Generation of atomic NOON states via adiabatic passage*

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We propose a scheme for generating atomic NOON states via adiabatic passage. In the scheme, a double Λ -type three-level atom is trapped in a bimodal cavity and two sets of Λ -type three-level atoms are translated into and outside of two single mode cavities respectively. The three cavities connected by optical fibres are always in vacuum states. After a series of operations and suitable interaction time, we can obtain arbitrary large-*n* NOON states of two sets of Λ -type three-level atoms in distant cavities by performing a single projective measurement on the double Λ type three-level atom. Due to adiabatic elimination of atomic excited states and the application of adiabatic passage, our scheme is robust against the spontaneous emissions of atoms, the decays of fibres and cavities photon leakage. So the scheme has a high fidelity and feasibility under the current available techniques.

Keywords: NOON states · Adiabatic passage · Cavity quantum electrodynamics

1. Introduction

Quantum entanglement, an interesting and attractive phenomenon in quantum mechanics, plays a significant role not only in testing quantum nonlocality, but also in processing a variety of quantum information tasks [1–8]. Multi-particle entangled states, such as GHZ states, W states, cluster states, NOON states, etc, are the fundamental resource of quantum information processing (QIP). The NOON states, as an intereting multi-particle entangled states, have the form as

$$|\text{NOON}\rangle = \frac{1}{\sqrt{2}}(|n,0\rangle + |0,n\rangle),\tag{1}$$

which contain n indistinguishable particles in an equal superposition of all being in one of the two possible modes. It's well established that the NOON states have significant applications either in lithography [9–11] or in quantum metrology [12, 13]. Especially, it

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can be used to obviously improve the phase sensitivity in quantum interferometry and beat the classical diffraction limit in quantum lithography [13–16]. Recently, much attention has been paid to prepare the NOON states. Many theoretical and experimental schemes have been proposed for generating the NOON states via optical components [11, 12, 17–23], cold atomic ensemble [24], superconducting circuits [25, 26] and cavity quantum electrodynamics (QED) [27–35]. However, the success probabilities of the proposals based on the linear optical components, are very low as the number of particles increasing [19]. Some people suggested using the optical nonlinear process for the generation of NOON states. Yet, it is difficult to carry out experimentally so far [27]. The QED system, as a suitable candidate for demonstrating QIP and quantum state engineering [36], has been studied extensively. Many proposals for preparing NOON states based on QED have been put forward, as previously mentioned. However, all of these systems are always affected by the various of external factors which will induce the decoherence and affect the probability of success, even destroy the entanglement. Although many kinds of physical systems are used to avoid or reduce the decoherence, it is always imperfect yet. So the preparation of the entangled multi-particle NOON states is still a severe challenge in the state of the art, though great progress has been made in recent decades. On the other hand, the adiabatic passage [37-40], as an useful tool for realizing QIP, is becoming more and more powerful and charming owing to its robust against the spontaneous emission of the excited states, cavity photon leakage and some experimental parameter errors. As a result, the technique of adiabatic passage has been extensively studied for the entanglement generation [41–51].

In this paper, we propose a scheme for generating the NOON states of two sets of the atoms via adiabatic passage. In the scheme, a double Λ -type three-level atom is trapped in a bimodal cavity and two sets of Λ -type three-level atoms are translated into and outside of two single mode cavities respectively. After a series of operations and suitable interaction time, the arbitrary large-*n* entangled NOON states of two sets of Λ -type three-level atoms in distant cavities can be obtained by performing a single projective measurement on the double Λ -type three-level atom. Our scheme has the following characteristics: (1) Due to along the dark state, the cavity modes are unpopulated during the whole interaction process, hence the scheme is robust against the decays of the cavities. (2) Owing to adiabatical eliminations of atomic excited states, the spontaneous emission rate can be regarded as zero. (3) Under certain conditions, the probabilities of fibre modes populated can be negligible safely, thus



Fig. 1: The schematic setup for generating atomic NOON states. The two sets of Λ -type three-level atoms are stored in two transverse optical lattices respectively and translated into and outside of the cavity 1 and 3 respectively for interacting with the cavity modes and the classical fields. The level configurations of the atoms are shown in Fig. 2. A double Λ -type three-level atom with level configuration shown in Fig. 3 is trapped in cavity 2. The three cavities are connected by two fibres A and B. The atom detector is used to detect the double Λ -type three-level atom.

the decays of fibres are effectively suppressed. (4) Taking advantages of adiabatic passage, the scheme is insensitive to small fluctuation of experimental parameters. (5) The scheme can be used to generate arbitrary large-n NOON states in theory.

