

# Efficient Deterministic Secure Quantum Communication protocols using multipartite entangled states

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**Abstract** We propose two deterministic secure quantum communication (DSQC) protocols employing three-qubit GHZ-like states and five-qubit Brown states as quantum channels for secure transmission of information in units of two bits and three bits using multipartite teleportation schemes developed here. In these schemes, the sender's capability in selecting quantum channels and the measuring bases leads to improved qubit efficiency of the protocols.

**Keywords** Deterministic Secure Quantum Communication · GHZ-like state · Five-qubit Brown state · Qubit efficiency

## 1 Introduction

The field of quantum cryptography which offers unconditional security in communication between legitimate users has emerged as an important area of research in this information age. In 1984, Bennett and Brassard[1] proposed the first Quantum Key Distribution (QKD) protocol for secure communication. Following this, several protocols[2][3] were suggested all of which required a unique secret key to be shared between the users before communication. Later developments showed how secure quantum communication could be achieved without the need for shared secret keys. Among the two group of protocols, the deterministic secure quantum communication (DSQC) protocols[4] the receiver can read out the message only after the exchange of at least one bit of classical information per qubit. But, in quantum secure direct communication(QSDC) the message is read out directly. Compared with QSDC, the DSQC protocols are more secure in that the message-carrying qubits need not be transmitted through external channels. Both protocols,

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however, require classical communication during the error checking and eavesdropping detection processes.

In 1999, Shimizu and Imoto[5] proposed the first DSQC protocol using EPR pairs and Bell measurements. Later, Beige *et.al.*[6] proposed a scheme based on single photons. In 2004, Yan and Zhang[7] found the first secure DSQC protocol based on teleportation. Following this work, several schemes employing multipartite entangled channels and teleportation were proposed[8][9][10][11][12][13][14] Many DSQC protocols based on entanglement swapping[15][16][17][18][19] and order rearrangement of particles[20][21] were also suggested. The advantage of teleportation based protocols is that these are more secure even in noisy channel[22] and are suitable for quantum error correction[23].

In this paper we propose two new DSQC protocols one of these using the three-qubit GHZ-like state and the other using the five-qubit Brown states for secure transmission of information in units two bits and three bits employing multipartite teleportation techniques. In these schemes, it is the sender who decides which entanglement channel and measurement basis are to be used. The receiver and the possible eavesdroppers are ignorant of the choices. The receiver performs unitary operations based on the classical information obtained from the sender and finally performs joint measurements to read out the secret message.

This paper is organized as follows. In section 2, we present the teleportation scheme of a special class of two particle state using GHZ-like state and a special class of three particle state using five-qubit Brown state in 3. The details of our new DSQC protocols using GHZ-like state and Brown state are given in 3. In section 4, we discuss the possible eavesdropping attacks. In section 5, we discuss and compare the efficiency of our protocol with other existing protocols. The final section contains our conclusions.

## 2 Teleportation of special classes of two and three particle states

### 2.1 Teleportation of a two particle state using GHZ-like state

In 2014, Nandi and Mazumdar[24] proposed a scheme for teleportation of a special form of two particle state,  $|\tau\rangle_{12} = \alpha(|00\rangle + |11\rangle)_{12} + \beta(|01\rangle + |10\rangle)_{12}$  using GHZ-like state

$$|\Phi_G\rangle_{345} = \frac{1}{\sqrt{2}}\{|0\rangle_3 |\psi^+\rangle + |1\rangle_3 |\phi^+\rangle_{45}\} \quad (1)$$

as entanglement channel. Here, the states  $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}\{|01\rangle \pm |10\rangle\}$  and  $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}\{|00\rangle \pm |11\rangle\}$  represent the Bell basis states. A three particle joint measurement is performed in the measurement basis:  $|\zeta^\pm\rangle = \frac{1}{2}\{(|001\rangle |111\rangle) \pm (|010\rangle - |100\rangle)\}$  and  $|\eta^\pm\rangle = \frac{1}{2}\{(|000\rangle - |110\rangle) \pm (|011\rangle - |101\rangle)\}$ , to achieve this teleportation. In the DSQC protocol proposed in section:3, we need a

