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CORRECTION

Correction to: Learning nonlinear input–output maps with dissipative quantum systems

Jiayin Chen[1] · Hendra I. Nurdin[1]

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The original version of this article unfortunately contained errors in the proofs of Lemma 1, Lemma 5 and Proposition 1. Corrections to the proofs of Lemmas 1 and 5 and Proposition 1 are given below:

1 Correction to [1, Lemma 5]

In the proof of Lemma 5, it was incorrectly claimed that $(T^{(1)} \otimes T^{(2)})|_{H_0(\mathbb{C}^{2^n})} = T^{(1)}|_{H_0(\mathbb{C}^{2^{n_1}})} \otimes T^{(2)}|_{H_0(\mathbb{C}^{2^{n_2}})}$. However, since the constituting subsystems are taken to be non-interacting and initialized in a product state [1, §4, paragraph 1], this erroneous argument in the proof is unnecessary. A correct and simpler argument showing that $T^{(1)} \otimes T^{(2)}$ is again convergent when restricted to product states of the subsystems, and the polynomial algebra $\mathscr{F}$ consists of fading memory maps, is the following. To show the convergence property when restricted to product states of the subsystems, given any two initial product states $\rho_{1,0} \otimes \rho_{2,0}$ and $\sigma_{1,0} \otimes \sigma_{2,0}$, we have

$$\begin{aligned}
&\|\rho_{1,k} \otimes \rho_{2,k} - \sigma_{1,k} \otimes \sigma_{2,k}\|_2 \\
&\quad \leq \left\| \overleftarrow{\prod}_{j=1}^{k} \left(T^{(1)}(u_j) \otimes T^{(2)}(u_j)\right) \rho_{1,0} \otimes (\rho_{2,0} - \sigma_{2,0}) \right\|_2 \\
&\qquad + \left\| \overleftarrow{\prod}_{j=1}^{k} \left(T^{(1)}(u_j) \otimes T^{(2)}(u_j)\right) (\rho_{1,0} - \sigma_{1,0}) \otimes \sigma_{2,0} \right\|_2
\end{aligned}$$

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✉ Hendra I. Nurdin
h.nurdin@unsw.edu.au

1 School of Electrical Engineering and Telecommunications, The University of New South Wales (UNSW), Sydney, NSW 2052, Australia

$$
\begin{aligned}
&= \left\|\rho_{1,k}\right\|_2 \left\| \left(\overleftarrow{\prod}_{j=1}^{k} T^{(2)}(u_j) \right) (\rho_{2,0} - \sigma_{2,0}) \right\|_2 \\
&\quad + \left\|\sigma_{2,k}\right\|_2 \left\| \left(\overleftarrow{\prod}_{j=1}^{k} T^{(1)}(u_j) \right) (\rho_{1,0} - \sigma_{1,0}) \right\|_2 \\
&\leq 2(1-\varepsilon_2)^k \|\rho_{1,k}\|_2 + 2(1-\varepsilon_1)^k \|\sigma_{2,k}\|_2 \leq 2(1-\varepsilon_2)^k + 2(1-\varepsilon_1)^k,
\end{aligned}
$$

where the last two inequalities follow from the property that for any density operator ρ, $\|\rho\|_2 \leq 1$. Furthermore, the subsystems are initialized in a product state $\rho_{-\infty}^{(1)} \otimes \rho_{-\infty}^{(2)}$. Therefore, the terms in the output functionals $F^{T^{(1)}} + \lambda F^{T^{(2)}}$ and $F^{T^{(1)}} F^{T^{(2)}}$ are products of quantum expectations of the form $\mathrm{Tr}\left(Z^{(j_1)} \left(\overrightarrow{\prod}_{k=0}^{\infty} T^{(1)}(u_{-k}) \right) \rho_{-\infty}^{(1)} \right)$ for $j_1 = 1, \ldots, n_1$ or $\mathrm{Tr}\left(Z^{(j_2)} \left(\overrightarrow{\prod}_{k=0}^{\infty} T^{(2)}(u_{-k}) \right) \rho_{-\infty}^{(2)} \right)$ for $j_2 = 1, \ldots, n_2$. Since $T^{(1)}$ and $T^{(2)}$ satisfy the conditions in Lemma 3, these quantum expectations are continuous with respect to $\|\cdot\|_w$. The fading memory property follows from the fact that finite sums and products of continuous elements are again continuous.

2 Correction to [1, Proposition 1]

In the argument showing that $T_K(x)$ satisfies the conditions in Lemma 3 for all $x \in [0, 1]$, $\tilde{T}$ was incorrectly claimed to be a CPTP map. However, the proof only requires $\tilde{T}$ to be bounded. This is automatically satisfied since $\tilde{T}$ is a linear operator on a finite dimensional normed space.

3 Weaker condition for [1, proof of Lemma 1]

The original proof of Lemma 1 requires the conditions of Theorem 1 to hold. However, the authors noticed that Lemma 1 still holds under the weaker requirement of the convergence property defined in Definition 1. This implies that a convergent CPTP map induces a unique filter. To see this, for any $\rho \in \mathscr{D}(\mathbb{C}^{2^n})$ and $j \leq m$,

$$
\begin{aligned}
\|S_j - S_m\|_2 &= \|T(u_k)T(u_{k-1}) \cdots T(u_{k-j})(\rho - T(u_{k-j-1}) \cdots T(u_{k-m})\rho)\|_2 \\
&= \|T(u_k)T(u_{k-1}) \cdots T(u_{k-j})(\rho - \rho')\|_2.
\end{aligned}
$$

By the convergence property, for any $\varepsilon > 0$, there exists $N(\varepsilon) \in \mathbb{N}$ such that for all $j, m \geq N$, $\|S_j - S_m\|_2 < \varepsilon$. Therefore, S_j is Cauchy and $\lim_{j\to\infty} S_j$ exists due to the completeness of $(\mathscr{D}(\mathbb{C}^{2^n}), \|\cdot\|_2)$. It also follows from this argument that $\lim_{j\to\infty} S_j$ is independent of the initial choice of $\rho \in \mathscr{D}(\mathbb{C}^{2^n})$.

References

1. Chen, J., Nurdin, H.I.: Learning nonlinear input–output maps with dissipative quantum systems. Quantum Inf. Process. **18**(7), 198 (2019)

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