

Quantum gravity and the square of Bell operators

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Abstract The Bell's inequality is a strong criterion to distinguish classical and quantum mechanical aspects of reality. Its violation is the net effect of the existence of non-locality in systems, an advantage for quantum mechanics (QM) over classical physics. The quantum mechanical world is under the control of the Heisenberg uncertainty principle (HUP) that is generalized by quantum gravity (QG) scenarios, called generalized uncertainty principle (GUP). Here, the effects of GUP on the square of Bell operators of qubits and qutrits are studied. The achievements claim that the violation quality of the square of Bell inequalities may be a tool to get a better understanding of the quantum features of gravity. In this regard, it is obtained that the current accuracy of the Stern-Gerlach experiments implies upper bounds on the values of the GUP parameters.

Keywords Quantum gravity · Quantum non-locality

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1 Introduction

The pioneering work of Einstein-Podolski-Rosen (EPR) reveals the non-local feature of physical realities [1], a property which leads to the violation of Bell's inequality [2]. One cornerstone of the EPR thought experiment is the uncertainty principle arisen from the non-commutativity of position (x) and momentum (p) operators [1, 3, 4, 5]. In order to make this point more clear, let us consider the CHSH (Clauser-Horne-Shimony) form of Bell's inequality [6]

$$\begin{aligned}\langle \hat{B}_{CHSH} \rangle &= \langle \hat{a}\hat{b} + \hat{a}\hat{b}' + \hat{a}'\hat{b} - \hat{a}'\hat{b}' \rangle \\ &= \langle \hat{a}(\hat{b} + \hat{b}') + \hat{a}'(\hat{b} - \hat{b}') \rangle \\ &= \pm 2,\end{aligned}\tag{1}$$

in which \hat{a}, \hat{a}' and \hat{b}, \hat{b}' are operators, employed by **Alice** and **Bob**, respectively, with eigenvalues ± 1 that satisfy $\hat{a}^2 = \hat{a}'^2 = \hat{b}^2 = \hat{b}'^2 = \hat{1}$. It is also obvious that operators of different particles commute with each other. The square of Eq. (1) ($\equiv \hat{B}_{CHSH}^2 = \hat{B}_{CHSH} \cdot \hat{B}_{CHSH}$) is also obtained as [5]

$$\hat{B}_{CHSH}^2 = 4\hat{1}_a \otimes \hat{1}_b - [\hat{a}, \hat{a}'] \otimes [\hat{b}, \hat{b}'],\tag{2}$$

which exposes the vital role of commutators of employed operators in \hat{B}_{CHSH}^2 . Indeed, if operators commute with each other, then we have $\hat{B}_{CHSH}^2 = 4\hat{1}_a \otimes \hat{1}_b$ which happens whenever there is not any non-locality [5]. Otherwise for non-commutative variables, it is not correct. For example, by considering the Pauli matrices, where their commutator is $[\hat{\sigma}_j, \hat{\sigma}_k] = 2i\epsilon_{jkl}\hat{\sigma}_l$, the maximum violation of Bell's inequality is $\langle \hat{B}_{CHSH} \rangle = \sqrt{\langle \hat{B}_{CHSH}^2 \rangle} = \sqrt{8} = 2\sqrt{2}$. This value violates the inequality $\langle \hat{B}_{CHSH} \rangle \leq 2$, a strong signal to non-locality. Here, ϵ_{jkl} denotes the antisymmetric Levi-Civita tensor.

In fact, it is Heisenberg who has firstly noted that the uncertainty relations constrain the knowledge stored in a quantum mechanical system [7]. This property causes non-locality and thus the violation of CHSH inequality [3, 8]. Finally, it is worth to mention that there are numerous attempts to investigate the violation of CHSH inequality theoretically [6, 9] and experimentally [10], and for a general review, one can see Refs. [11, 12]. It also seems that this inequality is beneficial in the relativistic regimes [13, 14, 15, 16, 17, 18]. In fact, the problem of the effects of relativity on non-locality and entanglement goes back to the pioneering work of Peres et al. [19].

