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Hybrid Quantum Cloning Machine

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Abstract

In this work, we introduce a special kind of quantum cloning machine called Hybrid quantum cloning machine. The introduced Hybrid quantum cloning machine or transformation is nothing but a combination of pre-existing quantum cloning transformations. In this sense it creates its own identity in the field of quantum cloners. Hybrid quantum cloning machine can be of two types: (i) State dependent and (ii) State independent or Universal. We study here the above two types of Hybrid quantum cloning machines. Later we will show that the state dependent hybrid quantum-cloning machine can be applied on only four input states. We will also find in this paper another asymmetric universal quantum cloning machine constructed from the combination of optimal universal B-H quantum cloning machine and universal anti-cloning machine. The fidelities of the two outputs are different and their values lie in the neighborhood of $\frac{5}{6}$.

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1 Introduction

A fundamental restriction in quantum theory is that quantum information cannot be copied perfectly [1] in contrast with the information we talk about in classical world. Similarly, it is known that quantum information cannot be deleted against a copy [2, 21, 22]. But if we pay some price, then approximate or exact cloning and deletion operations are possible. For example, it does not prohibit the possibility of approximate cloning of an arbitrary state of a quantum mechanical system. The existence of Universal Copying Machine' (UCM) created a class of approximate cloning machines which are independent of the amplitude of the input state [3, 4, 5]. The optimality of such cloning transformations has been verified [4]. There also exists another class of copying machines which are state dependent. The original proof of the no-cloning theorem was based on the linearity of the evolution. Later it was shown that the unitarity of quantum theory also forbids us from accurate cloning of non-orthogonal states with certainty [17, 18]. But non-orthogonal states secretly chosen from a set can be faithfully cloned with certain probabilities [6, 7] or can evolve into a linear superposition of multiple-copy states together with a failure term described by a composite state [19] if and only if the states are linearly independent.

The usual scheme of cloning consists of sending a single photon into an amplifying medium. If there is no photon in the medium, it spontaneously emit photon of any polarization but if the photon is present, the amplifying medium stimulates the emission of another photon in the same polarization . The quality of the amplification process is never perfect because spontaneous emission can never be suppressed [28]. The $1 \rightarrow 2$ quantum cloning machine can be implemented optically when we take into account the fact that there is a bridge between stimulated emission and quantum cloning. One of the first optical experiments using only linear optics by Huang et.al [23] that implemented the Buzek-Hillery cloning.

Most optical implementations of the $1 \rightarrow 2$ cloning machine use parametric down conversion as the amplification phenomenon. The cloning fidelities obtained in experiments with parametric down conversion are 0.81 ± 0.01 [24] and 0.810 ± 0.08 [25, 26].

As it is not possible to realize a perfect U-NOT gate which would flip an arbitrary qubit state, it is necessary to investigate what is the best approximation to this gate [27]. Martini et.al. reported the experimental realization of universal quantum machine that performs the best possible approximation to the universal NOT transformation. The optimal U-NOT transformation for flipping a single qubit is given by,
$U|\psi\rangle_a \otimes |X\rangle_{bc} = \sqrt{\frac{2}{3}}|\psi\psi\rangle_{ab}|\psi^+\rangle_c - \sqrt{\frac{1}{3}}(|\psi,\psi^+\rangle_{ab} + |\psi^+,\psi\rangle_{ab})|\psi\rangle_c$
where the gate prepared in the state $|X\rangle_{bc}$, independently of the input state $|\psi\rangle$. The above transformation describes a process when the original qubit is encoded in the system 'a', while the flipped qubit is in the system c. The density operator describing the output state of the system c is
$\rho^{(out)} = \frac{2}{3}|\psi^+\rangle\langle\psi^+| + \frac{1}{3}|\psi\rangle\langle\psi|$
Therefore, the average fidelity of the universal NOT gate is $F = \langle\psi^+|\rho^{(out)}|\psi^+\rangle = \frac{2}{3}$, which is exactly same as the fidelity of the optimal state estimation for single qubit. In the case where a qubit is encoded into a physical system to utilize the polarization states of the photon, the U-Not gate can be realized via stimulated emission. Martini's et.al experiment was based on the proposal that universal quantum machine such as quantum cloner can be realized with the help of stimulated emission in parametric down conversion. The reported experimental fidelity for the optimal U-NOT transformation is 0.630 ± 0.008.

In quantum world it is very important to know various limitations imposed by quantum theory on quantum information. Recently, some general impossible operations are studied by Pati [20] in detail. This unifies the no-cloning, no-compelementing and no-conjugating theorems in quantum information theory. Among all these impossible operations, the impossibility of 'cloning-cum-complementing' quantum machines attracts much attention here in the sense that it is a combination of cloning machine and complementing machine where the probabilities of separately existing cloning machines are

λ and $1-\lambda$, respectively. In the same spirit, we can imagine a hybrid cloning machine which is a superposition of two cloning machines with appropriate amplitudes [20]. When the corresponding probability λ takes value between 0 and 1 the resulting combined cloning machines can be identified as a 'Hybrid Cloning Machine' (HCM). Therefore, one can construct hybrid cloning machine by combining different existing cloning transformations. Our objective is to study the behavior of such types of Hybrid cloning machines. Also, we would like to see if there is any improvements in the fidelity or average fidelity of cloning under some special combinations. The present work is organized as follows. In section 2, for the sake of completeness we recapitulate all the different existing quantum cloning machines like Wootters-Zurek (WZ) quantum cloning machine, Buzek-Hillery (BH) quantum cloning machine, Phase Covariant quantum cloning machine, Pauli Asymmetric quantum cloning machines and Universal Anti cloning machine. In section 3, we study the combination of such types of cloning machines which gives state dependent hybrid quantum cloning machine. We show here that the state dependent hybrid quantum cloning machine produces better quality copy for only four input states. In section 4, we study the state independent hybrid quantum cloning machines. Interestingly, we are able to construct here an universal asymmetric hybrid quantum cloning machine whose fidelity of copying lie in the neighborhood of the optimal fidelity $\frac{5}{6}$. Then the conclusion follows.

2 Descriptions of existing quantum cloning machines

For the sake of completeness, in this section we briefly discuss about some existing quantum cloning machines. Then we study the different combinations of these quantum cloners known as hybrid quantum cloners in the next section.

2.1 The Wootters-Zurek (W-Z) Cloning Machine:

The Wootters and Zurek (W-Z) quantum cloning machine is a state- dependent one because it works perfectly for some inputs and badly for some other. It is defined by the following transformations. In the computational basis states $|0\rangle$ and $|1\rangle$ it is given by

$$|0\rangle|Q\rangle \longrightarrow |0\rangle|0\rangle|Q_0\rangle \tag{1}$$

$$|1\rangle|Q\rangle \longrightarrow |1\rangle|1\rangle|Q_1\rangle. \tag{2}$$

Unitarity of the transformation gives

$$\langle Q|Q\rangle = \langle Q_0|Q_0\rangle = \langle Q_1|Q_1\rangle = 1. \tag{3}$$

Let us now consider purely superposition state given by

$$|\chi\rangle = \alpha|0\rangle + \beta|1\rangle. \tag{4}$$

For simplicity, we will assume that the probability amplitudes are real and $\alpha^2 + \beta^2 = 1$. The density matrix of the state $|\chi\rangle$ in the input mode is given by

$$\rho^{id} = |\chi\rangle\langle\chi| = \alpha^2|0\rangle\langle 0| + \alpha\beta|0\rangle\langle 1| + \alpha\beta|1\rangle\langle 0| + \beta^2|1\rangle\langle 1|. \tag{5}$$

