

# Discrimination of Highly Entangled Z-states in IBM Quantum Computer

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Measurement-based quantum computation (MQC) is a leading paradigm for building a quantum computer. Cluster states being used in this context act as one-way quantum computers. Here, we consider Z-states as a type of highly entangled states like cluster states, which can be used for one-way or measurement based quantum computation. We define Z-state basis as a set of orthonormal states which are as equally entangled as the cluster states. We design new quantum circuits to non-destructively discriminate these highly entangled Z-states. The proposed quantum circuits can be generalized for N-qubit quantum system. We confirm the preservation of Z-states after the performance of the circuit by quantum state tomography process.

## I. INTRODUCTION

Entangled states have a wide range of application in the field of quantum computation and quantum information [1]. Using highly entangled states like Bell states, GHZ states, cluster states and Brown *et al.* states quantum information processing tasks such as quantum teleportation [2–7], quantum secret sharing [4–6, 8, 9], quantum information splitting [4, 6, 7, 10, 11], super dense coding [4, 7, 12], quantum cheque [13, 14] etc. have been performed. Recently, a set of schemes discriminating orthogonal entangled states [15–19] have been proposed.

Non-destructive discrimination of orthogonal entangled states is a significant variant of discrimination of orthogonal entangled states. Using this protocol, we can discriminate orthogonal entangled states without disturbing them using indirect measurements on ancillary qubits, which contain information about the entangled states. This protocol plays a pivotal role in quantum information processing and quantum computation [1]. This has been proposed for generalized orthonormal qudit Bell state discrimination [16] and has also been experimentally achieved for 2-qubit Bell states using both NMR [15, 20] and five qubit IBM quantum computer [21]. Some of the schemes have also been realized in optical medium [22] by using Kerr type nonlinearity.

This Distributed measurement technique finds a number of applications in photonic systems, measurement-based quantum computation, quantum error correction, Bell state discrimination across a quantum network involving multiple parties and optimization of the quantum communication complexity for performing measurements in distributed quantum computing. Non-destructive discrimination of entangled states has found applications in secure quantum conversation as proposed by Jain *et al.* [8], which involves two communicating parties. A more general scenario involving multi-parties has also been demonstrated by Luo *et al.* [23].

On the other hand, cluster states also have many impressive applications, mainly in areas like measurement-based quantum computation (MQC) [24–33] or one-way quantum computation. MQC provides a promising conceptual framework which can face the theoretical and experimental challenges for developing useful and practical quantum computers that can perform daily-life computational tasks and solve real world problems. The exciting feature of MQC is that it allows quantum information processing by cascaded measurements on qubits stored in a highly entangled state. The scheme of a one-way quantum computer was first introduced by Raussendorf and Briegel [34], whose key physical resource was the cluster state [35]. The scheme used measurements which were central elements to perform quantum computation [36–38]. In the proposed scheme, the cluster state can be used only once as it does not preserve entanglement after one-qubit measurements, hence the name one-way quantum computation.

Researchers are currently using IBM Quantum Experience to perform various quantum computational and quantum informational tasks [39–54]. Test of Leggett-Garg [55] and Mermin inequality [56], non-Abelian braiding of surface code defects [57], entropic uncertainty and measurement reversibility [58] and entanglement assisted invariance [59] have been illustrated. Estimation of molecular ground state energy [60] and error correction with 15 qubit repetition code [61] have also been implemented using 16 qubit IBM quantum computer *ibmqx5*. Experimental realization of discrimination of Bell states has motivated us to define a new set of highly entangled orthogonal states as Z-states and to demonstrate their discrimination using IBM's five-qubit real quantum processor *ibmqx4*. In the present work, we define Z-states which also form an orthonormal basis for the corresponding N-qubit quantum system. We then propose new quantum circuits to distinguish between those orthogonal states and experimentally verify our theoretical results using *ibmqx4*. We also generalize the quantum circuit for discriminating N-qubit Z-states.

The rest of the paper is organized as follows. In Sec. ??, we define Z-states and design quantum circuits for creating these states. In Sec. ??, we propose a new quantum circuit for non-destructively discriminating Z-states. Following

which, we explicate the experimental process by taking a particular Z-state and verify the results by quantum state tomography. Finally, we conclude our paper by discussing some future applications of Z-states which can be realized in a real quantum computer.