Alice's measurement result	Classical information	State of Bob's particles (4,5)	Unitary operation
$ \zeta'^+\rangle$	00	$\alpha( 00\rangle -  11\rangle) + \beta( 01\rangle -  10\rangle)$	$(I \otimes I)$
$ \zeta'^-\rangle$	01	$\alpha( 00\rangle -  11\rangle) - \beta( 01\rangle -  10\rangle)$	$(\sigma_z \otimes \sigma_z)$
$ \eta'^+\rangle$	10	$\alpha( 01\rangle -  10\rangle) + \beta( 00\rangle -  11\rangle)$	$(I \otimes \sigma_x)$
$ \eta'^-\rangle$	11	$\alpha( 01\rangle -  10\rangle) - \beta( 00\rangle -  11\rangle)$	$(\sigma_z \otimes i\sigma_y)$

**Table 1** Classical information corresponding to Alice's measurement result, state of particles (4,5) and unitary operations performed by Bob.

teleportation scheme for a related two particle state.

$$|\tau'\rangle_{12} = \alpha(|00\rangle - |11\rangle)_{12} + \beta(|01\rangle - |10\rangle)_{12} \quad (2)$$

We show, here, that this can be done by utilizing the Nandi-Mazumdar method by choosing a different GHZ-like state

$$|\Phi'_G\rangle_{345} = \frac{1}{\sqrt{2}}\{|0\rangle_3 |\phi^-\rangle + |1\rangle_3 |\psi^-\rangle_{45}\} \quad (3)$$

as the entanglement channel. Then the total wave function is  $\Gamma_{12345} = |\tau'\rangle_{12} \otimes |\Phi'_G\rangle_{345}$  and Alice performs a three particle joint measurement on particles in her possession (1, 2, 3) in a new measurement basis:  $|\zeta'^{\pm}\rangle = \frac{1}{2}\{(|001\rangle - |111\rangle) \pm (|010\rangle - |100\rangle)\}$  and  $|\eta'^{\pm}\rangle = \frac{1}{2}\{(|000\rangle - |110\rangle) \pm (|011\rangle - |101\rangle)\}$ . After getting the information about measurement result from Alice, Bob performs unitary operations on his particles (4, 5) as given in Table:1 and recovers the teleported state. It may be noted that the table:1 coincides with the table given in [24] except for the changes in labels representing measurement result and the state of Bob's particles (4,5).

## 2.2 Teleportation of a three particle state using five-qubit Brown state

We extend the two-qubit teleportation scheme given above for the teleportation of a three particle state

$$|\psi\rangle_{123} = \{\alpha(|000\rangle - |111\rangle) + \beta(|001\rangle + |110\rangle) + \gamma(|010\rangle + |101\rangle) + \delta(|011\rangle - |100\rangle)\}_{123} \quad (4)$$

where  $\alpha, \beta, \gamma$  and  $\delta$  are unknown coefficients satisfying the relation  $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$ .

The quantum channel is chosen as the maximally entangled five-qubit Brown state[25]. Brown *et.al.*, initially obtained this state via a numerical optimization procedure and later Muralidharan and Panigrahi[26] proposed

a simple theoretical method for the physical realization of this state. The five-qubit Brown state is given by

$$|\Psi\rangle_{45678} = \frac{1}{2}\{|001\rangle|\phi^-\rangle + |010\rangle|\psi^-\rangle + |100\rangle|\phi^+\rangle + |111\rangle|\psi^+\rangle\}_{45678} \quad (5)$$

This can be rewritten in equivalent form in the following order as given in [27].

$$|\Psi\rangle_{67458} = \frac{1}{2}\{|00\rangle|G_{010}\rangle - |01\rangle|G_{111}\rangle + |10\rangle|G_{001}\rangle - |11\rangle|G_{100}\rangle\}_{67458} \quad (6)$$

Here, the three-qubit states defined by

$$|G_{ijk}\rangle = \frac{1}{\sqrt{2}}\{|0\rangle|j\rangle|k\rangle + (-1)^i|1\rangle|j\oplus 1\rangle|k\oplus 1\rangle\} \quad (7)$$

where  $i, j, k$  takes values  $\{0, 1\}$  and  $\oplus$  represents addition modulo 2, are the GHZ[28] states. The equation:4 can be expressed in terms of these states as

$$|\psi\rangle_{123} = \sqrt{2}(\alpha|G_{100}\rangle + \beta|G_{001}\rangle + \gamma|G_{010}\rangle + \delta|G_{111}\rangle)_{123}$$