The rest of the paper is organized as follows. In Sect. 2, the fundamental model and Hamiltonian are introduced. In Sect. 3, we propose a scheme to generate atomic NOON states via adiabatic passage. Finally, we discuss the fidelity of the scheme and summarize the conclusion in Sect. 4.

2. The fundamental model and Hamiltonian

The schematic setup for generating atomic NOON states is shown in Fig. 1. There are three distant optical cavities connected by two fibres (fibres A and B). The two sets of Λ -type three-level atoms are stored in two transverse optical lattices respectively and translated into and outside of the cavity 1 and cavity 3 simultaneously and respectively [52, 53]. The level



Fig. 2: The level configurations of the Λ -type three-level atoms translated into and outside of cavities 1 and 3. The states $|s\rangle_{L(R)}$ and $|k\rangle_{L(R)}$ are two ground levels and $|r\rangle_{L(R)}$ is an excited level of the atoms interacting with cavity 1 (3). $g_{1(3)}$ is the coupling constant between the transition $|s\rangle_{L(R)} \leftrightarrow |r\rangle_{L(R)}$ and corresponding cavity mode. $\Omega_{L(R)}$ is the time-dependent Rabi frequency of classical field driving the atomic transition $|k\rangle_{L(R)} \leftrightarrow |r\rangle_{L(R)}$. The classical field with Rabi frequency $\Omega_{L(R)}$ and the mode of cavity 1(3) are detuned from the respective transitions by $\Delta_{1(3)}$.

configurations of the atoms interacting with cavities 1 and 3 respectively are shown in Fig. 2(a) and Fig. 2(b). The states $|s\rangle_{L(R)}$ and $|k\rangle_{L(R)}$ are two ground levels and $|r\rangle_{L(R)}$ is an excited level of the atoms interacting with cavity 1 (3). The transition $|s\rangle_{L(R)} \leftrightarrow |r\rangle_{L(R)}$ is coupled to the mode of cavity 1 (3) with the coupling constant $g_{1(3)}$. The transition $|k\rangle_{L(R)} \leftrightarrow |r\rangle_{L(R)}$ is driven by the classical field with time-dependent Rabi frequency $\Omega_{L(R)}$. The frequency detunings between the atomic transitions $|s\rangle_{L(R)} \leftrightarrow |r\rangle_{L(R)}, |k\rangle_{L(R)} \leftrightarrow |r\rangle_{L(R)}$ and the relevant cavity mode and classical cavity are the same and denoted as $\Delta_{1(3)}$. They satisfy the corresponding two-photon resonance conditions. The atom in the bimodal cavity 2 is a double Λ -type three-level atom. The relevant atomic levels and transitions are depicted in Fig. 3. Such level structure can be achieved in ${}^{40}Ca^+$ [35, 54–56]. Two degenerate ground states $|g_L\rangle$ and $|g_R\rangle$ correspond to ${}^{40}Ca^+$ atom hyperfine levels $|F = 1/2, m = -1/2\rangle$ and $|F = 1/2, m = -1/2\rangle$ of the level $4S_{1/2}$, while two degenerate excited states $|e_L\rangle$ and $|e_R\rangle$ correspond to $|F = 1/2, m = -1/2\rangle$ and $|F = 1/2, m = 1/2\rangle$ of the level $4P_{1/2}$. On the other hand, two intermediate semi-stable states $|f_L\rangle$ and $|f_R\rangle$ correspond to $|F = 3/2, m = -1/2\rangle$ of the level $3D_{3/2}$, respectively. The transition