## II. Z- STATES

The expression for N-qubit cluster state is given below.

$$|C_N\rangle = \frac{1}{\sqrt{2^N}} \otimes_{a=1}^N (|0\rangle_a Z_{a+1} + |1\rangle_a) \quad (1)$$

2-qubit, 3-qubit and 4-qubit cluster states are expressed as,

$$|C_2\rangle = \frac{1}{\sqrt{2}} (|0\rangle|+\rangle + |1\rangle|-\rangle), \quad (2)$$

$$|C_3\rangle = \frac{1}{\sqrt{2}} (|+\rangle|0\rangle|+\rangle + |-\rangle|1\rangle|-\rangle), \quad (3)$$

$$|C_4\rangle = \frac{1}{\sqrt{2}} (|+\rangle|0\rangle|+\rangle|0\rangle + |+\rangle|0\rangle|-\rangle|1\rangle + |-\rangle|1\rangle|-\rangle|0\rangle + |-\rangle|1\rangle|+\rangle|1\rangle) \quad (4)$$

A circuit creating 3-qubit cluster state is shown in Fig. 1.

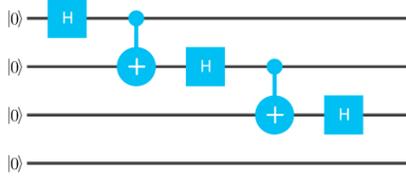


FIG. 1: *Circuit depicting the creation of  $C_3$  State*

For any given number of qubits, the cluster state is not the only highly entangled state. By applying an appropriate number of Z gates (phase flip gates) on different qubits we can generate Z-states. These states are as entangled as their corresponding cluster state. The Z-states are in an equal superposition of all the computational basis states, which form an orthonormal basis for the Hilbert space. The 2 qubit Z-states in the computational basis are as follows.

$$|Z_2^0\rangle = [1, 1, 1, -1]^T \quad (5)$$

$$|Z_2^1\rangle = [1, 1, -1, 1]^T \quad (6)$$

$$|Z_2^2\rangle = [1, -1, 1, 1]^T \quad (7)$$

$$|Z_2^3\rangle = [1, -1, -1, -1]^T \quad (8)$$

The Z-states for three-qubit case are as shown below.

