

Entanglement production by the magnetic dipolar interaction dynamics

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We consider two qubits prepared in a product state and evolved under the magnetic dipolar interaction (MDI). We describe the dependence of the entanglement generated by the MDI with time, with the interaction parameters, and with the system's initial state, identifying the symmetry and coherence aspects of those initial configurations that yield the maximal entanglement. We also show how one can obtain maximum entanglement from the MDI applied to some families of partially entangled initial states.

Keywords: Quantum coherence; quantum entanglement; magnetic dipolar interaction

I. INTRODUCTION

In view of its possible application as a quantum channel for quantum communication and as a resource for quantum computation tasks, the quantum correlations in the Gibbs thermal state [1] associated with the magnetic dipolar interaction (MDI) [2–4] have been receiving considerable attention in the quantum information science literature [5–8]. The dynamic behavior of entanglement and of others quantum correlations has been investigated too [9–13]. Besides, the MDI was used to simulate spin systems [14] and to obtain an Ising interaction [15] from which CNOT gates (an essential ingredient for universal quantum computation [16]) can be implemented. Due to the creation of quantum correlations between system and environment [17, 18], which leads to the classicality of the first, the MDI is the source of noise in several physical systems [19–25]. So it is important, from the fundamental and practical points of view, to investigate the dynamics of quantum coherence and of quantum correlations due to the MDI.

Incoherent operations, i.e., quantum operations that cannot generate superpositions of orthogonal states from their mixtures, are one of the basic elements of the resource theories of coherence that have been developed in the last few years [26]. One crucial aspect of this development is the interplay between coherence of subsystems and the quantum correlations of their composites, and interesting tradeoff relations for the transformation of coherence into entanglement by incoherent operations have been identified [27]. Nevertheless, although incoherent operations are the natural ones to consider from the resource theory perspective, for practical purposes it is also relevant to investigate the capabilities of some common physical operations to convert initial coherence into entanglement. In this article we shall perform that kind of investigation by regarding the MDI.

We organized the remainder of this article in the following manner. After presenting the regarded MDI Hamiltonian in Sec. II, we consider the evolved states generated by this interaction for initial product pure (Sec. III A) or mixed (Sec. III B) states and investigate the dependence of the transformation of local quantum coherence into quantum entanglement by the MDI on the interaction parameters, on time, and on the system initial states. In Sec. IV we show how one can obtain maximum entanglement from partially entangled states using the MDI. Our conclusions are presented in Sec. V.

II. HAMILTONIAN FOR THE MAGNETIC DIPOLAR INTERACTION

The Hamiltonian for the magnetic dipolar interaction (MDI) reads (see [28] and references therein):

$$H = D[(\vec{\sigma} \otimes \sigma_0) \cdot (\sigma_0 \otimes \vec{\sigma}) - 3\hat{n} \cdot \vec{\sigma} \otimes \hat{n} \cdot \vec{\sigma}], \quad (1)$$

with r being the distance between the dipoles centers and \hat{n} is a unit vector in \mathbb{R}^3 pointing from one dipole to the other, σ_0 is the 2x2 identity matrix, and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the vector of Pauli matrices. The strength of the MDI is given by the distance-related parameter $D = \mu_0 \hbar^2 \gamma_a \gamma_b / 16\pi r^3$, with μ_0 being the vacuum permeability and γ_s is the particle $s = a, b$ gyromagnetic ratio. Throughout this paper we use Planck's constant $\hbar = 1$ and set $D = 1$, which for this Hamiltonian is equivalent to measure time in units of D/\hbar .

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When we deal with two-level systems, it follows that $V\hat{n}\cdot\vec{\sigma}V^\dagger = (O\hat{n})\cdot\vec{\sigma} = \hat{n}'\cdot\vec{\sigma}$, where $V \in SU(2)$ and $O \in SO(3)$ (see e.g. [29, 30]). So, as $\sum_j V\sigma_j V^\dagger \otimes V\sigma_j V^\dagger = \sum_j \sigma_j \otimes \sigma_j$ we shall have

$$V \otimes V H V^\dagger \otimes V^\dagger = (\vec{\sigma} \otimes \sigma_0) \cdot (\sigma_0 \otimes \vec{\sigma}) - 3\hat{n}' \cdot \vec{\sigma} \otimes \hat{n}' \cdot \vec{\sigma}. \quad (2)$$

