

# Quantum Meet-in-the-Middle Attack on Feistel Construction

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**Abstract.** Inspired by Hosoyamada *et al.*'s work [14], we propose a new quantum meet-in-the-middle (QMITM) attack on  $r$ -round ( $r \geq 7$ ) Feistel construction to reduce the time complexity. Similar to Hosoyamada *et al.*'s work, our attack on 7-round Feistel is also based on Guo *et al.*'s classical meet-in-the-middle (MITM) attack [13]. The classic MITM attack consumes a lot of time mainly in three aspects: construct the lookup table, query data and find a match. Therefore, parallel Grover search processors are used to reduce the time of constructing the lookup table. And we adjust the truncated differentials of the 5-round distinguisher proposed by Guo *et al.* to balance the complexities between constructing the lookup table and querying data. Finally, we introduce a quantum claw finding algorithm to find a match for reducing time. The subkeys can be recovered by this match. Furthermore, for  $r$ -round ( $r > 7$ ) Feistel construction, we treat the above attack on the first 7 rounds as an inner loop and use Grover's algorithm to search the last  $r - 7$  rounds of subkeys as an outer loop. In summary, the total time complexity of our attack on  $r$ -round ( $r \geq 7$ ) is only  $O(2^{2n/3+(r-7)n/4})$  less than classical and quantum attacks. Moreover, our attack belongs to Q1 model and is more practical than other quantum attacks.

**Keywords:** Quantum meet-in-the-middle attack · Feistel construction · Quantum claw finding algorithm · Grover's algorithm.

## 1 Introduction

A Feistel network is a scheme that builds  $n$ -bit permutations from smaller, usually  $n/2$ -bit permutations or functions [10]. The Feistel-based design approach is widely used in block ciphers. In particular, a number of current and former international or national block cipher standards such as DES [5], Triple-DES [17] and Camellia [1] are Feistel ciphers.

Feistel ciphers have many constructions, and the analyzed target construction in this paper is displayed in Fig. 1. An  $n$ -bit state is divided into  $n/2$ -bit

halves denoted by  $a_i$  and  $b_i$ , and the state is updated by iteratively applying the following two operations:

$$a_{i+1} \leftarrow a_i \oplus F(k_i \oplus b_i), \quad b_{i+1} \leftarrow a_i,$$

where  $F$  is a public function and  $k_i$  is a subkey with  $n/2$  bits. Note that, the target Feistel construction is also called Feistel-2 in Ref. [13], or called Feistel-KF in Ref. [18]. For brevity, we simply call it the Feistel construction.

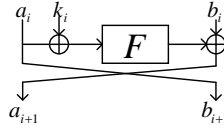


Fig. 1: Our target Feistel construction.

For the target Feistel construction, Guo *et al.* [13] absorbed the idea of meet-in-the-middle attack proposed by Demirci and Selçuk [6], and applied it on the attack of the Feistel construction. They firstly computed all the possible sequences constructed from a  $\delta$ -set such that a pair of messages satisfy the proposed 5-round distinguisher. Then, they collected enough pairs of plaintexts and corresponding ciphertexts, and guessed the subkeys out of the distinguisher to compute the corresponding sequence for each pair. Finally, if the computed sequence from each pair of collected plaintexts belongs to the precomputed sequences from the distinguisher, the guessed subkeys are correct; otherwise, they are wrong. In their attack on 6-round Feistel construction, its time complexity is  $O(2^{3n/4})$ . Later, Zhao *et al.* combined Guo *et al.*'s work and pairs sieve procedure to attack on 7-round Feistel construction with  $O(2^n)$  time. Besides, some other attacks penetrate up to 6 rounds, such as impossible differentials [24], all-subkey recovery [15, 16] and integral-like attacks [28].

In the quantum setting, cryptanalysts hope to take advantage of quantum computing to further reduce attack complexity. For symmetric cryptosystems, Grover's algorithm [12] can provide a quadratic speedup over exhaustive search on keys. In addition, some other quantum algorithms have been applied to the analysis of block ciphers and achieved good results, such as Simon's algorithm [27] and Bernstein-Vazirani algorithm [2]. Many block ciphers have been evaluated for the security in the quantum setting, e.g. against Even-Mansour cipher [22], MACs [19, 25], AEZ [26], AES-COPA [30], FX construction [23] and so on. To evaluate the adversary's ability, Kaplan divides these quantum attacks into two attack models: Q1 model and Q2 model [20].

1. **Q1 model.** The adversary is allowed only to make classical online queries and performs quantum offline computation.
2. **Q2 model.** The adversary can make quantum superposition online queries for cryptographic oracle and performs quantum offline computation.

Obviously, the adversary is more practical in Q1 model, and more powerful in Q2 model.

Many researches have analyzed the security of the Feistel construction in Q1 or Q2 model. In 2010, Kuwakado *et al.* [21] proposed a quantum 3-round Feistel distinguisher and used Simon's algorithm to recover subkeys in Q2 model. Its time complexity only needs  $O(n)$  because of Simon's algorithm. Based on Kuwakado *et al.*'s work, Dong *et al.* [8, 9] used Grover's algorithm to search the last  $r - 3$  rounds subkeys of  $r$ -round Feistel construction. Its time complexity is  $O(2^{0.25nr-0.75n})$ .

Different from Kuwakado *et al.*'s quantum 3-round Feistel distinguisher, Xie *et al.* [29] and Ito *et al.* [18] proposed new quantum Feistel distinguishers respectively. Xie *et al.* [29] used Bernstein-Vazirani algorithm instead of Simon's algorithm to recover subkeys. However, this modification cause a slight increase in complexity with  $O(n^2)$  time. Ito *et al.* [18] proposed a new 4-round Feistel distinguisher, and used Grover's algorithm to search the last  $r - 4$  rounds subkeys similarly to Dong *et al.*'s attack. Besides, Dong *et al.* [7] and Bonnetain *et al.* [3] proposed quantum slide attacks on Feistel construction by Simon's algorithm respectively.

The quantum attacks listed above are all Q2 models, Hosoyamada *et al.* [14] proposed a new quantum attack on 6-round Feistel construction that belongs to Q1 model. They gave a quantum claw finding algorithm to find a match between two tables, where two tables are constructed by two phases of Guo *et al.*'s attack: one table from precomputation and the other one from collected pairs. If there exists one match, the guessed subkeys are correct.

**Contribution.** Inspired by Hosoyamada *et al.*'s work [14], we firstly propose a new quantum meet-in-the-middle attack (QMITM) on 7-round Feistel construction to reduce time complexity in Q1 model. Similar to Guo *et al.*'s attack, we divide our attack into two phases: pre-computation phase and queried-data analysis phase. In the pre-computation phase, we use parallel Grover search processors to compute all possible sequence through the 5-round distinguisher, which are stored in a classical lookup table. To balance the complexity between pre-computation phase and queried-data analysis phase, we adjust truncated differentials of the 5-round distinguisher. Then, we transform these classical queried-data to a superposition state by quantum random access memory, and compute corresponding sequence for each pair by quantum operations. Finally, we use quantum claw finding algorithm [4] to find a claw between the table and the superposition state and recover subkeys through this found claw. In summary, the time complexity of our attack is only  $O(2^{2n/3})$ , less than classical attacks. And its data complexity is  $O(2^{2n/3})$  with  $O(2^{5n/6})$  classical memory and qubits.

