# Multipartite entangled magnon states as quantum communication channels

Sriram Prasath E.<sup>1</sup>, Sreraman Muralidharan<sup>2</sup>, Chiranjib Mitra<sup>3</sup> and Prasanta K. Panigrahi<sup>3,4</sup>

<sup>1</sup>Maulana Azad National Institute of Technology, Bhopal - 462051, India

<sup>2</sup>Loyola College, Chennai - 600034, India

<sup>3</sup>Indian Institute of Science Education and Research - Kolkata,

Mohanpur, BCKV Campus Main Office, Mohanpur - 741252, India and

<sup>4</sup> Physical Research Laboratory, Navrangpura, Ahmedabad-380 009, India\*

We explicate conditions under which, the two magnon state becomes highly entangled and is useful for several quantum communication protocols. This state, which is experimentally realizable in quantum dots using Heisenberg exchange interaction, is found to be suitable for carrying out deterministic teleportation of an arbitrary two qubit composite system. Further, conditions for which the channel capacity reaches "Holevo bound", allowing maximal amount of classical information to be encoded, are derived. Later, an explicit protocol for the splitting and sharing of a two qubit entangled state among two parties, using this state as an entangled resource, is demonstrated.

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Entanglement, an entirely quantum mechanical phenomenon, has been a subject of intense research, since the days of Schrödinger [1] and Einstein [2]. In recent times, it has received renewed attention with the advent of quantum information processing [3]. The amalgamation of the principles of quantum entanglement and information theory has led to the possibility of carrying out tasks, which would have been otherwise impossible in the classical world [4, 5]. The fact that some of the finite dimensional spin states naturally occur in physical systems and these can be experimentally realized makes them the states of choice for explicating quantum protocols [6]. The search for physically occurring entangled systems, which can be used for carrying out these quantum tasks, is of interest to both theorists and experimentalists.

A natural system attracting considerable interest is that of spin chains, where the Heisenberg exchange interaction can create desired entangled states [7]. Quantum dots have also shown considerable promise in the realization of these states, which can be manipulated by varying the voltage applied through gate electrodes between adjacent quantum dots [8]. Interestingly, entanglement has been found to be retained even up to certain non-vanishing temperature. Entangled states e.g., Bell states  $|\phi_{\pm}\rangle = \frac{1}{\sqrt{2}}, (|01\rangle \pm |10\rangle)$  multipartite cluster states [7, 9]:

$$|C_N\rangle = \frac{1}{2^{N/2}} \otimes_{a=1}^N (|0\rangle_a \sigma_z^{a+1} + |1\rangle_a)$$
(1)

have found application in carrying out various quantum tasks like error correction [9] and state sharing [10].

In a spin chain, low lying magnon excitations of the ground state can show robust entanglement properties. In fact, the well known Bell states and W states of the form [11, 12, 13]

$$|W'\rangle = (\alpha|100\rangle + \beta|010\rangle + \gamma|001\rangle),\tag{2}$$

are natural one magnon states, above the ferromagnetic ground states formed of two and three particles respectively. Although Bell states can be used for achieving perfect teleportation of,  $|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle$ ,  $(\alpha, \beta \in C, |\alpha|^2 + |\beta|^2 = 1)$  the symmetric W states cannot be used. However,  $|W'\rangle$  can be used for the perfect teleportation of  $|\psi_1\rangle$  in the case where  $|\alpha|^2 + |\beta|^2 = |\gamma|^2$ . Hence, the characterization of different multipartite magnon states need careful investigation for diverse quantum tasks. In this paper, we systematically investigate the four qubit, two magnon state for its ability to carry out various quantum tasks like teleportation, state sharing and dense coding. We start with the non trivial task of teleporting  $|\psi_2\rangle = \sum_0^1 \alpha_{i_1 i_2} |i_1 i_2\rangle$   $(\alpha_{i_1 i_2} \in C, \sum |\alpha_{i_1 i_2}|^2 = 1)$ , for which one needs two ebits of entanglement between the parties. The conditions for achieving the same are also discussed. It is worth mentioning that in case of four particles, only one of the nine distinct classes, characterized by LOCC [14], can be used for the teleportation of an arbitrary two qubit state.

