

# Quantum Conference

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## Abstract

A notion of quantum conference is introduced in analogy with the usual notion of a conference that happens frequently in today's world. Quantum conference is defined as a multiparty secure communication task that allows each party to communicate their messages simultaneously to all other parties in a secure manner using quantum resources. Two efficient and secure protocols for quantum conference have been proposed. The security and efficiency of the proposed protocols have been analyzed critically. It is shown that the proposed protocols can be realized using a large number of entangled states and group of operators. Further, it is shown that the proposed schemes can be easily reduced to protocol for multiparty quantum key distribution and some earlier proposed schemes of quantum conference, where the notion of quantum conference was different.

Keywords: quantum conference, quantum cryptography, secure quantum communication, multiparty quantum communication.

## 1 Introduction

In 1984, an unconditionally secure key distribution protocol using quantum resources was proposed by Bennett and Brassard [1]. The scheme, which is now known as BB84 protocol drew considerable attention of the cryptography community by its own merit as it offered unconditional security, which was unachievable by any classical protocol of key distribution. However, the relevance of BB84 quantum key distribution (QKD) protocol and a set of other schemes of QKD were actually established very strongly in 1994, when the seminal work of Shor [2] established that RSA [3] and a few other schemes of classical cryptography [4] would not remain secure if a scalable quantum computer is built. The BB84 protocol, not only established the possibility of obtaining unconditional security, but also manifested enormous power of quantum resources that had been maneuvered since then. Specifically, this attempt at the unconditional security of QKD was followed by a set of protocols for the same task [5–7]. Interestingly, the beautiful applications of quantum mechanics in secure communication did not remain restricted to key distribution. In fact, it was realized soon that the messages can be sent in a secure manner without preparing a prior key [8]. Exploiting this idea various such schemes were proposed which fall under the category of secure direct quantum communication ([8–12] and references therein).

The schemes for secure direct quantum communication can be categorized into two classes on the basis of additional classical communication required by the receiver (Bob) to decode each bit of the transmitted message- (i) quantum secure direct communication (QSDC) [8–10] and (ii) deterministic secure quantum communication (DSQC) [11]. In the former, Bob does not require an additional classical communication to decode the message, while such a classical communication is involved in the latter (see [13] for review). It is worth noting that in a scheme of QSDC/DSQC meaningful information flows in one direction as it only allows Alice to send a message to Bob in an unconditionally secure manner using quantum resources and without generation of a key. However, in our daily life, we often require two way communication (say, when we speak on a telephone). Interestingly, a modification of one of the first few QSDC schemes (i.e., ping-pong scheme [8]) led to a new type of protocol that allows both Alice and Bob to communicate simultaneously using the same quantum channel. This scheme for simultaneous two way communication was first proposed by Ba An [14] and is known as quantum dialogue (QD). Due to its similarity with the task performed by telephones, a scheme for QD are also referred as quantum telephone [?, 16]

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or quantum conversation<sup>1</sup> [17] scheme, but in what follows, we will refer to them as QD. Due to its practical relevance, schemes of QD received much attention and several new schemes of QD have been proposed in the last decade [18–21]. However, all these schemes of QD, and also the schemes of QSDC and DSQC, mentioned here are restricted to the two-party scenario. This observation led to two simple questions- (i) Do we need a multiparty QD for any practical purpose? and (ii) If answer of the previous question is yes, can we construct such a scheme? It is easy for us (specially for the readers of this paper and the authors of the similar papers who often participate in conferences and meet as members of various committees) to recognize that conferences and meetings provide examples of situation where multiparty dialogue happens. Specifically, in a conference a large number of participants can exchange their thoughts (inputs, which may be viewed as classical information). Although, usually participants of the conference/meeting are located in one place, but with the advent of new technologies, tele-conferences, webinar, and similar ideas that allow remotely located users to get involved in multiparty dialogue, are becoming extremely popular. For the participants of such a conference or meeting that allows users to be located at different places, desirable characteristics of the scheme for the conference should be as follows- (A) A participant must be able to communicate directly with all other participants, or in other words, every participant must be able to listen the talk/opinion delivered by every speaker as it happens in a real conference. (B) A participant should not be able to communicate different opinion/message to different users or user groups. (C) Illegitimate users or unauthorized parties (say those who have not paid conference registration fees) will not be able to follow the proceedings of the conference. It is obvious that criterion (C) requires security and a secure scheme for multiparty quantum dialogue satisfying (A)-(C) is essential for today's society. We refer to such a scheme for multiparty secure communication that satisfies (A)-(C) as a scheme for quantum conference (QC) because of its analogy with the traditional conferences (specially with the tele-conferences). The analogy between the communication task performed here and the traditional conference can be made clearer by noting that Wikipedia defines conference as “a conference is a meeting of people who "confer" about a topic” [22]. Similarly, Oxford dictionary describes a conference as “a linking of several telephones or computers, so that each user may communicate with the others simultaneously” [23]. This is exactly the task that the proposed protocol for QC is aimed to perform using quantum resources and in a secure manner. Thus, QC is simply a conference, which is an  $n$ -party communication, where each participant can communicate his/her inputs (classical information) using quantum resources to remaining  $(n - 1)$  participants. However, it should be made clear that it is neither a multi-channel QSDC nor a multi-channel QD scheme. To be precise, one may assume that each participant maintains private quantum channels with all other participants and uses those to communicate his/her input to others via QSDC or QD. This is against the idea of a conference, as in this arrangement, a participant may send different information/opinion to different participants, in violation of Criterion (B) listed above. The fact that to the best of our knowledge, no such scheme for multiparty secure quantum communication exists has motivated us to introduce the notion of QC and to aim to design a scheme for the same.

Here it would be apt to note that although no scheme for QC is yet proposed, various schemes for other multiparty quantum communication tasks have already been proposed. For example, quantum schemes for voting [24], auction [25, 26], and e-commerce [27] are necessarily expected to be multiparty quantum communication schemes. Interestingly, there are a few schemes for all these tasks proposed in the past ([24–27] and references therein). Another recently discussed multiparty task is quantum key agreement (QKA) ([28] and references therein), where the final key is generated by the contribution of all the parties involved, and a single or a few parties can not decide the final key. For instance, a multiparty QKA scheme [28] was proposed in the recent past, in which encoded qubits travel in a circular manner among all the parties. In fact, most of these multiparty quantum communication schemes, except QKA, can be intrinsically viewed as a (many) sender(s) sending some useful information in a secure manner to a (many) receiver(s) under the control of a third party. Further, all these schemes can be broadly categorized as secure multiparty quantum communication and secure multiparty quantum computation. Though the line between the two is very faint to distinguish and categorize a scheme among one of them, QKA and e-commerce may be considered in the former, while voting and auction fall under the latter. Some efforts have also been made to introduce a notion of QC as a multiparty quantum communication task. However, earlier ideas of QC can be viewed as special cases of the notion of QC presented here and they are not sufficient to perform a conference as defined above in analogy with the definition provided in Oxford dictionary and other sources.