Furthermore, we treat the above attack on 7-round as a new distinguisher. Similar to Dong *et al.*'s attack [9], we use Grover's algorithm to search the last  $r - 7$  rounds of subkeys as an outer loop with running time  $2^{(r-7)n/4}$ , and the above attack on the first 7 rounds as an inner loop with time  $2^{2n/3}$ . In total, the time complexity is only  $O(2^{(r-7)n/4+2n/3})$  less than classical attacks

and quantum attacks. Besides, our attack belongs to Q1 model, which is more practical. The detailed comparison with other attacks is shown in Table 1.

Table 1: Comparison with classical and quantum attacks on Feistel construction

Ref.	Setting	Round	Time	Data	Classical memory	Qubits
[13]	Classical	6	$O(2^{3n/4})$	$O(2^{3n/4})$	$O(2^{3n/4})$	-
[31]	Classical	7	$O(2^n)$	$O(2^n)$	$O(2^{3n/4})$	-
[7]	Q2	4	$O(n)$	$O(2^{n/2})$	-	$O(n)$
[3]	Q2	4	$O(n^2)$	$O(2^{n/2})$	-	$O(n)$
[9]	Q2	$r$ ( $r \geq 3$ )	$O(n2^{0.25nr-0.75n})$	$O(2^{n/2})$	-	$O(n^2)$
[18]	Q2	$r$ ( $r \geq 4$ )	$O(n2^{0.25nr-n})$	$O(2^{n/2})$	-	$O(n^2)$
[29]	Q2	3	$O(n^2)$	$O(2^{n/2})$	-	$O(n)$
[14]	Q1	6	$O(2^{n/2})$	$O(2^{n/2})$	-	$O(2^{n/2})$
Ours	Q1	$r$ ( $r \geq 7$ )	$O(2^{2n/3+(r-7)n/4})$	$O(2^{2n/3})$	$O(2^{5n/6})$	$O(2^{5n/6})$

This paper is organized as follows. Sect. 2 provides a brief description of related quantum algorithms. And an overview of Guo *et al.*'s work is presented in Sect. 3. We propose a new quantum meet-in-the-middle attack on 7-round Feistel construction in Sect. 4. Then, the attack is furthered on  $r$ -round ( $r > 7$ ) as shown in Sect. 5, followed by a conclusion in Sect. 6.

## 2 Related Quantum Algorithms

**Grover's Algorithm.** Grover's algorithm, or the Grover search, is one of the most famous quantum algorithms, with which we can obtain quadratic speed up on database searching problems compared to the classical algorithms. It was originally developed by Grover [12].

*Problem 1.* Suppose a function  $f : \{0, 1\}^n \rightarrow \{0, 1\}$  is given as a black box, with a promise that there is  $x$  such that  $f(x) = 1$ . Then, find  $x$  such that  $f(x) = 1$ .

The process of solving above problem is presented simply as below by Grover's algorithm.

1. Start with a uniform superposition  $|\varphi\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle$ .
2. The unitary transformation  $(2|\varphi\rangle\langle\varphi| - I)\mathcal{O}_f$  applied on  $|\varphi\rangle$  is iterated  $\sqrt{2^n}$  times, where the quantum oracle  $\mathcal{O}_f$  defined by the function  $f$  is a quantum amplitude flip operation followed as bellow:

$$\mathcal{O}_f |x\rangle \rightarrow \begin{cases} -|x\rangle, & \text{if } f(x) = 1, \\ |x\rangle, & \text{otherwise.} \end{cases} \quad (1)$$

3. The final measurement gives the  $x$  such that  $f(x) = 1$  with an overwhelming probability.

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**Algorithm 1** Quantum Claw Finding between Two Arbitrary Functions  $f$  and  $g$  [4]

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**Input:** Sets  $X$  and  $Y$  of size  $N$  and  $M$ , respectively.

**Output:** A pair of  $(x, y) \in X \times Y$  such that  $f(x) = g(y)$  if they exist.

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1. Select a random subset  $A \subseteq [N]$  of size  $l$  ( $l \leq \min\{N, \sqrt{M}\}$ ).
  2. Select a random subset  $B \subseteq [M]$  of size  $l^2$ .
  3. Sort the elements in  $A$  according to their  $f$ -value.
  4. For a specific  $b \in B$ , check if there is an  $a \in A$  such that  $(a, b)$  is a claw by using classical binary search on the sorted version of  $A$ . Combine this with quantum search on the elements of  $B$  to find a claw in  $A \times B$ .
  5. Apply amplitude amplification on steps 1-4.
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In summary, Grover's algorithm can solve Problem 1 with  $O(2^{n/2})$  evaluations of  $f$  using  $O(n)$  qubits. If we have  $2^p$  independent small quantum processors with  $O(n)$  qubits, by parallel running  $O(\sqrt{\frac{2^n}{2^p}})$  iterations on each small quantum processor, we can find  $x$  such that  $f(x) = 1$  with high probability. This parallelized algorithm runs in time  $O(\sqrt{\frac{2^n}{2^p}})$ .

**Amplitude Amplification.** Suppose there is a quantum algorithm  $QSearch$  with the following properties. Let  $\mathcal{A}$  be any quantum algorithm that uses no measurements, and boolean function  $f : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$ .  $p$  represents the success probability of  $\mathcal{A}$  of finding a solution (i.e., the probability of outputting  $x$  s.t.  $f(x) = 1$ ). The algorithm  $QSearch$  needs to execute  $O(1/\sqrt{p})$  times  $\mathcal{A}$  and  $\mathcal{A}^{-1}$  to find one solution if  $p > 0$ , otherwise it keeps running.

Generally speaking,  $QSearch$  needs to iterate a certain number of unitary transformation  $Q = -\mathcal{A}\mathcal{S}_0\mathcal{A}\mathcal{S}_f$  on the initial state  $\mathcal{A}|0\rangle$ , where  $\mathcal{S}_f|x\rangle = (-1)^{f(x)}|x\rangle$ ,  $\mathcal{S}_0|0\rangle = -|0\rangle$  and  $\mathcal{S}_0|x\rangle = |x\rangle$  ( $x \neq 0$ ). The algorithm  $QSearch$  needs to iterate  $O(1/\sqrt{p})$  times  $Q$  to get a solution with high probability a least  $\max\{p, 1-p\}$ . We can obviously find that amplitude amplification can accelerate the search algorithm, and Grover's algorithm is a special case of amplitude amplification.