$$|Z_3^0\rangle = [1, 1, 1, -1, 1, 1, -1, 1]^T \quad (9)$$

$$|Z_3^1\rangle = [1, -1, 1, 1, 1, 1, -1, -1, -1]^T \quad (10)$$



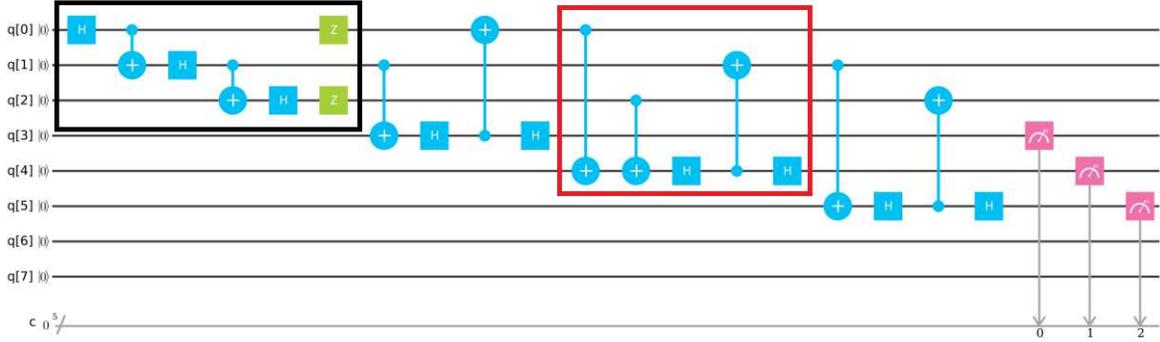


FIG. 4: A three qubit Z-state discrimination circuit

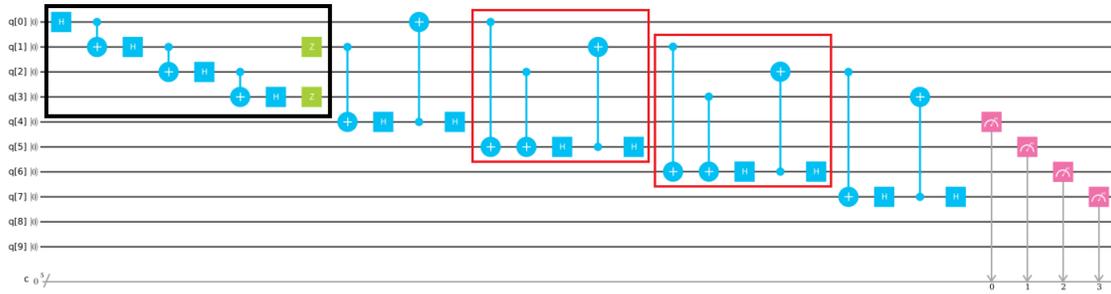


FIG. 5: A four qubit Z-state discrimination circuit

S.No.	Z-state	Ancilla
1.	$ 00\rangle +  01\rangle +  10\rangle -  11\rangle$	00
2.	$ 00\rangle +  01\rangle -  10\rangle +  11\rangle$	01
3.	$ 00\rangle -  01\rangle +  10\rangle +  11\rangle$	10
4.	$ 00\rangle -  01\rangle -  10\rangle -  11\rangle$	11

TABLE I: 2 qubit Z-states and corresponding ancilla states

In Fig. 5, the part of the circuit shown in black refers to the creation of one of the sixteen 4-qubit Z-states. From the circuits discriminating 3 qubit and 4 qubit Z-states, we can see a pattern arising. It is clear that the methodology for getting information for the first and the last ancillary qubits remains the same. The pattern that emerges applies for rest of the ancillary qubits. We can see that for all the ancillary qubits except the first and the last, two CNOTs act on it with control qubits as qubits exactly one above and below the ancillary qubit's corresponding Z-state qubit. Then we apply a Hadamard on the ancillary qubit, CNOT on the corresponding qubit (which appears to be sandwiched between the controls of previous two CNOTs) with the ancillary qubit as control, followed by a Hadamard. This “sandwich” pattern is enveloped in the red box.

For discriminating between the Z-states of higher number of qubits we can simply keep repeating this sandwich pattern for the ancillary qubits other than the first and the last, while the circuit for the first and the last ancillary qubits remains the same.

#### IV. RESULTS

We initially prepare the two qubit Z-state,  $|Z_2^1\rangle$  with two ancillas in state  $|00\rangle$ . Then the necessary single qubit and two qubit quantum gates are applied on the Z-state and the ancillas. After the performance of the quantum circuit the above Z-state is measured to check the non destructiveness of the proposed protocol. For this purpose quantum