In the above case, including two-dimensional systems such as the electron spin, the preassumption of ± 1 for the outcomes of measurements is a vital condition. In the case of d -dimensional systems such as those include orbital angular momentum, the story becomes more complicated. In these cases, it seems that the probabilistic versions of Bell's inequality are more suitable [20, 21]. In fact, the measurement outcomes are not essentially ± 1 which limits the applicability of the square of the existent operational versions (in comparison

with Eq. (2)) [5]. For simplicity, let us consider Bell's inequalities for three outcomes $\{0, 1, 2\}$, proposed by Collins et al. [20],

$$\begin{aligned}\hat{C}_{223} = & 2 - 3(\hat{a}^2 + \hat{b}'^2) + \frac{3}{4}(\hat{a}\hat{b} + \hat{a}^2\hat{b} - \hat{a}'\hat{b} - \hat{a}'^2\hat{b} - \hat{a}\hat{b}^2 \\ & + \hat{a}'\hat{b}^2 + \hat{a}\hat{b}' - \hat{a}^2\hat{b}' + \hat{a}'\hat{b}' + \hat{a}'^2\hat{b}' + \hat{a}\hat{b}'^2 - \hat{a}'\hat{b}'^2) \\ & + \frac{9}{4}(\hat{a}^2\hat{b}^2 - \hat{a}'^2\hat{b}^2 + \hat{a}^2\hat{b}'^2 + \hat{a}'^2\hat{b}'^2),\end{aligned}\quad (3)$$

where the notation \hat{C}_{223} denotes 2 parties, 2 settings, and 3 outcomes. The value is given by $\langle \hat{C}_{223} \rangle = 2(5 - \gamma^2)/3 \approx 2 \cdot 9149$ for the optimal state $|\psi\rangle = (|00\rangle + \gamma|11\rangle + |22\rangle)/\sqrt{(2 + \gamma^2)}$, where $\gamma = (\sqrt{11} - \sqrt{3})/2 \approx 0.7923$ [22]. The z component of angular momentum can be considered as an operator for which the states $|0\rangle, |1\rangle, |2\rangle$ are correspond to $m = 0, m = 1$, and $m = 2$, respectively (three outcomes $\{0, 1, 2\}$). The square of the operator \hat{C}_{223} can easily be found as [5]

$$\hat{C}_{223}^2 = 3 + (1 + \{\{\hat{a}, \hat{a}'\}\})(1 + \{\{\hat{b}, \hat{b}'\}\}), \quad (4)$$

in which $\{\{\hat{a}, \hat{a}'\}\}$ denotes the complex anti-commutator $\{\{\hat{a}, \hat{a}'\}\} = \hat{a}\hat{a}'^\dagger + \hat{a}'\hat{a}^\dagger$. In summary, all of the above cases authenticate the role of the commutation relations, or equally the uncertainty principles, in emerging the non-locality.

The quality of the violation of Bell inequality in the presence of a gravitational field is firstly studied in Ref. [23]. Additionally, the effects of curved spacetimes and also the presence of acceleration on entanglement and non-locality are investigated in various articles such as Refs. [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34]. In this regard, it is worthwhile to mention that the quantum features of gravity propose modified forms of ordinary HUP [35, 36, 37] and signal us to a minimal length [38]. Such modified forms are also proposed in optics [39].

Therefore, it is expected that QG affects our understanding of non-locality which may even give us a way to test the quantum gravity scenarios. There are several phenomenological studies on GUP which leads to modifications in several areas of QM [36]. Due to GUP, it is proposed that the commutation relations such as angular momentum operators [40] and spin algebra [40, 41] are modified which may give us a possibility to verify the Planck scale effects in low energy quantum systems [36, 37].

The aim of paper is to address the effects of quantum aspects of gravity (GUP) on the square of Bell inequalities for the systems including observables with two, and three outcomes. In the next section, we provide an introductory note on GUP, and its implications on the algebra of angular momentum, studied in Ref. [40]. The square of Bell inequality for spin-1/2 systems in the presence of GUP shall be investigated in the third section. Three-level systems together with a conclusion are also presented in the subsequent sections, respectively.