After applying the cloning transformation (1-2) the arbitrary quantum state (4) takes the form

$$|\psi^{out}\rangle = \alpha|0\rangle|0\rangle|Q_0\rangle + \beta|1\rangle|1\rangle|Q_1\rangle. \tag{6}$$

If it is assumed that two copying machine states $|Q_0\rangle$ and $|Q_1\rangle$ are orthonormal, then the reduced density operator $\rho_{ab}^{(out)}$ is given by

$$\rho_{ab}^{(out)} = Tr_x[\rho_{abx}^{(out)}] = \alpha^2|00\rangle\langle 00| + \beta^2|11\rangle\langle 11|. \tag{7}$$

The reduced density operators describing the original and the copy mode are given by

$$\rho_a^{(out)} = Tr_b[\rho_{ab}^{(out)}] = \alpha^2|0\rangle\langle 0| + \beta^2|1\rangle\langle 1|, \tag{8}$$

$$\rho_b^{(out)} = Tr_a[\rho_{ab}^{(out)}] = \alpha^2|0\rangle\langle 0| + \beta^2|1\rangle\langle 1|. \tag{9}$$

The copying quality, i.e, the distance between the density matrix of the input state $\rho_a^{(id)}$ and the reduced density matrices $\rho_a^{(out)}$, ($\rho_b^{(out)}$) of the output states can be measured by Hilbert-Schmidt norm. The Hilbert-Schmidt norm is defined as

$$D_a = Tr[\rho_a^{(id)} - \rho_a^{(out)}]^2. \tag{10}$$

In spite of having other measures of distance between two pure states Hilbert-Schmidt norm is easier to calculate and also it serves as a good measure of quantifying the distance between the pure states. Therefore, we have

$$D_a = 2\alpha^2\beta^2 = 2\alpha^2(1-\alpha^2) \tag{11}$$

Since D_a depends on α^2, so we have to calculate the average distortion over all input states, i.e., over all α^2 lying between 0 and 1. Thus, the average distortion is given by

$$\overline{D_a} = \int_0^1 D_a(\alpha^2)d\alpha^2 = \frac{1}{3}. \tag{12}$$

2.2 The Buzek-Hillery (B-H) Cloning Machine

The Buzek-Hillery cloning machine is a state independent one. This performs equally well for all input system hence it is a universal cloner. The BH transformation is given by

$$|0\rangle|Q\rangle \longrightarrow |0\rangle|0\rangle|Q_0\rangle + [|0\rangle|1\rangle + |1\rangle|0\rangle]|Y_0\rangle, \tag{13}$$

$$|1\rangle|Q\rangle \longrightarrow |1\rangle|1\rangle|Q_1\rangle + [|0\rangle|1\rangle + |1\rangle|0\rangle]|Y_1\rangle. \tag{14}$$

To maintain the unitarity of the transformation, the following conditions must hold:

$$\langle Q_i|Q_i\rangle + 2\langle Y_i|Y_i\rangle = 1, \quad (i=0,1) \tag{15}$$

$$\langle Y_0|Y_1\rangle = \langle Y_1|Y_0\rangle = 0. \tag{16}$$

It is further assumed that

$$\langle Q_i|Y_i\rangle = 0, \quad (i=0,1) \tag{17}$$

$$\langle Q_0|Q_1\rangle = 0. \tag{18}$$

The density operator of the output state after copying procedure is given by

$$\begin{aligned}\rho_{ab}^{(out)} &= \alpha^2|00\rangle\langle 00|\langle Q_0|Q_0\rangle + \sqrt{2}\alpha\beta|00\rangle\langle +|\langle Y_1|Q_0\rangle + \sqrt{2}\alpha\beta|+\rangle\langle 00|\langle Q_0|Y_1\rangle \\ &+[2\alpha^2\langle Y_0|Y_0\rangle + 2\beta^2\langle Y_1|Y_1\rangle]\langle +|+\rangle + \sqrt{2}\alpha\beta|+\rangle\langle 11|\langle Q_1|Y_0\rangle \\ &+\sqrt{2}\alpha\beta|11\rangle\langle +|\langle Y_0|Q_1\rangle + \beta^2|11\rangle\langle 11|\langle Q_1|Q_1\rangle, \qquad (19)\end{aligned}$$

where $|+\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$. The reduced density operator describing the original mode can be obtained by taking partial trace over the copy mode and it reads as

$$\rho_a^{(out)} = [\alpha^2 + \xi(\beta^2 - \alpha^2)]|0\rangle\langle 0| + \alpha\beta\gamma|0\rangle\langle 1| + \alpha\beta\gamma|1\rangle\langle 0| + [\beta^2 + \xi(\beta^2 - \alpha^2)]|1\rangle\langle 1|, \quad (20)$$

where $\langle Y_0|Y_0\rangle = \langle Y_1|Y_1\rangle \equiv \xi$ and $\langle Y_0|Q_1\rangle = \langle Q_0|Y_1\rangle = \langle Q_1|Y_0\rangle = \langle Y_1|Q_0\rangle = \frac{\eta}{2}$. The density operator $\rho_b^{(out)}$ describing the copy mode is exactly same as the density operator $\rho_a^{(out)}$ describing the original mode. Now the Hilbert Schmidt norm for the density operators $\rho_a^{(id)}$ and $\rho_a^{(out)}$ is given by

$$D_a = 2\xi^2(4\alpha^4 - 4\alpha^2 + 1) + 2\alpha^2\beta^2(\eta - 1)^2 \quad (21)$$

with $0 \le \xi \le \frac{1}{2}$ and $0 \le \eta \le 2\sqrt{\xi(1-2\xi)} \le \frac{1}{\sqrt{2}}$ which follows from Schwarz inequality. The main criterion in their work was to look out for a copying machine such that all input states are copied equally well, i.e, the Hilbert Schmidt norm D_a must be independent of the parameter α^2. Thus, the relation between the parameters ξ and η can be determined from the condition

$$\frac{\delta D_a}{\delta\alpha^2} = 0 \Longrightarrow \eta = 1 - 2\xi. \quad (22)$$

Using equation (22), equation (21) reduces to

$$D_a = 2\xi^2. \quad (23)$$

The value of the parameter ξ can be determined from the second condition assumed for the universality criterion of cloning machine, i.e., the distance between two mode density operators $\rho_{ab}^{(id)}$ and $\rho_{ab}^{(out)}$ is input state independent. Mathematically,

$$\frac{\delta D_{ab}^2}{\delta\alpha^2} = 0, \quad (24)$$

where $D_{ab}^2 = Tr[\rho_{ab}^{(out)} - \rho_{ab}^{(id)}]^2$. Solving the equation (24) we find $\xi = \frac{1}{6}$. For this value of ξ the norm D_{ab}^2 is independent of α^2 and its value is equal to $\frac{2}{9}$. For $\xi = \frac{1}{6}$, the deviation of the output from the input is given by

$$D_a = \frac{1}{18}. \tag{25}$$

2.3 Phase-covariant quantum cloning machine

Phase covariant quantum cloning machine [13] can be defined as

$$|0\rangle|\Sigma\rangle|Q\rangle| \longrightarrow ((\frac{1}{2} + \frac{1}{\sqrt{8}})|0\rangle|0\rangle + (\frac{1}{2} - \frac{1}{\sqrt{8}})|1\rangle|1\rangle)|\uparrow\rangle + \frac{1}{2}|+\rangle|\downarrow\rangle, \tag{26}$$

$$|1\rangle|\Sigma\rangle|Q\rangle| \longrightarrow ((\frac{1}{2} + \frac{1}{\sqrt{8}})|1\rangle|1\rangle + (\frac{1}{2} - \frac{1}{\sqrt{8}})|0\rangle|0\rangle)|\downarrow\rangle + \frac{1}{2}|+\rangle|\uparrow\rangle. \tag{27}$$