Fig. 3: The level configuration of the double Λ -type three-level atom [35]. $|g_L\rangle$ and $|g_R\rangle$, $|e_L\rangle$ and $|e_R\rangle$ are two degenerate ground states, two degenerate excited states respectively and $|f_L\rangle$, $|f_R\rangle$ are two intermediate semi-stable states. Ω_1 is the Rabi frequency of classical field F_1 driving the transitions $|f_L\rangle \leftrightarrow |e_L\rangle$ and $|f_R\rangle \leftrightarrow |e_R\rangle$; $g_{2l(r)}$ is the coupling constant between the transition $|e_L\rangle \leftrightarrow |g_L\rangle$ ($|e_R\rangle \leftrightarrow |g_R\rangle$) and the left (right) circular polarized cavity mode; Ω_2 is the Rabi frequency of classical field F_2 driving the transitions $|g_L\rangle \leftrightarrow |f_L\rangle$ and $|g_R\rangle \leftrightarrow |f_R\rangle$. The frequency detunings of the cavity modes and classical cavity F_1 from the respective atomic transitions are the same and denoted as Δ_2 .

 $|f_{L(R)}\rangle \leftrightarrow |e_{L(R)}\rangle$ is driven by a classical field F_1 with Rabi frequency Ω_1 ; $|e_{L(R)}\rangle \leftrightarrow |g_{L(R)}\rangle$ is coupled to the left(right) circular polarized cavity mode with the coupling constant $g_{2l(r)}$; the transition $|g_{L(R)}\rangle \leftrightarrow |f_{L(R)}\rangle$ is driven by another classical field F_2 with Rabi frequency Ω_2 . The frequency detunings of the cavity modes and classical cavity F_1 from the respective atomic transitions are the same and denoted as Δ_2 . Now, we consider the case both cavity 1 and cavity 3 have one atom respectively, thus the Hamiltonian of atom-cavity system under the rotating-wave approximation can be written as ($\hbar = 1$)

$$H_{ac} = \frac{\Omega_L^2}{\Delta_1} |k\rangle_L \langle k| + \frac{\Omega_1^2}{\Delta_2} |f_L\rangle \langle f_L| + \frac{\Omega_1^2}{\Delta_2} |f_R\rangle \langle f_R| + \frac{\Omega_R^2}{\Delta_3} |k\rangle_R \langle k|$$

+ $\frac{g^2}{\Delta_1} a_1^{\dagger} a_1 |s\rangle_L \langle s| + \frac{g^2}{\Delta_2} a_{2l}^{\dagger} a_{2l} |g_L\rangle \langle g_L| + \frac{g^2}{\Delta_2} a_{2r}^{\dagger} a_{2r} |g_R\rangle \langle g_R| + \frac{g^2}{\Delta_3} a_3^{\dagger} a_3 |s\rangle_R \langle s|$
+ $\left(\frac{g\Omega_L}{\Delta_1} a_1^{\dagger} |s\rangle_L \langle k| + \frac{g\Omega_1}{\Delta_2} a_{2l}^{\dagger} |g_L\rangle \langle f_L| + \frac{g\Omega_1}{\Delta_2} a_{2r}^{\dagger} |g_R\rangle \langle f_R| + \frac{g\Omega_R}{\Delta_3} a_3^{\dagger} |s\rangle_R \langle k| + \text{H.c.}\right), (2)$

where $a_{1(3)}^{\dagger}$ and $a_{1(3)}$ are the creation and annihilation operators of the cavity 1(3); $a_{2l(r)}^{\dagger}$ and $a_{2l(r)}$ are the creation and annihilation operators of left(right) circular polarization of the cavity 2. For convenience, here we have set $g_{2l(r)} = g_{1(3)} = g$. The first four terms on the right-hand side in Eq. (2) represent the atom level shifts induced by classical fields. By using the nonresonant coupling of other lasers with the corresponding atom levels, these energy level shifts can be compensated straightforwardly [57]. Considering the cavities are initially in vacuum states, the Hamiltonian H_{ac} can be further simplified into

$$H_{ac}^{\prime} = \Omega_{Le}(t)a_{1}^{\dagger}|s\rangle_{L}\langle k| + \Omega_{1e}(t)a_{2l}^{\dagger}|g_{L}\rangle\langle f_{L}| + \Omega_{1e}(t)a_{2r}^{\dagger}|g_{R}\rangle\langle f_{R}| + \Omega_{Re}(t)a_{3}^{\dagger}|s\rangle_{R}\langle k| + \text{H.c., (3)}$$