Alice, now, performs a five-particle joint measurement on her particles (1, 2, 3, 6, 7) in the measurement basis  $\{|\Phi_{1,2\dots 16}\rangle\}$  given below:

$$\begin{aligned} |\Phi_{1,2}\rangle &= \frac{1}{2}\{|G_{010}\rangle|00\rangle - |G_{111}\rangle|01\rangle \pm |G_{001}\rangle|10\rangle \mp |G_{100}\rangle|11\rangle\} \quad (8) \\ |\Phi_{3,4}\rangle &= \frac{1}{2}\{|G_{010}\rangle|00\rangle + |G_{111}\rangle|01\rangle \pm |G_{001}\rangle|10\rangle \pm |G_{100}\rangle|11\rangle\} \\ |\Phi_{5,6}\rangle &= \frac{1}{2}\{|G_{010}\rangle|01\rangle - |G_{111}\rangle|00\rangle \pm |G_{001}\rangle|11\rangle \mp |G_{100}\rangle|10\rangle\} \\ |\Phi_{7,8}\rangle &= \frac{1}{2}\{|G_{010}\rangle|01\rangle + |G_{111}\rangle|00\rangle \pm |G_{001}\rangle|11\rangle \pm |G_{100}\rangle|10\rangle\} \\ |\Phi_{9,10}\rangle &= \frac{1}{2}\{|G_{010}\rangle|10\rangle - |G_{111}\rangle|11\rangle \pm |G_{001}\rangle|00\rangle \mp |G_{100}\rangle|01\rangle\} \\ |\Phi_{11,12}\rangle &= \frac{1}{2}\{|G_{010}\rangle|10\rangle + |G_{111}\rangle|11\rangle \pm |G_{001}\rangle|00\rangle \pm |G_{100}\rangle|01\rangle\} \\ |\Phi_{13,14}\rangle &= \frac{1}{2}\{|G_{010}\rangle|11\rangle - |G_{111}\rangle|10\rangle \pm |G_{001}\rangle|01\rangle \mp |G_{100}\rangle|00\rangle\} \\ |\Phi_{15,16}\rangle &= \frac{1}{2}\{|G_{010}\rangle|11\rangle + |G_{111}\rangle|10\rangle \pm |G_{001}\rangle|01\rangle \pm |G_{100}\rangle|00\rangle\} \end{aligned}$$

Alice then communicates her measurement result to Bob via four bits of classical information. In order to recover the teleported state, appropriate unitary operations are performed by Bob on his particles (4, 5, 8) as given in table: 2.

In a similar fashion another three-qubit state  $|\psi'\rangle_{123} = \{\alpha(|000\rangle + |111\rangle) + \beta(|001\rangle - |110\rangle) + \gamma(|010\rangle - |101\rangle) + \delta(|011\rangle + |100\rangle)\}_{123}$  needed in the DSQC protocol proposed in section:3 can be teleported. The appropriate five-qubit