**Quantum Claw Finding Algorithm.** Quantum claw finding algorithm aims to solve the claw finding problem which is defined in detail as below.

*Problem 2.* Suppose there are 2 functions  $f : X \rightarrow Z$  and  $g : Y \rightarrow Z$ , where  $|X| = M$  and  $|Y| = N$ . If there is a pair  $(x, y) \in X \times Y$  such that  $f(x) = g(y)$ , then the pair is called a claw of the functions  $f$  and  $g$ .

To solve above problem, Buhrman et al. [4] gave the first quantum claw finding algorithm based on amplitude amplification, which is shown in Algorithm 1.

In Algorithm 1, Step 3 requires classical sorting with  $O(2^l \cdot l)$  comparisons and Step 4 needs  $O(\sqrt{|B|} \log |A|) = O(l \log l)$  comparisons, since checking if there is an  $A$ -element colliding with a given  $b \in B$  takes  $O(\log |A|) = O(\log l)$  comparisons via binary search on the sorted  $A$ , and quantum search needs  $O(\sqrt{|B|}) = O(l)$  to find a  $B$ -element that collides with an element occurring

in  $A$ . In general, the time complexity of steps 1-4 is  $O(l \log l)$ . The probability of getting one claw  $(x, y) \in A \times B$  in the first four steps is  $p = l^3/2NM$ . Therefore, the amplitude amplification of Step 5 requires an expected  $O(\sqrt{NM/l^3})$  iterations of steps 1-4. In total, the time complexity of steps 1-5 is  $O(\sqrt{\frac{NM}{l}} \log l)$ . When  $N \leq M \leq N^2$ ,  $O(\sqrt{\frac{NM}{l}} \log l) \approx O(N^{1/2} M^{1/4} \log N)$ . And when  $M > N^2$ ,  $O(\sqrt{\frac{NM}{l}} \log l) \approx O(M^{1/2} \log N)$ .

**Quantum Random Access Memory.** A quantum random access memory (QRAM) is a quantum analogue of a classical random access memory (RAM), which uses  $n$ -qubit to address any quantum superposition of  $2^n$  (quantum or classical) memory cells [11]. The QRAM is modeled as an unitary transformation  $U_{QRAM}$  such that

$$\sum_i a_i |i\rangle_{addr} \xrightarrow{U_{QRAM}} \sum_i a_i |i\rangle_{addr} |D_i\rangle_{data}, \quad (2)$$

where  $\sum_i a_i |i\rangle_{addr}$  is a superposition of addresses and  $D_i$  is the content of the  $i$ th memory cell.

### 3 Overview of Classical Meet-in-the-Middle Attack on 6-Round Feistel Constructions

Our proposed quantum meet-in-the-middle attack on Feistel constructions is based on Guo's work [13], so we first briefly introduce the framework of Guo's attack.

#### 3.1 Attack Idea

The MITM attack generally consists of the distinguisher and the key-recovery parts as illustrated in Fig. 2. Suppose that a truncated differential is specified to the entire cipher and the plaintext difference  $\Delta P$  propagates to the input difference  $\Delta X$  of the distinguisher with probability  $p_1$ . Similarly, from the other direction, the ciphertext difference  $\Delta C$  propagates to the output difference  $\Delta Y$  of the distinguisher with probability  $p_2$ . Generally, the attack consists of two phases: precomputation and queried-data analysis.

In the pre-computation phase, the adversary firstly enumerates all the possible differential characteristics that can satisfy the truncated differential of the distinguisher. Suppose that there exit  $N_c$  such characteristics. Let  $(X, X')$  be the input pair values for each characteristic with difference  $\Delta X$ . Then, the adversary generates a  $\delta$ -set that contains  $X_1, X_2, \dots, X_\delta$ , where  $X_i = X \oplus i$  ( $i = 1, 2, \dots, \delta$ ). Let  $Y_1, Y_2, \dots, Y_\delta$  be the corresponding values at the output of the distinguisher. And the differences  $\Delta_i$  between  $Y$  and  $Y_i$  for  $i = 1, 2, \dots, \delta$  make up a sequence, called  $\Delta$ -sequence ( $\Delta$ -sequence =  $\Delta_1 || \Delta_2 || \dots || \Delta_\delta$ ). Note that the size of the

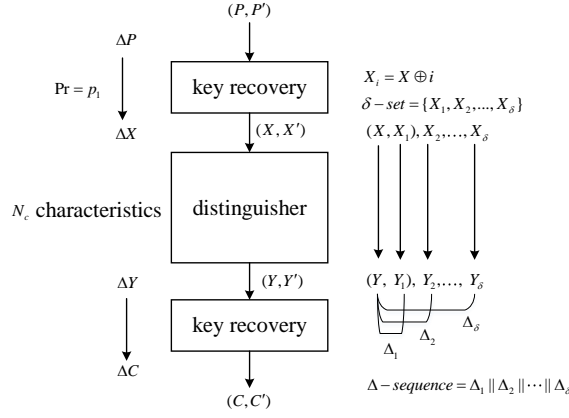


Fig. 2: Overview of the MITM attack.

difference  $\Delta_i$  is  $\lambda$  bits. In the end,  $N_c$   $\Delta$ -sequences of the size  $\lambda\delta$  bits would be stored in the Table  $T_\delta$ .

In the queried-data analysis phase, the adversary collects  $(p_1 p_2)^{-1}$  pairs of plaintexts with the difference  $\Delta P$  and their corresponding ciphertexts with the difference  $\Delta C$ . One pair of collected pairs, with high probability, satisfies  $\Delta X$  and  $\Delta Y$  at the input and output of the distinguisher, respectively. Thus, for each of  $(p_1 p_2)^{-1}$  paired values, the adversary guesses subkeys for the key recovery rounds such that  $\Delta X$  and  $\Delta Y$  appear after the first and the last key recovery parts, respectively. Then, for each pair and the guessed subkeys,  $P$  is modified to  $P_0$  and the other  $P_1, P_2, \dots, P_\delta$  are computed by generated  $\delta$ -set and the guessed subkeys. And these plaintexts are queried to obtain their corresponding ciphertexts. Next, the adversary partially decrypt these ciphertexts with the guessed subkeys, and the  $\Delta$ -sequence is computed at the output of the distinguisher. Finally, those  $\Delta$ -sequences are matched the table  $T_\delta$ , if the analyzed pair is a right pair and the guessed subkeys are correct, then a match will be found. Otherwise, a match will not be found as long as  $(p_1 p_2)^{-1} N_c \times 2^{-\lambda\delta} \ll 1$ .

