ρ, such that, if the orthogonal states $|1\rangle$ and $|2\rangle$ correspond to the particle passing through the slit number 1 or number 2, respectively, the probability of detecting the particle passing through the slit number 1 (2) is $p(1) = \langle 1|\rho|1\rangle$ ($p(2) = \langle 2|\rho|2\rangle$). Notice that ρ can be a pure state, as in the original quantum eraser proposal $\rho = |+\rangle\langle +|$, with $|+\rangle = 1/\sqrt{2}\,(|1\rangle + |2\rangle)$.

If nothing is in between the slits and the collecting screen at the end, fringes can be observed in the interference pattern, coming from the non-null off-diagonal terms $\langle 1|\rho|2\rangle$ and $\langle 2|\rho|1\rangle$. But if we place a probe, as in Fig. 1, consisting of two resonant cavities initialized in the vacuum state $|0\rangle_p$, then interference fringes disappear, since the atom, while relaxing to its ground state, leaves a photon in one out of the two cavities, depending of the slit it passed through. The interaction of the atom with the probe can be described by means of a controlled unitary U of the form (11), namely

$$U|i\rangle \otimes |0\rangle_p = |i\rangle \otimes |i\rangle_p, \qquad i = 1, 2, \tag{17}$$

where $|1\rangle_p$ and $|2\rangle_p$ are the orthogonal states of the electromagnetic field corresponding to the situations "one photon in cavity 1" and "one photon in cavity 2", respectively. Since $|1\rangle_p$ and $|2\rangle_p$ are orthogonal, the input state ρ instantaneously collapses to its decohered final state ρ_∞, and off-diagonal terms are annihilated. This fact is usually interpreted as saying that the probe, by means of the interaction (17), keeps track of the *which-way* information about the atom's path, in such a way that such an information can be in principle extracted by the experimenter. Nevertheless, it is still possible to *erase* the which-path from the probe by measuring on it the Fourier-conjugate observable $\{|+\rangle\langle+|, |-\rangle\langle-|\}$, where $|-\rangle = 1/\sqrt{2}\,(|1\rangle - |2\rangle)$. Experimentally, this can be done long *after* the atom passed through the cavities, by removing at once both mirrors in Fig. 1, in such a way that the detector between the two cavities is coupled with the symmetric state of the radiation inside them [16]. Then, separating the two subensembles of events corresponding to the measurement outcomes $+$ and $-$, it is possible to retrieve the original interference fringes.

We interpret the whole double-slit setup as being a realization of a completely decohering process described by the channel

$$\mathcal{E}(\rho) = \sum_{i=1}^{2} |i\rangle\langle i|\rho|i\rangle\langle i| = I \circ \rho. \tag{18}$$

Such a channel is actually random-unitary (it is a decohering process in dimension two), and hence is correctable by an environment-assisted control procedure. In particular, for the atom-radiation interaction given by Eq. (17), by measuring the observable $\{|+\rangle\langle+|, |-\rangle\langle-|\}$ on the probe, we obtain a realization of the random-unitary Kraus representation

$$\mathcal{E}(\rho) = \frac{1}{2}\rho + \frac{1}{2}\sigma_z\rho\sigma_z, \tag{19}$$

In conclusion, conditionally on the probe outcomes, both atom final states conserve the original off-diagonal terms (a part of an innocuous unitary rotation), and fringes appear on the intereference pattern on the screen. Moreover, from Eq. (16), since $\xi = I$, we know that the erasure process picks up from the probe $S(I/2) = 1$ bit of information.

The quantum eraser can be simply generalized to the case of instantaneous decoherence of $d-$dimensional quantum system. This situation can be thought of as a kind of "d-slits" interference experiment, where an excited atom emits a photon in one out of d possible cavities. Analogously to the two-dimensional situation, the correlation matrix describing the intantaneous decoherence channel is $\xi = I$, namely one has $\mathcal{E}(\rho) = \sum_{i=1}^{d} \langle i|\rho|i\rangle\ |i\rangle\langle i| = I \circ \rho$. The channel itself admits the random-unitary representation

$$\mathcal{E} = \frac{1}{d}\sum_{j=1}^{d} Z_j\rho Z_j^\dagger, \qquad Z_j = \sum_{k=1}^{d} e^{2\pi i\frac{kj}{d}}|k\rangle\langle k|\ , \tag{20}$$

where the unitary operators Z_j's generalize I and σ_z to the d-dimensional case. In this case, for the system-probe interaction given by $U|i\rangle \otimes |0\rangle_p = |i\rangle \otimes |i\rangle_p,\ i =$

$1, \ldots, d$, the terms of the random-unitary decomposition can be isolated by measuring the probe observable $\{|\tilde{e}_j\rangle\langle\tilde{e}_j|\}$, where the vectors $|\tilde{e}_j\rangle$ are the Fourier transform

$$|\tilde{e}_j\rangle = \frac{1}{\sqrt{d}} \sum_k e^{2\pi i \frac{jk}{d}} |k\rangle \tag{21}$$

of the elements of the decoherence basis B. Once the measurement outcome "j" is known, it is enough to undo the unitary Z_j to retrieve any unknown initial state ρ. The amount of classical information to be erased from the probe is then equal to $H(p_i) = \log d$.

An equivalent way of presenting the $d-$dimensional eraser is by stating that *any von Neumann measurement on a system can be erased by its Fourier complementary measurement on the environment.* The istantaneous decoherence $\mathcal{E}(\rho)$ $= \sum_i \langle i|\rho|i\rangle\ |i\rangle\langle i|$ can be indeed considered as the effect of the von Neumann measurement of the observable $\{|i\rangle\langle i|\}$, while the interaction $U : |i\rangle \otimes |0\rangle_p \mapsto |i\rangle \otimes |i\rangle_p$ can be viewed as the transfer of classical information from the system to a quantum register. On the other hand, the Fourier-complementary measurement $\{|\tilde{e}_j\rangle\langle\tilde{e}_j\}$ allows one to extract from the classical register the information needed to restore coherence in the system. Quite naturally, this amount of information is exactly the same amount that was stored into the register, maximized over all possible unknown states ρ, i.e. $\log d$.

Acknowledgments

Stimulating and enjoyable discussions with K Życzkowski are gratefully acknowledged. This work has been supported by Ministero Italiano dell'Università e della Ricerca (MIUR) through PRIN 2005. F B acknowledges Japan Science and Technology Agency for support through the ERATO-SORST Project on Quantum Computation and Information.

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