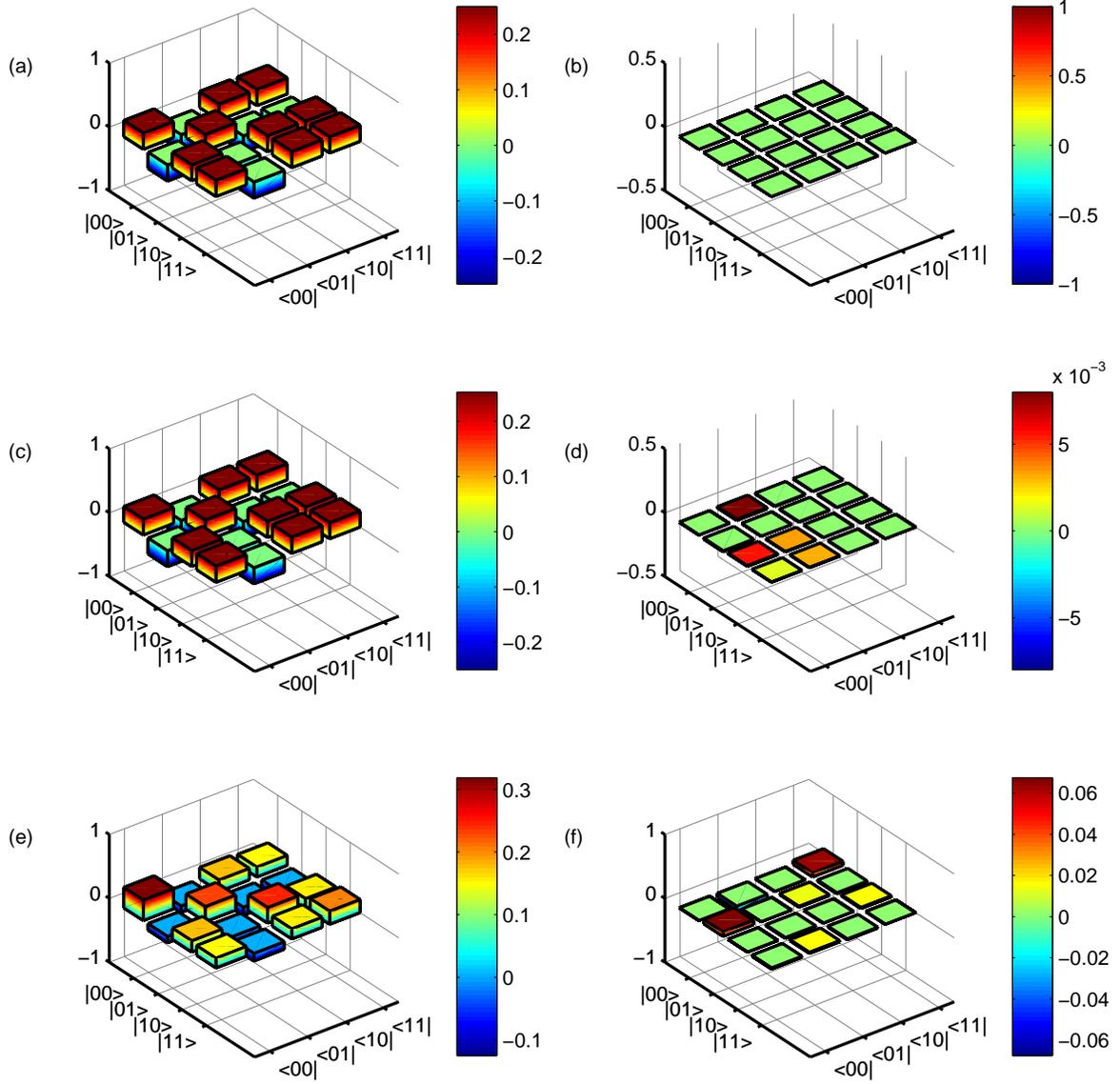


FIG. 7: LHS: Real part of the density matrix for the Z-state  $|Z_2^1\rangle$ . RHS: Imaginary part of the density matrix for the Z-state  $|Z_2^1\rangle$ . (a) and (b): ideal density matrix, (c) and (d): simulated density matrix, (e) and (f): experimentally obtained density matrix

We measure the ancilla qubits for obtaining the information about the Z-state. Fig. 8 illustrates the the quantum circuit for measuring the ancilla states. As the input Z-state is  $|Z_2^1\rangle$ , from the Table 1, it is predicted that the ancilla state should be in  $|01\rangle$  state. Hence, the theoretical density matrix for ancilla state ( $\rho^T$ ) is given as,  $\rho^T = |01\rangle\langle 01|$ . The experimental density matrices (simulational- $\rho_S^E$ , and run- $\rho_R^E$ ) for the same are provided in the following. The comparison of density matrices is depicted in the Fig. 9.