The quantum cloning machine defined above can copy the equatorial states such as $\frac{|0\rangle + e^{i\phi}|1\rangle}{\sqrt{2}}$ with a fidelity $F = \frac{1}{2} + \frac{1}{\sqrt{8}}$ which is slightly higher than the optimal bound achievable for universal quantum cloning. The important property of this class that allows for this higher fidelity is that the coefficients have equal norm. Due to this property a state dependent term in the final density matrix of the clones in the cloning transformation becomes automatically state independent, hence no need for making its coefficient vanish by tuning the parameters of the cloning transformation. It had been already shown that if the input state contains only one unknown parameter, then we are able to construct a cloning machine which improves the fidelity.

2.4 Universal asymmetric Pauli cloning machine

Asymmetric cloning transformation [11, 12] is given by

$$|0\rangle|\Sigma\rangle|Q\rangle| \longrightarrow (\frac{1}{\sqrt{1+p^2+q^2}})(|0\rangle|0\rangle|\uparrow\rangle + (p|0\rangle|1\rangle + q|1\rangle|0\rangle)|\downarrow\rangle, \tag{28}$$

$$|1\rangle|\Sigma\rangle|Q\rangle| \longrightarrow (\frac{1}{\sqrt{1+p^2+q^2}})(|1\rangle|1\rangle|\downarrow\rangle + (p|1\rangle|0\rangle + q|0\rangle|1\rangle)|\uparrow\rangle. \tag{29}$$

Pauli cloning machines (transformations) is nothing but a family of asymmetric cloning machines that generates two non-identical approximate copies of a single quantum bit, each output qubits emerging from a Pauli channel [12]. The asymmetric quantum cloning machine play an important role in the situation in which one of the clones need to be a bit better than the other.

parameter (p)	$(F_1)_{PCM} = \frac{(p^2+1)}{2(p^2-p+1)}$	$(F_2)_{PCM} = \frac{(p^2-2p+2)}{2(p^2-p+1)}$	Difference between qualities of the two copies $(F_1)_{PCM} \sim (F_2)_{PCM}$
0.0	0.50	1.00	0.50
0.1	0.55	0.99	0.44
0.2	0.62	0.98	0.36
0.3	0.69	0.94	0.25
0.4	0.76	0.89	0.13
0.5	0.83	0.83	0.00 (Symmetric copies)
0.6	0.89	0.76	0.13
0.7	0.94	0.69	0.25
0.8	0.98	0.62	0.36
0.9	0.99	0.55	0.44
1.0	1.00	0.50	0.50

The above table represents the quality of the two different outputs from asymmetric Pauli cloning machine in terms of the fidelity for different values of the parameter p. We find that when $p = 0$ or $p = 1$, one of the output is totally undisturbed i.e. contains full information of the quantum state but the other output contains just 50 percent of the total information. For $p = 0.5$, the Pauli cloning machine reduces to B-H symmetric quantum cloning machine. We also observe here that the Pauli quantum cloning machine gives better quality asymmetric copies when $p = 0.4$ and $p = 0.6$.

2.5 Universal anti- cloning machine

Few years earlier, Gisin and Popescu [10] discovered an important fact that quantum information is better stored in two anti-parallel spins as compared to two parallel spins. This fact gave birth to a new type of cloning machine called anti-cloning machine [9, 10] which generates two outputs, one of the output has the same direction as the input and the other output has direction opposite to the input. Song and Hardy [9] constructed a universal quantum anti-cloner which takes an unknown quantum state just as in quantum cloner but its output as one with the same copy while the second one with opposite spin direction to the input state. For the Bloch vector, an input $\mathbf{n}$, quantum anti-cloner would have the input as $\frac{1}{2}(\mathbf{1}+\mathbf{n}.\sigma)$, then it generates two outputs,$\frac{1}{2}(\mathbf{1}+\eta\mathbf{n}.\sigma)$ and $\frac{1}{2}(\mathbf{1}-\eta\mathbf{n}.\sigma)$, where $0 \leq \eta \leq 1$ is the shrinking factor and the fidelity is defined as $F = \langle\mathbf{n}|\rho^{out}|\mathbf{n}\rangle = \frac{1}{2}(1+\eta)$. If spin flipping were allowed then anti-cloner would have the same fidelity as the regular cloner since one could clone first then flip the spin of the second copy. However spin flipping of an unknown state is not allowed in quantum mechanics. They also showed that the quantum state can be anti-cloned exactly with non-zero probability.

The universal anti-cloning transformation is given by

$$
\begin{aligned}
|0\rangle|\Sigma\rangle|Q\rangle \quad \longrightarrow \quad & \sqrt{\frac{1}{6}}|0\rangle|0\rangle|\uparrow\rangle + ((\frac{1}{\sqrt{2}})e^{icos^{-1}(\frac{1}{\sqrt{3}})}|0\rangle|1\rangle - \frac{1}{\sqrt{6}}|1\rangle|0\rangle)|\rightarrow\rangle + \\
& \frac{1}{\sqrt{6}}|1\rangle|1\rangle|\leftarrow\rangle, \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
|1\rangle|\Sigma\rangle|Q\rangle \quad \longrightarrow \quad & \sqrt{\frac{1}{6}}|1\rangle|1\rangle|\rightarrow\rangle + ((\frac{1}{\sqrt{2}})e^{icos^{-1}(\frac{1}{\sqrt{3}})}|1\rangle|0\rangle - \frac{1}{\sqrt{6}}|0\rangle|1\rangle)|\uparrow\rangle + \\
& \frac{1}{\sqrt{6}}|0\rangle|0\rangle|\downarrow\rangle, \qquad (31)
\end{aligned}
$$

where$|\uparrow\rangle,|\downarrow\rangle,|\rightarrow\rangle,|\leftarrow\rangle$ are orthogonal machine states. The fidelity of universal anti-cloner is same as the fidelity of measurement which is equal to $\frac{2}{3}$ [8].

3 State dependent hybrid cloning transformation

In this section, we study two state dependent cloning machines and later we find that their average fidelities are greater than the fidelity of the optimal universal quantum-cloning machine. Since the quality of the state dependent cloning machine depends on the input state given to the cloning machine so naturally one may ask a question why this type of cloning machine is important for study? Here we give two reasons for this question. First, the importance of the state dependent cloner lies in the eavesdropping strategy on some quantum cryptographic system. For example, if the quantum key distribution protocol is based on two non-orthogonal states [14], the optimal state dependent cloner can clone the qubit in transit between a sender and a receiver. The original qubit can then be re-sent to the receiver and the clone can stay with an eavesdropper who by measuring it can obtain some information about the bit value encoded in the original. The eavesdropper may consider storing the clone and delaying the actual measurement until any further public communication between the sender and the receiver takes place. This eavesdropping strategy has been discussed in Ref. [15, 16]. Second, the state dependent cloning machines may play an important role when the cloning machine produces a copy of an arbitrary input state with better fidelity on average than the optimal universal quantum cloning machine. Thus an interesting problem would be to construct a state dependent cloning machine whose average fidelity of copying is greater than the optimal value $\frac{5}{6}$.