The most general two magnon four qubit state is given by,

$$|4;2\rangle = \sum_{i=0}^{1} (W_{ii\tilde{i}}|ii\tilde{i}\tilde{i}\rangle + W_{i\tilde{i}i}|i\tilde{i}\tilde{i}\tilde{i}\rangle + W_{\tilde{i}ii}|i\tilde{i}\tilde{i}\tilde{i}\rangle)_{1234}$$
(3)

<sup>\*</sup>Electronic address: prasanta@prl.res.in

where  $\tilde{i}$  is the negation of i.

Here the first index on the left hand side denotes the number of qubits in the channel and the second one the number of magnons. From an experimental point of view, spins of electrons in quantum dots are proposed to be good building blocks for obtaining the above type of states [8, 15, 16]. With a high level of control over the number of electrons [17], and the applied external fields, desirable initial states can be achieved. The exchange interaction between any two electronic spins can be controlled by the applied gate voltages [15, 18]. Specific multi-qubit entangled states can be realized by switching on the exchange interaction, generated by the following Hamiltonian:

$$H = JS_A.S_B + JS_B.S_C + J\Delta S_A.S_C, \tag{4}$$

where J is the strength of interaction. States having definite spin, e.g., generalised W states can be created using different values of the parameter  $\Delta$ , at different points in time due to exchange interaction induced dynamics. For example,  $|W'\rangle$  can be obtained when  $\Delta=1$  and at time  $t=\frac{2}{3J}cos^{-1}\left(\frac{1}{8}\right)$  [13]. N-qubit W states can, in general, be formed by varying the exchange interaction using the gate voltage in an array of quantum dots and also allowing that variation to happen on a fixed time scale. The amplitudes  $W_{ijk}$ 's in Eq. (3) can be varied by suitably choosing the time scales over which, the gate electrodes can be switched on and off. The generalized W states can also be generated from other interactions which conserve the  $\hat{z}$  component of the total spin of qubits. The exchange interaction induced quantum gates, for example, SWAP and CNOT gates have been implemented at picosecond time scales [8].

This procedure can be generalized to N-magnon entangled channels. For specificity, we concentrate here on the two magnon state and explore its utility for various quantum tasks, starting from teleportation in the following section.

## I. TELEPORTATION OF $|\psi_2\rangle$

In this protocol, Alice possesses qubits 1 and 3, and Bob qubits 2 and 4 of  $|4;2\rangle$  respectively. Alice performs a joint four qubit von-Neumann type measurement on  $|\psi_2\rangle$  and on her qubits. Let the qubits in  $|\psi_2\rangle$  be represented by subscripts p and q and the four qubits from  $|4;2\rangle$  be represented by subscripts 1, 2, 3 and 4. The combined state of  $|\psi_2\rangle_{pq}$  and  $|4;2\rangle_{1234}$  can be written as

$$\sum_{i=1}^{16} (M_i \otimes N_i \otimes I \otimes I)|4;2\rangle_{pq13}')(N_i \otimes M_{(i+8)mod(16)}|\psi_2\rangle_{24}), \tag{5}$$

where  $N_i$ ,  $M_i$  are given in table 1 and

$$|4;2\rangle' = \sum (W_{\tilde{i}\tilde{i}\tilde{i}}|i\tilde{i}\tilde{i}\tilde{i}\rangle + W_{\tilde{i}\tilde{i}\tilde{i}}|i\tilde{i}\tilde{i}\tilde{i}\rangle + W_{\tilde{i}\tilde{i}\tilde{i}}|i\tilde{i}\tilde{i}\tilde{i}\rangle). \tag{6}$$

The set of operators  $M_i \otimes N_i \otimes I \otimes I$ , for values of *i* from 1 to 16, when operated on  $|4;2\rangle'$  generate the measurement basis for Alice. Since these sixteen states need to be distinguishable, they have to be mutually orthogonal. The orthogonality condition results in the following relations,

$$W_{011}W_{110}^* + W_{001}W_{100}^* = 0,$$

$$|W_{101}|^2 = |W_{110}|^2 + |W_{100}|^2 = |W_{011}|^2 + |W_{010}|^2 = |W_{001}|^2.$$
(7)