2 GUP formalism

Whenever the quantum features of gravity are considered, the generalized coordinates $\hat{\mathbf{X}}$ and $\hat{\mathbf{P}}$ emerge instead of canonical coordinates $\hat{\mathbf{x}}$, $\hat{\mathbf{p}}$, and in a quadratic model of GUP, proposed in Ref. [35], the HUP modification takes a momentum dependent quadratic term as

$$\Delta\hat{X}\Delta\hat{P} \geq \frac{\hbar}{2} \left(1 + \beta\Delta\hat{P}^2\right), \quad (5)$$

where β denotes the GUP parameter and a fundamental minimal length is obtained as $\Delta X_{min} = \hbar\sqrt{\beta}$, which is of the order of Planck's length ($l_p = \sqrt{\hbar G/c^3}$). The above GUP is obtained by using the modified Heisenberg algebra [35,36,37]

$$[\hat{X}, \hat{P}] = i\hbar(1 + \beta\hat{P}^2). \quad (6)$$

The quantum mechanical commutators are replaced by the Poisson bracket (PB) for corresponding classical variables by considering the classical limit (i.e., $\hbar \rightarrow 0$). It means that [37]

$$\frac{1}{i\hbar}[\hat{X}, \hat{P}] \rightarrow \left\{ \hat{X}, \hat{P} \right\}_{PB}, \quad (7)$$

and thus [37]

$$\left\{ \hat{X}, \hat{P} \right\}_{PB} = 1 + \beta\hat{P}^2. \quad (8)$$

To construct a general framework to study the GUP effects, we introduce a representation, called coordinate representation, in the form of

$$\begin{aligned} \hat{X} &= \hat{x}, \\ \hat{P} &= \hat{p} (1 + \beta\hat{p}^2), \end{aligned} \quad (9)$$

where $\hat{x} = (\hat{x}, \hat{y}, \hat{z})$ and $\hat{p} = (\hat{p}_x, \hat{p}_y, \hat{p}_z)$ represent the position and momenta operators in QM, respectively. This representation can connect the $(\hat{\mathbf{X}}, \hat{\mathbf{P}})$ space near Planck scale (QG) to the $(\hat{\mathbf{x}}, \hat{\mathbf{p}})$ space of QM, employed in the various works [36,37,42].

It is shown that Eq. (6) modifies the angular momentum algebra, including the orbital angular momentum and spin algebra [40,41]. In the presence of GUP, using Eq. (9) one can easily find out the modified algebra of orbital angular momentum (spin) as [40]

$$[\hat{L}_i, \hat{L}_j] = i\epsilon_{ijk}\hat{L}_k(1 + \beta\hat{P}^2). \quad (10)$$

Here, $\hat{L} = \hat{X} \times \hat{P}$. By bearing Eq. (9) and $\hat{\mathcal{L}} = \hat{x} \times \hat{p}$ in mind, one can use the above result to find out $\hat{L} = \hat{X} \times \hat{P} = \hat{\mathcal{L}}(1 + \beta\hat{p}^2)$, and thus $[\hat{L}_i, \hat{L}_j] = i\epsilon_{ijk}\hat{\mathcal{L}}_k(1 + \beta\hat{p}^2)(1 + \beta\hat{p}^2(1 + \beta\hat{p}^2)^2) \approx i\epsilon_{ijk}\hat{\mathcal{L}}_k(1 + 2\beta\hat{p}^2)$ where the latter is written by considering only the terms of the order of β , and for simplicity, we set $\hbar = 1$. It is obvious that the standard commutation relation of angular momentum is recovered as $\beta \rightarrow 0$.

3 Two-qubit systems

Now, considering the set of operators $\{\hat{A}, \hat{A}', \hat{B}, \hat{B}'\}$, with eigenvalues ± 1 satisfying the condition $\hat{A}^2 = \hat{A}'^2 = \hat{B}^2 = \hat{B}'^2 = \hat{1}$. But, here, they obey relation (10) instead of purely quantum mechanical commutator (obtained for $\beta = 0$), and by rewriting Bell's operator in Eq. (1) with these operators, one finds

$$\hat{B}_{GUP} = \hat{A} \otimes \hat{B} + \hat{A} \otimes \hat{B}' + \hat{A}' \otimes \hat{B} - \hat{A}' \otimes \hat{B}'. \quad (11)$$