We see thus that by changing the relative spacial orientation of the dipoles centers ($\hat{n} \rightarrow \hat{n}'$) we will not affect the entanglement of the MDI Hamiltonian eigenstates nor of its associated Gibbs thermal state¹, because

$$e^{cV \otimes V H V^\dagger \otimes V^\dagger} = V \otimes V e^{cH} V^\dagger \otimes V^\dagger \quad (3)$$

for $c \in \mathbb{C}$. But, as we will show in this article, the dynamical generation of entanglement by the MDI is affected by the change in spacial orientation $\hat{n} \rightarrow \hat{n}'$, which corresponds to a general local rotation of the dipoles initial states before their original MDI is turned on. For simplicity, all results we shall present hereafter are for $\hat{n} = (0, 0, 1)$, so that the dipoles centers lie in the z axis. In this case

$$\begin{aligned} H &= 2^{-1}(\sigma_1 \otimes \sigma_1 + \sigma_2 \otimes \sigma_2 - 2\sigma_3 \otimes \sigma_3) \\ &= 0|\Psi_-\rangle\langle\Psi_-| + 2|\Psi_+\rangle\langle\Psi_+| - |\Phi_-\rangle\langle\Phi_-| - |\Phi_+\rangle\langle\Phi_+|, \end{aligned} \quad (4)$$

with $|\Psi_\pm\rangle = 2^{-1/2}(|01\rangle \pm |10\rangle)$ and $|\Phi_\pm\rangle = 2^{-1/2}(|00\rangle \pm |11\rangle)$ being the Bell's states. Throughout this article we use the notation $|j\rangle \otimes |k\rangle = |jk\rangle$, where $\{|j\rangle\}_{j=0}^1$ is the standard basis for \mathbb{C}^2 . In the next sections, the dynamics generated by this Hamiltonian is studied with particular focus on its capabilities to transform local quantum coherence into quantum entanglement or partial entanglement into maximal entanglement.

III. ENTANGLEMENT PRODUCTION BY THE MAGNETIC DIPOLAR INTERACTION FOR PRODUCT INITIAL STATES

A. Initial product-pure states

In this subsection we consider the two dipoles prepared in a product-pure state $|\psi_a\rangle \otimes |\psi_b\rangle$, with

$$|\psi_s\rangle = \alpha_s|0\rangle + \beta_s|1\rangle = \cos\frac{\theta_s}{2}|0\rangle + \sin\frac{\theta_s}{2}|1\rangle \quad (6)$$

and $\theta_s \in [0, 2\pi]$ for $s = a, b$, i.e., we consider two coaxial rings in the Bloch's sphere picture for the initial states. With reference to the standard basis $\{|0\rangle, |1\rangle\}$, the l_1 -norm quantum coherence [31] of such a state is:

$$C_{l_1}(|\psi_s\rangle) = 2|\alpha_s|\sqrt{1 - |\alpha_s|^2} = |\sin\theta_s|. \quad (7)$$

For the aforementioned initial state, the evolved state under the magnetic dipolar interaction (MDI) is given (up to a global phase) by:

$$\begin{aligned} |\Psi_t\rangle &= U_t|\psi_a\rangle \otimes |\psi_b\rangle = e^{-iHt}|\psi_a\rangle \otimes |\psi_b\rangle \\ &= (\alpha_a\beta_b \cos t - i\beta_a\alpha_b \sin t)|01\rangle + (\beta_a\alpha_b \cos t - i\alpha_a\beta_b \sin t)|10\rangle + e^{i2t}(\alpha_a\alpha_b|00\rangle + \beta_a\beta_b|11\rangle). \end{aligned} \quad (8)$$

In this subsection we shall compute the entanglement of the evolved state in Eq. (8) using the concurrence [32], which for the pure state above is:

$$E_C(|\Psi_t\rangle) = |\langle\Psi_t|\sigma_2 \otimes \sigma_2|\Psi_t^*\rangle| = \sqrt{f^2 + g^2}, \quad (9)$$

with $f = 2\alpha_a\beta_a\alpha_b\beta_b(\cos 2t - \cos 4t)$ and $g = (\alpha_a^2\beta_b^2 + \beta_a^2\alpha_b^2)\sin 2t + 2\alpha_a\beta_a\alpha_b\beta_b\sin 4t$, where $|\Psi_t^*\rangle$ is the complex conjugate of $|\Psi_t\rangle$ represented in the standard basis. Some examples of the time and initial state dependence of the entanglement created by the MDI are shown graphically in Fig. 1. If it would be possible to experimentally turn off the MDI at any given instant of time, we could choose the moment at which the two dipoles share the greater

¹ The Gibbs thermal state has the form: $\rho_{th} = Z^{-1}e^{-\beta H}$, where $Z = \text{Tr}(e^{-\beta H})$ is the partition function and $\beta = (k_B T)^{-1}$, with T being the bath temperature and k_B is the Boltzmann constant.

value of entanglement. So, in Fig. 1 we present also the dependence of the entanglement generated by the MDI on the angles θ_a and θ_b for some fixed values of time.