The resulting state  $|4;2\rangle$  fulfilling these conditions possesses two ebits of entanglement between the subsystems (1,3) and (2,4) respectively. Alice, now conveys the outcome of her measurement to Bob via four classical bits. After receiving the outcome of Alice's measurement, Bob can perform an appropriate unitary operation  $(N_i \otimes M_{(i+8)mod(16)})^{-1}$  on his qubits and obtain  $|\psi_2\rangle$ .

From the earlier conditions, it can be seen that the members of Alice's measurement basis are mutually orthogonal to each other. Hence these states can be perfectly distinguished making the present scheme deterministic. It is worth mentioning that teleportation can also be successfully implemented if we redistribute the qubits between Alice and Bob. We shall now turn our attention towards the usefulness of  $|4;2\rangle$  for dense coding.

#### II. DENSE CODING

The general condition for a given 2N qubit entangled channel  $|\xi_{AB}\rangle$  to be used for sending 2N classical bits by sending N quantum bits is that there has to be N ebits of entanglement between A and B. In the dense coding

e operators for generating rince s				
i	operator $M_i$	operator $N_i$		
1	$\sigma_0$	$\sigma_0$		
2	$\sigma_0$	$\sigma_3$		
3	$\sigma_3$	$\sigma_0$		
4	$\sigma_3$	$\sigma_3$		
5	$\sigma_1$	$\sigma_0$		
6	$\sigma_1$	$\sigma_3$		
7	$\sigma_2$	$\sigma_3$		
8	$\sigma_2$	$\sigma_0$		
9	$\sigma_0$	$\sigma_1$		
10	$\sigma_0$	$\sigma_2$		
11	$\sigma_3$	$\sigma_1$		
12	$\sigma_3$	$\sigma_2$		
13	$\sigma_1$	$\sigma_1$		
14	$\sigma_2$	$\sigma_1$		
15	$\sigma_1$	$\sigma_2$		
16	$\sigma_2$	$\sigma_2$		

TABLE I: Required operators for generating Alice's measurement basis.

scenario, we let Alice possess qubits 1 and 3, while Bob is left with 2 and 4. Alice applies a set of unitary operators from  $(\sigma_0, \sigma_1, i\sigma_2, \sigma_3)$  and encodes her classical information in operators and sends her qubits to Bob. The sixteen states obtained by Bob are

$$(M_i \otimes I \otimes N_i \otimes I)|4;2\rangle. \tag{8}$$

The amount of classical bits that can be encoded into a given quantum state  $\rho^{AB}$ , shared by Alice and Bob, is given by [19],

$$X(\rho^{AB}) = \log_2 d_A + S(\rho^B) - S(\rho^{AB}) \tag{9}$$

Here,  $d_A$  refers to the dimension of the Alice's system. The maximal amount of classical information that could be encoded in a four qubit quantum state is four. When  $X(\rho^{AB})$  reaches the maximum value, the protocol is said to reach the "Holevo bound".  $|4;2\rangle$  reaches the "Holevo bound" when all the sixteen states obtained by Bob are distinguishable i.e., all of them are orthogonal. Subjecting the states to be orthogonal, we have the following conditions:

$$|W_{101}|^2 = |W_{010}|^2 = |W_{001}|^2 = |W_{110}|^2 + |W_{011}|^2$$
  
and  $W_{100} = 0$ . (10)

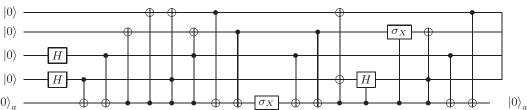


FIG. 1: Circuit diagram to experimentally generate  $|4;2\rangle$ , satisfying relations for dense coding.

An experimental procedure, to generate  $|4;2\rangle$  satisfying these relations, is shown as a quantum circuit in Fig.1. In order to obtain the classical information encoded by Alice, Bob performs a joint von-Neumann type measurement. Since, all these states are orthogonal to each other, they can be distinguished perfectly. Thus, Alice sends four classical bits by sending only two quantum bits making the superdense coding capacity reach the "Holevo bound".