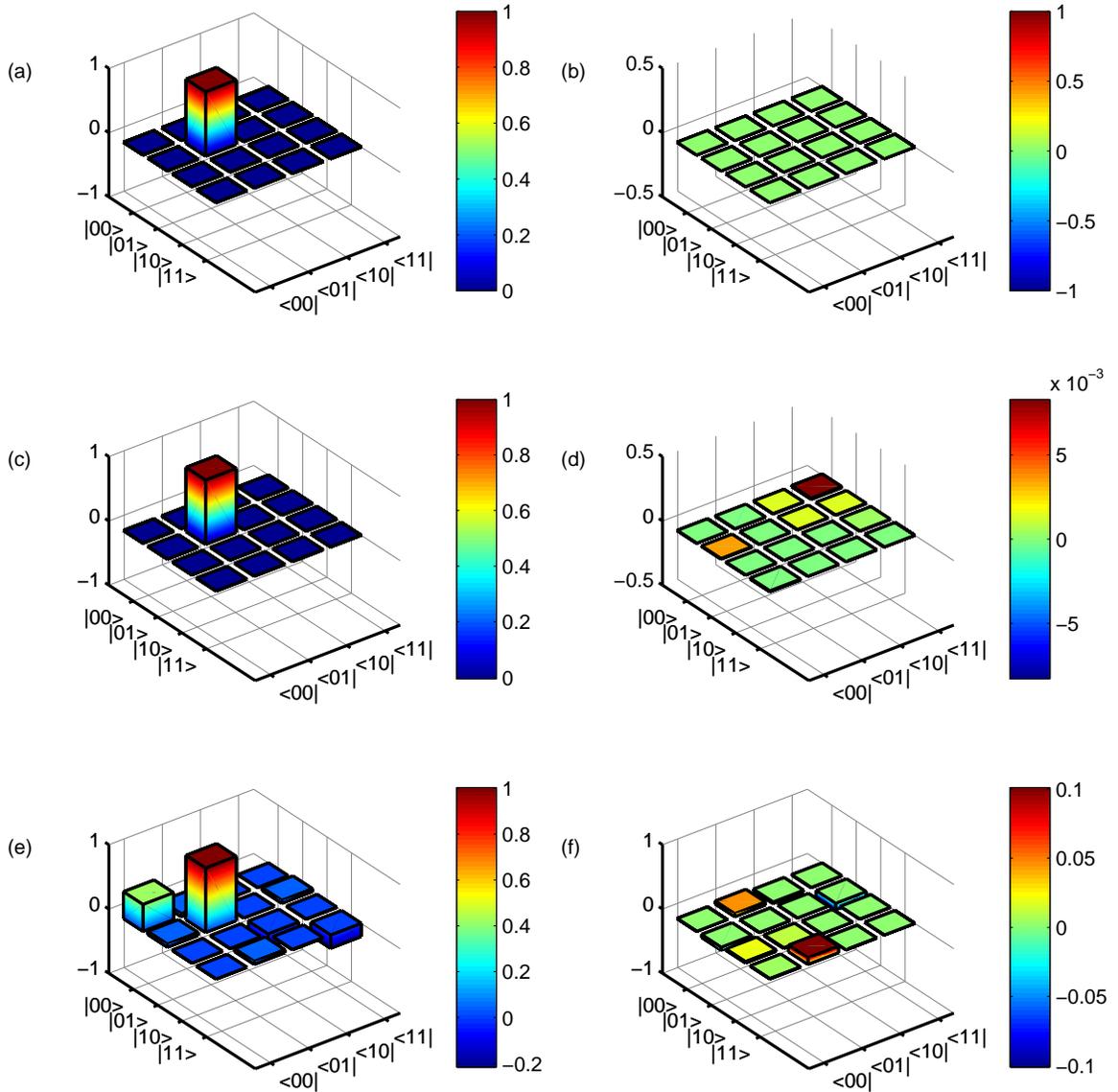


FIG. 9: LHS: Real part of the density matrix for the ancillary qubits corresponding to the Z-state  $Z_2^1$ . RHS: Imaginary part of the density matrix for the ancillary qubits corresponding to the Z-state  $Z_3^1$ . (a) and (b): ideal density matrix, (c) and (d): simulated density matrix, (e) and (f): experimentally obtained density matrix.

## V. DISCUSSION AND CONCLUSION

To conclude, we have defined here a new type of highly entangled state named as Z-state. We have proposed a new quantum circuit for non-destructively discriminating Z-states. We run the quantum circuit in the IBM quantum computer and verify the results by quantum state tomography. It is found that we have experimentally prepared the nondestructive Z-states with a fidelity of 0.815. We hope, Z-states can be applied to the branch of measurement-based quantum computation. We also could find the application of nondestructive discrimination of orthogonal entangled

states in distributed quantum computing in a quantum network.

**Acknowledgements.** B.K.B. is financially supported by DST Inspire Fellowship. SS and KS acknowledge the support of HBCSE and TIFR for conducting National Initiative on Undergraduate Sciences (NIUS) Physics camp. We are extremely grateful to IBM team and IBM QE project. The discussions and opinions developed in this paper are only those of the authors and do not reflect the opinions of IBM or IBM QE team.