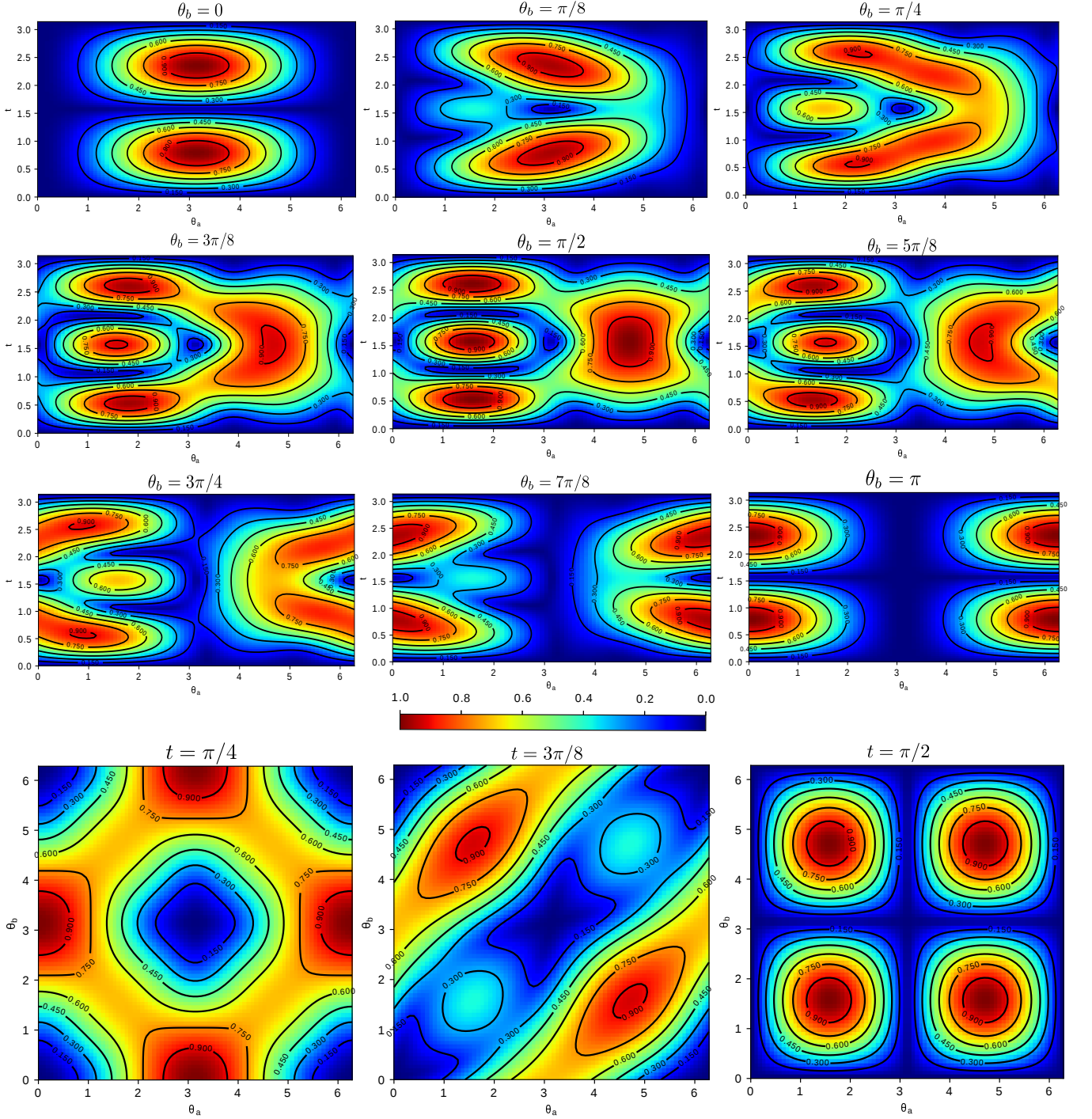


Figure 1: (color online) In the *first three rows* of plots is shown the entanglement as a function of time (in units of D/\hbar) and of dipole a initial state for some initial states of dipole b . We verified that entanglement is a periodic function in time with period π . Besides, the plots for $\theta_b = \pi + \phi$, for $\phi \in [0, \pi]$, look just like those for $\theta_b = \pi - \phi$ reflected in relation to the $\theta_a = \pi$ axis. In the *last row* of plots is shown the entanglement generated by the MDI as a function of the angles that determine the dipoles initial states for some values of time. The figure for $t = \pi/8$ is equal to that for $t = 3\pi/8$ rotated clockwise in the $\theta_a \times \theta_b$ plane by $\pi/2$. For the other values of time, the values of entanglement are equal or lesser than the corresponding values in these four figures. Overall, the dependence of the entanglement generated by the MDI on the initial local coherences is far from simple, and we highlight its most important characteristics in the main text.