## III. QUANTUM INFORMATION SPLITTING OF AN ENTANGLED STATE

Entanglement can be used for splitting and sharing of both classical and quantum information. Sharing of quantum information between a group of parties such that none of them can reconstruct the unknown information by operating on their own qubits is referred to as "Quantum information splitting".

Quantum information splitting of a single qubit state  $|\psi_1\rangle$  was first achieved using a three qubit GHZ state,  $|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle)$ , as a shared entangled resource [20, 21]. In this scheme the three parties involved, Alice, Bob and Charlie, possess one qubit each. Alice has an unknown qubit  $|\psi_1\rangle$ , that she wants Bob and Charlie to share. To achieve this purpose, Alice performs a Bell measurement on her qubits and conveys its outcome to Charlie via two classical bits. Now, the unknown qubit is locked between Bob and Charlie such that none of them can obtain it completely by operating on their own qubits. Now, if Bob can perform a measurement in a suitable basis and can convey his outcome to Charlie via one classical bit, then Charlie can reconstruct  $|\psi_1\rangle$  by operating on his qubits. This will complete the QIS protocol of  $|\psi_1\rangle$ .

The feasibility of realizing QIS experimentally was discussed by making use of a pseudo GHZ state [22]. Experimental schemes which used single photon sources to split  $|\psi_1\rangle$  have been demonstrated [23]. Recently, attention has turned towards the usage of genuine multipartite entangled channels which can be realized experimentally [24, 25]. Further, it was proved that one can devise (N-2n) protocols for the QIS of an arbitrary n qubit state using a genuinely entangled N qubit state as an entangled channel among two parties, in the case where the two parties need not meet [26]. According to this theorem, one cannot use a four qubit channel for QIS of an arbitrary two qubit state. Interestingly, here we show that  $|4;2\rangle$  can be used for the QIS of an entangled state of the type  $A|00\rangle + B|11\rangle$ , if we devise an unconventional QIS protocol. The normalized  $|4;2\rangle$  state fulfilling the following conditions,

$$|W_{110}|^2 = |W_{100}|^2 + |W_{010}|^2, (11)$$

$$|W_{001}|^2 = |W_{101}|^2 + |W_{011}|^2, \tag{12}$$

$$W_{110}W_{001}^* = W_{100}W_{011}^* + W_{010}W_{101}^* = 0,$$

$$(12)$$

can be used for QIS of  $A|00\rangle + B|11\rangle$ . In this scenario, Alice has the first qubit, Bob has the second qubit and Charlie is left with the last two qubits. Alice performs a joint measurement on her qubits and conveys its outcome to Charlie via two classical bits. Bob performs a measurement on his qubit in  $(|0\rangle, |1\rangle)$  basis and conveys his outcome to Charlie via one classical bit. The outcome of Alice's and Bob's measurement and the state obtained by Charlie are shown in table 2.

	D 11	G: 1 11 G1 11
Alice's measurement	Bob's measurement	State obtained by Charlie
$\frac{ 001\rangle+ 110\rangle}{\sqrt{2}}$	$ 1\rangle$	$ AW_{110} 00\rangle + B(W_{100} 10\rangle + W_{010} 01\rangle)$
	$ 0\rangle$	$A(W_{101} 10\rangle + W_{011} 01\rangle) + BW_{001} 11\rangle$
$\frac{ 001\rangle -  110\rangle}{\sqrt{2}}$	$ 1\rangle$	$ AW_{110} 00\rangle - B(W_{100} 10\rangle + W_{010} 01\rangle)$
	$ 0\rangle$	$A(W_{101} 10\rangle - W_{011} 01\rangle) + BW_{001} 11\rangle$
$\frac{ 111\rangle+ 000\rangle}{\sqrt{2}}$	$ 1\rangle$	$ BW_{110} 00\rangle + A(W_{100} 10\rangle + W_{010} 01\rangle)$
·	$ 0\rangle$	$B(W_{101} 10\rangle + W_{011} 01\rangle) + AW_{001} 11\rangle$
$\frac{ 111\rangle -  000\rangle}{\sqrt{2}}$	$ 1\rangle$	$ BW_{110} 00\rangle - A(W_{100} 10\rangle + W_{010} 01\rangle)$
	$ 0\rangle$	$B(W_{101} 10\rangle + W_{011} 01\rangle) - AW_{001} 11\rangle$

TABLE II: QIS of an entangled state using [4; 2]. Normalization factors have been removed for convenience.