Alice's result and Classical information	State of qubits (6,7,8)	Unitary operation
$ \Phi_1\rangle 0000$	$\sqrt{2}(\alpha  G_{100}\rangle + \beta  G_{001}\rangle + \gamma  G_{010}\rangle + \delta  G_{111}\rangle)$	$I \otimes I \otimes I$
$ \Phi_2\rangle 0001$	$\sqrt{2}(-\alpha  G_{100}\rangle - \beta  G_{001}\rangle + \gamma  G_{010}\rangle + \delta  G_{111}\rangle)$	$\sigma_z \otimes \sigma_z \otimes I$
$ \Phi_3\rangle 0010$	$\sqrt{2}(-\alpha  G_{100}\rangle + \beta  G_{001}\rangle + \gamma  G_{010}\rangle - \delta  G_{111}\rangle)$	$\sigma_z \otimes I \otimes \sigma_z$
$ \Phi_4\rangle 0011$	$\sqrt{2}(\alpha  G_{100}\rangle - \beta  G_{001}\rangle + \gamma  G_{010}\rangle - \delta  G_{111}\rangle)$	$I \otimes \sigma_z \otimes \sigma_z$
$ \Phi_5\rangle 0100$	$\sqrt{2}(-\alpha  G_{001}\rangle - \beta  G_{100}\rangle - \gamma  G_{111}\rangle - \delta  G_{010}\rangle)$	$I \otimes I \otimes \sigma_x$
$ \Phi_6\rangle 0101$	$\sqrt{2}(\alpha  G_{001}\rangle + \beta  G_{100}\rangle - \gamma  G_{111}\rangle - \delta  G_{010}\rangle)$	$\sigma_z \otimes \sigma_z \otimes \sigma_x$
$ \Phi_7\rangle 0110$	$\sqrt{2}(\alpha  G_{001}\rangle - \beta  G_{100}\rangle - \gamma  G_{111}\rangle + \delta  G_{010}\rangle)$	$\sigma_z \otimes I \otimes i\sigma_y$
$ \Phi_8\rangle 0111$	$\sqrt{2}(-\alpha  G_{001}\rangle + \beta  G_{100}\rangle - \gamma  G_{111}\rangle + \delta  G_{010}\rangle)$	$I \otimes \sigma_z \otimes i\sigma_y$
$ \Phi_9\rangle 1000$	$\sqrt{2}(\alpha  G_{111}\rangle + \beta  G_{010}\rangle + \gamma  G_{001}\rangle + \delta  G_{100}\rangle)$	$I \otimes \sigma_x \otimes I$
$ \Phi_{10}\rangle 1001$	$\sqrt{2}(-\alpha  G_{111}\rangle - \beta  G_{010}\rangle + \gamma  G_{001}\rangle + \delta  G_{100}\rangle)$	$\sigma_z \otimes i\sigma_y \otimes I$
$ \Phi_{11}\rangle 1010$	$\sqrt{2}(-\alpha  G_{111}\rangle + \beta  G_{010}\rangle + \gamma  G_{001}\rangle - \delta  G_{100}\rangle)$	$\sigma_z \otimes \sigma_x \otimes \sigma_z$
$ \Phi_{12}\rangle 1011$	$\sqrt{2}(\alpha  G_{111}\rangle - \beta  G_{010}\rangle + \gamma  G_{001}\rangle - \delta  G_{100}\rangle)$	$I \otimes i\sigma_y \otimes \sigma_z$
$ \Phi_{13}\rangle 1100$	$\sqrt{2}(-\alpha  G_{010}\rangle - \beta  G_{111}\rangle - \gamma  G_{100}\rangle - \delta  G_{001}\rangle)$	$\sigma_x \otimes I \otimes I$
$ \Phi_{14}\rangle 1101$	$\sqrt{2}(\alpha  G_{010}\rangle + \beta  G_{111}\rangle - \gamma  G_{100}\rangle - \delta  G_{001}\rangle)$	$i\sigma_y \otimes \sigma_z \otimes I$
$ \Phi_{15}\rangle 1110$	$\sqrt{2}(\alpha  G_{010}\rangle - \beta  G_{111}\rangle - \gamma  G_{100}\rangle + \delta  G_{001}\rangle)$	$i\sigma_y \otimes I \otimes \sigma_z$
$ \Phi_{16}\rangle 1111$	$\sqrt{2}(-\alpha  G_{010}\rangle + \beta  G_{111}\rangle - \gamma  G_{100}\rangle + \delta  G_{001}\rangle)$	$\sigma_x \otimes \sigma_z \otimes \sigma_z$

**Table 2** Alice's measurement results and corresponding operations performed by Bob

channel  $|\Psi'\rangle$  and the measurement basis  $|\Phi'_{1,2,\dots,16}\rangle$  are obtained by making the replacements ( $|G_{100}\rangle \rightarrow |G_{000}\rangle$ ,  $|G_{001}\rangle \rightarrow |G_{101}\rangle$ ,  $|G_{010}\rangle \rightarrow |G_{110}\rangle$  and  $|G_{111}\rangle \rightarrow |G_{011}\rangle$ ) in equations: 6 and 8 respectively. *i.e.*,  $|\Psi\rangle_{67458}$  changes to  $|\Psi'\rangle_{67458}$  and  $|\Phi_{1,2,\dots,16}\rangle$  changes to  $|\Phi'_{1,2,\dots,16}\rangle$ . The recovery operations to be performed by Bob, corresponding to Alice's classical information, are same as in table:2 with the labels representing measurement results and state of Bob's particles changed accordingly.

### 3 New Deterministic Secure Quantum Communication protocols

Applying the teleportation schemes in the preceding section, we propose two novel DSQC protocols for communication of information in units of 2 bits or 3 bits.

#### 3.1 DSQC protocol using GHZ-like state

Our protocol for sending information in units of 2 bits makes use of the GHZ-like state as the quantum channel and involves the following steps.

**1. Preparation of GHZ-like states:** The sender, Alice, prepares copies of two orthogonal GHZ-like states  $|\Phi_G\rangle$  and  $|\Phi'_G\rangle$ , their

number depending on the size of secret messages to be sent. The ordered sequence of three particle GHZ-like state  $|\Phi_G\rangle_{345}$  is given by  $[(P_3^1, P_4^1, P_5^1), (P_3^2, P_4^2, P_5^2), \dots, (P_3^n, P_4^n, P_5^n)]$  and the sequence of  $|\Phi'_G\rangle_{345}$  is given by  $[(P_3^{\prime 1}, P_4^{\prime 1}, P_5^{\prime 1}), (P_3^{\prime 2}, P_4^{\prime 2}, P_5^{\prime 2}), \dots, (P_3^{\prime n}, P_4^{\prime n}, P_5^{\prime n})]$ . The entangled states in each of these sets are given a unique number, with the superscripts representing the order in the sequence and the subscript representing particle labels.