where $\Omega_{Le}(t) = \Omega_L g / \Delta$, $\Omega_{Re}(t) = \Omega_R g / \Delta$, $\Omega_{1e}(t) = \Omega_1 g / \Delta$ are the effective Rabi frequencies for the corresponding Raman transitions $|k\rangle_L \to |s\rangle_L$, $|k\rangle_R \to |s\rangle_R$, $|f_{L(R)}\rangle \to |g_{L(R)}\rangle$, respectively. Here we have assumed $\Delta_{1,2,3} = \Delta$.

In our scheme, three cavities are connected by two optical fibres A and B. In the short fibre limit, only one (resonant) mode of a fibre interacts with corresponding cavity modes. The interaction Hamiltonian of fibre-cavity system can be approximated to

$$H_{cf} = \eta_A [b_A (a_1^{\dagger} + a_{2l}^{\dagger})] + \eta_B [b_B (a_{2r}^{\dagger} + a_3^{\dagger})] + \text{H.c..}$$
(4)

where b_A and b_B are the annihilation operators of the resonant modes of fibre A and B respectively and the polarizations of the fibre modes A and B have been chosen as the left circular and the right circular polarizations, respectively. $\eta_{A(B)}$ is the coupling strength between the fibre mode A(B) and corresponding cavity modes.

Lastly, in the interaction picture, the total Hamiltonian of the system can be written as

$$H_{eff} = H'_{ac} + H_{cf}$$

$$= \Omega_{Le}(t)a_1^{\dagger}|s\rangle_L \langle k| + \Omega_{1e}(t)a_{2l}^{\dagger}|g_L\rangle \langle f_L| + \Omega_{1e}(t)a_{2r}^{\dagger}|g_R\rangle \langle f_R| + \Omega_{Re}(t)a_3^{\dagger}|s\rangle_R \langle k|$$

$$+ \eta_A[b_A(a_1^{\dagger} + a_{2l}^{\dagger})] + \eta_B[b_B(a_{2r}^{\dagger} + a_3^{\dagger})] + \text{H.c.}.$$
(5)

3. Generation of the NOON states

In this section, we will show how to deterministically prepare the multi-particle NOON states. Considering that both the cavity modes and the fiber modes are all in vacuum states, initially, and the double Λ -type three-level atom is prepared in the superposition

state $1/\sqrt{2}(|f_L\rangle + |f_R\rangle)$ with the method of Ref. [58], while atoms interacting with cavities 1 and 3 respectively are in the state $|s\rangle_L|s\rangle_R$, so the initial state of the whole compound system is

$$\frac{1}{\sqrt{2}}(|f_L\rangle + |f_R\rangle)|s\rangle_L|s\rangle_R|000\rangle_c|00\rangle_f.$$
(6)

For the initial state $|f_L\rangle|s\rangle_L|s\rangle_R|000\rangle_c|00\rangle_f$, dominated by Hamiltonian (5), the evolution of the system state remains in the subspace with one excitation number spanned by the basis vectors

$$\begin{aligned} |\psi_1\rangle &= |f_L\rangle |s\rangle_L |s\rangle_R |000\rangle_c |00\rangle_f, \\ |\psi_2\rangle &= |g_L\rangle |s\rangle_L |s\rangle_R |01^l 0\rangle_c |00\rangle_f, \\ |\psi_3\rangle &= |g_L\rangle |s\rangle_L |s\rangle_R |000\rangle_c |1^l 0\rangle_f, \\ |\psi_4\rangle &= |g_L\rangle |s\rangle_L |s\rangle_R |1^l 00\rangle_c |00\rangle_f, \\ |\psi_5\rangle &= |g_L\rangle |k\rangle_L |s\rangle_R |000\rangle_c |00\rangle_f. \end{aligned}$$

$$(7)$$

The Hamiltonian (5) has a dark state (i.e., zero energy eigenstate of Hamiltonian (5))

$$|\Psi_{D1}\rangle = \frac{1}{K_0} (\Omega_{Le} \eta_A |\psi_1\rangle - \Omega_{Le} \Omega_{1e} |\psi_3\rangle + \Omega_{1e} \eta_A |\psi_5\rangle), \tag{8}$$

where $K_0 = \sqrt{\Omega_{Le}^2 \eta_A^2 + \Omega_{Le}^2 \Omega_{1e}^2 + \Omega_{1e}^2 \eta_A^2}$.