### 3.2 Application on 6-Round Feistel Construction

**Pre-computation phase.** Guo *et al.* [13] give the 5-round distinguisher, which is illustrated in Fig. 3(a). The input and output differences of the 5-round distinguisher are defined as  $0||X$  and  $X'||0$ , where  $X, X' \in \{0, 1\}^{n/2}$ ,  $X \neq X'$  and the block size is  $n$ . For a given  $X, X'$ , the 5-round differential characteristics can be fixed to

$$(0||X) \xrightarrow{1stR} (X||0) \xrightarrow{2ndR} (Y||X) \xrightarrow{3rdR} (X'||Y) \xrightarrow{4thR} (0||X') \xrightarrow{5thR} (0||X'),$$

where  $Y$  represents the output difference of the 2nd round-function  $F_{i+2}$  and has  $2^{n/2}$  possible values. And the output difference of the 3rd round-function  $F_{i+3}$  is  $X''(X'' = X \oplus X')$ .

For one choice of  $X, X'$ , the number of the 5-round differential characteristics satisfying such input and output differences is  $2^{n/2}$ , i.e., the number of corresponding  $\Delta$ -sequence for the left-half of the distinguisher's output is  $2^{n/2}$  proved by the Proposition 1 in Ref. [13]. To compute  $\Delta$ -sequence, we need to know the input values of the middle 3 rounds round-functions  $F_{i+2}, F_{i+3}$ , and  $F_{i+4}$ . According to the proof of the Lemma 1 in Ref. [13], for each  $Y$ , both input and output differences of  $F_{i+2}, F_{i+3}$ , and  $F_{i+4}$  are fixed, which suggests that the paired values during the round-function are fixed to one choice on average. Thus, the adversary needs to construct three tables that record the input values of  $F_{i+2}, F_{i+3}$ , and  $F_{i+4}$ , for each  $Y$  and one given  $X, X'$ , respectively. Finally, the computed  $\Delta$ -sequences are stored in Table  $T_\delta$  for each  $Y$  and one given  $X, X'$ , whose complexity is  $2^{n/2}$  in both time and memory.

To balance the complexities between the pre-computation phase and the queried-data analysis phase, Guo *et al.* iterate the above analysis for  $2^{n/4}$  different choices of  $X'$ . They assume that the values of  $X'$  differ in the last  $n/4$  bits and are the same in the remaining  $n/4$  bits. Hence, the entire complexity of the pre-computation phase is  $O(2^{3n/4})$  in both time and memory.

**Queried-data analysis phase.** Guo *et al.* append 1-round before the 5-round distinguisher to attack the 6-round Feistel construction, as illustrated in the Fig. 3(b). Firstly, the adversary prepares 2 sets of plaintexts in the form of  $\{(m||0), (m||1), \dots, (m||2^{n/2-1})\}$  and  $\{(m \oplus X||0), (m \oplus X||1), \dots, (m \oplus X||2^{n/2-1})\}$  respectively, where  $m$  is a randomly chosen  $n/2$ -bit constant. These plaintexts can compose  $2^n$  pairs, and only  $2^{n/4}$  pairs satisfy  $\Delta C$  in the corresponding ciphertexts for  $2^{n/4}$  choices of  $X'$ . By iterating this procedure  $2^{n/4}$  times for different choice of  $m$ ,  $2^{n/2}$  pairs satisfying  $\Delta C$  are collected. Due to that the probability  $p_1$  that a randomly chosen plaintext pair with the difference  $X||*$  satisfies the difference  $0||X$  after 1 round is  $2^{-n/2}$ , one pair will satisfy the entire differential characteristic.  $*$  can be any  $n/2$ -bit difference.

Since we assume that the input difference of the distinguisher is  $0||X$  for each pair, the input and output differences of  $F_0$  are fixed,  $X$  and  $*$ , which will fix one key candidate for  $k_0$  uniquely. The adversary computes a set of plaintexts by the  $\delta$ -set and the key candidate for each pair, and queries their corresponding ciphertexts. Finally, The adversary computes the corresponding  $\Delta$ -sequence and matches  $T_\delta$ . If there is a match, the key candidate for  $k_0$  is correct and the  $Y$  can be known. The other keys are trivially recovered from the second round one by one. Note that  $p_1 = 2^{-n/2}$ ,  $p_2 = 1$ ,  $N_c = 2^{n/2}$ ,  $\lambda = n/2$ , hence,  $\delta = 3$  is sufficient to filter out all the wrong candidates.

## 4 Quantum Meet-in-the-Middle Attack on 7-round Feistel Construction

To attack 7-round Feistel, we append 1-round before and after the 5-round distinguisher respectively, and the differential characteristic of the 7-round Feistel that we need is illustrated in the Fig. 3 (c). Similar to classic meet-in-the-middle



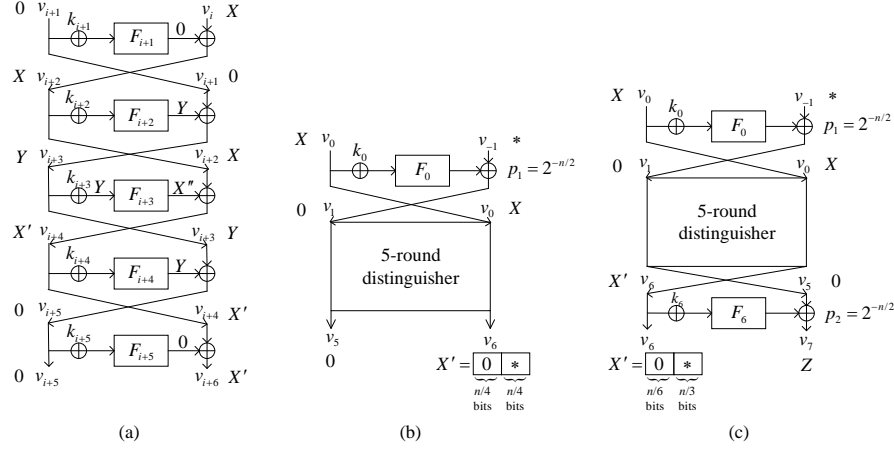


Fig. 3: (a) The 5-round distinguisher. (b) 6-round Feistel construction for the key recovery. (c) 7-round Feistel construction for the key recovery.  $v_i$  and  $F_i$  represent the input value and the round-function of each round, respectively.  $X'' = X' \oplus X$  and  $X' \neq X$ .  $Z$  is the difference at  $v_7$ .

attack, we also divide our attack into two phase: pre-computation phase and queried-data analysis phase.

#### 4.1 Pre-computation Phase

Similar to the classical MITM on 6-round Feistel, we need to calculate the  $\Delta$ -sequence based on the distinguisher and choose a fixed  $X$  value. But,  $2^{n/3}$  choices of  $X'$  are considered, to balance the complexities between the pre-computation phase and the queried-data analysis phase. Without loss of generality, assume that the values of  $X'$  differ in the last  $n/3$  bits and are zero in the remaining  $n/6$  most significant bits (MSBs). We define a function  $\mathcal{F} : X' \times Y \rightarrow \Delta$ -sequence, where  $X', Y \in \{0, 1\}^{n/2}$ ,  $0 \leq X' \leq 2^{n/3} - 1$ ,  $0 \leq Y \leq 2^{n/2} - 1$  and  $\Delta$ -sequence  $\in \{0, 1\}^{\delta n/2}$ .