B-H type cloning transformation: B-H cloning transformation generally indicates the optimal universal quantum cloning transformation but in this paper, we relax one condition of universality of B-H cloning transformation and hence we rename the B-H cloning transformation as B-H type cloning transformation. Therefore, although B-H type cloning transformation is structurally same as the universal B-H cloning transformation but it is different in the sense that this type of transformation is state dependent. State dependent ness of the cloning machine arises because of the relaxation of the condition $\frac{\partial D_{ab}}{\partial \alpha^2} = 0$.

3.1 Hybridization of two B-H type cloning transformation:

Here we investigate a new kind of cloning transformation that can be obtained by combining two different BH type cloning transformations. This may be given by

$$\begin{aligned}|\psi\rangle|\Sigma\rangle|Q\rangle \otimes |n\rangle \longrightarrow \sqrt{\lambda}[|\psi\rangle|\psi\rangle|Q_{\psi}\rangle + (|\psi\rangle|\overline{\psi}\rangle + |\overline{\psi}\rangle|\psi\rangle)|Y_{\psi}\rangle]|i\rangle \\ +(\sqrt{1-\lambda})[|\psi\rangle|\psi\rangle|Q'_{\psi}\rangle + (|\psi\rangle|\overline{\psi}\rangle + |\overline{\psi}\rangle|\psi\rangle)|Y'_{\psi}\rangle]|j\rangle.\end{aligned} \quad (32)$$

Unitarity of the transformation gives

$$\lambda(\langle Q_{\psi}|Q_{\psi}\rangle + 2\langle Y_{\psi}|Y_{\psi}\rangle) + (1-\lambda)(\langle Q'_{\psi}|Q'_{\psi}\rangle + 2\langle Y'_{\psi}|Y'_{\psi}\rangle) = 1, \quad (33)$$

$$2\lambda(\langle Y_{\psi}|Y_{\bar{\psi}}\rangle) + 2(1-\lambda)(\langle Y'_{\psi}|Y'_{\bar{\psi}}\rangle) = 0. \quad (34)$$

Equations (33) and (34) is satisfied for all values of $\lambda(0 < \lambda < 1)$ if

$$\langle Q_{\psi}|Q_{\psi}\rangle + 2\langle Y_{\psi}|Y_{\psi}\rangle = \langle \acute{Q}_{\psi}|\acute{Q}_{\psi}\rangle + 2\langle \acute{Y}_{\psi}|\acute{Y}_{\psi}\rangle = 1 \quad (35)$$

$$\langle Y_{\psi}|Y_{\overline{\psi}}\rangle = \langle \acute{Y}_{\psi}|\acute{Y}_{\overline{\psi}}\rangle = 0 \quad (36)$$

Further we assume that

$$\langle Q_{\psi}|Y_{\psi}\rangle = 0 = \langle Q_{\psi}|Q_{\overline{\psi}}\rangle. \quad (37)$$

Let $|\chi\rangle = \alpha|0\rangle + \beta|1\rangle$ with $\alpha^2 + \beta^2 = 1$, be the input state. The cloning transformation (32) copy the information contained in the input state $|\chi\rangle$ approximately into two identical states described by the density operators $\rho_a^{(out)}$ and $\rho_b^{(out)}$, respectively. The reduced density operator $\rho_a^{(out)}$ is given by

$$\begin{aligned}\rho_a^{(out)} = {} & |0\rangle\langle 0|[\alpha^2 + (\beta^2\langle Y'_1|Y'_1\rangle - \alpha^2\langle Y'_0|Y'_0\rangle) + \lambda(\beta^2\langle Y_1|Y_1\rangle - \alpha^2\langle Y_0|Y_0\rangle - \beta^2\langle Y'_1|Y'_1\rangle + \\ & \alpha^2\langle Y'_0|Y'_0\rangle)] + |0\rangle\langle 1|[\alpha\beta(\langle Q'_1|Y'_0\rangle + \langle Y'_1|Q'_0\rangle) + \\ & \lambda\alpha\beta(\langle Q_1|Y_0\rangle + \langle Y_1|Q_0\rangle - \langle Q'_1|Y'_0\rangle - \langle Y'_1|Q'_0\rangle)] + \\ & |1\rangle\langle 0|[\alpha\beta(\langle Q'_1|Y'_0\rangle + \langle Y'_1|Q'_0\rangle) + \lambda\alpha\beta(\langle Q_1|Y_0\rangle +\end{aligned}$$

$$\langle Y_1|Q_0\rangle - \langle Q_1'|Y_0'\rangle - \langle Y_1'|Q_0'\rangle)] +$$
$$|1\rangle\langle 1|[\beta^2 - (\beta^2\langle Y_1'|Y_1'\rangle - \alpha^2\langle Y_0'|Y_0'\rangle) + \lambda(\beta^2\langle Y_1|Y_1\rangle - \alpha^2\langle Y_0|Y_0\rangle -$$
$$\beta^2\langle Y_1'|Y_1'\rangle + \alpha^2\langle Y_0'|Y_0'\rangle)]. \quad (38)$$

The other output state described by the density operator $\rho_b^{(out)}$ looks exactly the same as $\rho_a^{(out)}$.

Let $\langle Y_0|Y_0\rangle = \langle Y_1|Y_1\rangle = \xi$, $\langle Q_1|Y_0\rangle = \langle Y_0|Q_1\rangle = \langle Q_0|Y_1\rangle = \langle Y_1|Q_0\rangle = \frac{\eta}{2}$,
$\langle Y_0'|Y_0'\rangle = \langle Y_1'|Y_1'\rangle = \xi'$ and $\langle Q_1'|Y_0'\rangle = \langle Y_0'|Q_1'\rangle = \langle Q_0'|Y_1'\rangle = \langle Y_1'|Q_0'\rangle = \frac{\eta'}{2}$
with $0 \le \xi(\xi') \le 1$ and $0 \le \eta(\eta') \le 2\sqrt{\xi(1-2\xi)}(2\sqrt{\xi'(1-2\xi')}) \le \frac{1}{\sqrt{2}}$.
Using above conditions, equation (38) can be rewritten as

$$\rho_a^{(out)} = |0\rangle\langle 0|[\alpha^2 + \xi'(\beta^2-\alpha^2) + \lambda(\xi-\xi')(\beta^2-\alpha^2)] + |0\rangle\langle 1|[\alpha\beta(\eta' + \lambda(\eta-\eta'))]$$
$$+|1\rangle\langle 0|[\alpha\beta(\eta' + \lambda(\eta-\eta'))] + |1\rangle\langle 1|[\beta^2 - \xi'(\beta^2-\alpha^2) - \lambda(\xi-\xi')(\beta^2-\alpha^2)]. \quad (39)$$

To investigate how well our hybrid cloning machine copy the input state, we have to calculate the fidelity. Therefore, the fidelity F_{HCM} is defined by

$$F_{HCM} = \langle\chi|\rho_a^{(out)}|\chi\rangle = \alpha^4[(1-\xi') - \lambda(\xi-\xi')] + \beta^4[(1-\xi') - \lambda(\xi-\xi')]$$
$$+2\alpha^2\beta^2[\xi' + \lambda(\xi-\xi') + \eta' + \lambda(\eta-\eta')]. \quad (40)$$