We see in Fig. 1 that only the following set of initial states $\{|01\rangle, |10\rangle, |++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$, with $|\pm\rangle = 2^{-1/2}(|0\rangle \pm |1\rangle)$, yields the maximum possible value of entanglement. While the last four initial states are maximally coherent for the two dipoles, the first two initial configurations have zero local coherence. We notice then that the MDI is not an incoherent operation, since it can produce entanglement from incoherent states (see e.g. Ref. [27]). On the other hand, we can understand the non-equivalence between the two pairs of incoherent states ($|01\rangle, |10\rangle$) and ($|00\rangle, |11\rangle$) with regard to entanglement generation by noticing that as $[H, \sigma_3 \otimes \sigma_0 + \sigma_0 \otimes \sigma_3] = 0$ the dynamics under the MDI conserves the total number of excitations of the system. So, the later pair of states remain confined to their subspaces, which involves only the product states, while the first pair can superpose to produce entanglement.

The commutation relation above can be used also to show that U_t commutes with $R_z(\delta) \otimes R_z(\delta)$, where $R_z(\delta) = \exp(-i\delta\sigma_3/2)$. Once $E_C(R_z(\delta) \otimes R_z(\delta)U_t|\psi_a\rangle \otimes |\psi_b\rangle) = E_C(U_tR_z(\delta)|\psi_a\rangle \otimes R_z(\delta)|\psi_b\rangle)$, our main conclusions about the coherence-entanglement conversion by the MDI shall be the same for any orientation we use for the two coaxial rings of initial states. We emphasize e.g. that any pair of “parallel” or “anti-parallel” states in the equator of the Bloch sphere shall lead to maximal entanglement if evolved under the MDI.

Now that we have presented these general results for the entanglement generated by the MDI for the initial spacial orientation $\hat{n} = (0, 0, 1)$, we can show explicitly that although the MDI eigenstates and thermal entanglement do not change by changing the dipole centers spatial orientation, the dynamical generation of non-separable states by the MDI can be greatly affected by this kind of operation. The main point here is that the change $\hat{n} \rightarrow \hat{n}'$ is equivalent to modifying the evolution operator as $e^{-iHt} \rightarrow V \otimes V e^{-iHt} V^\dagger \otimes V^\dagger$, which is effectively equivalent, with respect to entanglement generation, to change the initial state to $V^\dagger|\psi_a\rangle \otimes V^\dagger|\psi_b\rangle$. As an extreme example, let us consider the change $\hat{n} \rightarrow \hat{n}'$ corresponding to the unitary operation V^\dagger that leads to a rotation of the initial states Bloch vectors by $\pi/2$ around the y axis. This rotation applied to the initial state $|00\rangle$ returns the state $|++\rangle$, and in this case we would go from a situation where no entanglement is created to another initial state that gives us maximal entanglement by the MDI. Of course, this issue will appear also for the initial mixed-product states we study in the next subsection.

The results presented in this section indicate the non-existence of a direct-general temporal correlation between the values of coherence and entanglement. But, for completeness, we present in the Appendix the time evolution of local quantum coherence in this case.

B. Initial product-mixed states

In order to investigate the effect of the purity of the initial state on the entanglement produced by MDI, we regard as initial states the following product states of the two dipoles: $\rho_{ja} \otimes \rho_{jb}$, where $\rho_{js} = 2^{-1}(\sigma_0 + r_{js}\sigma_j)$ with $r_{js} = \text{Tr}(\rho_s\sigma_j) \in [-1, 1]$ and $j = 1$ or $j = 3$ (these are, respectively, the x and z axis in the Bloch's ball). For these local states, the l_1 -norm coherence is given by $C_{l_1}(\rho_{1s}) = |r_{1s}|$ and $C_{l_1}(\rho_{3s}) = 0$, i.e., we use a generally coherent or an incoherent initial state. The local purities read $P(\rho_{js}) = \text{Tr}(\rho_{js}^2) = 2^{-1}(1 + r_{js}^2)$. Notice that for both states ρ_{js} the purity is a monotonously increasing function of $|r_{js}|$.