Bose, Vedral and Knight [29] proposed a generalized entanglement-swapping-based scheme for multiparty quantum communication that led to a set of quantum communication schemes related to QC, viz., cryptographic conference [29], conference key agreement and conference call [30], and a scheme where many senders send their messages to single receiver via generalized superdense coding [29]. In cryptographic conference, all parties share a multipartite entangled state. They perform measurement in the computational or diagonal basis, and the results of those measurements in which the bases chosen by all the users coincide are used to establish the secret key which will be known to all the users within the group. A similar notion of conference key agreement was used in [30], where a generalized notion of dense coding was used. Clearly the notion of conference is weaker here, and in our version of conference such keys can be distributed easily if all the users communicate random bits instead of meaningful messages. Recent success of designing the above mentioned schemes for

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<sup>1</sup>It may be noted that in an ideal scheme of QD, information encoded by two parties exist simultaneously in a channel, but in the protocol for quantum conversation introduced in [17], it was not the case. However, the communication task in hand was equivalent.

multiparty quantum communication further motivated us to look for a scheme for QC.

A two party analogue of QC can be considered as QD, where both parties can communicate simultaneously. The group theoretic structure of Ba-An-type QD schemes has been discussed in Ref. [31]. The group theoretic structure discussed in [28, 31, 32] will be exploited here to introduce the concept of QC. Further, an asymmetric counterpart of the Ba-An-type QD scheme is proposed in the recent past [32]. Following which we will also introduce and briefly discuss an asymmetric QC (AQC), where all the parties involved need not to send an equal amount of information. With the recent interest of quantum communication community on quantum internet [33, 34] and experimental realization of multiparty quantum communication schemes [35], the motivation for introducing a QC or AQC scheme can be established.

Remaining part of the paper is organized as follows. Sec. 2 is dedicated to a brief review of QD and the group theoretic approach of QD for the sake of completeness of the paper, which has been used in the forthcoming sections to develop the idea of QC. Two general schemes for the task of QC have been introduced in Sec. 3. In the next section, we have considered a few specific examples of both these schemes. The feasibility of an AQC scheme has also been discussed in Sec. 4. Finally, the security and efficiency of the proposed schemes have been discussed in Sec. 5 before concluding the paper in Sec. 6.

## 2 Ba An protocol of QD and its generalization using modified Pauli group

It would be relevant to mention that some of the present authors had presented the general structure of QD protocols in [31] and established that the set of unitary operators used by Alice and Bob must form a group under multiplication. The group structure has also been found to be suitable for the asymmetric QD schemes [32], where Alice and Bob use encoding operations from different subgroups of a modified Pauli group, like  $G_1 = \{I, X, iY, Z\}$ . This particular Abelian group ( $G_1$ ) is of order 4 under multiplication and is called a modified Pauli group as we neglect the global phase in the product of any two elements of this group, which is consistent with the quantum mechanics (for detail see [31, 32]). The generalized group  $G_n$  can be formed by  $n$ -fold tensor products of  $G_1$ , i.e.,  $G_n = G_1^{\otimes n}$ . In the original QD protocol [14], the encoding is done by Alice and Bob, respectively, using the same set of operations  $\{U_i\}$  from the modified Pauli group  $G_1$ . The entire scheme of Ba An [14] can be summed up in the formula  $|\psi_j\rangle_{final} = U_B U_A |\psi_i\rangle_{initial}$  :  $\langle\psi_i|\psi_j\rangle_{final} = \delta_{i,j}$ , where  $|\psi_k\rangle$  are the Bell states. It is required that all the possible final states obtained after Alice's and Bob's encoding operations should remain orthonormal to each other and also with the initial state. Once the initial and final states are known to both the legitimate users, they can exploit knowledge of their own encoding operation to extract each other's message.

Interestingly, Alice and Bob encode information with the same operators, say,  $I$  for 00,  $X$  for 01,  $iY$  for 10, and  $Z$  for 11. In this scenario, Alice obtains a unique bijective mapping from the composite encoding of Alice and Bob ( $U_B.U_A$ ) to Bob's operation ( $U_B$ ) using her unitary operation ( $U_A$ ). This is obvious where there are only 2 parties, we may ask, is it possible to extend this scheme for QD to design a scheme for multiparty conference? Let us examine two cases with 3 parties: in Case 1: when all the parties encode the same bits say, 00 i.e., they apply  $U_{C_I}$ ,  $U_{B_I}$  and  $U_{A_I}$ ; and in Case 2: when one of them encodes the same bits used in Case 1, i.e., 00 and other two will encode the similar bits but other than 00, say 01 (or 10, 11), i.e., they apply  $U_{C_X}$ ,  $U_{B_X}$  and  $U_{A_I}$ , respectively. In these two cases, the resultant state is always the same as what was prepared initially, and none of the parties can deterministically conclude each others encoding. In fact, there will be many such cases, hence, Ba An's original protocol for QD cannot be generalized directly to design a scheme for multiparty conference.

To design a scheme for QC, we will use the idea of disjoint subgroups introduced by some of the present authors in the recent past [28]. Disjoint subgroups refer to subgroups, say  $g_i$  and  $g_j$ , of a group  $G_n$  such that they satisfy  $g_i \cap g_j = \{I\}$ . Thus, except Identity  $g_i$  and  $g_j$  do not contain any common element. The modified Pauli group  $G_1$  has 3 mutually disjoint subgroups:  $g_1 = \{I, X\}$ ,  $g_2 = \{I, iY\}$  and  $g_3 = \{I, Z\}$ . Whenever there are more than two parties, we can encode using disjoint subgroups of operators, i.e., each party may be allowed to encode with a unique disjoint subgroup. For example, if Alice, Bob and Charlie want to set up a QC among them, then Alice can encode using  $g_1$ , Bob can encode using  $g_2$  and Charlie can encode using  $g_3$ . The use of disjoint subgroups circumvents the limitations of the original two-party QD scheme and provides a unique mapping required for multiparty conversation.

In what follows, we have proposed two protocols to accomplish the task of a QC scheme.

## 3 Quantum conference

Here, we have designed two multiparty quantum communication schemes where prior generation of key is not required. These schemes may be used for QC, i.e., for multiparty communication of meaningful information among the users. Additionally, it is easy to observe that these schemes naturally reduce to the schemes for multiparty key distribution if the parties send random bits instead of meaningful messages.

### 3.1 Protocol 1: Multiparty QSDC scheme for QC

Let us start with the simplest case, where  $(N - 1)$  parties send their message to  $N$ th party. This can be thought of as a multiparty QSDC. Suppose all the parties decide to encode or communicate  $k$ -bit classical messages. In this case, each user would require a subgroup of operators with at least  $2^k$  operators. In other words, each party would need at least a subgroup  $g_i$  of order  $2^k$  of a group  $\mathcal{G}$ . Here, we would like to propose one such multiparty QSDC scheme.