Now we get relationship between ξ, ξ', η, η' by solving the equation $\frac{\delta F_{HCM}}{\delta\alpha^2} = 0$
Therefore $\frac{\delta F_{HCM}}{\delta\alpha^2} = 0$ implies that we must have

$$\eta'(1-\lambda) + \eta\lambda = 1 - 2\xi' - 2\lambda(\xi-\xi'). \quad (41)$$

Using (41), equation (40) reduces to

$$F_{HCM} = (1-\xi') - \lambda(\xi-\xi'). \quad (42)$$

Now the distance between the two mode density operators $\rho_{ab}^{(out)}$ and $\rho_{ab}^{(id)} = \rho_a^{(id)} \otimes \rho_b^{(id)}$ is given by

$$D_{ab} = Tr[\rho_{ab}^{(out)} - \rho_{ab}^{(id)}]^2$$
$$= U_{11}^2 + 2U_{12}^2 + 2U_{13}^2 + U_{22}^2 + 2U_{23}^2 + U_{33}^2, \quad (43)$$

where

$$\begin{aligned}
U_{11} &= \alpha^4 - \alpha^2[\lambda(1-2\xi) + (1-\lambda)(1-2\xi')], \\
&U_{12} = U_{21} = \sqrt{2}\alpha^3\beta - \sqrt{2}\alpha\beta(\eta\frac{\lambda}{2} + (1-\lambda)\frac{\eta'}{2}), \\
&U_{13} = U_{31} = \alpha^2\beta^2, \\
&U_{22} = 2\alpha^2\beta^2 - (2\xi\lambda + 2\xi'(1-\lambda)), \\
&U_{23} = U_{32} = \sqrt{2}\alpha\beta^3 - \sqrt{2}\alpha\beta(\eta\frac{\lambda}{2} + (1-\lambda)\frac{\eta'}{2}), \\
&U_{33} = \beta^4 - \beta^2[\lambda(1-2\xi) + (1-\lambda)(1-2\xi')].
\end{aligned} \tag{44}$$

It is interesting to see that the transformation (32) can behave as a state dependent cloner if we relax the condition $\frac{\delta D_{ab}}{\delta\alpha^2} = 0$. Therefore, it is natural to expect that the machine parameters depends on the input state. Thus, our prime task is to find the relationship between the machine parameters and the input state that minimizes the distortion D_{ab}. Now we will get an interesting result if we fix any one of the machine parameters ξ or ξ' as $\frac{1}{6}$. Without any loss of generality we can fix $\xi' = \frac{1}{6}$. In doing so, the cloning transformation (32) reduces to the combination of B-H optimal universal cloning machine and the B-H type cloning machine.
Now, substituting $\xi' = \frac{1}{6}$ in (44) and using (41), equation (43) can be rewritten as

$$D_{ab} = V_{11}^2 + 2V_{12}^2 + 2V_{13}^2 + V_{22}^2 + 2V_{23}^2 + V_{33}^2, \tag{45}$$

where

$$\begin{aligned}
V_{11} &= \alpha^4 - \alpha^2[\lambda(1-2\xi) + (1-\lambda)(\frac{2}{3})], \\
&V_{12} = V_{21} = \sqrt{2}\alpha^3\beta - \sqrt{2}\alpha\beta(\frac{1}{3} - \lambda(\xi - \frac{1}{6})), \\
&V_{13} = V_{31} = \alpha^2\beta^2, \\
&V_{22} = 2\alpha^2\beta^2 - (2\xi\lambda + (\frac{1}{3})(1-\lambda)), \\
&V_{23} = V_{32} = \sqrt{2}\alpha\beta^3 - \sqrt{2}\alpha\beta(\frac{1}{3} - \lambda(\xi - \frac{1}{6})), \\
&V_{33} = \beta^4 - \beta^2[\lambda(1-2\xi) + (1-\lambda)(\frac{2}{3})].
\end{aligned} \tag{46}$$

Now we are in a position to determine the relationship between the machine parameter and the input state that minimizes the distortion D_{ab}. To obtain the minimum value of D_{ab} for given α and λ, we solve the equation

$$\frac{\delta D_{ab}}{\delta \xi} = 0 \Longrightarrow \xi = \frac{(9\alpha^2\beta^2 - 2(1-\lambda))}{12\lambda}, \quad provided \;\; \lambda \neq 0. \tag{47}$$

Now, the cloning machine defined by those parameters common to the whole family of state that one wants to clone. Therefore, it is clear from equation (47) that the quantum cloning machine can be applied on the family of states such that $\alpha^2\beta^2 = constant$. That means the cloning machine can be applied on just four states $|\psi^{\pm}\rangle_1 = \alpha|0\rangle \pm \beta|1\rangle, |\psi^{\pm}\rangle_2 = \alpha|1\rangle \pm \beta|0\rangle$.

Since the value of the machine parameter ξ cannot be negative, so the parameter λ take values lying in the interval $[1 - \frac{9\alpha^2(1-\alpha^2)}{2}] < \lambda < 1$.

Also

$$\frac{\delta^2 D_{ab}}{\delta \xi^2} = 16\lambda^2 > 0. \tag{48}$$

Therefore, the equation (47) represents the required relationship between the machine parameter and the input state which minimizes D_{ab} and the minimum value of D_{ab} is given by

$$(D_{ab})_{min} = 2\alpha^2\beta^2 - \frac{9\alpha^4\beta^4}{2} \tag{49}$$

which depends on α^2 but not on λ.

Substituting $\xi = \frac{(9\alpha^2(1-\alpha^2)-2(1-\lambda))}{12\lambda}$ and $\xi' = \frac{1}{6}$ in equation (42), we get $F_{HCM} = 1 - \frac{3\alpha^2\beta^2}{4}$.

Input state (α^2)	Parameter λ	Machine parameter (ξ)	$(D_{ab})_{min}$	F_{HCM}
0.1 or 0.9	(0.595, 1.0)	(0.0, 0.0675)	0.14	0.93
0.2 0r 0.8	(0.280, 1.0)	(0.0, 0.1200)	0.21	0.88
0.3 or 0.7	(0.055, 1.0)	(0.0, 0.1575)	0.22	0.84
0.4 or 0.6	(0.000, 1.0)	(0.0, 0.1800)	0.22	0.82
0.5	(0.000, 1.0)	(0.0, 0.1875)	0.22	0.81

The above table shows that there exists several quantum cloning machines (for different values of ξ) which can clone the four states $\{|\psi^{\pm}\rangle_1, |\psi^{\pm}\rangle_2\}$ with the same fidelity. For example, If the input states are chosen from the set $\{\sqrt{0.1}|0\rangle \pm \sqrt{0.9}|1\rangle, \sqrt{0.9}|0\rangle \pm \sqrt{0.1}|1\rangle\}$, then corresponding to different values of the machine parameter ξ $(0 < \xi < 0.0675)$, there exists different quantum cloners which clone the above states, each with the fidelity 0.93.

3.2 Hybridization of B-H type cloning transformation and phase-covariant quantum cloning transformation

Now, we show that the combination of B-H type cloning transformation and the phase-covariant quantum cloning transformation gives a state dependent quantum cloning transformation which copy the input state having two unknown parameters with average fidelity greater than $\frac{1}{2} + \sqrt{\frac{1}{8}}$.