Under the condition of

$$\eta_A \gg \Omega_{Le}, \Omega_{1e}, \tag{9}$$

the dark state (8) reduces to

$$|\Psi'_{D1}\rangle = \frac{1}{\sqrt{\Omega_{Le}^2 + \Omega_{1e}^2}} (\Omega_{Le} |\psi_1\rangle + \Omega_{1e} |\psi_5\rangle).$$
(10)

If pulse shapes are designed such that

$$\lim_{t \to -\infty} \frac{\Omega_{1e}}{\Omega_{Le}} = 0, \ \lim_{t \to +\infty} \frac{\Omega_{Le}}{\Omega_{1e}} = 0, \tag{11}$$

the initial state $|\psi_1\rangle$ of the system is adiabatically transferred to $|\psi_5\rangle$.

For the initial state $|f_R\rangle|s\rangle_L|s\rangle_R|000\rangle_c|00\rangle_f$, dominated by Hamiltonian (5) also, the evolution of the system state remains in the subspace with one excitation number spanned by the basis vectors

$$|\psi_6\rangle = |f_R\rangle|s\rangle_L|s\rangle_R|000\rangle_c|00\rangle_f,$$

$$\begin{aligned} |\psi_{7}\rangle &= |g_{R}\rangle|s\rangle_{L}|s\rangle_{R}|01^{r}0\rangle_{c}|00\rangle_{f}, \\ |\psi_{8}\rangle &= |g_{R}\rangle|s\rangle_{L}|s\rangle_{R}|000\rangle_{c}|01^{r}\rangle_{f}, \\ |\psi_{9}\rangle &= |g_{R}\rangle|s\rangle_{L}|s\rangle_{R}|001^{r}\rangle_{c}|00\rangle_{f}, \\ |\psi_{10}\rangle &= |g_{R}\rangle|s\rangle_{L}|k\rangle_{R}|000\rangle_{c}|00\rangle_{f}. \end{aligned}$$
(12)

The Hamiltonian (5) has a dark state

$$|\Psi_{D2}\rangle = \frac{1}{K_1} (\Omega_{Re} \eta_B |\psi_6\rangle - \Omega_{Re} \Omega_{1e} |\psi_8\rangle + \Omega_{1e} \eta_B |\psi_{10}\rangle), \tag{13}$$

where $K_1 = \sqrt{\Omega_{Re}^2 \eta_B^2 + \Omega_{Re}^2 \Omega_{1e}^2 + \Omega_{1e}^2 \eta_B^2}$.

Under the condition of

$$\eta_B \gg \Omega_{Re}, \Omega_{1e}, \tag{14}$$

the dark state (13) reduces to

$$|\Psi_{D2}'\rangle = \frac{1}{\sqrt{\Omega_{Re}^2 + \Omega_{1e}^2}} (\Omega_{Re} |\psi_6\rangle + \Omega_{1e} |\psi_{10}\rangle).$$
(15)

If pulse shapes are designed such that

$$\lim_{t \to -\infty} \frac{\Omega_{1e}}{\Omega_{Re}} = 0, \ \lim_{t \to +\infty} \frac{\Omega_{Re}}{\Omega_{1e}} = 0, \tag{16}$$

the initial state $|\psi_6\rangle$ of the system is adiabatically transferred to $|\psi_{10}\rangle$.