**2. Splitting Home Block and Travel Block:** The block of all the first particles in the sequences  $|\Phi_G\rangle$  and  $|\Phi'_G\rangle$ ,  $[P_3^1, P_3^{\prime 1}, P_3^2, P_3^{\prime 2}, \dots, P_3^n, P_3^{\prime n}]$  are kept at sender's location and we refer to them as the home block. The block of all second and third particles from both sequences  $[(P_4^1, P_5^1), (P_4^2, P_5^2), \dots, (P_4^n, P_5^n)]$ , and  $[(P_4^{\prime 1}, P_5^{\prime 1}), (P_4^{\prime 2}, P_5^{\prime 2}), \dots, (P_4^{\prime n}, P_5^{\prime n})]$  are referred to as the travel block.

**3. Arrangement of particles for Error checking and Message transmission:** Alice selects  $\frac{N}{2}$  channels for error checking, where N represents the even number of secret information to be transmitted. The order and position of corresponding particles in the home block are noted by Alice and the leftover particles gets reserved for message transmission. In order to send information bits 00 or 01 Alice uses the channel  $|\Phi_G\rangle$  and for sending 10 or 11 she uses  $|\Phi'_G\rangle$ . Alice arranges all the particles in both travel and home block according to the particular message to be sent. Alice sends the travel block to Bob and retains the home block with herself.

**4. Error checking:** On receiving the travel block, Bob sends a signal back to Alice confirming the reception. Alice, then, initiates the error checking process by performing single particle measurement on her particles initially chosen for error checking. After the measurement, Alice announces the positions of particles that were chosen for error checking. Bob performs either Bell measurement or single particle measurement on the corresponding particles in his possession according to the information received from Alice and he, in turn, announces his measurement outcomes. Alice, now, checks for correlations in the measurement results by comparing it with equations: 1 and 3. If the error rate is high they abort the communication. Otherwise they proceed to the next step.

**5. Teleportation:** In the scheme of communicating in units of two bit the users make an agreement to encode the bits 00, 01, 10 and 11 in the Bell states  $|\psi^+\rangle$ ,  $|\phi^+\rangle$ ,  $|\psi^-\rangle$ , and  $|\phi^-\rangle$  respectively. With appropriate choice the coefficients  $\alpha$  and  $\beta$  in  $|\tau\rangle_{12}$  and  $|\tau'\rangle_{12}$  (effectively reducing these to the Bell states) information can be conveyed securely using the teleportation scheme described earlier. For instance, if Alice wants to send 10 or 11, she choose the state  $|\tau'\rangle_{12}$  and channel  $|\Phi'_G\rangle_{345}$ . The coefficient  $\alpha$  or  $\beta$  is made zero depending on whether the information is 11 or 10. Alice, then, follows the teleportation procedure given in section:2. Finally, Bob performs a Bell measurement to read out the secret information.

Our protocol works efficiently when the sender prepares the secret message beforehand. For example, consider the case where Alice wants to communicate a four bit secret information ‘0011’ to Bob. She prepares two channels,  $|\Phi_G\rangle$ ,  $|\Phi'_G\rangle$  and arrange them in the same order as that of the messages *i.e.*,  $|\Phi_G\rangle$  taken first and  $|\Phi'_G\rangle$  second. The steps 2, 3 and 4 are followed. Finally, the state  $|\tau\rangle_{12}$  with coefficient  $\beta = 0$  or  $|\psi^+\rangle$  and  $|\tau'\rangle_{12}$  with  $\alpha = 0$  or  $|\phi^-\rangle$ , corresponding to information 00 and 11, are teleported using the appropriate schemes given in section:2. In this scheme, it may be noted that the participants (receiver and eavesdropper) are not aware of the different entanglement channels and measurement basis employed by the sender. The receiver performs unitary operations based on the classical information obtained from sender and read out the secret messages by performing Bell measurements.