The Hybrid cloning transformation is given by

$$|0\rangle|\Sigma\rangle|Q\rangle|n\rangle \longrightarrow \sqrt{\lambda}[|0\rangle|0\rangle|Q_0\rangle + (|0\rangle|1\rangle + |1\rangle|0\rangle)|Y_0\rangle]|i\rangle$$
$$+(\sqrt{1-\lambda})[((\frac{1}{2} + \frac{1}{\sqrt{8}})|0\rangle|0\rangle + (\frac{1}{2} - \frac{1}{\sqrt{8}})|1\rangle|1\rangle)|\uparrow\rangle + \frac{1}{2}|+\rangle|\downarrow\rangle)]|j\rangle, \quad (50)$$
$$|1\rangle|\Sigma\rangle|Q\rangle|n\rangle \longrightarrow \sqrt{\lambda}[|1\rangle|1\rangle|Q_1\rangle + (|0\rangle|1\rangle + |1\rangle|0\rangle)|Y_1\rangle]|i\rangle$$
$$+(\sqrt{1-\lambda})[((\frac{1}{2} + \frac{1}{\sqrt{8}})|1\rangle|1\rangle + (\frac{1}{2} - \frac{1}{\sqrt{8}})|0\rangle|0\rangle)|\downarrow\rangle + \frac{1}{2}|+\rangle|\uparrow\rangle)]|j\rangle. \quad (51)$$

When $\lambda = 1$ cloning transformation reduces to B-H type cloning transformation and when $\lambda = 0$ it takes the form of phase-covariant quantum cloning transformation.

The cloning machine (52-53) approximately copy the information of the input state $|\chi\rangle$ given in (4) into two identical states described by the reduced density operator

$$\rho = \lambda[(1-\xi)|\chi\rangle\langle\chi| + \xi|\overline{\chi}\rangle\langle\,\overline{\chi}|] + (1-\lambda)[(\frac{1}{2} + \frac{1}{\sqrt{8}})|\chi\rangle\langle\chi| + (\frac{1}{2} - \frac{1}{\sqrt{8}})|\overline{\chi}\rangle\langle\,\overline{\chi}|] \quad (52)$$

where $|\bar{\chi}\rangle$ is an orthogonal state to $|\chi\rangle$. Now, the fidelity is given by

$$F_1 = \langle\chi|\rho|\chi\rangle = (\frac{1}{2} + \frac{1}{\sqrt{8}}) + \lambda(\frac{1}{2} - \frac{1}{\sqrt{8}} - \xi) \quad (53)$$

The hybrid quantum cloning machine constructed by combining the B-H type cloning transformation and phase-covariant quantum cloning transformation is state dependent. State dependent ness condition arises from the fact that B-H type cloning transformation is state dependent. Consequently, the fidelity F_1 depends on the input state as it depends on the machine parameter $\xi(\alpha^2)$. We can get the relationship between the machine parameter ξ associated with the B-H type cloning machine and the input state α^2 by putting $\lambda = 1$ in equation (47). Therefore, the dependence of ξ on α^2 can be expressed as $\xi(\alpha^2) = \frac{3\alpha^2(1-\alpha^2)}{4}$.

From the argument given in section (3.1), we find that the hybrid quantum cloning machine (B-H type cloning transformation + phase covariant quantum cloning transformation) clone the same four states $\{|\psi^{\pm}\rangle_1, |\psi^{\pm}\rangle_2\}$. Also there is no improvement in the quality of cloning of these four states. Therefore, this hybrid quantum cloning machine does not give anything new because it neither involve in cloning of new family of states nor it gives any improvement in the fidelity of cloning.

4 State independent hybrid cloning transformation

In this section, we study one symmetric and two asymmetric universal hybrid quantum cloning machines.

4.1 Hybridization of two BH type cloning transformations

In the preceding section, we find that the quantum cloning machine obtained by combining two BH type cloning transformations is state dependent but in this section we will observe that a proper combination of two BH type cloning transformations can serve as a state independent cloner also. A hybrid quantum cloning machine (32) becomes state independent or universal if the fidelity F_{HCM} and the deviation D_{ab}, defined in section 3, both are state independent. From equation (42), it is clear that F_{HCM} is state independent. Therefore, the only remaining task is to show the independence of the deviation D_{ab}. We will find that the deviation D_{ab} is state independent if there exists a relationship between the parameter λ and the machine parameters ξ, ξ'. D_{ab} is input state independent if,

$$\frac{\delta D_{ab}}{\delta \alpha^2} = 0 \Longrightarrow [2(\lambda(1-2\xi)+(1-\lambda)(1-2\xi'))-3]^2$$
$$-[2(\eta\lambda-(1-\lambda)\eta')-2]^2+8[2\xi\lambda+2\xi'(1-\lambda)]-5=0. \tag{54}$$

Using equation (41) in equation (54), we get

$$\lambda = \frac{(6\xi'-1)}{6(\xi'-\xi)}, \tag{55}$$

provided $\xi \neq \xi'$.
Using the value of λ in (42), we get

$$F_{HCM} = \frac{5}{6}. \tag{56}$$

If $\xi = \xi'$, then there is nothing special about the transformation (32) because if $\xi = \xi'$ holds then the transformation (32) simply reduces to B-H cloning machine. The special feature of the equation (55) is that it makes the transformation (32) state independent for all values of ξ and ξ'(provided $\xi \neq \xi'$). This characteristic of the newly defined cloning machine takes it into the field of universal cloning machines and creates its identification as a universal cloner. The introduced universal cloning machine is optimal also in the sense that the fidelity of the cloning machine is equal to $\frac{5}{6}$. Although the machine is universal and optimal for an unknown quantum state but it is different from B-H cloning machine. It is different in the sense that B-H cloning machine is state independent for just only one value of the machine parameter $\xi = \frac{1}{6}$ while the cloning machine defined by (32) works as a universal cloner for all values of ξ and ξ'(provided $\xi \neq \xi'$).

4.2 Hybridization of optimal universal symmetric B-H cloning transformation and optimal universal asymmetric Pauli cloning transformation

Another asymmetric quantum cloning machine can be constructed by applying hybridization technique. Therefore using the hybridization procedure we can construct universal asymmetric quantum cloning machine by combining universal symmetric B-H cloning transformation and optimal universal asymmetric Pauli cloning transformation. The Hybrid cloning transformation is given by

$$\begin{aligned}
&|0\rangle|\Sigma\rangle|Q\rangle|n\rangle \longrightarrow \sqrt{1-\lambda}[\sqrt{\frac{2}{3}}|0\rangle|0\rangle|\uparrow\rangle + \sqrt{\frac{1}{6}}(|0\rangle|1\rangle + |1\rangle|0\rangle)|\downarrow\rangle]|i\rangle \\
&+\sqrt{\lambda}[(\frac{1}{\sqrt{1+p^2+q^2}})(|0\rangle|0\rangle|\uparrow\rangle + (p|0\rangle|1\rangle + q|1\rangle|0\rangle)|\downarrow\rangle)]|j\rangle, \qquad (57)\\
&|1\rangle|\Sigma\rangle|Q\rangle|n\rangle \longrightarrow \sqrt{1-\lambda}[\sqrt{\frac{2}{3}}|1\rangle|1\rangle|\downarrow\rangle + \sqrt{\frac{1}{6}}(|0\rangle|1\rangle + |1\rangle|0\rangle)|\uparrow\rangle]|i\rangle \\
&+\sqrt{\lambda}[(\frac{1}{\sqrt{1+p^2+q^2}})(|1\rangle|1\rangle|\downarrow\rangle + (p|1\rangle|0\rangle + q|0\rangle|1\rangle)|\uparrow\rangle)]|j\rangle, \qquad (58)
\end{aligned}$$

where p + q =1.