Therefore, the corresponding square of Bell's operator is obtained as

$$\begin{aligned} \hat{B}_{GUP}^2 &= \hat{A}^2 \otimes \hat{B}^2 + \hat{A}^2 \otimes \hat{B}'^2 + \hat{A}'^2 \otimes \hat{B}^2 + \hat{A}'^2 \otimes \hat{B}'^2 \\ &\quad + (\hat{A}^2 - \hat{A}'^2) \otimes \{\hat{B}, \hat{B}'\}_{PB} \\ &\quad + \{\hat{A}, \hat{A}'\}_{PB} \otimes (\hat{B}^2 - \hat{B}'^2) \\ &\quad - [\hat{A}, \hat{A}'] \otimes [\hat{B}, \hat{B}'], \end{aligned} \quad (12)$$

that finally leads to

$$\hat{B}_{GUP}^2 = 4\hat{1}_A \otimes \hat{1}_B - [\hat{A}, \hat{A}'] \otimes [\hat{B}, \hat{B}'], \quad (13)$$

similar to Eq. (2). In order to get more detailed analysis, let us firstly consider a quantum state $|\varphi\rangle$, which includes spin and momentum information of the system, and the unit vectors $\vec{a} = (a_x, a_y, a_z)$ and $\vec{a}' = (a'_x, a'_y, a'_z)$ for the directions of operators \hat{A} and \hat{A}' , respectively, and $\vec{b} = (b_x, b_y, b_z)$ and $\vec{b}' = (b'_x, b'_y, b'_z)$ for \hat{B} and \hat{B}' , respectively. Based on Eq. (10) and Ref. [40], we have $\hat{S} = \hat{s}(1 + \beta\hat{p}^2)$, where \hat{s} is the spin operator in the pure quantum mechanical regime, and \hat{S}_i (spin operator in the presence of minimal length with eigenvalues $\lambda = \pm \frac{1}{2}(1 + \beta p^2)$) obey algebra (10). Therefore, defining the operator $\hat{O} \equiv \frac{\hat{S} \cdot \vec{n}}{|\vec{n}|}$, where $\vec{n} = \{\vec{a}, \vec{a}', \vec{b}, \vec{b}'\}$, we can get operators $\hat{A}, \hat{A}', \hat{B}, \hat{B}'$, respectively. In this manner, bearing in mind that spin and momentum commute with each other, $[\hat{p}_i, \hat{P}_i] = 0$, and by using Eq. (10), we easily find

$$\langle \hat{B}_{GUP}^2 \rangle = 4 + \frac{(1 + \beta P_1^2)(1 + \beta P_2^2)}{\lambda_1^2 \lambda_2^2} \langle (\sum_{i,j} a_i a'_j \epsilon_{ijk} \hat{S}_k^1) (\sum_{l,m} b_l b'_m \epsilon_{lmn} \hat{S}_n^2) \rangle, \quad (14)$$

where P_i and \hat{S}^i denote the momentum and the spin operators of the i -th particle in the QG regime, respectively. Moreover, $\lambda_i = \frac{1}{2}(1 + \beta p_i^2)$ (corresponding to the i -th particle), one finally obtains

$$\langle \hat{B}_{GUP}^2 \rangle \simeq 4 + \frac{(1 + \beta P^2)^2}{\lambda^2} \langle (\sum_{i,j} a_i a'_j \epsilon_{ijk} \hat{\sigma}_k^1) (\sum_{l,m} b_l b'_m \epsilon_{lmn} \hat{\sigma}_n^2) \rangle, \quad (15)$$

in which we considered $p_a = p_b \equiv p$ (particles have the same momentum), and $\hat{s}_i = \frac{1}{2} \hat{\sigma}_i$ (Pauli matrices) has also been used. Now, bearing Eq. (9) in mind, since $\frac{(1 + \beta P^2)^2}{\lambda^2} \approx 4 \frac{1 + 2\beta P^2}{1 + 2\beta p^2} \approx 4[1 + 4\beta^2 p^4]$, we have $\langle \hat{B}_{GUP}^2 \rangle \simeq \langle \hat{B}_{CHSH}^2 \rangle$ up to the first order of β , and

$$\langle \hat{B}_{GUP}^2 \rangle \simeq \langle \hat{B}_{CHSH}^2 \rangle + 16\beta^2 p^4 \langle (\sum_{i,j} a_i a'_j \epsilon_{ijk} \hat{\sigma}_k^1) (\sum_{l,m} b_l b'_m \epsilon_{lmn} \hat{\sigma}_n^2) \rangle, \quad (16)$$