For a given  $X'$  and  $Y$ , assume that a pair of plaintexts  $(m, m \oplus 0||X)$  satisfy the differential characteristic of the distinguisher,  $t_{i+2}$ ,  $t_{i+3}$  and  $t_{i+4}$  are the input values of  $F_{i+2}$ ,  $F_{i+3}$  and  $F_{i+4}$ . Let us consider a new pair of plaintexts  $(m, m \oplus 0||j)$  ( $j \in \{1, 2, \dots, \delta\}$ ) and compute the corresponding  $\Delta_j$  that is the difference at  $v_{i+5}$ .

Since, the output difference  $\Delta F_{i+1}^{\mathcal{O}}$  of the first round-function  $F_{i+1}$  in the distinguisher is always 0. So,  $\Delta v_{i+2} = \Delta v_i = j$ . In the second round, due to the input value  $F_{i+2}^{\mathcal{I}} = t_{i+2}$  and  $\Delta F_{i+2}^{\mathcal{I}} = j$ , then,  $\Delta F_{i+2}^{\mathcal{O}} = F_{i+2}(t_{i+2}) \oplus F_{i+2}(t_{i+2} \oplus j)$ . And in the 3rd round,  $\Delta F_{i+3}^{\mathcal{O}} = F_{i+3}(t_{i+3}) \oplus F_{i+3}(t_{i+3} \oplus \Delta F_{i+2}^{\mathcal{O}})$  because of  $F_{i+3}^{\mathcal{I}} = t_{i+3}$  and  $\Delta F_{i+3}^{\mathcal{I}} = \Delta F_{i+2}^{\mathcal{O}}$ . In the next round,  $\Delta v_{i+4} = \Delta F_{i+3}^{\mathcal{O}} \oplus j$ , the output difference of 4th round is  $\Delta F_{i+4}^{\mathcal{O}} = F_{i+4}(t_{i+4}) \oplus F_{i+4}(t_{i+4} \oplus \Delta F_{i+3}^{\mathcal{O}} \oplus j)$ .

Finally, we can get the difference  $\Delta_j$  at  $v_5$ ,

$$\begin{aligned}
\Delta_j &= \Delta v_{i+5}^j \\
&= \Delta F_{i+4}^{\mathcal{O}} \oplus \Delta F_{i+2}^{\mathcal{O}} \\
&= F_{i+4}(t_{i+4}) \oplus F_{i+4}(t_{i+4} \oplus \Delta F_{i+3}^{\mathcal{O}} \oplus j) \oplus \Delta F_{i+2}^{\mathcal{O}} \\
&= F_{i+4}(t_{i+4}) \oplus F_{i+4}(t_{i+4} \oplus \Delta F_{i+3}^{\mathcal{O}} \oplus j) \oplus F_{i+2}(t_{i+2}) \oplus F_{i+2}(t_{i+2} \oplus j) \\
&= F_{i+4}(t_{i+4}) \oplus F_{i+4}(t_{i+4} \oplus F_{i+3}(t_{i+3}) \oplus F_{i+3}(t_{i+3} \oplus F_{i+2}(t_{i+2}) \\
&\quad \oplus F_{i+2}(t_{i+2} \oplus j)) \oplus j) \oplus F_{i+2}(t_{i+2}) \oplus F_{i+2}(t_{i+2} \oplus j).
\end{aligned} \tag{3}$$

By repeating this procedure for different choices of  $j$ , we can get

$$\Delta - \text{sequence} = \Delta_1 || \Delta_2 || \dots || \Delta_\delta \tag{4}$$

for a given  $X'$  and  $Y$ .

From Eq. 3, to calculate  $\Delta_j$ , we need to know the values of  $t_{i+2}$ ,  $t_{i+3}$  and  $t_{i+4}$  corresponding to the given  $(X, X', Y)$ . According to the Lemma 1 in Ref. [13], there exists one state value (one solution) that satisfies such input-output difference in each of the middle three rounds for a given  $(X, X', Y)$ . As  $Y$  takes at most  $2^{n/2}$  different values,  $t_{i+2}$ ,  $t_{i+3}$  and  $t_{i+4}$  can assume only  $2^{n/2}$  different values. Therefore, we need to search for eligible  $t_{i+2}$ ,  $t_{i+3}$  and  $t_{i+4}$  from a space of size  $2^{n/2}$  for a given  $(X, X', Y)$ , where  $t_{i+2}$ ,  $t_{i+3}$  and  $t_{i+4}$  satisfy

$$\begin{aligned}
F_{i+2}(t_{i+2}) \oplus F_{i+2}(t_{i+2} \oplus X) &= Y, \\
F_{i+3}(t_{i+3}) \oplus F_{i+3}(t_{i+3} \oplus Y) &= X \oplus X', \\
F_{i+4}(t_{i+4}) \oplus F_{i+4}(t_{i+4} \oplus X') &= Y.
\end{aligned} \tag{5}$$

So far, the above process is the calculation process of the function  $\mathcal{F}$ . In the classical setting, the final  $\Delta$ -sequences are stored in a table  $T_\delta$  with  $O(2^n)$  computations and  $O(2^n)$  classical memory. With quantum parallelism and superposition, we can reduce the time complexity to  $O(2^{n/4})$  with  $O(n)$  qubits, which is shown as below.

1. Prepare the superposition state

$$|\varphi_1\rangle = \sum_{X'=0}^{2^{n/3}-1} \frac{1}{\sqrt{2^{n/3}}} |X'\rangle_1 \sum_{Y=0}^{2^{n/2}-1} \frac{1}{\sqrt{2^{n/2}}} |Y\rangle_2 |X\rangle_3, \tag{6}$$

where the bit length of  $X'$  is  $n/2$  and  $X' \neq X$ .