Here the evolved states, $\rho_j = e^{-iHt}(\rho_{ja} \otimes \rho_{jb})e^{iHt}$, read

$$\begin{aligned} 4\rho_3 = & (1 + r_{3a})(1 + r_{3b})|00\rangle\langle 00| + [1 - r_{3a}r_{3b} + (r_{3a} - r_{3b})\cos 2t]|01\rangle\langle 01| + i(r_{3a} - r_{3b})\sin 2t|01\rangle\langle 10| \\ & - i(r_{3a} - r_{3b})\sin 2t|10\rangle\langle 01| + [1 - r_{3a}r_{3b} - (r_{3a} - r_{3b})\cos 2t]|10\rangle\langle 10| + (1 - r_{3a})(1 - r_{3b})|11\rangle\langle 11| \end{aligned} \quad (10)$$

and

$$\begin{aligned} 4\rho_1 = & (1 - r_{1a}r_{1b})(|\Psi_-\rangle\langle\Psi_-| + |\Phi_-\rangle\langle\Phi_-|) + (1 + r_{1a}r_{1b})(|\Psi_+\rangle\langle\Psi_+| + |\Phi_+\rangle\langle\Phi_+|) \\ & + (r_{1b} - r_{1a})(e^{it}|\Phi_-\rangle\langle\Psi_-| + e^{-it}|\Psi_-\rangle\langle\Phi_-|) + (r_{1b} + r_{1a})(e^{i3t}|\Phi_+\rangle\langle\Psi_+| + e^{-i3t}|\Psi_+\rangle\langle\Phi_+|). \end{aligned} \quad (11)$$

For bipartite mixed states of two qubits the entanglement concurrence is computed using [32]:

$$E_C(\rho) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \quad (12)$$

with $\{\lambda_j\}_{j=1}^4$ being the eigenvalues of $\rho\sigma_2 \otimes \sigma_2\rho^*\sigma_2 \otimes \sigma_2$ indexed in decreasing order and ρ^* is the complex conjugate of the system's density matrix ρ . In Fig. 2 we show the numerical results for the entanglement concurrence of ρ_3 and of ρ_1 as a function of time and of dipoles a and b initial states.

As one can observe in Fig. 2, the entanglement has an oscillatory behavior with time and E_C generally increases with the total purity of the dipoles initial states. This proportionality is confirmed by the maximum values of the entanglement as a function of the dipoles initial states. We observe that for $r_{ja} = r_{jb} = 0$ the initial state is proportional to the identity and no entanglement is created. Besides, there is a minimal total purity of the dipoles below which we get no entanglement. Of course, in the limiting cases of maximum purity, coinciding with those of the last subsection, the MDI produce the maximum possible amount of entanglement. Notwithstanding, as we have

shown here, although purity is a important figure to consider regarding the entanglement of the evolved state, the symmetry of the initial state with relation to the Hamiltonian generating the evolution is also relevant for analyzing the dynamical creation of entanglement.

Once more, because U_t commutes with $R_z(\delta) \otimes R_z(\delta)$, the results presented in this subsection shall be valid for all initial states of the two qubits corresponding to parallel axes in the xy plane of the Bloch sphere.