After taking $|\chi\rangle$ given in (4) as input state by the cloning machine, the two asymmetric clones emerges as output which are described by the reduced density operators ρ_1 and ρ_2

$$\rho_1 = \lambda[(\frac{1}{1+p^2+q^2})((1-q^2+p^2)|\chi\rangle\langle\chi| + q^2 I)] + (1-\lambda)[\frac{5}{6}|\chi\rangle\langle\chi| + \frac{1}{6}|\overline{\chi}\rangle\langle\overline{\chi}|], \qquad (59)$$

$$\rho_2 = \lambda[(\frac{1}{1+p^2+q^2})((1-p^2+q^2)|\chi\rangle\langle\chi| + p^2 I)] + (1-\lambda)[\frac{5}{6}|\chi\rangle\langle\chi| + \frac{1}{6}|\overline{\chi}\rangle\langle\overline{\chi}|]. \qquad (60)$$

Let F_1 and F_2 denote the fidelities of the two asymmetric clones.

$$F_1 = \frac{5}{6} + (\frac{\lambda}{2})[\frac{(p^2+1)}{(p^2-p+1)} - \frac{5}{3}], \qquad (61)$$

$$F_2 = \frac{5}{6} + (\frac{\lambda}{2})[\frac{(p^2-2p+2)}{(p^2-p+1)} - \frac{5}{3}]. \qquad (62)$$

From equation (61) and (62), we can observe that the Hybrid quantum cloning machine reduces to B-H state independent quantum cloning machine if $\lambda \to 0$ and $0 \leq p \leq 1$ or if $\lambda \to 1$ and $p = \frac{1}{2}$.

Next our task is to show that if $F_1 > \frac{5}{6}$ then $F_2 < \frac{5}{6}$ for all $\lambda' s$ lying between 0 and 1 and vice-versa. Therefore, for $0 < \lambda < 1$, we can find $F_1 > \frac{5}{6}$ if $\frac{(p^2+1)}{(p^2-p+1)} > \frac{5}{3}$

$$\begin{aligned} &\text{i.e. if } (2p-1)(p-2) < 0 \\ &\text{i.e. if } (2p-1) > 0 \\ &\text{i.e. if } p > \frac{1}{2}. \end{aligned}$$

Now we are going to show that if $p > \frac{1}{2}$ then $F_2 < \frac{5}{6}$. If possible, let $F_2 > \frac{5}{6}$ for $p > \frac{1}{2}$. Therefore, we have

$$\begin{aligned} F_2 > \frac{5}{6} &\implies \frac{(p^2-2p+2)}{(p^2-p+1)} > \frac{5}{3} \\ &\implies (2p-1)(p+1) < 0 \\ &\implies (2p-1) < 0, \text{ Since } p+1 > 0 \\ &\implies p < \frac{1}{2} \end{aligned}$$

which contradicts our assumption. Hence $F_2 < \frac{5}{6}$ for $p > \frac{1}{2}$. Therefore, we can conclude that the fidelities given in (61) and (62) cannot cross the optimal limit $\frac{5}{6}$ simultaneously. Next we construct a table below in which we show that if we made the quality of one of the output better than the optimal quality then how much far away the quality of the other copy from the optimal one.

p	λ	$F_1 = \frac{5}{6} + \frac{\lambda}{2} [(\frac{p^2+1}{2(p^2-p+1)}) - \frac{5}{3}]$	$F_2 = \frac{5}{6} + \frac{\lambda}{2} [(\frac{p^2-2p+2}{p^2-p+1}) - \frac{5}{3}]$	Difference between qualities of the two copies
[0.0,1.0]	0.0	0.83	0.83	0.00 (symmetric copies)
0.0	[0.1,0.9]	[0.80,0.53]	[0.85,0.98]	[0.05,0.45]
0.1	[0.1,0.9]	[0.81,0.58]	[0.85,0.98]	[0.04,0.40]
0.2	[0.1,0.9]	[0.81,0.64]	[0.85,0.96]	[0.04,0.32]
0.3	[0.1,0.9]	[0.82,0.70]	[0.84,0.93]	[0.02,0.23]
0.4	[0.1,0.9]	[0.83,0.77]	[0.84,0.89]	[0.01,0.12]
0.5	[0.1,0.9]	0.83	0.83	0.0 (Symmetric copies)
0.6	[0.1,0.9]	[0.84,0.89]	[0.83,0.77]	[0.01,0.12]
0.7	[0.1,0.9]	[0.84,0.93]	[0.82,0.70]	[0.02,0.23]
0.8	[0.1,0.9]	[0.85,0.96]	[0.81,0.64]	[0.04,0.32]
0.9	[0.1,0.9]	[0.85,0.98]	[0.81,0.58]	[0.04,0.40]
[0.0,1.0]	1.0	$(F_1)_{PCM}$	$(F_2)_{PCM}$	$(F_1)_{PCM} \sim (F_2)_{PCM}$

The above table represents the qualities of the asymmetric copies of the hybrid cloning machine. We note that the fidelity of the hybrid quantum cloning machine (B-H cloner + Pauli cloner) depends on the parameter p and λ . From table we observe that one of the output $(F_1)_{HCM}$ behave as a decreasing function for $p = 0.0$ to $p = 0.4$ and for all values of λ lying between 0 and 1. At the same time, another output of the asymmetric cloning machine $(F_2)_{HCM}$ behaves as an increasing function for $p = 0.0$ to $p = 0.4$ and for all values of λ lying between 0 and 1. The role of the fidelities $(F_1)_{HCM}$ and $(F_2)_{HCM}$ are swapped for $p = 0.6$ to $p = 0.9$ and for all values of λ lying between 0 and 1. Here we observe that the asymmetric hybrid cloning machine reduces to B-H symmetric cloning machine in two cases: (i) when $\lambda = 0$ and $0 \le p \le 1$ and (ii)when $p = 0.5$ and $0.1 \le \lambda \le 0.9$. Our asymmetric hybrid cloner also reduces to asymmetric Pauli cloner when $\lambda = 1.0$ and $0 \le p \le 1$.