2. Do Grover search for  $t_{i+2}$ ,  $t_{i+3}$  and  $t_{i+4}$  which satisfy Eq. 5 for each  $(X', Y)$ , and store them in the new registers, i.e.

$$\begin{aligned}
& \sum_{Y=0}^{2^{n/2}-1} \frac{1}{\sqrt{2^{n/2}}} |Y\rangle_2 |X\rangle_3 \sum_{t_{i+2}=0}^{2^{n/2}-1} |t_{i+2}\rangle_4 \xrightarrow{\text{Grover}} \sum_{Y=0}^{2^{n/2}-1} \frac{1}{\sqrt{2^{n/2}}} |Y\rangle_2 |X\rangle_3 |t_{i+2}\rangle_4, \\
& \sum_{X'=0}^{2^{n/3}-1} \frac{1}{\sqrt{2^{n/3}}} |X'\rangle_1 \sum_{Y=0}^{2^{n/2}-1} \frac{1}{\sqrt{2^{n/2}}} |Y\rangle_2 |X\rangle_3 \sum_{t_{i+3}=0}^{2^{n/2}-1} |t_{i+3}\rangle_5 \xrightarrow{\text{Grover}} \\
& \sum_{X'=0}^{2^{n/3}-1} \frac{1}{\sqrt{2^{n/3}}} |X'\rangle_1 \sum_{Y=0}^{2^{n/2}-1} \frac{1}{\sqrt{2^{n/2}}} |Y\rangle_2 |X\rangle_3 |t_{i+3}\rangle_5 \\
& \sum_{X'=0}^{2^{n/3}-1} \frac{1}{\sqrt{2^{n/3}}} |X'\rangle_1 \sum_{Y=0}^{2^{n/2}-1} \frac{1}{\sqrt{2^{n/2}}} |Y\rangle_2 \sum_{t_{i+4}=0}^{2^{n/2}-1} |t_{i+4}\rangle_6 \xrightarrow{\text{Grover}} \\
& \sum_{X'=0}^{2^{n/3}-1} \frac{1}{\sqrt{2^{n/3}}} |X'\rangle_1 \sum_{Y=0}^{2^{n/2}-1} \frac{1}{\sqrt{2^{n/2}}} |Y\rangle_2 |t_{i+4}\rangle_6.
\end{aligned} \tag{7}$$

And we obtain

$$|\varphi_2\rangle = \sum_{X', Y=0}^{X'=2^{n/3}-1, Y=2^{n/2}-1} 2^{-5n/12} |X'\rangle_1 |Y\rangle_2 |X\rangle_3 |t_{i+2}\rangle_4 |t_{i+3}\rangle_5 |t_{i+4}\rangle_6 \tag{8}$$

after Grover search with time  $O(2^{n/4})$ .

3. Compute  $\Delta$ -sequence according to Eq. 3 with  $\bigotimes_{j=1}^{\delta} |j\rangle$  as input, and store them in a new register of  $\delta n/2$  qubits.

$$\begin{aligned}
|\varphi_3\rangle &= \sum_{X', Y=0}^{X'=2^{n/3}-1, Y=2^{n/2}-1} 2^{-5n/12} |X'\rangle_1 |Y\rangle_2 |X\rangle_3 |\Delta - \text{sequence}\rangle_7 \\
&= \sum_{X', Y=0}^{X'=2^{n/3}-1, Y=2^{n/2}-1} 2^{-5n/12} |X'\rangle_1 |Y\rangle_2 |X\rangle_3 |\mathcal{F}(X', Y)\rangle_7,
\end{aligned} \tag{9}$$

where we discard some useless registers finally.

4. Do Grover search parallelly on  $|\varphi_3\rangle$  for each  $(X', Y)$  with  $2^{5n/6}$  parallel single quantum processors, and store the measured  $|\Delta - \text{sequence}\rangle_7$  in the table  $T_\delta$  indexed by  $(X', Y)$ .

In summary, the entire quantum computation for the table  $T_\delta$  will cost  $O(2^{2n/3})$  time with  $O(2^{5n/6})$  qubits and  $O(2^{5n/6})$  classical memory.

## 4.2 Queried-data analysis phase

Firstly, we choose two plaintext sets in the form of  $\{m||0, m||1, \dots, m||2^{n/2} - 1\}$  and  $\{m \oplus X||0, m \oplus X||1, \dots, m \oplus X||2^{n/2} - 1\}$ , where  $m$  is a randomly chosen  $n/2$ -bit constant.  $2^n$  pairs of plaintexts can be generated,  $2^{5n/6}$  pairs of them will satisfy  $\Delta C = X'||Z$  in the corresponding ciphertexts for  $2^{n/3}$  choices of  $X'$  and  $2^{n/2}$  choices of  $Z$ . By iterating this procedure  $2^{n/6}$  times for different choices of  $m$ ,  $2^n$  pairs satisfying  $\Delta P = X||*$  and  $\Delta C = X'||Z$  are collected. These pairs are stored in the table  $T_{PC}$  indexed by the item  $I$  ( $0 \leq I \leq 2^n - 1$ ). We can see that, the probability of that the plaintext difference  $\Delta P$  propagates to the input difference  $0||X$  of the distinguisher is  $p_1 = 2^{-n/2}$ , and the probability of the ciphertext difference  $\Delta C$  propagates to the output difference  $X'||0$  is  $p_2 = 2^{-n/2}$ . Therefore, one pair of  $2^n$  pairs would satisfy the entire differential characteristic of 7-round Feistel.

In this phase, we aim to compute the  $\Delta$ -sequence for each pair, and we define this computing process as a function  $\mathcal{G} : I \rightarrow \Delta\text{-sequence}$ , where  $0 \leq I \leq 2^n - 1$  and  $\Delta\text{-sequence} \in \{0, 1\}^{\delta n/2}$ .

For each pair, we should guess subkeys  $k_0$  and  $k_6$  to ensure that the differences  $\Delta P$  and  $\Delta C$  propagate to the input difference  $0||X$  and the output difference  $X'||0$  of the distinguisher respectively. In other words, we assume that the differences  $\Delta P$  and  $\Delta C$  for each pair propagate to  $0||X$  and  $X'||0$  respectively. In this case, we can find the input values of the two round-functions  $F_0$  and  $F_6$  according to the known input and output differences, and then use the input values with the known plaintext and ciphertext values to deduce the guessed subkeys, called subkey candidates.

Suppose that one pair of  $2^n$  pairs is  $(P, P') = (v_0||v_{-1}, v_0 \oplus X||v_{-1} \oplus *)$  and their corresponding ciphertexts  $(C, C') = (v_6||v_7, v_6 \oplus X'||v_7 \oplus Z)$ . Then, we need to search the input values  $F_0^{\mathcal{I}}$  and  $F_6^{\mathcal{I}}$  according to the input and output differences. Therefore, we compute all  $2^{n/2}$  possible values of  $F_0^{\mathcal{I}}$  and  $F_6^{\mathcal{I}}$  by Eq. 10, and store them in two tables  $T_0$  and  $T_6$  indexed by  $(X, *)$  and  $(X', Z)$  respectively. This step will cost  $O(2^{5n/6})$  time classically.

$$F_0(F_0^{\mathcal{I}}) \oplus F_0(F_0^{\mathcal{I}} \oplus X) = *, \quad F_6(F_6^{\mathcal{I}}) \oplus F_6(F_6^{\mathcal{I}} \oplus X') = Z. \quad (10)$$

When we get  $F_0^{\mathcal{I}}$  and  $F_6^{\mathcal{I}}$ , we can obtain two subkeys candidates  $k_0$  and  $k_6$  by

$$k_0 = F_0^{\mathcal{I}} \oplus v_0, \quad k_6 = F_6^{\mathcal{I}} \oplus v_6. \quad (11)$$