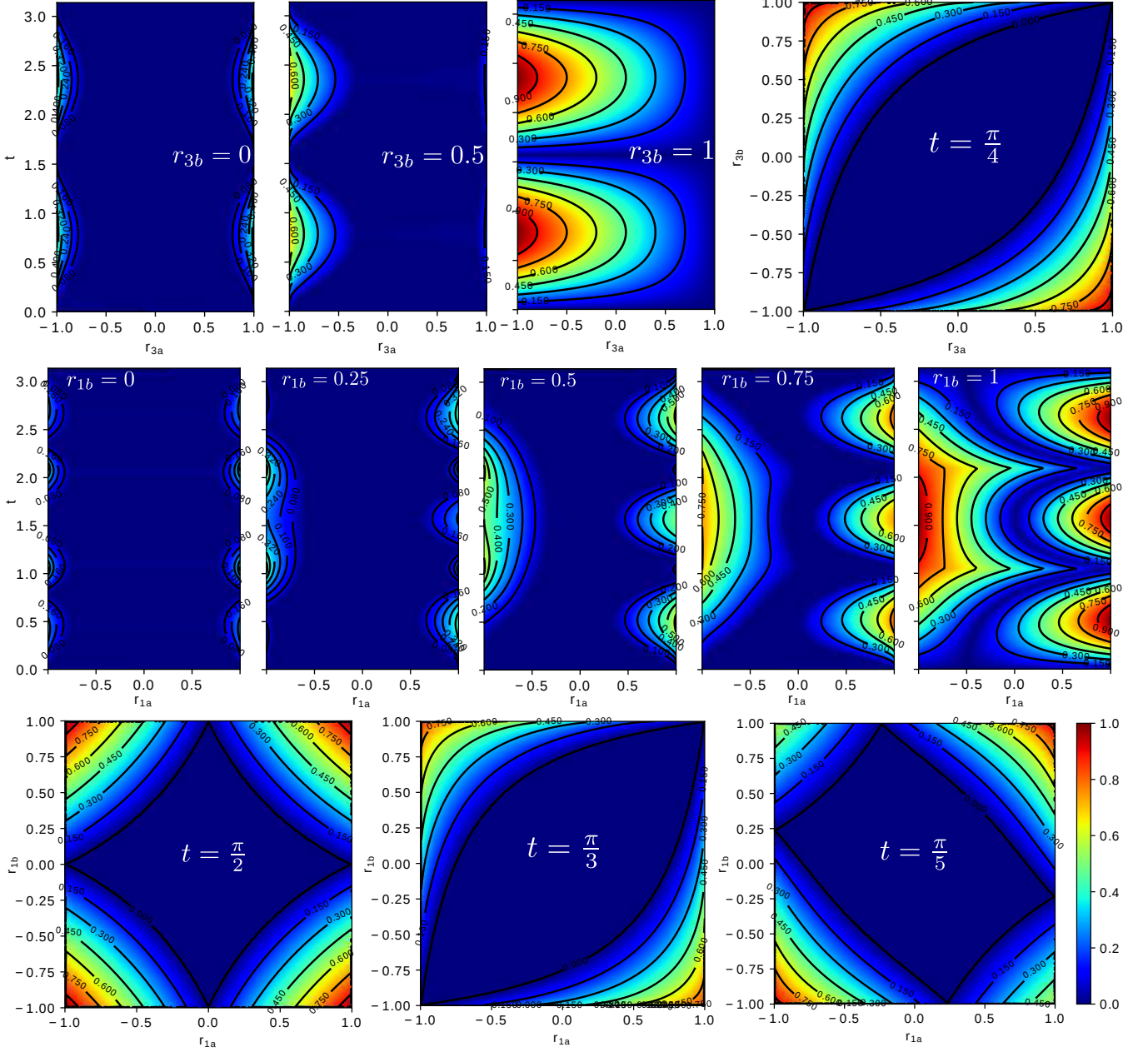


Figure 2: (color online) In the *first row* of plots is presented the entanglement of ρ_3 . The first three figures show the simple temporal dependence of E_C with time in this case. The figures for $r_{3b} = -x$ look just like those for $r_{3b} = x$ reflected in relation to the $r_{3a} = 0$ axis. As in the last subsection, the period of E_C in t is equal to π . In the last plot in this row is shown the entanglement generated by the MDI for $t = \pi/4$. Actually, maximum entanglement is obtained in this case for all $t = (2n + 1)\pi/4$ with $n \in \mathbb{N}$. In the *second and third rows* of plots is shown the entanglement of the state ρ_1 . The temporal dependence of E_C in this case (figures in the second row) is more involved than for ρ_3 . However, here also we have the pattern for E_C for $r_{1b} = -x$ equivalent to that for $r_{1b} = x$ reflected in relation to the $r_{1a} = 0$ axis. Although for ρ_1 we cannot identify instants in time giving the maximum entanglement in general, in the last three figures we show E_C as a function of the initial states for three times that should contribute the most for that general maximum.

IV. ENTANGLEMENT PRODUCTION BY THE MAGNETIC DIPOLAR INTERACTION FOR PARTIALLY ENTANGLED INITIAL STATES

In this section we shall study partially entangled states evolving under the magnetic dipolar interaction (MDI). As the computational base states $|00\rangle$ and $|11\rangle$ gain the same phase when evolved under the MDI, we shall start by regarding the initial pure state

$$|\Psi_0\rangle = \sqrt{w}|01\rangle + \sqrt{1-w}|10\rangle \quad (13)$$

with $w \in [0, 1]$. Actually, we can get $|\Psi_0\rangle$ from superpositions of $|00\rangle$ and $|11\rangle$ by applying the flip operation σ_1 to one of the dipoles before they interact. For the initial state $|\Psi_0\rangle$, the evolved state reads, up to a global phase, as follows

$$|\Psi_t\rangle = (\sqrt{w}\cos t - i\sqrt{1-w}\sin t)|01\rangle + (\sqrt{1-w}\cos t - i\sqrt{w}\sin t)|10\rangle. \quad (14)$$

The entanglement concurrence of this pure state is given by

$$E_C(|\Psi_t\rangle) = \sqrt{\sin^2 2t + 4w(1-w)\cos^2 2t}, \quad (15)$$

and is shown graphically in Fig. 3. We notice in this figure that for any value of the entanglement of the initial state, there will be points in time for which the maximum value for the entanglement is attained. Actually, we see that the equation $E_C(|\Psi_t\rangle) = 1$ is satisfied for any value of w if $t = (2n+1)\pi/4$ with $n \in \mathbb{N}$. So, if the MDI between the qubits is turned off in any of these instants of time, we shall have prepared a maximally entangled state from any of the partially entangled or product states investigated in this section.