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- [1] M. Nielsen, I. Chuang  
**Quantum Computation and Quantum Information**  
Cambridge University Press (2000)
- [2] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W.K. Wootters  
**Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels**  
Phys. Rev. A, 70 (1993), p. 1895
- [3] S. Ghosh, G. Kar, A. Roy, D. Sarkar, U. Sen  
**Entanglement teleportation through GHZ-class states**  
New J. Phys., 4 (2002), p. 48
- [4] S. Muralidharan, P.K. Panigrahi  
**Perfect teleportation, quantum-state sharing, and superdense coding through a genuinely entangled five-qubit state**  
Phys. Rev. A, 77 (2008), p. 032321
- [5] S. Choudhury, S. Muralidharan, P.K. Panigrahi  
**Quantum teleportation and state sharing using a genuinely entangled six-qubit state**  
J. Phys. A: Math. Theor., 42 (2009), p. 115303
- [6] S. Muralidharan, S. Karumanchi, S. Jain, R. Srikanth, and P.K. Panigrahi  
**2N qubit “mirror states” for optimal quantum communication**  
Eur. Phys. J. D, 61 (2011), pp. 757-763
- [7] N. Paul, J.V. Menon, S. Karumanchi, S. Muralidharan, P.K. Panigrahi  
**Quantum tasks using six qubit cluster states**  
Quantum Inf. Process., 10 (2011), pp. 619-632
- [8] S. Jain, S. Muralidharan, P.K. Panigrahi  
**Secure quantum conversation through non-destructive discrimination of highly entangled multipartite states**  
Euro. Phys. Lett., 87 (2009), Article 60008
- [9] E.S. Prasath, S. Muralidharan, C. Mitra, P.K. Panigrahi  
**Multipartite entangled magnon states as quantum communication channels**  
Quantum Inf. Process., 11 (2012), pp. 397-410
- [10] P.K. Panigrahi, S. Karumanchi, S. Muralidharan  
**Minimal classical communication and measurement complexity for quantum information splitting of a two-qubit state**  
Pramana - J. Phys., 73 (2009), pp. 499-504
- [11] S. Muralidharan, P.K. Panigrahi  
**Quantum-information splitting using multipartite cluster states**  
Phys. Rev. A, 78 (2008), p. 062333
- [12] P. Agrawal, A. Pati  
**Perfect teleportation and superdense coding with W states**  
Phys. Rev. A, 74 (2006), p. 062320
- [13] S.R. Moullick, P.K. Panigrahi  
**Quantum cheques**  
Quantum Inf. Process., 15 (2016), pp. 2475–2486
- [14] B.K. Behera, A. Banerjee, P.K. Panigrahi  
**Experimental realization of quantum cheque using a five-qubit quantum computer**  
Quantum Inf. Process., 16 (2017), p. 312
- [15] M. Gupta, A. Pathak, R. Srikanth, P.K. Panigrahi  
**General circuits for indirecting and distributing measurement in quantum computation**  
Int. J. Quantum Inf., 5 (2007), pp. 627-640
- [16] P.K. Panigrahi, M. Gupta, A. Pathak, R. Srikanth  
**Circuits for distributing quantum measurement**  
AIP Conf. Proc., 864 (2006), pp. 197-207
- [17] W.J. Munro, K. Nemoto, T.P. Spiller, *et al.*  
**Efficient optical quantum information processing**  
J. Opt. B, Quantum Semiclass. Opt., 7 (2005), p. S135