Then, we construct a  $\delta$ -set  $\{v_1||v_0 \oplus 1, v_1||v_0 \oplus 2, \dots, v_1||v_0 \oplus \delta\}$ , where  $v_1||v_0$  is corresponding to the plaintext  $v_0||v_{-1}$  and can be obtained by encrypting with the subkey candidate  $k_0$ . So, with the knowledge of subkey candidate  $k_0$ , we compute the corresponding  $\Delta F_0^{\mathcal{O}}$ , modify  $v_{-1}$  so that the value of  $v_1$  stays unchanged, and obtain the plaintexts corresponding to the  $\delta$ -set, i.e.,

$$\begin{aligned} \Delta F_0^{\mathcal{O}} &= F_0(v_0 \oplus k_0) \oplus F_0(v_0 \oplus k_0 \oplus j), \quad j \in \{1, 2, \dots, \delta\}, \\ P_j &= v_0 \oplus j||v_{-1} \oplus \Delta F_0^{\mathcal{O}}. \end{aligned} \quad (12)$$

And these plaintexts are queried to get ciphertexts  $\{C_1, C_2, \dots, C_\delta\}$ . With the knowledge of subkey candidate  $k_6$ , these ciphertexts are decrypted partially to get the values  $\{v_5^1, v_5^2, \dots, v_5^\delta\}$  at  $v_5$ . Finally, the differences between  $v_5$  (corresponding to ciphertext  $C$ ) and  $v_5^j$  make up a sequence  $\Delta$ -sequence. If this  $\Delta$ -sequence can be found in Table  $T_\delta$ , the subkey candidates  $k_0$  and  $k_6$  are correct. Note that,  $p_1 = 2^{-n/2}$ ,  $p_2 = 2^{-n/2}$ ,  $N_c = 2^{n/2}$ ,  $\lambda = n/2$ , hence,  $\delta = 4$  is sufficient to filter out all the wrong candidates.

In the classical setting, the computing process of  $\mathcal{G}$  needs  $O(2^n)$  computations with  $O(2^n)$  classical memory. To reduce complexity, we do the following quantum operations.

1. Prepare the superposition state

$$|\varphi_1\rangle = \sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1. \quad (13)$$

2. Use  $|\varphi_1\rangle$  as the address to query the paired plaintexts and ciphertexts  $(P, P', C, C')$  from the table  $T_{PC}$  by QRAM,

$$\begin{aligned} |\varphi_2\rangle &= \sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |P, P', C, C'\rangle_2 \\ &= \sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |PC\rangle_2, \end{aligned} \quad (14)$$

where  $P, P', C, C'$  is abbreviated as  $PC$ .

3. Compute  $(X, *, X', Z)$  for each pair to obtain

$$|\varphi_3\rangle = \sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |PC\rangle_2 |X, *, X', Z\rangle_3 \quad (15)$$

4. Do Grover search to find  $F_0^{\mathcal{I}}$  and  $F_6^{\mathcal{I}}$  satisfying Eq. 10 for each pair

$$\begin{aligned} &\sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |PC\rangle_2 |X, *, X', Z\rangle_3 \sum_{F_0^{\mathcal{I}}=0}^{2^{n/2}-1} |F_0^{\mathcal{I}}\rangle_4 \xrightarrow{\text{Grover}} \\ &\sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |PC\rangle_2 |X, *, X', Z\rangle_3 |F_0^{\mathcal{I}}\rangle_4 \\ &\sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |PC\rangle_2 |X, *, X', Z\rangle_3 \sum_{F_6^{\mathcal{I}}=0}^{2^{n/2}-1} |F_6^{\mathcal{I}}\rangle_5 \xrightarrow{\text{Grover}} \\ &\sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |PC\rangle_2 |X, *, X', Z\rangle_3 |F_6^{\mathcal{I}}\rangle_5, \end{aligned} \quad (16)$$

which costs  $O(2^{n/4})$  time, and obtain

$$|\varphi_4\rangle = \sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |PC\rangle_2 |X, *, X', Z\rangle_3 |F_0^I\rangle_4 |F_6^I\rangle_5. \quad (17)$$

5. Compute the subkey candidates  $k_0$  and  $k_6$  with known plaintext  $P$  and ciphertext  $C$  according to Eq. 11, to get

$$|\varphi_5\rangle = \sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |PC\rangle_2 |X, *, X', Z\rangle_3 |F_0^I\rangle_4 |F_6^I\rangle_5 |k_0\rangle_6 |k_6\rangle_7. \quad (18)$$

6. Compute the plaintexts corresponding to the  $\delta$ -set according to Eq. 12, to get

$$|\varphi_6\rangle = \sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |PC\rangle_2 |X, *, X', Z\rangle_3 |F_0^I\rangle_4 |F_6^I\rangle_5 |k_0\rangle_6 |k_6\rangle_7 |P_1, P_2, \dots, P_\delta\rangle_8. \quad (19)$$

7. Query the corresponding ciphertexts  $\{C_1, C_2, \dots, C_\delta\}$  for  $\{P_1, P_2, \dots, P_\delta\}$  by QRAM,

$$|\varphi_7\rangle = \sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |PC\rangle_2 |X, *, X', Z\rangle_3 |F_0^I\rangle_4 |F_6^I\rangle_5 |k_0\rangle_6 |k_6\rangle_7 \otimes |P_1, P_2, \dots, P_\delta\rangle_8 |C_1, C_2, \dots, C_\delta\rangle_9. \quad (20)$$

Note that another table  $T'_{PC}$  stores  $2^{2n/3+1+\delta}$  plaintexts and corresponding ciphertexts, which is indexed by the plaintexts.

8. Compute the  $\Delta$ -sequence with the knowledge of subkey candidate  $k_6$  and ciphertexts  $\{C_1, C_2, \dots, C_\delta\}$ , to obtain

$$\begin{aligned} |\varphi_8\rangle &= \sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |\Delta - sequence\rangle_{10} \\ &= \sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |\mathcal{G}(I)\rangle_{10}, \end{aligned} \quad (21)$$

where we discard some useless registers.

For now, we need to find a claw  $((X', Y), I)$  such that  $\mathcal{F}(X', Y) = \mathcal{G}(I)$  from the table  $T_\delta$  and the superposition state  $\sum_{I=0}^{2^n-1} \frac{1}{\sqrt{2^n}} |I\rangle_1 |\mathcal{G}(I)\rangle_{10}$ , which can be implemented by Algorithm 2. When we find the claw  $((X', Y), I)$ , the corresponding subkeys candidates  $k_0$  and  $k_6$  are correct, and the other subkeys can be deduced with the knowledge of  $X$ ,  $X'$  and  $Y$ .