up to the second order of β . Hence, the existence of non-zero minimum length affects the square of Bell inequality, due to the fact that the commutation relations are modified in the presence of a non-zero minimum length. In general, for quantum states like the Bell states, the maximum value of $\langle \hat{B}_{CHSH}^2 \rangle$ is achieved, as we have $\langle (\sum_{i,j} a_i a'_j \epsilon_{ijk} \hat{\sigma}_k^1) (\sum_{l,m} b_l b'_m \epsilon_{lmn} \hat{\sigma}_n^2) \rangle = 1$, that finally leads to $\langle \hat{B}_{GUP}^2 \rangle \simeq 8 + 16\beta^2 p^4$. In summary, when the quantum features of gravity become non-ignorable, \hat{B}_{GUP}^2 is the true square of the Bell operator. The difference between \hat{B}_{GUP}^2 and \hat{B}_{CHSH}^2 is mathematically due to the effects of QG on the commutation relations. It means that we may get a better understanding of spin in high energy physics by using much more accurate apparatus in the future, a result in line with previous studies (see [43] and references therein).

Here, it is worthwhile to focus on a more general GUP framework [44] in which

$$\hat{P} = \hat{p}(1 + f(\hat{p})), \quad (17)$$

where $f(\hat{p}) = \alpha \hat{p} + \beta \hat{p}^2$, α and β also denote the corresponding GUP parameters in the linear and quadratic terms, respectively. By considering this general form and following the approach of Ref. [40], the alternative of angular momentum algebra described in Eq. (10) is obtained as [40]

$$[\hat{L}_i, \hat{L}_j] = i\epsilon_{ijk} \hat{L}_k (1 + f(\hat{P})), \quad (18)$$

that finally leads to

$$\begin{aligned}
\langle \hat{B}_{GUP}^2 \rangle &\simeq 4 + 4 \frac{(1 + f(P))^2}{(1 + f(p))^2} \langle (\sum_{i,j} a_i a'_j \epsilon_{ijk} \hat{\sigma}_k^1) (\sum_{l,m} b_l b'_m \epsilon_{lmn} \hat{\sigma}_n^2) \rangle, \\
&\simeq \langle \hat{B}_{CHSH}^2 \rangle \\
&+ \left(8\alpha^2 p^2 + 16\beta^2 p^4 + \dots \right) \langle (\sum_{i,j} a_i a'_j \epsilon_{ijk} \hat{\sigma}_k^1) (\sum_{l,m} b_l b'_m \epsilon_{lmn} \hat{\sigma}_n^2) \rangle.
\end{aligned} \tag{19}$$

Therefore, up to the leading order, one easily finds $\langle \hat{B}_{GUP}^2 \rangle \simeq 8 + 8\alpha^2 p^2 + 16\beta^2 p^4$, for the Bell states leading to $\langle (\sum_{i,j} a_i a'_j \epsilon_{ijk} \hat{\sigma}_k^1) (\sum_{l,m} b_l b'_m \epsilon_{lmn} \hat{\sigma}_n^2) \rangle = 1$.

Now, to examine Eq. (19), let us consider the Stern-Gerlach experiment described in [45, 46]. This experiment includes atoms of $p^2 = 2.8 \times 10^{-26} (\frac{\text{kg.m}}{\text{s}})^2$ combined with $|\langle \frac{\hat{B}_{GUP}^2 - \hat{B}_{CHSH}^2}{\hat{B}_{CHSH}^2} \rangle| \simeq \alpha^2 p^2 + 2\beta^2 p^4$ to provide two upper bounds on α (β) as $\alpha_0 \ll 10^{13}$ ($\beta_0 \ll 10^{26}$), and $\alpha_0 \ll 10^{11}$ ($\beta_0 \ll 10^{24}$) for the splitting accuracies 10^{-1} [45], and 10^{-3} [46], respectively. Here, $\alpha_0 \equiv \alpha M_p c^2$ ($\beta_0 \equiv \beta M_p^2 c^4$), M_p denotes the Planck mass, and the obtained upper bounds are well comparable with previous reports [36, 40, 47].