### 3.2 DSQC protocol using five-qubit maximally entangled Brown states.

By a straight foreword generalization of the procedure given above, Alice can transmit information in units of 3 bits by using the five-qubit Brown states as quantum channels and employing the teleportation scheme given in section:2.2. Before transmitting messages Alice and Bob agree that the states  $|G_{ijk}\rangle$ , where  $i, j, k$  take values  $\{0, 1\}$  correspond to 8 different messages (000, 001, 010, 011, 100, 101, 110, 111) respectively. For instance, if Alice wants to transmit a six bit secret information ‘100 110’ to Bob, she chooses two channels  $|\Psi\rangle$  and  $|\Psi'\rangle$  and then arrange them in the order of messages *i.e.*,  $|\Psi\rangle$  taken first and  $|\Psi'\rangle$  second.  $|G_{100}\rangle$  and  $|G_{110}\rangle$  are the corresponding messages and they are prepared by setting the coefficient  $\alpha = 1$ , other coefficients to zero in equation:4 and  $\gamma = 1$ , other coefficients to zero in its conjugate equation  $|\psi'\rangle$  respectively. Alice follows the steps discussed in the previous section and finally teleport the states as given in section:2. Here, 8 bits of classical information is used up in this process.

## 4 Security Analysis

In teleportation-based DSQC protocols, the information carrying qubits do not travel and thus the security of information solely depends on the security of quantum channel shared between Alice and Bob. The most common attack strategies of an eavesdropper for DSQC using GHZ-like is discussed below.

**Measure and resend Attack:** In this type of attack, the eavesdropper Eve may capture the travel block from Alice and measure all the qubits in either Bell basis or perform single particle measurement on them and resend it to Bob. After this process, neither Alice-Bob pair nor any Alice-Eve pair shares entanglement. Therefore it is impossible for Eve to gain any useful information from Alice’s side using this type of attack, as long as she employs teleportation procedure to transmit the secret information. Further, in the first

error checking process the correlation results itself will reveal the presence of Eve and the communication is abandoned.

**Intercept and resend Attack:** The eavesdropper may try to intercept the travel block and replace it with particles (6, 7) which is a part of random GHZ-like states prepared by Eve and resend them to Bob. Now, Eve establishes an entanglement relation with Alice's particle and may receive the teleported message. But, on checking the correlations given in equations: 1 and 3, Alice detects the presence of Eve. If the error rate is high, she informs Bob to stop the entire process.

**Entanglement Attack:** In this attack[7][12], the eavesdropper may capture the travel block and entangle the ancilla particles (6,7) in the state  $|00\rangle_{67}^n$  with the  $n^{th}$  order of particles (4,5) in the travel block, where  $n$  represents the order of entangled GHZ-like particles in the travel block. Eve uses CNOT gate to entangle ancilla qubits with travel block. The particles (4,5) of GHZ-like state acts as control bits and (6,7) as target bits respectively. This changes the total state to

$$\begin{aligned} |\Psi\rangle_{34567} &= CNOT(5; 7)CNOT(4; 6) |\Phi_G\rangle_{345} \otimes |00\rangle_{67} \quad (9) \\ &= \frac{1}{2} [|0\rangle_3 (|\phi^+\rangle_{45} |\phi^+\rangle_{67} + |\phi^-\rangle_{45} |\phi^-\rangle_{67}) \\ &\quad + |1\rangle_3 (|\psi^+\rangle_{45} |\psi^+\rangle_{67} + |\psi^-\rangle_{45} |\psi^-\rangle_{67})] \end{aligned}$$

The same operation changes the state of  $|\Phi'_G\rangle_{345}$  to

$$\begin{aligned} |\Psi'\rangle_{34567} &= \frac{1}{2} [|0\rangle_3 (|\phi^+\rangle_{45} |\phi^-\rangle_{67} + |\phi^-\rangle_{45} |\phi^+\rangle_{67}) \quad (10) \\ &\quad + |1\rangle_3 (|\psi^+\rangle_{45} |\psi^-\rangle_{67} + |\psi^-\rangle_{45} |\psi^+\rangle_{67})] \end{aligned}$$

Equation: 9 shows, the state of (4,5) and (6,7) are correlated and equation: 10 shows that they are anti-correlated. By entangling ancillary particles to travel block Eve gets access to Alice's information carrying qubits. But, during the error checking process, Alice can detect the presence of Eve 50% of times by checking relations given in equations 1 and 3. Alice informs Bob to continue the process if the error rate is less than 50% and otherwise to abandon. In the case where Eve goes undetected, the probability of secret message getting leaked is still 50% due to the superposition of states existing between particles (4, 5, 6, 7).