**Step 1.1** First party Alice be given one subgroup  $g_A = \{A_1, A_2, \dots, A_{2^k}\}$  to encode her  $k$ -bit information. Similarly, other parties (say Bob and Charlie) can encode using subgroups  $g_B = \{B_1, B_2, \dots, B_{2^k}\}$ , and  $g_C = \{C_1, C_2, \dots, C_{2^k}\}$ , and so on for  $(N - 1)$ th party Diana, whose encoding operations are  $g_D = \{D_1, D_2, \dots, D_{2^k}\}$ .

All these subgroups are pairwise disjoint subgroups, i.e., they are chosen in such a way that  $g_i \cap g_j = \{\mathbb{I}\} \forall i, j \in \{1, 2, \dots, N - 1\}$ . As the requirement for encoding operations to be from disjoint subgroups has been already established beforehand.

Additionally, here we assume that all the parties do nothing (equivalent to operator Identity) on their qubits for encoding a string of  $k$  zeros. As Identity is the common element in the set of encoding operations to be used by each party it will be convenient to consider this as a convention in the rest of the paper.

**Step 1.2** Nathan (the  $N$ th party) prepares an  $n$ -qubit entangled state  $|\psi\rangle$  (with  $n \geq (N - 1)k$ ).

It is noteworthy that maximum information that can be encoded on the  $(N - 1)k$ -qubit quantum channel is  $(N - 1)k$  bits and here  $(N - 1)$  parties are sending  $k$  bits each. In other words, after encoding operation of all the  $(N - 1)$  parties the quantum states should be one of the  $2^{(N-1)k}$  possible orthogonal states.

**Step 1.3** Nathan sends  $m$  qubits ( $m < n$ ) of the entangled state  $|\psi\rangle$  to Alice in a secure manner, who applies one of the operations  $A_i$  (which is an element of the subgroup of operators available with her) on the travel qubits to encode her message. This will transform the initial state to  $|\psi_A\rangle = A_i|\psi\rangle$ . Subsequently, Alice sends all these encoded qubits to the next user Bob.

**Step 1.4** Bob encodes his message which will transform the quantum state to  $|\psi_B\rangle = B_j A_i |\psi\rangle$ . Finally, he also sends the encoded qubits to Charlie in a secure manner.

**Step 1.5** Charlie would follow the same strategy as followed by Alice and Bob. In the end, Diana receives all the encoded travel qubits and she also performs the operation corresponding to her message to transform the state into  $|\psi'_{i,j,k,\dots,l}\rangle = D_l \dots C_k B_j A_i |\psi\rangle$ . She returns all the travel qubits to Nathan.

**Step 1.6** Nathan can extract the information sent by all  $(N - 1)$  parties by measuring the final state using an appropriate basis set.

It may be noted that Nathan can decode messages sent by all  $(N - 1)$  parties, if and only if the set of all the encoding operations gives orthogonal states after their application on the quantum state, i.e.,  $\{|\psi'_{i,j,k,\dots,l}\rangle\}$  are orthogonal for all  $i, j, k, \dots, l \in \{1, \dots, 2^k\}$ . In other words, after the encoding operation of all the  $(N - 1)$  parties the quantum states should be a part of a basis set with  $2^{(N-1)k}$  orthogonal states for unique decoding of all possible encoding operations.

This scheme can be viewed as the generalization of ping-pong protocol [8] to a multiparty scenario, where multiple sender's can simultaneously send their information to a receiver. In a similar way, if all the senders wish to send and receive the same amount of information, then all of them can also choose to prepare their initial state  $|\psi\rangle$  independently and send it to all other parties in a sequential manner. Subsequently, all of them may follow the above protocol faithfully to perform  $N$  simultaneous multiparty QSDC protocols.

In fact,  $N$  simultaneous multiparty QSDC schemes of the above form will perform the task required in an ideal QC scheme. However, as each sender has to encode his secret multiple times  $(N - 1)$  times, it would allow him to encode different information in each round. Though it may be advantageous in some communication schemes, where a sender is allowed to send different bit values to different receivers, but is undesirable in a scheme for QC. Specifically, to stress on the relevance of a scheme that allows each sender to encode different bits to all the receivers, we may consider a situation where each party (or a few of them) publicly asks a question, and the receivers answer the question independently (for an analogy think of a panel discussion in television). In this case, all the receivers may have different opinions (say one may agree with some of them and may not with the remaining) about various questions being asked. As far as a scheme for QC is concerned, Protocol 1 described here would work under the assumption of semi-honesty. Specifically, a semi-honest party may try to cheat, but he/she would follow the protocol faithfully. This assumption would enable us to consider that each party is encoding the same information every time. In what follows, we will establish that such an assumption is not required. Specifically, in Protocol 2, we aim to design a genuine QC scheme, which does not require the semi-honesty assumption to restrict a user from sending different information to different receivers.

### 3.2 Protocol 2: Multiparty QD-type scheme for QC

Here, we will attempt to design an efficient QC scheme, which can be thought of as a generalized QD scheme. In analogy of the original Ba-An-type QD scheme, we will need the set of encoding operations for the  $N$ th party (Nathan). Here, firstly we propose the protocol which is followed by a prescription to obtain the set of operations for  $N$ th party, assuming a working scheme designed for the Protocol 1.

**Step 2.1** Same as that of Step 1.1 of Protocol 1 with a simple modification that also provide Nathan a subgroup  $g_N = \{N_1, N_2, \dots, N_{2^k}\}$  which enables him to encode a  $k$ -bit message at a later stage. The mathematical structure of this subgroup will be discussed after the protocol.

**Step 2.2** Same as Step 1.2 of Protocol 1.

**Step 2.3** Same as Step 1.3 of Protocol 1.

**Step 2.4** Same as Step 1.4 of Protocol 1.

**Step 2.5** Same as Step 1.5 of Protocol 1.

**Step 2.6** Nathan applies unitary operation  $N_m$  to encode his secret and the resulting state would be  $|\psi''_{i,j,k \dots l,m}\rangle = N_m D_l \dots C_k B_j A_i |\psi\rangle$ .

**Step 2.7** Nathan measures  $|\psi''_{i,j,k \dots l,m}\rangle$  using the appropriate basis as was done in Step 1.6 of Protocol 1 and announces the measurement outcome. Now, with the information of the initial state, final state and one's own encoding all parties can extract the information of all other parties.

It is to be noted that the information can be extracted only if the set of all the encoding operations gives orthogonal states after their application on the quantum state, i.e., all the elements of  $\{|\psi''_{i,j,k \dots l,m}\rangle\}$  are required to be mutually orthogonal for  $i, j, k \dots l, m \in \{1, \dots, 2^k\}$ . In other words, after the encoding operation of all the  $N$  parties the set of all possible quantum states should form a  $2^{(N-1)k}$  dimensional basis set.