4.3 Hybridization of universal B-H cloning transformation and universal anti- cloning transformation

Now we introduce an interesting hybrid quantum-cloning machine, which is a combination of universal B-H cloning machine and a universal anti-cloning machine. The introduced cloning machine is interesting in the sense that it acts like anti-cloning machine. That means the spin direction of the outputs of the cloner are antiparallel. We will show later that the newly introduced Hybrid cloning machine (B-H cloner + Anti-cloner) serve as a better anti-cloner than the existing quantum anti-cloning machine [9]. Also we show that if the values of the machine parameter λ is in the neighborhood of 1 then the values of the two non-identical fidelities lies in the neighborhood of $\frac{5}{6}$
Therefore the introduced anti-cloning transformation is defined by

$$\begin{aligned}
&|0\rangle|\Sigma\rangle|Q\rangle|n\rangle \longrightarrow \sqrt{\lambda}[\sqrt{\frac{2}{3}}|0\rangle|0\rangle|\uparrow\rangle + \sqrt{\frac{1}{6}}(|0\rangle|1\rangle + |1\rangle|0\rangle)|\downarrow\rangle]|i\rangle + (\sqrt{1-\lambda}) \\
&[\sqrt{\frac{1}{6}}|0\rangle|0\rangle|\uparrow\rangle + ((\frac{1}{\sqrt{2}})e^{icos^{-1}(\frac{1}{\sqrt{3}})}|0\rangle|1\rangle - \frac{1}{\sqrt{6}}|1\rangle|0\rangle)|\rightarrow\rangle + \frac{1}{\sqrt{6}}|1\rangle|1\rangle|\leftarrow\rangle]|j\rangle, \quad (63) \\
&|1\rangle|\Sigma\rangle|Q\rangle|n\rangle \longrightarrow \sqrt{\lambda}[\sqrt{\frac{2}{3}}|1\rangle|1\rangle|\downarrow\rangle + \sqrt{\frac{1}{6}}(|0\rangle|1\rangle + |1\rangle|0\rangle)|\uparrow\rangle]|i\rangle + (\sqrt{1-\lambda}) \\
&[\sqrt{\frac{1}{6}}|1\rangle|1\rangle|\rightarrow\rangle + ((\frac{1}{\sqrt{2}})e^{icos^{-1}(\frac{1}{\sqrt{3}})}|1\rangle|0\rangle - \frac{1}{\sqrt{6}}|0\rangle|1\rangle)|\uparrow\rangle + \frac{1}{\sqrt{6}}|0\rangle|0\rangle|\downarrow\rangle]|j\rangle, \quad (64)
\end{aligned}$$

where $|\uparrow\rangle$, $|\downarrow\rangle$, $|\rightarrow\rangle$, $|\leftarrow\rangle$ are orthogonal machine states.
The above defined cloning machine (63-64) produces two copies of the input state (4) which are described by the reduced density operator in mode 'a' and mode 'b' is given by

$$\begin{aligned}
\rho_a = &|0\rangle\langle 0|[\lambda(\frac{5\alpha^2}{6} + \frac{\beta^2}{6}) + (1-\lambda)(\frac{2\alpha^2}{3} + \frac{\beta^2}{3})] + |0\rangle\langle 1|[\lambda\frac{2\alpha\beta}{3} + (1-\lambda)\frac{\alpha\beta}{3}] \\
&+|1\rangle\langle 0|[\lambda\frac{2\alpha\beta}{3} + (1-\lambda)\frac{\alpha\beta}{3}] + |1\rangle\langle 1|[\lambda(\frac{5\beta^2}{6} + \frac{\alpha^2}{6}) + (1-\lambda)(\frac{\alpha^2}{3} + \frac{2\beta^2}{3})], \quad (65) \\
\rho_b = &|0\rangle\langle 0|[\lambda(\frac{5\alpha^2}{6} + \frac{\beta^2}{6}) + (1-\lambda)(\frac{\alpha^2}{3} + \frac{2\beta^2}{3})] + |0\rangle\langle 1|[\lambda\frac{2\alpha\beta}{3} - (1-\lambda)\frac{\alpha\beta}{3}] \\
&+|1\rangle\langle 0|[\lambda\frac{2\alpha\beta}{3} - (1-\lambda)\frac{\alpha\beta}{3}] + |1\rangle\langle 1|[\lambda(\frac{5\beta^2}{6} + \frac{\alpha^2}{6}) + (1-\lambda)(\frac{2\alpha^2}{3} + \frac{\beta^2}{3})]. \quad (66)
\end{aligned}$$

Let F_a and F_b denotes the fidelities of the two copies with opposite spin direction. Therefore, the fidelities for two outputs are given by

$$F_a = \frac{5\lambda}{6} + \frac{2(1-\lambda)}{3},\ F_b = \frac{5\lambda}{6} + \frac{(1-\lambda)}{3}. \qquad (67)$$

It is clear from equation (67) that the introduced anti- cloning machine is asymmetric in nature, i.e., the hybrid quantum cloning machine resulting from Universal B-H cloning machine and universal anti-cloning machine behaves as a asymmetric quantum cloning machine for all values of the parameter λ lying between 0 and 1. The two different fidelities given in (67) of the anti-cloning machine (63-64) can approaches to the optimal value $\frac{5}{6}$ when the parameter λ approaches to one. Here we should note an important fact that both the fidelities tends to $\frac{5}{6}$ but not equal to $\frac{5}{6}$ unless $\lambda = 1$. Hence the fidelities F_a and F_b takes different values in the neighborhood of $\frac{5}{6}$ when the values of λ lying in the neighborhood of 1. For further illustration we construct a table below:

parameter (λ)	$F_a = \frac{5\lambda}{6} + \frac{2(1-\lambda)}{3}$	$F_b = \frac{5\lambda}{6} + \frac{(1-\lambda)}{3}$	Difference between qualities of the two copies $F_a \sim F_b$
0.0	0.67	0.33	0.34
0.1	0.68	0.38	0.30
0.2	0.70	0.43	0.27
0.3	0.72	0.48	0.24
0.4	0.73	0.53	0.20
0.5	0.75	0.58	0.17
0.6	0.77	0.63	0.14
0.7	0.78	0.68	0.10
0.8	0.80	0.73	0.07
0.9	0.82	0.78	0.04
1.0	0.83	0.83	0.00 (Symmetric copies)

It is clear that both the fidelities of output copies with opposite spins are increasing function of the parameter λ. Therefore, as λ increases, the values of the fidelities F_a and F_b also increases and approaches towards the optimal cloning fidelity 0.83. The above Table shows that when $\lambda = 0$, our Hybrid anti-cloner reduces to anti-cloner introduced by Song and Hardy [9]. Also when $\lambda = 1$, we observe that the copies with opposite spin direction changes into the copies with same spin direction with optimal fidelity.Therefore, we can conclude that the hybrid anti-cloner performs better than the existing quantum anti-cloning machine.

5 Conclusion

In this paper we have studied two state dependent hybrid quantum-cloning machine and three state independent hybrid quantum-cloning machine. We get few interesting results after studying the hybrid quantum-cloning machine in detail. First, the combination of a universal B-H quantum cloning machine and B-H type quantum cloning machine gives a state dependent hybrid quantum cloning machine which copy only four input states with maximum fidelity 0.93. Another hybrid state dependent quantum cloning machine introduced in this paper is the combination of B-H type quantum cloning transformation and phase-covariant quantum cloning transformation. But this type of hybrid quantum cloning machine does not perform better than other state dependent quantum cloning machine. Second, the hybridization of two B-H type cloning transformation also serve as a state independent cloner with optimal fidelity 5/6 for all values of the machine parameters. This result is interesting in the sense that the original B-H quantum cloning machine serve as a universal cloner for just only one value of the machine parameter but the introduced hybrid cloner (32) acts as a state independent cloner for all values of the machine parameters lying in the given range. Third, we construct here an universal hybrid anti-cloning machine by combining the universal B-H cloning transformation and universal anti-cloning transformation. This machine copies an arbitrary input state with different fidelities of the copies with opposite spin direction. Although the fidelities are different but the values of the fidelities lie in the neighborhood of the optimal value 5/6 provided the machine is constructed in such a way that the parameter λ takes the value close to 1. Thus, our hybrid anti-cloner can clone an arbitrary input state into two copies with antiparallel spin direction and improves the quality of copy upto the optimal quality. Hence collecting all the given arguments above, we can say that Hybrid quantum cloner performs better than any other existing individual cloners.

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