- [18] X.-W. Wang, D.-Y. Zhang, S.-Q. Tang, L.-J. Xie  
**Nondestructive Greenberger-Horne-Zeilinger-state analyzer**  
 Quantum Inf. Process., 12 (2013), pp. 1065-1075
- [19] C. Zheng, Y. Gu, W. Li, Z. Wang, J. Zhang  
**Complete distributed hyper-entangled-bell-state analysis and quantum super dense coding**  
 Int. J. Theor. Phys., 55 (2016), pp. 1019-1027
- [20] J.R. Samal, M. Gupta, P.K. Panigrahi, A. Kumar  
**Non-destructive discrimination of Bell states by NMR using a single ancilla qubit**  
 J. Phys. B: Atom. Molec. Opt. Phys., 43 (2010), p. 095508
- [21] M. Sisodia, A. Shukla, A. Pathak  
**Experimental realization of nondestructive discrimination of Bell states using a five-qubit quantum computer**  
 Phys. Lett. A, 381 (2017), pp. 3860-3874
- [22] J. Li, B.-S. Shi, Y.-K. Jiang, X.-F. Fan, G.-C. Guo  
**A non-destructive discrimination scheme on 2n-partite GHZ bases**  
 J. Phys. B, At. Mol. Opt. Phys., 33 (2000), p. 3215
- [23] Q.-b. Luo, G.-w. Yang, K. She, W.-n. Niu, Y.-q. Wang  
**Multi-party quantum private comparison protocol based on d-dimensional entangled states**  
 Quantum Inf. Process., 13 (2014), pp. 2343-2352
- [24] M.A. Nielsen  
**Quantum computation by measurement and quantum memory**  
 Phys. Lett. A, 308 (2003), pp. 96-100
- [25] D.W. Leung  
**Quantum computation by measurements**  
 Int. J. Quantum Inf., 2 (2004), pp. 33-43
- [26] R. Raussendorf, H.J. Briegel  
**Computational model underlying the one-way quantum computer**  
 Quantum Inf. Comput., 2 (2002), pp. 443-486
- [27] R. Raussendorf, D.E. Browne, H.J. Briegel  
**Measurement-based quantum computation using cluster states**  
 Phys. Rev. A, 68 (2003), p. 022312
- [28] P. Aliferis, D.W. Leung  
**Computation by measurements: A unifying picture**  
 Phys. Rev. A, 70 (2004), p. 062314
- [29] V. Danos, E. Kashefi, P. Panangaden  
**The measurement calculus**  
 J. Assoc. Comput. Mach., 54 (2007), p. 8
- [30] A.M. Childs, D.W. Leung, M.A. Nielsen  
**Unified derivations of measurement-based schemes for quantum computation**  
 Phys. Rev. A, 71 (2005), p. 032318
- [31] D.E. Browne, E. Kashefi, M. Mhalla, S. Perdrix  
**Generalized flow and determinism in measurement-based quantum computation**  
 New J. Phys., 9 (2007), p. 250
- [32] F. Verstraete, J.I. Cirac  
**Valence-bond states for quantum computation**  
 Phys. Rev. A, 70 (2004), p. 060302
- [33] D. Gross, J. Eisert  
**Novel schemes for measurement-based quantum computation**  
 Phys. Rev. Lett., 98 (2007), p. 220503
- [34] R. Raussendorf, H.J. Briegel  
**A One-Way Quantum Computer**  
 Phys. Rev. Lett., 86 (2001), p. 5188
- [35] H.J. Briegel, R. Raussendorf  
**Persistent Entanglement in Arrays of Interacting Particles**  
 Phys. Rev. Lett., 86 (2001), p. 910
- [36] M.A. Nielsen, I.L. Chuang  
**Programmable Quantum Gate Arrays**  
 Phys. Rev. Lett., 79 (1997), p. 321
- [37] D. Gottesman, I.L. Chuang  
**Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations**  
 Nature, 402 (1999), pp. 390-393
- [38] E. Knill, R. Laflamme, G.J. Milburn  
**A scheme for efficient quantum computation with linear optics**