After the above adiabatic processes, the system state

$$|\Psi_{1}\rangle = \frac{1}{\sqrt{2}} (|g_{L}\rangle|k\rangle_{L}|s\rangle_{R} + |g_{R}\rangle|s\rangle_{L}|k\rangle_{R})|000\rangle_{c}|00\rangle_{f}$$

$$= \frac{1}{\sqrt{2}} (|g_{L}\rangle|1,0\rangle + |g_{R}\rangle|0,1\rangle)|000\rangle_{c}|00\rangle_{f}$$
(17)

can be achieved. Here, $|1,0\rangle(|0,1\rangle)$ denotes $|k\rangle_L|s\rangle_R(|s\rangle_L|k\rangle_R)$. After that, we turn off the classical field F_1 and apply the classical field F_2 with Rabi frequency Ω_2 on the double Λ -type three-level atom to drive the transition $|g_L\rangle \rightarrow |f_L\rangle(|g_R\rangle \rightarrow |f_R\rangle)$. After the interaction time τ which satisfies $\Omega_2\tau = \pi/2$, the state of the whole system becomes

$$\frac{1}{\sqrt{2}}(|f_L\rangle|k\rangle_{L1}|s\rangle_{R1} + |f_R\rangle|s\rangle_{L1}|k\rangle_{R1})|000\rangle_c|00\rangle_f.$$
(18)

Here, in order to describe the process for preparing the atomic NOON states clearly, we introduce symbol $|x\rangle_{L(R)i}(x = s, k; i = 1, 2, 3, \cdots)$ which denotes the *i*-th atom through the cavity 1(3) is in the state $|x\rangle$. The state in Eq. (18) is the new initial state again. Then turn

off the classical field F_2 and turn on the classical field F_1 simultaneously. After repeating the above adiabatic process and choosing suitable interaction time each time, the whole system evolves successively into the states

$$\begin{split} |\Psi_{2}\rangle &= \frac{1}{\sqrt{2}} \langle |g_{L}\rangle|k\rangle_{L1}|k\rangle_{L2}|s\rangle_{R1}|s\rangle_{R2} + |g_{R}\rangle|s\rangle_{L1}|s\rangle_{L2}|k\rangle_{R1}|k\rangle_{R2}\rangle|000\rangle_{c}|00\rangle_{f} \\ &= \frac{1}{\sqrt{2}} \langle |g_{L}\rangle|2,0\rangle + |g_{R}\rangle|0,2\rangle\rangle|000\rangle_{c}|00\rangle_{f}, \\ |\Psi_{3}\rangle &= \frac{1}{\sqrt{2}} \langle |g_{L}\rangle|k\rangle_{L1}|k\rangle_{L2}|k\rangle_{L3}|s\rangle_{R1}|s\rangle_{R2}|s\rangle_{R3} + |g_{R}\rangle|s\rangle_{L1}|s\rangle_{L2}|s\rangle_{L3}|k\rangle_{R1}|k\rangle_{R2}|k\rangle_{R3}\rangle|000\rangle_{c}|00\rangle_{f}, \\ &= \frac{1}{\sqrt{2}} \langle |g_{L}\rangle|3,0\rangle + |g_{R}\rangle|0,3\rangle\rangle|000\rangle_{c}|00\rangle_{f}, \\ |\Psi_{4}\rangle &= \frac{1}{\sqrt{2}} \langle |g_{L}\rangle|4,0\rangle + |g_{R}\rangle|0,4\rangle\rangle|000\rangle_{c}|00\rangle_{f}, \\ |\Psi_{5}\rangle &= \frac{1}{\sqrt{2}} \langle |g_{L}\rangle|5,0\rangle + |g_{R}\rangle|0,5\rangle\rangle|000\rangle_{c}|00\rangle_{f}, \\ &\vdots \\ |\Psi_{n}\rangle &= \frac{1}{\sqrt{2}} \langle |g_{L}\rangle|n,0\rangle + |g_{R}\rangle|0,n\rangle\rangle|000\rangle_{c}|00\rangle_{f}, \end{split}$$

$$(19)$$

Here, in the sign $|n,0\rangle(|0,n\rangle)(n = 1, 2, 3, \cdots)$, $|n\rangle$ donates there are *n* atoms through the cavity 1(3) in the state $|k\rangle_{L(R)}$; $|0\rangle$ donates there are *n* atoms in the state $|s\rangle_{R(L)}$. At last, we use a classical field F_3 to implement a Hadamard operation

$$|g_L\rangle \to \frac{1}{\sqrt{2}} (|g_L\rangle + |g_R\rangle), |g_R\rangle \to \frac{1}{\sqrt{2}} (|g_L\rangle - |g_R\rangle),$$
(20)

and then, the state $|\Psi_n\rangle$ becomes

$$\frac{1}{2}[|g_L\rangle(|n,0\rangle + |0,n\rangle) + |g_R\rangle(|n,0\rangle - |0,n\rangle)].$$
(21)