4 Two qutrits

The corresponding operator for three outcomes, and its square, introduced in Eq. (3), are rewritten in the GUP framework as

$$\begin{aligned}
(\hat{C}_{223})_{GUP} &= 2 - 3(\hat{A}^2 + \hat{B}'^2) \\
&+ \frac{3}{4}(\hat{A}\hat{B} + \hat{A}^2\hat{B} - \hat{A}'\hat{B} - \hat{A}'^2\hat{B} - \hat{A}\hat{B}^2 + \hat{A}'\hat{B}^2 \\
&+ \hat{A}\hat{B}' - \hat{A}^2\hat{B}' + \hat{A}'\hat{B}' + \hat{A}'^2\hat{B}' + \hat{A}\hat{B}'^2 - \hat{A}'\hat{B}'^2) \\
&+ \frac{9}{4}(\hat{A}^2\hat{B}^2 - \hat{A}'^2\hat{B}^2 + \hat{A}^2\hat{B}'^2 + \hat{A}'^2\hat{B}'^2),
\end{aligned} \tag{20}$$

and

$$(\hat{C}_{223}^2)_{GUP} = 3 + (1 + \{\{\hat{A}, \hat{A}'\}\})(1 + \{\{\hat{B}, \hat{B}'\}\}), \tag{21}$$

respectively. Here, the operator $\hat{O} \in \{\hat{A}, \hat{A}', \hat{B}, \hat{B}'\}$ contains three outcomes $\{0, 1, 2\}$. For example, consider a particle with quantum mechanical momentum p whose angular momentum meets algebra (10), the operator $\frac{\hat{L}_z}{1 + \beta p^2}$ leads to outcomes 0, 1, 2 for states $|0\rangle, |1\rangle, |2\rangle$, respectively. Now, following Ref. [40], one can find $\{\{\hat{L}_i, \hat{L}_j\}\} = (1 + \beta \hat{P}^2)^2 \{\{\hat{l}_i, \hat{l}_j\}\}$, and hence, $\{\{\hat{O}_i, \hat{O}_j\}\} = \frac{(1 + \beta \hat{P}^2)^2}{(1 + \beta p^2)^2} \{\{\hat{l}_i, \hat{l}_j\}\}$. Bearing all of these points in mind, after some calculations, we finally get

$$\begin{aligned}
\langle (\hat{C}_{223}^2)_{GUP} \rangle &= \langle \hat{C}_{223}^2 \rangle + 4\beta^2 p^4 \underbrace{\langle \{\{\hat{a}, \hat{a}'\}\} + \{\{\hat{b}, \hat{b}'\}\} + 2\{\{\hat{a}, \hat{a}'\}\}\{\{\hat{b}, \hat{b}'\}\} \rangle}_{\langle \hat{G} \rangle} \\
&+ \mathcal{O}(\beta^3),
\end{aligned} \tag{22}$$

whenever all particles have the same momentum. Here, it is again clear that $\langle (\hat{C}_{223}^2)_{GUP} \rangle \rightarrow \langle \hat{C}_{223}^2 \rangle$ as $\beta \rightarrow 0$. Although, the usefulness of $\langle \hat{C}_{223}^2 \rangle$ in studying the non-locality is challenging [22], Eq. (22) clearly shows that $\langle (\hat{C}_{223}^2)_{GUP} \rangle$ differs from $\langle \hat{C}_{223}^2 \rangle$ meaning that this operator may be employed to investigate the predictions of QG. As an example, consider $\hat{a} = \hat{a}' = \hat{l}_z$, $\hat{b} = \hat{b}' = \hat{l}_z$, and the state $|\psi\rangle = \frac{|00\rangle + |11\rangle + |22\rangle}{\sqrt{3}}|pp\rangle$ where two particles have the same momentum p (and thus P). In this manner, $\{\{\hat{l}_z, \hat{l}_z\}\} = 2\hat{l}_z^2$ leading to $\langle \hat{C}_{223}^2 \rangle \simeq 32 \cdot 7$, and $\langle \hat{G} \rangle = 52$, and we finally have $\langle \hat{C}_{223}^2 \rangle \neq \langle (\hat{C}_{223}^2)_{GUP} \rangle$.

5 Conclusion

The belief that the world is non-local comes from the amazing EPR paper [1] that motivated Bell to introduce his inequality. Indeed, a cornerstone of the EPR argument is the role of HUP (commutation relations) in the emergence of non-locality which emerges in the square of Bell operators. On the other hand, it is believed that the quantum aspects of gravity affect HUP, and according to this proposal, we tried to shed light on the relation between non-locality and QG. Related experiments and studies may help us achieve a better understanding of gravity, and its relation with non-locality, and quantum mechanics. Indeed, the hopes to test the QG scenarios via studying its relation with non-locality can be strengthened by increasing the accuracy of related experiments such as the Stern-Gerlach apparatus.

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