Nathan's unitary operation can be obtained using the fact that the remaining  $(N - 1)$  parties have already utilized the channel capacity. Hence, his encoding should be in such a way that after his encoding operation  $N_m$ , the final quantum state should remain an element of the basis set in which the initial state was prepared. However, the bijective mapping between the initial and final states present in Protocol 1 would disappear here. This is not a limitation. It is actually a requirement. This is so because, in contrast to Protocol 1 where the initial and final states are secret, in Protocol 2, the choice of the initial state and the final state are publicly broadcasted. Existence of a bijective mapping would have revealed all the secrets to Eve. This condition provides us a mathematical advantage. Specifically, it allows us to construct the set of unitary operations that Nathan can apply. To do so we need to use the information about the disjoint subgroups of operators that are used by other parties. The procedure for construction of Nathan's set of operations is described below.

For simplicity, let us write the encoding operations of all the parties as follows:

	$\tilde{0}$	$\tilde{1}$	$\dots$	$(2^k - 1)$
Alice	$A_1$	$A_2$	$\dots$	$A_{2^k}$
Bob	$B_1$	$B_2$	$\dots$	$B_{2^k}$
Charlie	$C_1$	$C_2$	$\dots$	$C_{2^k}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Diana	$D_1$	$D_2$	$\dots$	$D_{2^k}$
Nathan	$N_1$	$N_2$	$\dots \dots$	$N_{2^k}$

Here,  $\tilde{x}$  corresponds to the binary value of the decimal number  $x$ , and it represents the classical information to be encoded by user  $X^{***}$  (listed in Column 1) using the the operator  $X_{x+1}$  (listed in  $x + 1$ th column in the row corresponding to the user  $X^{***}$ ). For example, to encode  $01 = \tilde{1}$ , Alice would use the operator  $A_{1+1} = A_2$ , whereas for the same encoding Bob and Charlie would use  $B_2$  and  $C_2$ , respectively. Further, we would like to note that by construction operators  $X_i = X_i^{-1}$  as  $X_i$  is an element of the modified Pauli group, and it is assumed that the encoding operations of the different users are chosen from the disjoint subgroups of the modified Pauli groups in such a way that the product of operations listed in any column is Identity, i.e.,

$$A_i B_i C_i \dots D_i N_i = \mathbb{I} \forall i. \quad (1)$$

Number of parties ( $N$ )	cbits by each party ( $k$ )	Groups	Number of travel qubit ( $m$ )	Entangled states
3	1	$G_1$	1	Bell
4	1	$G_2$	2	4-qubit cluster or $ \Omega\rangle$ state
4	1	$G_2^1(8), G_2^2(8), G_2^4(8), G_2^5(8)$	2	GHZ
5	1	$G_2$	2	4-qubit cluster or $ \Omega\rangle$ state
2	2	$G_1$	1	Bell
3	2	$G_2$	2	4-qubit cluster or $ \Omega\rangle$ state
2	3	$G_2$	2	4-qubit cluster or $ \Omega\rangle$ state
2	3	$G_2^1(8), G_2^2(8), G_2^4(8), G_2^5(8)$	2	GHZ
2	4	$G_2$	2	4-qubit cluster or $ \Omega\rangle$ state

Table 1: Various possibilities of QC scheme with a maximum number of  $N$  parties each encoding  $k$  bits using a group of unitary operators with at least  $2^{(N-1)k}$  elements. The quantum states suitable in each case and corresponding number of travel qubits are also mentioned.

This implies that if all the parties encode the same secret then the final state and the initial state would be the same. To illustrate this we may consider following example

$$\begin{array}{rcc}
& \tilde{0} & \tilde{1} \\
\text{Alice} & \mathbb{I} & X \\
\text{Bob} & \mathbb{I} & iY \\
\text{Nathan} & \mathbb{I} & Z.
\end{array} \tag{2}$$

From Eqs. (1)-(2), it is clear that the choice of encoding operations of the other users (i.e.,  $A_i, B_i, C_i, \dots, D_i$ ) would uniquely determine  $N_i$ . Further, it is assumed that the encoding operations used by different users to encode  $\tilde{x}$  are selected in a particular order that ensures  $X_i X_j = X_k \forall X$  and particular choice of  $i, j$ . For example, this condition implies that if Alice's operators satisfy  $A_2 A_3 = A_5$ , then Bob and Charlie would be given the encoding operators in an order that satisfy  $B_2 B_3 = B_5$  and  $C_2 C_3 = C_5$ , respectively, and the same ordering of operators will be applicable to all other users. Now, using the above mentioned facts and convention, we need to establish that  $\{N_x\}$  forms a group under multiplication. Eq. (1) and the self reversibility of the elements  $X_i$  lead to following identity-  $N_i = D_i^{-1} \dots C_i^{-1} B_i^{-1} A_i^{-1} = D_i \dots C_i B_i A_i$ . This may be used to establish the closure property of the group  $\{N_x\}$  as  $N_i N_j = (D_i \dots C_i B_i A_i) (D_j \dots C_j B_j A_j) = (D_i D_j) \dots (C_i C_j) (B_i B_j) (A_i A_j) = (D_k \dots C_k B_k A_k) = N_k \in \{N_x\}$ . This is so because the Pauli operators commute with each other under the operational definition of multiplication used in defining the modified Pauli group. All the remaining properties of the group follows directly from the nature of Pauli operators used to design  $X_x$ . Thus, it is established that the generalized multiparty QSDC scheme can be modified to a generalized QD scheme. It will be interesting to obtain the original Ba An's QD scheme as a limiting case as follows.

$$\begin{array}{rcccc}
& \tilde{0} & \tilde{1} & \tilde{2} & \tilde{3} \\
\text{Alice} & \mathbb{I} & X & iY & Z \\
\text{Bob} & B_1 & B_2 & B_3 & B_4.
\end{array}$$

This particular case and all the discussions leave us with  $\{B_i\} = \{\mathbb{I}, X, iY, Z\}$ , which is identical with Alice's operations.

In Table 1, we have provided a list comprising of the number of participants in the QC and the number of cbits they want to encode. The table explicitly mentions different multipartite states or quantum channels that can be utilized for the same.

Finally, it is also worth mentioning here that this protocol is free from the individual participant's attack as each user is allowed to encode only once. The remaining attack strategies and security against them will be discussed in detail in Sec. 5. Before doing so, it may be noted that the message is extracted in different ways in Protocol 1 and 2. Specifically, in Protocol 1, the encoding of each sender is inferred from the bijective mapping between the initial and final states, in analogy to the QSDC protocols. In Protocol 2, the same task is achieved by each party by exploiting the bijective mapping between the final state and his/her own encoding, which is analogous to the QD protocols. Therefore, Protocol 1 (2) proposed here can be viewed as a generalized multiparty QSDC (QD) scheme.