From table 2, it can be observed that Charlie still does not possess a state, which can be converted to  $A|00\rangle + B|11\rangle$  through local operations on his individual qubits.

Hence, Charlie has to carry out further operations at his end to obtain  $A|00\rangle + B|11\rangle$ . If Bob measures  $|0\rangle$  then, Charlie needs to carry out a two-step process to obtain the desired state, on his qubits  $|C\rangle_1$  and  $|C\rangle_2$ . First, Charlie attaches two ancilla bits to his two qubit state. Subsequently, he needs to carry out required operations, as shown in Fig. 2 and given in table III. The disentangling operation performed by Charlie using two ancilla qubits is unconventional, after which Charlie ends up with a four qubit state. He then needs to perform a two qubit measurement with a basis, which depends on the input magnon state  $|4;2\rangle$ , as described in the table 3. The measurement is done over the ancilla qubits and the final state is obtained on the qubits  $|C\rangle_1$  and  $|C\rangle_2$ .

Here,  $|C\rangle_1$  and  $|C\rangle_2$  refer to Charlie's first and second qubits respectively, and  $|1\rangle_a$  refers to the ancilla used by Charlie. If Bob measures  $|1\rangle$ , then the same circuit diagram is implemented using  $|0\rangle_a$  as the ancilla bit. This completes the protocol for the QIS of an entangled state using  $|4;2\rangle$  as an entangled channel.

FIG. 2: Circuit diagram to retrieve the desired state if Bob measures  $|0\rangle$ .

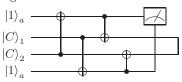


TABLE III: Charlie's disentangling operations.

Bob's measurement outcome	Charlie's measurement basis	final state
$ 1\rangle$	$ W_{110} 00\rangle + W_{100} 01\rangle + W_{010} 10\rangle$	$A 00\rangle + B 11\rangle$
	$ W_{110} 00\rangle - W_{100} 01\rangle - W_{010} 10\rangle$	$A 00\rangle - B 11\rangle$
$ 0\rangle$	$W_{101} 10\rangle + W_{011} 01\rangle + W_{001} 00\rangle$	$B 00\rangle + A 11\rangle$
	$ W_{101} 10\rangle + W_{011} 01\rangle - W_{001} 00\rangle$	$B 00\rangle - A 11\rangle$

#### IV. CONCLUSION

We have investigated the conditions under which the four qubit two magnon state  $|4;2\rangle$  gets maximally entangled and illustrated its usefulness for several quantum communication protocols. After establishing general conditions for which the state  $|4;2\rangle$  can be used for the deterministic quantum teleportation of a two qubit composite system, we showed that this state can also be used for required splitting up of a two qubit entangled state among two parties. Further, conditions are established for which the channel capacity of this state reaches the "Holevo bound", allowing four classical bits to be transmitted with just two qubits. Since teleportation of a two qubit composite system has been experimentally demonstrated using Bell pairs and  $|4;2\rangle$  can be realized using Heisenberg exchange interactions, all the presented schemes are experimentally feasible in either spin chains or in an array of coupled quantum dots. These protocols can be extended to other quantum channels in the future. Further, we wish to investigate the usefulness of magnon states for quantum secure direct communication. We also intend to find out the usefulness of higher particle magnon states for quantum communication protocols. It has been shown, that in the presence of decoherence, the fidelity of teleportation in a channel depends upon the coupling of the quibits to a common bath [27]. In future, we intend to investigate the effect of decoherence in the present system. We also plan to carry out these protocols by performing a non destructive discrimination scheme on multiple qubits instead of joint multiqubit entangled measurements.

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