Now, a single projective measurement should be performed on the double Λ -type three-level atom. If the double Λ -type three-level atom is detected in the state $|g_L\rangle$, Eq. (21) collapses to

$$|\text{NOON}\rangle_{+} = \frac{1}{\sqrt{2}}(|n,0\rangle + |0,n\rangle), \qquad (22)$$

and if the atom is detected in the state $|g_R\rangle$, Eq. (21) collapses to

$$|\text{NOON}\rangle_{-} = \frac{1}{\sqrt{2}}(|n,0\rangle - |0,n\rangle).$$
(23)



Fig. 4: (a) Time dependence of $\Omega_{\xi}(t)/\Omega_0$ of the laser fields for getting $|\Psi_n\rangle$. (b) Time evolutions of the populations of corresponding system states. Here, the system parameters are set to be $\Omega_0 = 1.5g, T = 100/g, \tau = 12/g, t_L = t_R = -15/g, t_1 = 15/g, g = 1$ GHz and $\Delta = 15g$.

It is worth noting that no matter what the measurement result is, the NOON states can be always achieved. That's to say, the successful probability of our protocol is unity in the ideal case.

4. Discussion and conclusion

In order to generate the NOON states, the conditions of Eq. (11) and Eq. (16) should be satisfied in our scheme. So we can design the pulse shape of the laser fields Ω_L , Ω_R and Ω_1 as the Gaussian [59–61],

$$\Omega_{\xi}(t) = \Omega_0 \exp[-\frac{(t - T/2 - t_{\xi})^2}{2\tau^2}],$$
(24)

where $\xi = L, R, 1; \Omega_0$ is the amplitude of $\Omega_{\xi}; T$ is the total adiabatic time and τ is the laser beam waist. t_{ξ} is the time we turn on the laser with Rabi frequency Ω_{ξ} on the corresponding atoms.

For showing expression (24) clearly, we plot the time dependence of $\Omega_{\xi}(t)/\Omega_0$ of the laser fields in Fig. 4(a) and the population curves of $|\psi_1\rangle + |\psi_6\rangle$ and $|\psi_5\rangle + |\psi_{10}\rangle$ with gt in Fig. 4(b). The system parameters are chosen as g = 1 GHz, $\Omega_0 = 1.5g$, T = 100/g, $\tau = 12/g$, $t_L = t_R = -15/g$, $t_1 = 15/g$ and $\Delta = 15g$. It can be seen from Fig. 4(a) that the conditions

$$\lim_{t \to -\infty} \frac{\Omega_{1e}}{\Omega_{Le}} = 0, \quad \lim_{t \to +\infty} \frac{\Omega_{Le}}{\Omega_{1e}} = 0,$$
$$\lim_{t \to -\infty} \frac{\Omega_{1e}}{\Omega_{Re}} = 0, \quad \lim_{t \to +\infty} \frac{\Omega_{Re}}{\Omega_{1e}} = 0,$$
(25)

can be satisfied during the whole evolution for realizing the NOON states. In addition, we can see from Fig. 4(b) that when $gt \to +\infty$, the population of $|\psi_5\rangle + |\psi_{10}\rangle$ is 1, which means the NOON states can be deterministically achieved via adiabatic evolution.

Certainly, all the above results are based on the ideal case because the influences of atomic spontaneous emission, photon leakage out of the cavities and fibres are not taken into account. In fact, the effects of these factors are inevitable, so we will study the effects of these factors on the fidelity below.