## 4 Examples and possible modifications

Let's elaborate the proposed idea by discussing a particular example of the proposed Protocol 2 for a 3 party case, where each party encodes only one bit. To begin with, let us assume that one of the parties, say, Nathan, prepares an entangled state in the

Party	c-bits	Quantum state	Set of encoding operations
3	1	Bell or GHZ	$\{P_1 : \{\mathbb{I}, X\}, P_2 : \{\mathbb{I}, iY\}, P_3 : \{\mathbb{I}, Z\}\}$
3	2	4-qubit cluster state	$\{P_1 : \{\mathbb{I} \otimes \mathbb{I}, \mathbb{I} \otimes X, X \otimes \mathbb{I}, X \otimes X\}, P_2 : \{\mathbb{I} \otimes \mathbb{I}, \mathbb{I} \otimes iY, iY \otimes \mathbb{I}, iY \otimes iY\}, P_3 : \{\mathbb{I} \otimes \mathbb{I}, \mathbb{I} \otimes Z, Z \otimes \mathbb{I}, Z \otimes Z\}\}$
4	1	GHZ	$\{P_1 : \{\mathbb{I} \otimes \mathbb{I}, X \otimes \mathbb{I}\}, P_2 : \{\mathbb{I} \otimes \mathbb{I}, X \otimes X\}, P_3 : \{\mathbb{I} \otimes \mathbb{I}, iY \otimes X\}, P_4 : \{\mathbb{I} \otimes \mathbb{I}, iY \otimes \mathbb{I}\}\}$
4	1	4-qubit cluster state	$\{P_1 : \{\mathbb{I} \otimes \mathbb{I}, X \otimes iY\}, P_2 : \{\mathbb{I} \otimes \mathbb{I}, X \otimes Z\}, P_3 : \{\mathbb{I} \otimes \mathbb{I}, iY \otimes Z\}, P_4 : \{\mathbb{I} \otimes \mathbb{I}, iY \otimes iY\}\}$

Table 2: We present some examples of the quantum states required for QC and corresponding encoding operations. In these examples, if one of the party do not encode (consider Identity) then Protocol 2 will reduce to Protocol 1.

Bell basis  $|\psi_{\text{in}}\rangle$  in Step 2.2 (the same is also illustrated through Figure 1, where the quantum state transforming in the various intermediate steps is mentioned). Nathan sends one of the qubits of the Bell state to Alice in Step 2.3, who encodes a unitary operation  $U_X : U_X \in \{\mathbb{I}, X\}$  corresponding to her secret and sends it to Bob. Similarly, Bob also encodes his message in Step 2.4 using a unitary operation  $U_{iY} : U_{iY} \in \{\mathbb{I}, iY\}$ . Finally, Nathan receives the encoded qubit in Step 2.5 and encodes his message using a unitary operation  $U_Z : U_Z \in \{\mathbb{I}, Z\}$  in Step 2.6. Finally, in Step 2.7, Nathan measures the final quantum state  $|\psi_{\text{fin}}\rangle = U_Z U_{iY} U_X |\psi_{\text{in}}\rangle$  in the Bell basis and announces the measurement outcome. From the knowledge of the encoding operation performed by himself/herself, and the initial and final Bell states all three parties would be able to decode the secrets sent by the remaining two parties, for which they have to use the bijective mapping present between his/her own encoding and the pair of encoding operations performed by the other two users. For instance, we may consider a particular case, where Nathan's choice of the initial state and measurement outcomes are the same, say  $|\psi^+\rangle$ . This announcement leaves only two possibilities, either  $U_Z = U_{iY} = U_X = \mathbb{I}$  or  $U_X = X, U_{iY} = iY$  and  $U_Z = Z$ . In this particular case, each party knows whether they have encoded 0 (i.e., applied Identity) or not. Using which they can extract the message sent by the remaining users. We may further note that if we restrict Nathan to always encode Identity, then this scheme (Protocol 2) will reduce to Protocol 1.

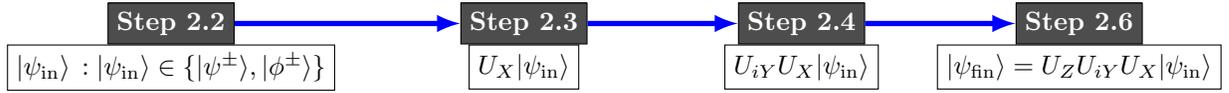


Figure 1: The evolution of the quantum channel with all the intermediate states and corresponding encoding in an example scheme are summarized. Here, the unitary operations  $U_X : U_X \in \{\mathbb{I}, X\}$ ,  $U_{iY} \in \{\mathbb{I}, iY\}$ , and  $U_Z \in \{\mathbb{I}, Z\}$ .

Further examples with the higher number of parties involved in the QC are summarized in Table 2. The examples listed are not the unique choices and similar set of unitary operators may be easily obtained using the prescription defined in the previous section.

The proposed QC scheme may also be extended to an asymmetric counterpart of the QC scheme, where each party may not be encoding the same amount of information. One such easiest example is a lecture, where the orator speaks most of the time while the remaining users barely speak. In such cases, the parties sending redundant bits to accommodate the QC scheme may choose an AQC scheme. To exploit the maximum benefit of such schemes a party encoding more information than others (say Alice) should prepare (and also measure) the quantum state (in other words, start the QC scheme). In this case, the choice of unitary operations by each party would also become relevant and Alice should use a subgroup of higher order than the remaining users. For instance, in a 3-party scenario, Alice may use a  $P_2$  from Row 2 of Table 2 to encode 2 bits message, while the remaining three users may choose  $P_1$  and  $P_3$ , respectively. It is worth noting here that the security of the QC scheme discussed in the following section ensures the security of the AQC scheme designed here as well.

Further, the proposed schemes can also be easily modified to obtain corresponding schemes for controlled QC, where an additional party (who is referred to as the controller) would prepare the quantum channel in such a way that the QC task can only be accomplished after the controller allows the other users to do so [12, 36]. Controlled QC can be achieved in various ways. For example, the controller may prepare the initial state and keep some of the qubits with himself, and in absence of the measurement outcome of the corresponding qubits the other legitimate parties would fail to accomplish the task [12]. The same feat can also be achieved by the controller without keeping a single qubit with himself by using permutation of particles [36]. Thus, it is easy to generalize the proposed schemes for QC to yield schemes for controlled QC. Such a scheme for controlled QC would have many applications. For example, a direct application to that scheme would be quantum telephone where the controller can be a Telephone company [16] that provides the channel to the respective users after authentication. Thus, the present scheme can be used to generalize the scheme proposed in [16] and thus to obtain a scheme for multiparty quantum telephone or quantum teleconference. Additionally, the multiparty communication schemes proposed here can be reduced to

schemes for secure multiparty quantum computation. Interestingly, a recently proposed secure multiparty computation scheme designed for quantum sealed-bid auction task [26] can be viewed as a reduction of the Protocol 1 proposed here. Therefore, we hope that the proposed schemes may also be modified to obtain solutions of various other real life problems.