- Nature, 409 (2001), pp. 46-52
- [39] R.P. Rundle, P.W. Mills, T. Tilma, J.H. Samson, M.J. Everitt  
**Simple procedure for phase-space measurement and entanglement validation**  
 Phys. Rev. A, 96 (2017), p. 022117
- [40] U. Alvarez-Rodriguez, M. Sanz, L. Lamata, E. Solano  
**Artificial Life in Quantum Technologies**  
 arXiv preprint  
 arxiv:1505.03775 (2016)
- [41] D. Grimaldi, M. Marinov  
**Distributed measurement systems**  
 Measurement, 30 (2001), pp. 279-287
- [42] A.R. Kalra, S. Prakash, B.K. Behera, P.K. Panigrahi  
**Experimental Demonstration of the No Hiding Theorem Using a 5 Qubit Quantum Computer**  
 arXiv preprint  
 arXiv:1707.09462 (2017)
- [43] D. Ghosh, P. Agarwal, P. Pandey, B.K. Behera, P.K. Panigrahi  
**Automated Error Correction in IBM Quantum Computer and Explicit Generalization**  
 arXiv preprint  
 arXiv:1708.02297 (2017)
- [44] S. Gangopadhyay, Manabputra, B.K. Behera, P.K. Panigrahi  
**Generalization and Partial Demonstration of an Entanglement Based Deutsch-Jozsa Like Algorithm Using a 5-Qubit Quantum Computer**  
 arXiv preprint  
 arXiv:1708.06375 (2017)
- [45] M. Schuld, M. Fingerhuth, F. Petruccione  
**Implementing a distance-based classifier with a quantum interference circuit**  
 arXiv preprint  
 arXiv:1703.10793 (2017)
- [46] A. Majumder, S. Mohapatra, A. Kumar  
**Experimental Realization of Secure Multiparty Quantum Summation Using Five-Qubit IBM Quantum Computer on Cloud**  
 arXiv preprint  
 arXiv:1707.07460 (2017)
- [47] R. Li, U. Alvarez-Rodriguez, L. Lamata, E. Solano  
**Approximate Quantum Adders with Genetic Algorithms: An IBM Quantum Experience**  
 arXiv preprint  
 arXiv:1611.07851 (2017)
- [48] M. Sisodia, A. Shukla, K. Thapliyal, A. Pathak  
**Design and experimental realization of an optimal scheme for teleportation of an n-qubit quantum state**  
 arXiv preprint  
 arXiv:1704.05294 (2017)
- [49] Vishnu P.K., D. Joy, B.K. Behera, P.K. Panigrahi  
**Experimental Demonstration of Non-local Controlled-Unitary Quantum Gates Using a Five-qubit Quantum Computer**  
 arXiv preprint  
 arXiv:1709.05697 (2017)
- [50] İ. Yalçınkaya, Z. Gedik  
**Optimization and experimental realization of quantum permutation algorithm**  
 arXiv preprint  
 arXiv:1708.07900 (2017)
- [51] A. Dash, S. Rout, B.K. Behera, P.K. Panigrahi  
**A Verification Algorithm and Its Application to Quantum Locker in IBM Quantum Computer**  
 arXiv preprint  
 arXiv:1710.05196 (2017)
- [52] S. Roy, B.K. Behera, P.K. Panigrahi  
**Demonstration of Entropic Noncontextual Inequality Using IBM Quantum Computer**  
 arXiv preprint  
 arXiv:1710.10717 (2017)
- [53] B.K. Behera, S. Seth, A. Das, P.K. Panigrahi  
**Experimental Demonstration of Quantum Repeater in IBM Quantum Computer**  
 arXiv preprint  
 arXiv:1712.00854 (2017)
- [54] R. Li, U. Alvarez-Rodriguez, L. Lamata, E. Solano

- Approximate Quantum Adders with Genetic Algorithms: An IBM Quantum Experience**  
Quantum Meas. Quantum Metrol., 4 (2017), pp. 1-7
- [55] E. Huffman, A. Mizel  
**Violation of noninvasive macrorealism by a superconducting qubit: Implementation of a Leggett-Garg test that addresses the clumsiness loophole**  
Phys. Rev. A, 95 (2017), p. 032131
- [56] D. Alsina, J.I. Latorre  
**Experimental test of Mermin inequalities on a five-qubit quantum computer** Phys. Rev. A, 94 (2016), p. 012314
- [57] J. R. Wootton  
**Demonstrating non-Abelian braiding of surface code defects in a five qubit experiment**  
Quantum Sci. Technol., 2 (2017), p. 015006
- [58] M. Berta, S. Wehner, M.M. Wilde  
**Entropic uncertainty and measurement reversibility**  
New J. Phys., 18 (2016), p. 073004
- [59] S. Deffner  
**Demonstration of entanglement assisted invariance on IBM's quantum experience**  
Heliyon, 3 (2016), p. e00444
- [60] A. Kandala, A. Mezzacapo, K. Temme, M. Takita, M. Brink, J.M. Chow, J.M. Gambetta  
**Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets**  
Nature, 549 (2017), pp. 242-246
- [61] J.R. Wootton, D. Loss  
**A repetition code of 15 qubits**  
arXiv preprint  
arXiv:1709.00990 (2017)