## 5 Efficiency Analysis

In 2000, Cabello[29] put forward a simple definition to find the qubit efficiency of a given protocol. Here the qubit efficiency  $\eta_2$  depends on the ratio of the number of messages transmitted  $b_s$ , to the sum of total number of classical

DSQC protocol	$\eta_2 = \text{Efficiency}(\%)$		Entanglement Channel
	Without decoy qubits [29]	With decoy [30] qubits	
YZ04 [7]	25.00	16.67	Bell pairs
CS06 [8]	16.67	09.52	W-State
DXG08 [9]	20.00	12.50	W-State
XGC09 [10]	30.00	18.75	Six-particle
QCY13 [11]	25.00	16.67	Four-qubit Cluster
PP-1	40.00	25.00	GHZ-like State
PP-2	33.33	21.43	Five-Qubit Brown

**Table 3** Comparing efficiency of Teleportation-based DSQC protocols. PP- Proposed protocol

bits  $b_t$  utilized to decode the message and the total number of qubits  $q_t$  used in the protocol. Note that, here the decoy qubits and the classical bits used for eavesdrop checking are not included to calculate the efficiency.

$$\eta_2 = \frac{b_s}{q_t + b_t} \quad (11)$$

In 2012 Banerjee and Pathak[30] proposed a modified measure of efficiency which considers the decoy qubits used for eavesdrop checking. Since the number of classical bits used in eavesdrop checking varies linearly with number of decoy qubits used, inclusion of the number of decoy qubits in efficiency calculations seems more appropriate. Using the definition of [30](including decoy qubits) and the one by [29] (excluding decoy qubits), we have calculated the qubit efficiency of different teleportation based protocols [7][8][9][10][11] and have tabulated them in table: 3. We find that the qubit efficiency of these protocols are  $\leq 20\%$ . In these calculations we have taken the number of decoy states as equal to the number of transmitted qubits following suggestion in [30]

In our protocol with GHZ-like state as quantum channel, to communicate a four bit message ( $b_s = 4$ ) we require two GHZ-like channels or 6 qubits *i.e.*, ( $q_t = 6$ ) and four bits of classical information ( $b_t = 4$ ). As the classical signal from Bob after the receipt of travel block becomes asymptotically (N large) insignificant, it is not included in  $b_t$ . By definition of [29], this leads to a qubit efficiency of  $\eta = 40\%$ . But, in order to ensure security, we used  $\frac{N}{2}$  GHZ-like channels for eavesdrop checking in the protocol. In our example, we used two GHZ-like states for eavesdrop checking. Hence the total number of qubits consumed becomes ( $q_t = 12$ ). The efficiency calculated according to [30] gives  $\eta = 25\%$ . In the case of sending a six bit message ( $b_s = 6$ ) given in section:3.2, using two five qubit channels ( $q_t = 10$ ) and 8 bits of classical information ( $b_t = 8$ ) the qubit efficiency is  $\eta = 33.33\%$  according to [29]. Using the definition of [30], it becomes 21.43%.

## 6 Conclusion

We have proposed an extension of Nandi-Mazumdar scheme for teleportation of a special class of two particle states and have also developed a method for the teleportation of special class of three-particle state using five-qubit Brown state as entanglement channel. The different teleportation schemes introduced here are then combined effectively to build novel DSQC protocols. These works provide new insight into the way in which teleportation can be used for efficient DSQC protocols. The comparison of qubit efficiency shows that the proposed protocol is better than the other DSQC protocols based on teleportation [7][8][9][10][11]. This increase in efficiency results from the sender's capability of choosing different channels and measurement basis independently. Even though our protocol requires the sender to prepare the secret message beforehand, the efficiency factor gives it an edge over other existing protocols. We suggest that the efficiency of using five-qubit Brown state can be further increased if the sender employs more entanglement channels and appropriate measurement basis.

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## References

1. C. H. Bennett and G. Brassard, in Proceedings of IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India - IEEE, New York, p. 175 (1984).
2. A.K. Ekert, Quantum cryptography based on Bell's theorem, Phys. Rev. Lett. 67, 661 (1991).
3. G.L. Long, X.S. Liu, Theoretically efficient high-capacity quantum-key-distribution scheme, Phys. Rev. A 65, 032302 (2002).
4. Long G.L, Deng F.G, Wang C, Li X.H, Wen K. and Wang W.Y, Quantum secure direct communication and deterministic secure quantum communication, Front. Phys. China, 2(3), 251-272 (2007).
5. K Shimizu and N Imoto, Communication channels secured from eavesdropping via transmission of photonic Bell states, Phys. Rev. A, 60, 157 (1999)
6. Beige A, Englert B.G, Kurtsiefer C and Weinfurter H, Secure Communication with a Publicly Known Key. Acta. Phys. Pol. A. 101(3), 357-368 (2002)
7. Yan F, Zhang X, A scheme for secure direct communication using EPR pairs and teleportation. Euro. Phys. J. B 41, 75-78 (2004)
8. H.J Cao and H.S Song, Quantum secure direct communication scheme using a W state and teleportation, Phys. Scr. 74, 572-575 (2006).
9. Dong L, Xiu X H and Gao Y J, Quantum secure direct communication with W-state, Commun. Theor. Phys. 49, 1495- 1498 (2008).
10. Xiu X M, Gao Y J and Chi F, Quantum secure direct communication using six particle maximally entangled states and Teleportation, Commun. Theor. Phys. 51, 429- 432 (2009)