## 5 Security analysis and efficiency

A QC protocol is expected to confront the disturbance attack (or denial of service attack), the intercept-and-resend attack, the entangle-and-measure attack, man-in-the-middle attack and Trojan-horse attack by implementing the BB84 subroutine strategy (for detail see [32, 37]), which allows senders to insert decoy qubits prepared randomly in  $X$ -basis or  $Z$ -basis in analogy with BB84 protocol and to reveal the traces of eavesdropping by comparing the initial states of the decoy qubits with the states of the same qubits after measured by the receivers randomly using  $X$ -basis or  $Z$ -basis. In fact, quantum communication of all the qubits from one party to other, as mentioned in both the protocols (for example, in Step 1.3), is performed in a secure manner. To accomplish the secure communication of message qubits using BB84 subroutine, an equal number of decoy qubits (the number of decoy qubits are required to be equal to the number of message qubits traveling through the channel) are inserted randomly in the string of travel qubits. On the authenticated receipt of this enlarged sequence of travel qubits, the sender discloses the positions of the decoy qubits and those qubits are then measured by the receiver randomly in  $X$ -basis or  $Z$ -basis. Subsequent comparison of the initial states and the measurement outcomes reveals the error rate. If the computed error rate is obtained below a tolerable limit, then the quantum communication of message qubits is considered to be accomplished in a secure manner [37, 38], and the steps thereafter are followed. Therefore, the above mentioned attacks on the proposed schemes can be defeated simply by adding decoy qubits and following BB84 subroutine.

Further, Bob's intimation by Alice that she has sent her qubits and Bob's acknowledgment of the receipt of qubits, via an authenticated classical channel, is necessary to avoid the unwanted circumstances under which Eve pretends as the desired party. There also exist some technical procedures to circumvent the Trojan-horse attack ([32] and references therein). As a scheme of QC incorporates multiusers we have discussed below the security in two scenarios where (1) an outsider (Eve) attacks the protocol, or (2) an insider (one or some of the legitimate users) attacks the protocol. Further, all the attacks and counter measures mentioned in this section are applicable on both the schemes, unless specified.

### Outsider's attacks

In the **entangle-and-measure attack**, Eve entangles her qubit  $\alpha|0\rangle + \beta|1\rangle$  with the travel qubit in the channel. Eve can extract the information by performing the  $Z$ -basis measurement on her ancillae. To counter this attack, the decoy states  $|0\rangle$ ,  $|1\rangle$ ,  $|+\rangle$  and  $|-\rangle$  are randomly inserted and when they are examined for security, then Eve is detected with probability  $|\beta|^2$  when she attacks  $|0\rangle$  and  $|1\rangle$  states, otherwise the states remain separable for  $|+\rangle$  and  $|-\rangle$ . Consequently, the total detection probability of Eve is  $\frac{|\beta|^2}{2}$  taking into account that the probability of generation of each decoy qubit state is  $\frac{1}{4}$ .

In the **intercept-and-resend attack**, Eve prepares some fresh qubits and swaps one of her qubits with the accessible qubit in the channel when  $(i - 1)$ th user sends it to  $i$ th user. Thereafter, Eve retrieves her qubit during their communication from  $i$ th user to  $(i + 1)$ th user and obtains the encoding of  $i$ th user by performing a measurement on her qubits. This attack will also be defended by incorporating decoy qubits. However, Eve may modify her strategy to measure the intercepted qubits randomly in either the computational or diagonal basis before sending the freshly prepared qubits corresponding to the measurement outcomes. It is evident that Eve's measurement of the decoy qubits will produce disturbance if she measures in the wrong basis. Let  $n$  be the total number of travel qubits such that  $\frac{n}{2}$  are decoy and message qubits each. Eve intercepts  $m$  qubits which will entail both decoy and message. Without a loss of generality, we assume that half of the  $m$  qubits are decoy and the other half are message qubits. Since the security check is performed on the decoy qubits alone, we are interested in the  $m/2$  decoy qubits which Eve measures in her lab out of the  $\frac{n}{2}$  decoy qubits in the channel. The fraction of qubits measured by Eve out of the total decoy qubits is given by  $f = \frac{m/2}{n/2} = \frac{m}{n}$ . From which the information gained by Eve is  $I(A : E) = f/2$ . This implies that  $f/2$  times the correct basis will be chosen by Eve. The error induced by Eve is observed by Alice and Bob only when Bob measures in the same basis as of Alice and is  $e = \frac{f/2}{2} = \frac{f}{4}$ . The amount of information Bob receives is given by  $I(A : B) = (1 - H[\frac{f}{4}])$ , where  $H[u]$  is the Shannon binary entropy. The security is ensured until  $I(A : B) \geq I(A : E)$ . One can calculate the fraction  $f \cong 0.68$  for secure communication with the tolerable error rate 17% ([39, 40] and references therein). Eve's success probability is  $\frac{3}{4}$  and it would decrease with the increasing value of  $m$  as  $(\frac{3}{4})^m$ .

**Information leakage attack** is inherent in the QD schemes, and consequently, is applicable to Protocol 2 proposed here as well. It refers to the information gained by Eve about the encoding of the legitimate parties by analyzing the classical channel only. In brief, the leakage can be thought of as the difference between the total information sent by both the legitimate users and the minimum information required by Eve to extract that information (i.e., Eve's ignorance). The mathematical prescription for an average gain of Eve's information is

$$I(A_i : E) = H_{\text{a priori}} - H_{\text{a posteriori}}, \quad (3)$$

where  $H_{a\text{ priori}}$  is the total classical information all the legitimate parties have encoded; and  $H_{a\text{ posteriori}}$  is Eve's ignorance after the announcement of the measurement outcome and is averaged over all the possible measurement outcomes as  $\sum_r P(r) H(i|r)$ , with the conditional entropy  $H(i|r) = -\sum_i P(i|r) \log(P(i|r))$ . If the party authorized to prepare and measure the quantum state selects the initial state randomly and sends it to all the remaining users by using a standard unconditionally secure protocol for QSDC or DSQC then the leakage can be avoided as it increases the  $H_{a\text{ posteriori}}$ , and thus decreases  $I(A_i : E)$  to zero corresponding to no leakage [32].

### Insider's attacks

**Participant attack** is possible in both the schemes proposed here. In the first scheme, a participant can send different cbits to different members unless we assume semi-honest parties. Although this scheme is advantageous in certain applications, like sealed bid auction (where this attack is detected in post-confirmation steps) [26] or where each participant wants to encode different values to respective participant, but in the conference scenario where it is required that each participant encodes the same message to all other participants then this attack is prominent, and it is wise to follow the second scheme, which is free from the assumption of semi-honest parties.