To give an example of the effect of decreasing the purity of the initial state, let us consider $|\Psi_0\rangle$ subject to the depolarization channel [16], whose action is leaving a state alone with probability p or turning it into the maximal uncertain state with probability $1-p$, i.e., $|\Psi_0\rangle \rightarrow \rho_d = (1-p)2^{-2}\sigma_0 \otimes \sigma_0 + p|\Psi_0\rangle\langle\Psi_0|$. For this initial state, the evolved state under the MDI reads:

$$\rho_t = U_t \rho_d U_t^\dagger = (1-p)2^{-2}\sigma_0 \otimes \sigma_0 + p|\Psi_t\rangle\langle\Psi_t|. \quad (16)$$

The entanglement of this state is shown in Fig. 3 for some instants of time. As expected, the entanglement of ρ_t is that of $|\Psi_t\rangle$ diminished proportionally to $1-p$; and there are values of p below which no entanglement is generated by the MDI.

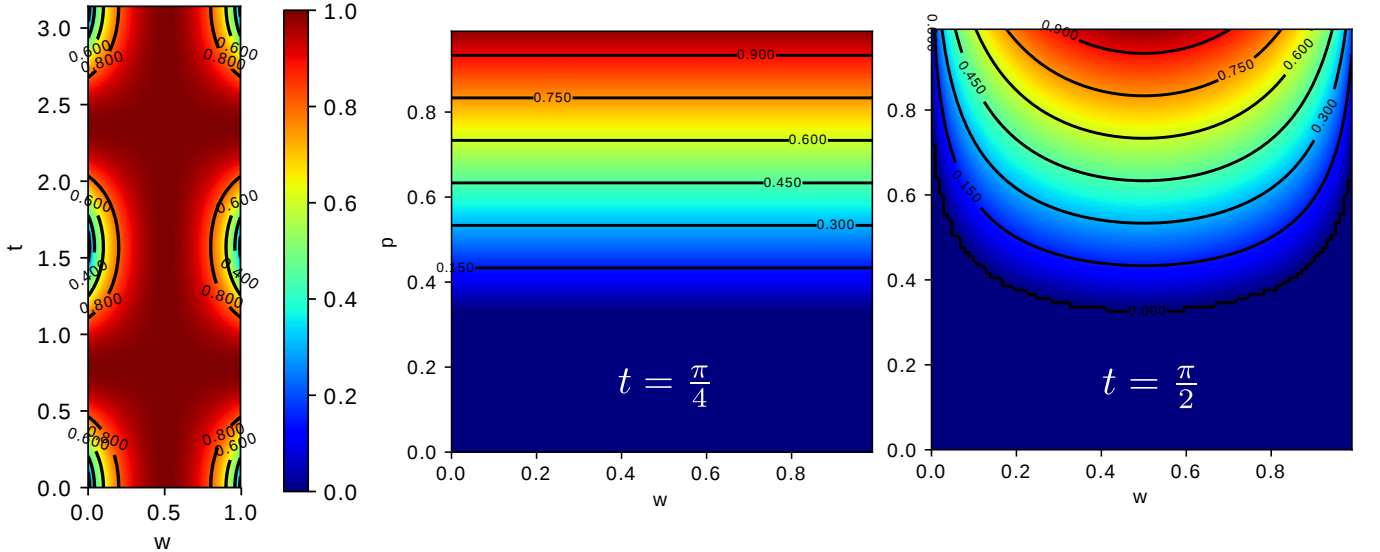


Figure 3: (color online) On the *first* plot is shown the entanglement generated by the MDI as a function of time (in units of D/\hbar) and of the parameter w , which determines the partially entangled initial state. The two plots on the *right* show the entanglement dependence on the depolarization parameter p for two instants of time.

V. CONCLUSIONS

Entanglement is an important resource in quantum information science, being essential for quantum teleportation [8, 33–35] and for its applications in quantum networks [36] and quantum computation [37]. In this article we investigated the capabilities of the magnetic dipolar interaction to generate entanglement. The MDI is a coherent operation that was shown to be capable of generating maximally entangled states from local maximally coherent or incoherent states. The symmetry of the initial state with relation to the MDI Hamiltonian was identified as a determinant property regarding entanglement production, besides the initial states coherences and/or purities. Finally, we identified conditions under which some classes of partially entangled initial states can be transformed into maximally entangled states by the MDI. We believe that the interesting dynamical properties of the MDI reported in this article can contribute to its deployment in quantum information science.