**Complexity.** Obviously, our attack belongs to Q1 model, which needs  $O(2^{2n/3})$  classical queries. And the time complexities of constructing the table  $T_\delta$  and Algorithm 2 are both  $O(2^{2n/3})$ . Therefore, the total time complexity of our attack is  $O(2^{2n/3})$ . Besides, our attack needs  $O(2^n)$  classical memory due to the tables  $T_\delta$ ,  $T_{PC}$  and  $T'_{PC}$ . And it consumes  $O(2^{5n/6})$  qubits.

**Algorithm 2** Quantum Claw Finding between Two Functions  $\mathcal{F}$  and  $\mathcal{G}$ 

1. Select a random subset  $A \subseteq \{(X', Y)\}$  of size  $l$  ( $l \leq 2^{5n/6}$ ), where  $|\{(X', Y)\}| = 2^{5n/6}$ .
2. Select a random subset  $B \subseteq \{I\}$  of size  $l^2$ .
3. Compute  $\sum_{b \in B} \frac{1}{l} |b\rangle |\mathcal{G}(b)\rangle$  according to Eqs. 13-21.
4. Sort the elements in  $A$  according to their  $\mathcal{F}$ -value according to the table  $T_\delta$ .
5. For a specific  $b \in B$ , check if there is an  $a \in A$  such that  $(a, b)$  is a claw by using classical binary search on the sorted version of  $A$ . Combine this with quantum search on the elements of  $B$  to find a claw in  $A \times B$ .
6. Apply amplitude amplification on steps 1-4 with  $\sqrt{\frac{2^{5n/6+n}}{l^3}}$  iterations. And output the claw  $((X', Y), I)$  such that  $\mathcal{F}(X', Y) = \mathcal{G}(I)$ .

## 5 Quantum Meet-in-the-Middle Attack on $r$ -round Feistel Construction

To attack  $r$ -round ( $r > 7$ ) Feistel construction, Dong *et al.* [9] guess the subkeys of the last  $r - 3$  rounds by Grover search, and use Simon's algorithm to recover the period satisfying the first 3-round distinguisher, which is inspired by Leander and May's work [23]. Furthermore, Ito *et al.* [18] continue this attack idea and propose a new 4-round distinguisher to reduce the time complexity. Similarly, we guess the subkeys of the last  $r - 7$  rounds by Grover search, and find a claw between functions  $\mathcal{F}$  and  $\mathcal{G}$  under each choice of the subkeys of the last  $r - 7$  rounds. Therefore, the time complexity of our attack is  $O(2^{(r-7)n/4+2n/3})$ .

**Pre-computation phase.** As same as the computing process in Sect. 4.1, we construct the table  $T_\delta$  classically without quantum computation. It will cost  $O(2^{5n/6})$  time and  $O(2^{5n/6})$  classical memory.

**Queried-data analysis phase.** We choose the same paired plaintexts in the Sect. 4.2, and query for their ciphertexts, which are also stored in the table  $T_{PC-all}$  indexed by  $I$  ( $0 \leq I \leq 2^{7n/6} - 1$ ). After querying, we do following operations.

1. Prepare the initial superposition state

$$|\varphi_1\rangle = \sum_{K=0}^{2^{(r-7)n/2}} \frac{1}{\sqrt{2^{(r-7)n/2}}} |K\rangle_1 \sum_{I=0}^{2^{7n/6}-1} \frac{1}{\sqrt{2^{7n/6}}} |I\rangle_2, \quad (22)$$

where  $K = (k_7, k_8, \dots, k_{r-1})$ .

2. Query the paired plaintexts and ciphertexts from the table  $T_{PC-all}$  by QRAM,

$$|\varphi_2\rangle = \sum_{K=0}^{2^{(r-7)n/2}} \frac{1}{\sqrt{2^{(r-7)n/2}}} |K\rangle_1 \sum_{I=0}^{2^{7n/6}-1} \frac{1}{\sqrt{2^{7n/6}}} |I\rangle_2 |PC-all\rangle_3. \quad (23)$$

3. Decrypt the ciphertexts partially to the values at  $v_7||v_6$  for each  $K$ ,

$$|\varphi_3\rangle = \sum_{K=0}^{2^{(r-7)n/2}} \frac{1}{\sqrt{2^{(r-7)n/2}}} |K\rangle_1 \sum_{I=0}^{2^{7n/6}-1} \frac{1}{\sqrt{2^{7n/6}}} |I\rangle_2 |PC - all\rangle_3 |C_{v_7||v_6}\rangle_4, \quad (24)$$

where  $C_{v_7||v_6}$  represents the values at  $v_7||v_6$ .

4. Do Grover search for that the difference  $X'$  of the corresponding  $C_{v_7||v_6}$  is all zero in the first  $n/6$  bits, with  $O(2^{n/12})$  time. This will increase the time complexity of step 2 in Algorithm 2 with  $O(2^{n/3}) = O(2^{n/4} \times 2^{n/12})$  time, but does not have an influence of the entire time complexity.

$$|\varphi_4\rangle = \sum_{K=0}^{2^{(r-7)n/2}} \frac{1}{\sqrt{2^{(r-7)n/2}}} |K\rangle_1 \otimes \sum_{0 \leq I' \leq 2^{7n/6}-1, |I'|=2^n} \frac{1}{\sqrt{2^n}} |I'\rangle_2 |PC - all^{I'}\rangle_3 |C_{v_7||v_6}^{I'}\rangle_4 \quad (25)$$

5. Do outer Grover search for  $K$  satisfying  $\mathcal{H}(K) = 1$  and apply Algorithm 2 in the inner for each  $K$ . We define the function  $\mathcal{H} : K \rightarrow \{0, 1\}$ .  $\mathcal{H}(K) = 1$  if and only if there is claw after Algorithm 2 for the corresponding  $K$ , otherwise  $\mathcal{H}(K) = 0$ .

Note that  $p_1 = 2^{-n/2}$ ,  $p_2 = 2^{-2n/3}$ ,  $N_c = 2^{n/2}$ , hence,  $\delta = 4$  is sufficient to filter out all the wrong candidates.

## 6 Conclusion

To reduce the time complexity of classic and quantum attacks on  $r$ -round ( $r \geq 7$ ) Feistel construction, we propose a new quantum meet-in-the-middle attack in Q1 model. We introduce a quantum claw finding algorithm and Grover's algorithm in Guo *et al.*'s meet-in-the-middle attack to reduce complexity. Its time complexity only needs  $O(2^{2n/3+(r-7)n/4})$  and is less than classical and quantum attacks. Moreover, it belongs to the Q1 model and is more practical than other quantum attacks.

Furthermore, we hope to carry out quantum meet-in-the-middle attacks on more multi-rounds Feistel constructions. Because there are not only 5-round distinguisher, but also 7-round distinguisher, 8-round distinguisher, etc. Combining these distinguishers with quantum claw finding algorithms, or even other quantum algorithms may achieve good attack results.

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