In the second scheme, the authorized party (authorized to prepare and measure the quantum state) encodes his information at the end just before performing the joint measurement and announcing the outcome. If he wants to cheat he can disclose an incorrect measurement outcome corresponding to his modified encoding once he comes to know others' encoding. This action can be circumvented, and we can implement this protocol either with a trusted party or we can randomly select any two participants and run the scheme twice considering that respective party encodes same information. Another solution would be that the initiator sends the hash value of his message at the beginning to all the remaining users, and if the hash value of his encoding revealed at the last do not match with that of the initially sent hash value, then he had cheated and will be certainly identified.

**Collusion attack** is a kind of illegal collaboration of more than one party who are not adjacent to each other, to cheat other members of a group to learn their encoding (precisely of those who are in between them). The proposed schemes are circular in nature. In this type of an attack, the attackers generate an entangled state and circulate the same number of fake qubits as that of the travel qubits. The attackers at the end already possess the home photons of the fake qubits circulated by the first attacker and performs a joint measurement to learn the encoding of the participants in between them. It will be more effective if  $i$ th and  $(i + \frac{n}{2})$ th participants collude. This is so, as both of them get the access of the travel particles at least once after knowing the secret of all the remaining parties. This attack can be averted by breaking the larger circle into  $l$  sub-circles such that if less than  $l$  attackers collude, they will not be able to cheat (see [26] for details). This attack and the solution are applicable in both the proposed schemes.

### Qubit efficiency:

The qubit efficiency of a quantum communication scheme is calculated as

$$\eta = \frac{c}{q + b},$$

where  $c$  bits of classical information is transmitted using  $q$  number of qubits, and an additional classical communication of  $b$  bits [?]. In the first QC scheme,  $c = Nk$ ,  $q = (n + mN)N$ , and  $b = 0$  as each party sends  $k$  bits and prepares  $n$ -qubit entangled state and  $m$  decoy qubits in each round of quantum communication. Therefore, the efficiency is calculated to be  $\eta_{\text{Protocol 1}} = \frac{k}{(mN+n)}$ .

Similarly, the qubit efficiency of the second QC scheme among  $N$  parties such that each party encodes  $k$  bits can be computed by noting that in this case  $c = Nk$ ,  $q = n + Nm$ , and  $b = n$ . Here,  $b \neq 0$ , as the classical communication of  $n$  cbits is associated with the broadcast of the measurement outcome by the authorized party. Thus, the qubit efficiency is obtained as  $\eta_{\text{Protocol 2}} = \frac{k}{m+(2n/N)}$ . From the  $\eta_{\text{Protocol 2}}$  one can easily calculate the qubit efficiency of various possible QC schemes detailed in Table 1. For example, one can check that the qubit efficiency of a two party QC with each party encoding 2 bits (which is Ba An's QD protocol) using Bell state as quantum channel is 67%. Similarly, the qubit efficiency for a QC scheme involving three parties sending 1 bit each with Bell state as the quantum channel can be obtained as 43%. Hence, we find that for the same initial state as quantum channel the efficiency decreases as the number of parties increases and/or the number of encoded bits decreases.

## 6 Conclusion

In summary, the notion of QC is introduced as a multiparty secure quantum communication task which is analogous with the notion of classical conference, and two protocols for secure QC are designed. The proposed protocols are novel in

the sense that they are the first set of protocols for QC, as the term QC used earlier were connected to communication tasks that were not analogous to classical conference. Further, it is shown that protocols proposed here can be reduced to protocols for QC proposed earlier considering much weaker notion of conference. One of the proposed protocols can be viewed as a generalization of the ping-pong protocol for QSDC, whereas the other one can be viewed as a generalization of the schemes for QD. It is noted that Protocol 1 composes number of rounds of multiple-sender to single receiver secure direct communication, which accomplishes the task of QC under the assumption of semi-honesty of the users. However, this semi-honesty assumption is not required for Protocol 2, which is proposed here as multiple-sender to multiple-receiver scheme, where the task is performed in a single round. Subsequently, both the proposed schemes are elaborated with the help of an explicit example.

We have discussed the utility and applications of these protocols in different scenarios. Specifically, the proposed schemes may be reduced to a set of multi-party QKD and QKA schemes, if the parties involved in QC send random bits instead of meaningful messages. Further, feasibility and significance of the controlled and asymmetric counterparts of the proposed QC schemes have also been established. The modified versions of the proposed schemes may also be found useful in accomplishing some real-life problems, whose primitive is secure multiparty computation. For example, one can employ the proposed schemes for voting among the five countries having power of veto in United Nations, where it is desired that the choice of a voter is not influenced by the choice of the others. The proposed scheme can also be extended to obtain a dynamic version of QC, where a participant can join the conference once it has started and leave it before its termination. Such a generalization is possible using the method introduced by some of the present authors in Ref. [42]. Further, the effect of various types of Markovian and non-Markovian noise on the schemes proposed here can be investigated easily using the approach adopted in [43,44].

Security of the proposed schemes has been established against various types of insider and outsider attacks. Further, the qubit efficiency analysis established that Protocol 2 is more efficient than Protocol 1. Further, one can easily observe that the proposed schemes are much more efficient compared to a simple minded scheme that performs the same task by using multiple two-party direct communication schemes, which will again work only under the assumption of semi-honest users.

Finally, we have also presented a set of encoding operations suitable with a host of quantum channels for performing the QC schemes for number of parties. This provides experimentalists a freedom to choose the encoding operations and the quantum state to be used as quantum channel as per convenience. Further, experimental realization of quantum secure direct communication scheme, which can demonstrate protocols, like quantum dialogue, quantum authentication, has been successfully performed in [45], and it paves way for experimental realization of QC. Keeping these facts in mind, we conclude this paper with a hope that the schemes proposed here and/or their variants will be realized in the near future.

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## References

- [1] Bennett, C. H., Brassard, G.: Quantum cryptography: Public key distribution and coin tossing. In Proceedings of the IEEE International Conference on Computers, Systems, and Signal Processing, Bangalore, India, pp. 175-179 (1984)
- [2] Shor, P. W. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer, In Proceedings of 35th Annual Symp. on Foundations of Computer Science, Santa Fe, IEEE Computer Society Press. (1994)
- [3] Rivest, R. L. Shamir, A., Adleman, L.: A method for obtaining digital signatures and public-key cryptosystems. *Comm. ACM* **21**, 120-126 (1978)
- [4] Diffie, W., Hellman, M.: New directions in cryptography, *IEEE Trans. Inf. Theor.* **22**, 644-654 (1976)
- [5] Ekert, A. K.: Quantum cryptography based on Bell's Theorem. *Phys. Rev. Lett.* **67**, 661 (1991)
- [6] Bennett, C. H.: Quantum cryptography using any two nonorthogonal states, *Phys. Rev. Lett.* **68**, 3121 (1992)
- [7] Goldenberg, L., Vaidman, L.: Quantum cryptography based on orthogonal states. *Phys. Rev. Lett.* **75**, 1239 (1995)
- [8] Bostrom, K. Felbinger, T.: Deterministic secure direct communication using entanglement. *Phys. Rev. Lett.* **89**, 187902 (2002)
- [9] Shukla, C., Banerjee, A., Pathak, A.: Improved protocols of secure quantum communication using W states. *Int. J. Theor. Phys.* **52**, 1914 (2013)