For purpose of generating atomic NOON states, we have used the large detuning condition and adiabatically eliminated the excited states of the atoms, as a result, setting the spontaneous emission rate to be zero is acceptable. While, the effect of photon loss out of the cavity can also be ignored safely in our scheme because the cavity modes are never populated in the whole process due to the adiabatic passage along dark state. The terms related to $|\psi_3\rangle$ and $|\psi_8\rangle$, however, have been discarded for above calculations since we have assumed that $|\eta_A|, |\eta_B| \gg |\Omega_{Le}|, |\Omega_{Re}|, |\Omega_{1e}|$ in Eq. (10) and Eq. (15). The practical situation is that the fibre modes may be excited, in other words, there is a probability that the state $(|\psi_3\rangle + |\psi_8\rangle)/\sqrt{2}$ are populated, and then they may evolve into $(|g_L\rangle|s\rangle_{L1}|s\rangle_{R1} + |g_R\rangle|s\rangle_{L1}|s\rangle_{R1}||000\rangle_c|00\rangle_f/\sqrt{2}$ due to the fibre decays, which will cause error for generating the NOON states. Considering the effect of photon leakage out of the fibres, the fidelity can be written as

$$F = 1 - \frac{\gamma_f}{2} \int_0^T \frac{\Omega_{Le}^2 \Omega_{1e}^2}{K_0^2} + \frac{\Omega_{Re}^2 \Omega_{1e}^2}{K_1^2} dt,$$
(26)

where K_0, K_1 are given in Eq. (8) and Eq. (13); γ_f denotes the decay rate of fibres (here we assume the decay rates of two fibres are the same); T is still the total adiabatic time.

We investigate the effect of fibre loss on the fidelity of getting $|\Psi_1\rangle$ for different Ω_0 values $(\Omega_0 = 0.75g, 1.5g, 2.25g)$ as shown in Fig. 5(a), where we set $\eta_{A,B} = \eta = 0.6g$. It is seen obviously from Fig. 5(a) that the fidelity decreases slightly with the increase of the γ_f/g . However, even if there exists a relatively large fibre decay rate $\gamma_f = 0.3g$ when $\Omega_0 \leq 1.5g$, we can still obtain NOON states with a high fidelity. Besides, we plot the fidelity as a function



Fig. 5: The effect of fibre loss γ_f and the coupling strength η of fibre-cavity on the fidelity of getting $|\Psi_1\rangle$ with T = 100/g, $\tau = 12/g$, $t_L = t_R = -15/g$, $t_1 = 15/g$, g = 1 GHz and $\Delta = 15g$. (a) Fidelities each as a function of γ_f/g for different Ω_0 with $\eta = 0.6g$. (b) Fidelities each as a function of η/g for different γ_f with $\Omega_0 = 1.5g$.

of η/g for different $\gamma_f(\gamma_f = 0.05g, 0.1g, 0.2g)$ in Fig. 5(b) with $\Omega_0 = 1.5g$. We can see that the fidelities increase gradually with the increase of η/g , while, with the increase of decay rate γ_f , the fidelity approaches 1 more slowly. But, when $\eta/g \ge 0.6$ the fidelity is higher than 0.99 even with $\gamma_f = 0.2g$.

In addition, we also plot the curves of fidelity of getting $|\Psi_n\rangle$ versus *n* for different decay rates of fibre modes with $\Omega_0 = 1.5g$ in Fig. 6. The other parameters are the same as those in Fig. 5. Obviously, it can be seen from Fig. 6 that the fidelity decreases with the increase of the particle number *n*. Nevertheless, the fidelity can still reach 0.934 even if the particle number *n* is up to 10 when $\gamma_f = 0.2g$ and up to 20 when $\gamma_f = 0.1g$.

In conclusion, we have proposed a scheme for generating arbitrary large-n NOON states via adiabatic passage. By using a sequence of pulse laser fields, our atom-cavity-fibre system is always in the dark states. In the whole process, the influence of atomic spontaneous emissions, photon leakage out of fibres and cavities are effectively compressed via adiabatic elimination of excited states and adiabatic passage. Furthermore, we make an estimation on the fidelities of the NOON states by considering different parameters and show that our scheme is insensitive to small fluctuations of experimental parameters. Anyway, the present scheme provides an efficient approach to realize arbitrary large-n atomic NOON states and



Fig. 6: Fidelities of getting $|\Psi_n\rangle$ each as a function of n for different γ_f when $\Omega_0 = 1.5g$, $\eta = 0.6g$ with T = 100/g, $\tau = 12/g$, $t_L = t_R = -15/g$, $t_1 = 15/g$, g = 1 GHz and $\Delta = 15g$.

we hope our work may be useful for the quantum information in the near future.

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