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Appendix: Dynamics of local quantum coherence for initial pure-product states

Here we use the l_1 -norm coherence [31], $C(\rho) = \sum_{j \neq k} |\langle j|\rho|k\rangle|$, to quantify quantum coherence. By taking the partial trace [38] over one of the two dipoles, whose composite state is (8), we obtain the reduced density operator $\rho_r = \text{Tr}_p(|\Psi_t\rangle\langle\Psi_t|)$. The quantum coherence of this state reads

$$\begin{aligned} 2^{-2}C^2(\rho_r) = & \alpha_a^2\beta_a^2(\alpha_b^4 + \beta_b^4)\cos^2 t + \alpha_b^2\beta_b^2(\alpha_a^4 + \beta_a^4)\sin^2 t + 2\alpha_a^2\beta_a^2\alpha_b^2\beta_b^2\cos 2t\cos 4t \\ & - \alpha_a\beta_a\alpha_b\beta_b(\alpha_a^2\beta_b^2 + \beta_a^2\alpha_b^2)\sin 2t\sin 4t. \end{aligned} \quad (\text{A.1})$$

In Fig. 4 we plot this quantity as a function of time and of the dipole a initial state for some initial states of dipole b . Comparison with Fig. 1 confirms the non-existence of a general temporal correlation between the values of coherence and entanglement.

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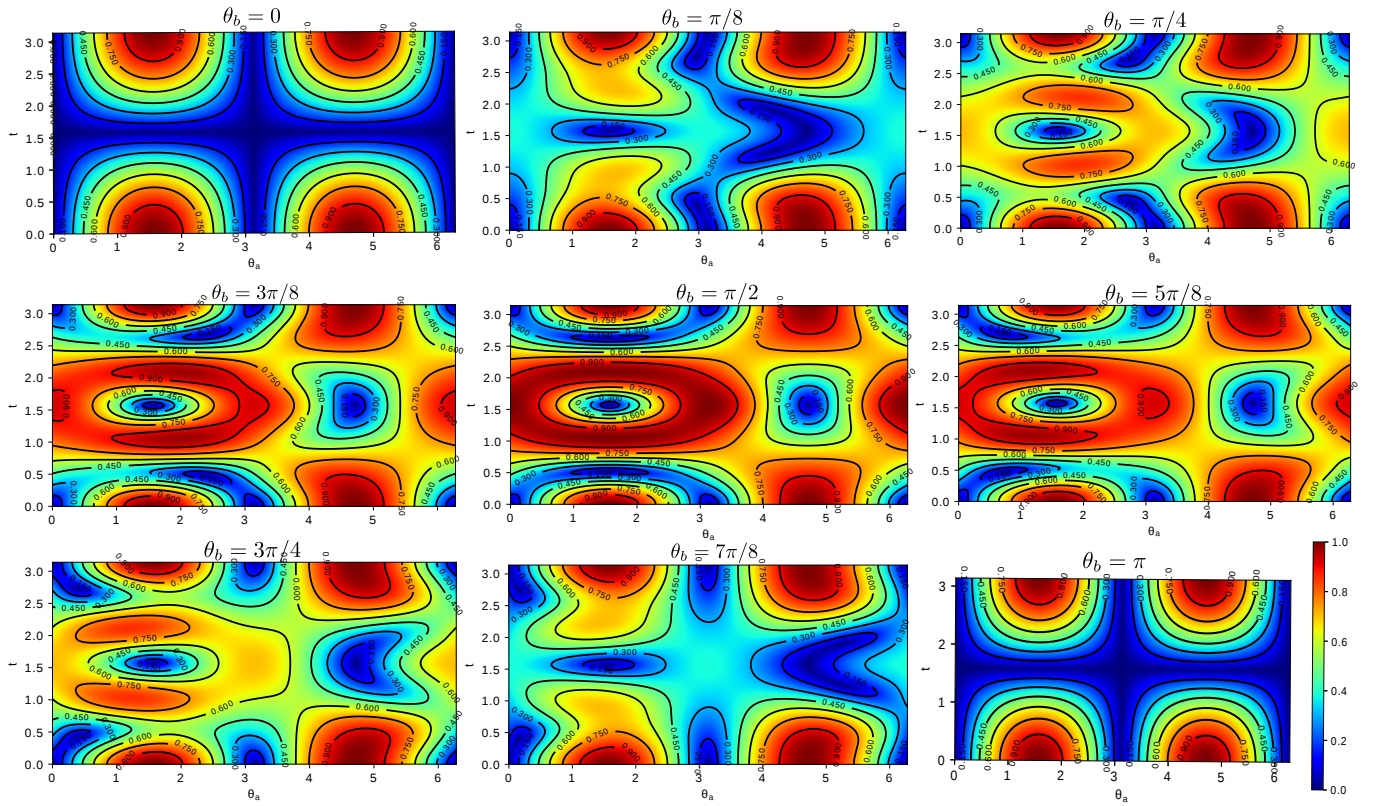


Figure 4: (color online) Local quantum coherence for initial pure-product states as function of time (in units of D/h) and of dipole a initial state for some initial states of dipole b .

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