- [10] Long, G.-l., Deng, F.-g., Wang, C., Li, X.-h., Wen, K., Wang, W.-y.: Quantum secure direct communication and deterministic secure quantum communication. *Front. Phys. China* **2**, 251 (2007)
- [11] Banerjee, A., Pathak, A.: Maximally efficient protocols for direct secure quantum communication. *Phys. Lett. A* **376**, 2944 (2012)
- [12] Pathak, A.: Efficient protocols for unidirectional and bidirectional controlled deterministic secure quantum communication: Different alternative approaches. *Quantum Inf. Process.* **14**, 2195 (2015)
- [13] Pathak, A.: *Elements of Quantum Computation and Quantum Communication*. CRC Press, Boca Raton, USA (2013)
- [14] An, N. B.: Quantum dialogue. *Phys. Lett. A* **328**, 6 (2004)
- [15] Wen, X., Liu, Y., Zhou, N.: Secure quantum telephone. *Opt. Commun.* **275**, 278 (2007)
- [16] Wen, X., Liu, Y., Zhou, N., Secure quantum telephone, *Opt. Comm.* **275**, 278–282 (2007)
- [17] Jain, S., Muralidharan, S., Panigrahi, P. K.: Secure quantum conversation through non-destructive discrimination of highly entangled multipartite states. *Eur. Phys. Lett.* **87**, 60008 (2009)
- [18] Wang, H., Zhang, Y. Q., Liu, X. F., Hu, Y. P. Efficient quantum dialogue using entangled states and entanglement swapping without information leakage. *Quantum Inf. Process.* **15**, 2593 (2016)
- [19] Zhang, L.-L., Zhan, Y.-B.: Quantum dialogue by using the two-qutrit entangled states. *Mod. Phys. Lett. B* **23**, 2993 (2009)
- [20] Yang, C. W., Hwang, T. Quantum dialogue protocols immune to collective noise. *Quantum Inf. Process.* **12**, 2131 (2013)
- [21] Chang, C. H., Yang, C. W., Hsu, G. R., Hwang, T., Kao, S. H.: Quantum dialogue protocols over collective noise using entanglement of GHZ state. *Quantum Inf. Process.* **15**, 2971-2991 (2016)
- [22] Wikipedia: Conference, <https://en.wikipedia.org/wiki/Conference> [Online; accessed 28-January-2017]
- [23] English Oxford Dictionaries: Conference, <https://en.oxforddictionaries.com/definition/conference> [Online; accessed 28-January-2017]
- [24] Thapliyal, K., Sharma, R. D., Pathak, A.: Protocols for quantum binary voting. *Int. J. Theor. Phys.* **15**, 1750007 (2016)
- [25] Liu, W. J., Wang, H. B., et al.: Multiparty quantum sealed-bid auction using single photons as message carrier. *Quantum Inf. Process.* **15**, 869-879 (2016)
- [26] Sharma, R. D., Thapliyal, K., and Pathak A.: Quantum sealed-bid auction using a modified scheme for multiparty circular quantum key agreement, arXiv:1612.08844v1 (2016)
- [27] Huang, W., Yang, Y.-H., Jia, H.-Y.: Cryptanalysis and improvement of a quantum communication-based online shopping mechanism. *Quantum Inf. Process.* **14**, 2211-2225 (2015)
- [28] Shukla, C., Alam, N., Pathak, A.: Protocols of quantum key agreement solely using Bell states and Bell measurement. *Quantum Inf. Process.* **13**, 2391 (2014)
- [29] Bose, S., Vedral, V., Knight, P.L.: Multiparticle generalization of entanglement swapping. *Phys. Rev. A* **57**, 822 (1998)
- [30] Chen, K., Lo, H.-W.: Multi-partite quantum cryptographic protocols with noisy GHZ states, *Quantum Inf. & Comp.* **7**, 689-715 (2007)
- [31] Shukla, C., Kothari, V., Banerjee, A., Pathak, A.: On the group-theoretic structure of a class of quantum dialogue protocols. *Phys. Lett. A* **377**, 518 (2013)
- [32] Banerjee, A., Shukla, C., Thapliyal, K., Pathak, A., Panigrahi, P. K.: Asymmetric quantum dialogue in noisy environment. *Quantum Inf. Process.* (2016) doi:10.1007/s11128-016-1508-4
- [33] Kimble, H. J.: The quantum internet. *Nature* **453**, 1023-1030 (2008)
- [34] Pirandola, S., Braunstein, S. L.: Physics: Unite to build a quantum Internet. *Nature* **532**, 169-171 (2016)
- [35] Smania, M., Elhassan, A. M., Tavakoli, A., and Bourennane, M.: Experimental quantum multiparty communication protocols. *NPJ Quantum Inf.* **2**, 16010 (2016)

- [36] Thapliyal, K., Pathak, A.: Applications of quantum cryptographic switch: Various tasks related to controlled quantum communication can be performed using Bell states and permutation of particles. *Quantum Inf. Process.* **14**, 2599-2616 (2015)
- [37] Sharma, R. D., Thapliyal, K., Pathak, A., Pan, A. K., De, A.: Which verification qubits perform best for secure communication in noisy channel? *Quantum Inf. Process.* **15**, 1703-1718 (2016)
- [38] Nielsen, M. A., Chuang, I. L.: *Quantum Computation and Quantum Information*. Cambridge University Press, New Delhi (2008)
- [39] Pathak, A., Thapliyal, K.: A comment on the one step quantum key distribution based on EPR entanglement. arXiv preprint arXiv:1609.07473 (2016)
- [40] Aravinda, S., Srikanth, R. Pathak A.: On the origin of nonclassicality in single systems. arXiv preprint arXiv:1607.01768 (2016)
- [41] Cabello, A.: Quantum key distribution in the Holevo limit. *Phys. Rev. Lett.* **85**, 5635 (2000)
- [42] Mishra, Sandeep, et al.: An integrated hierarchical dynamic quantum secret sharing protocol. *Int. J. Theor. Phys.* **54**, 3143-3154 (2015)
- [43] Sharma, V., Thapliyal, K., Pathak, A., Banerjee, S.: A comparative study of protocols for secure quantum communication under noisy environment: single-qubit-based protocols versus entangled-state-based protocols. *Quantum. Inf. Process.* **15**, 4681 (2016)
- [44] Thapliyal, K., Pathak, A., Banerjee, S.: Quantum cryptography over non-Markovian channels. arXiv preprint arXiv:1608.06071 (2016)
- [45] Hu., J.-Y. et al.: Experimental quantum secure direct communication with single photons. *Light: Science & Applications* **5**, e16144 (2016)