11. Qin N Z, Cui C L and Yuan H L, Quantum secure direct communication based on Four-qubit Cluster State, *Int J Theor Phys* 52, 22–27 (2013).
12. T. Gao, F. L. Yan, and Z. X. Wang, Quantum secure conditional direct communication via EPR pairs, *Int. J. Mod. Phys. C* 16, 1293 (2005).
13. Cao H.J, Song H.S, Quantum Secure Direct Communication with W State, *Chin. Phys. Lett.* 23, 290 (2006).
14. Q.N. Zhang, C.C Li, Y.H Li and Y.Y Nie, Quantum Secure Direct Communication Based on Four-Qubit Cluster States, *Int J Theor Phys* 52, 22–27 (2013).
15. T Gao, Yan F. L and Wang Z.X, Deterministic secure direct communication using GHZ states and swapping quantum entanglement. *J. Phys. A: Math. Gen.* 38(25), 5761 (2005).
16. Man Z, Xia, Y. and An, N. Quantum secure direct communication by using GHZ states and entanglement swapping. *J Phys B: At Mol Opt Phys.* 39, 3855–3863 (2006)
17. X.M Xiu, H.K Dong, L Dong, Y.J Gao, F Chi, Deterministic secure quantum communication using four-particle genuine entangled state and entanglement swapping, *Opt. Commun.* 282, 2457 (2009).
18. T Gao, F.L Yan, Z.X Wang, Quantum secure direct communication by Einstein-Podolsky-Rosen pairs and entanglement swapping, *arXiv:quant-ph/0406083v1* (2004)
19. Man Z.X, Zhang Z.J, L Yong, Deterministic secure direct communication by using swapping quantum entanglement and local unitary operations, *Chin. Phys. Lett.* 22, 18 (2005).
20. A.D Zhu, Y Xia, Q.B Fan, and S Zhang, Secure direct communication based on secret transmitting order of particles, *Phys. Rev. A*, 73, 022338 (2006).
21. X.H Li, F.G Deng, and H.Y Zhou, Improving the security of secure direct communication based on the secret transmitting order of particles, *Phys. Rev. A*, 74, 054302 (2006).
22. Shima Hassanpour, Monireh Houshmand, Efficient controlled quantum secure direct communication based on GHZ-like states, *Quantum Inf Process*, 14, 739–753 (2015).
23. X.H Li, F.G Deng, C.Y Li, Y.J Liang, P Zhou and H.Y Zhou, Deterministic Secure Quantum Communication Without Maximally Entangled States, *J. Korean Phys. Soc.*, 49, 1354 (2006)
24. Kaushik Nandi and Chandan Mazumdar, Quantum Teleportation of a Two Qubit State Using GHZ-Like State, *Int J Theor Phys*, 53,1322–1324 (2014).
25. Iain D K Brown, Susan Stepney, Anthony Sudbery and Samuel L Braunstein, Searching for highly entangled multi-qubit states, *J. Phys. A*, 38, 1119(2005)
26. Sreraman Muralidharan and Prasanta K. Panigrahi, Perfect teleportation, quantum-state sharing, and superdense coding through a genuinely entangled five-qubit state, *Physical Review A*, 77, 032321 (2008)
27. Lin S, Gao F, Liu X F, Quantum Secure Direct Communication with Five-Qubit Entangled State, *Chin Phys Lett*, 28, 030302(2011)
28. D. M. Greenberger, M. A. Horne, A. Shimony, and A. Zeilinger, Bell's Theorem without inequalities, *Am. J. Phys.* 58, 1131 (1990).
29. A Cabello, Quantum Key Distribution in the Holevo Limit, *Phys. Rev. Lett.* 85, 5635 (2000).
30. Anindita Banerjee and Anirban Pathak, Maximally efficient protocols for direct secure quantum communication, *Physics Letters A* 376, 2944–2950 (2012)