# Generalized uncertainty principle and quantum non-locality

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The emergence of the generalized uncertainty principle and the existence of a non-zero minimal length are intertwined. On the other hand, the Heisenberg uncertainty principle forms the core of the EPR paradox. Subsequently, here, the implications of resorting to the generalized uncertainty principle (or equally, the minimal length) instead of the Heisenberg uncertainty principle on the quantum non-locality are investigated through focusing on the Franson experiment in which the uncertainty relation is the backbone of understanding and explaining the results.

## I. INTRODUCTION

Certainly, quantum non-locality (QNL) is one of the most intriguing subjects in physics rooted in the famous paper by Einstein, Podolsky, and Rosen (EPR) [1]. Basically, there is a deep connection between QNL and the Heisenberg uncertainty principle (HUP) [1–4]. Indeed, this property has been obtained since people could reach quantum energy levels. The quality of the validity of the Schrödinger equation (or equally, the quantum mechanics) decreases as the energy of a system increases, and at high energy levels, quantum mechanical interpretations should be replaced by those of the quantum field theory. Therefore, it is significant challenge to study the quality of non-locality in high energy physics to answer the question of whether there is a change in QNL with increasing energy or not? In this regard, the effects of special relativity and curved spacetime on the behavior of QNL have extensively been studied [5–13].

Despite the success of general relativity, the relation between quantum mechanics and gravity is still mysterious [14], and attempts to find a quantum gravity scenario continue [15, 16]. One common feature of quantum gravity (QG) scenarios is the existence of a non-zero minimum length, also expectable in Newtonian gravity [17]. Additionally, the existence of such a non-zero minimum length naturally leads to the generalized uncertainty principle (GUP) [16] meaning that the quantum mechanical commutators of operators should change if one wants to recover GUP. Therefore, this correspondence is a great motivation to replace GUP with HUP, which stimulates us to study the implications of this replacement on QNL.

In general, when the quantum features of gravity are considered, canonical operators x and p are replaced with their generalized counterparts X and P, respectively, and up to the first order of the GUP parameter  $\beta$ , we can write  $P_i = p_i(1 + \beta f(p))$  in the position representation where X = x. In order to get an insight into the implications of QG (GUP) on the current physics, one may estimate the effects of QG as the perturbations to the quantum mechanics, classical mechanics, and quantum field theory [16, 18–26].

As it has been argued, QNL is the result of HUP (the result of the non-commutative of operators) [1-4]. Dependency of the square of Bell's operators to the commutators is a bright signal from it [27, 28]. This point has been addressed in Ref. [29] where, considering the effects of GUP on angular momentum algebra, it is shown that the Bell's operators square of two partite systems changes. Based on this paper, the Bell operator and thus its expectation value do not change as this operator consists of operators with eigenvalues  $\pm 1$ , and hence, it remains unchanged. Therefore, while the Bell's operator does not change, its square changes, an incompatibility. Indeed, compared to the square of Bell's operator, the role of commutators (or equally, HUP) in the original Bell's inequality is not obvious and it is figured in the coincidence rate version appeared in the Franson experiment [3, 30, 31]. Finally, the fundamental question of whether GUP affects Bell's inequality and QNL or not still needs to be studied.

Consequently, in order to answer the mentioned question, we focus on the Franson experiment where the role of HUP is vital to interpret the results. The paper is structured as follows. After providing a general remarks on the Franson experiment in section (II), the implications of the existence of a minimal length on its outcome are addressed in Sec. (III). A summary has also been presented in the last section.

## **II. FRANSON EXPERIMENT**

There is a three levels quantum mechanical system in the Franson setup, such that the highest energy level has energy  $E_1$  and relatively long lifetime  $\tau_1$ , the intermediate state of energy  $E_2$  and lifetime  $\tau_2 \ll \tau_1$ , and the ground state energy  $E_3$  of very long lifetime  $\tau_3$  ( $\tau_1 < \tau_3$ ) [3]. These states are labeled with states  $\varphi_1$ ,  $\varphi_2$ , and  $\varphi_3$ , respectively [3]. In the Franson experiment, uncertainty in the position of photons is reflected in their transit time difference  $\Delta T$  satisfying  $\tau_2 \ll \Delta T \ll \tau_1$  (See Ref. [3] for more info about the setup). Finally, the fields at the

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detectors  $D_1$  and  $D_2$  are written as [3]

$$\varphi_k(\mathbf{x}_1, t) = \frac{1}{2} \varphi_{k,0}(\mathbf{x}_1, t) + \frac{1}{2} e^{i\phi_1} \varphi_{k,0}(\mathbf{x}_1, t - \Delta T), \quad (1)$$

and

$$\varphi_k(\mathbf{x}_2, t) = \frac{1}{2} \varphi_{k,0}(\mathbf{x}_2, t) + \frac{1}{2} e^{i\phi_2} \varphi_{k,0}(\mathbf{x}_2, t - \Delta T), \quad (2)$$

respectively. Here,  $\phi_1$ , and  $\phi_2$  store the information related to the phase shifts due to the half-silvered mirrors  $D_1$  and  $D_2$ , respectively, and  $\Delta T$  is assumed to be the same for both photons [3]. In Ref. [3],  $R_c$  (the coincidence rate between two detectors) is evaluated by

$$R_{c} = (3)$$
  

$$\eta_{1}\eta_{2} \left\langle 0 \left| \varphi_{k}^{\dagger} \left( \mathbf{x}_{1}, t \right) \varphi_{k}^{\dagger} \left( \mathbf{x}_{2}, t \right) \varphi_{k} \left( \mathbf{x}_{2}, t \right) \varphi_{k} \left( \mathbf{x}_{1}, t \right) \right| 0 \right\rangle,$$

where  $\eta_1, \eta_2$  denote the efficiency of the corresponding detectors, and we can briefly write

$$R_c = \frac{1}{16} \eta_1 \eta_2 \langle 0 | A^{\dagger} A | 0 \rangle, \qquad (4)$$

in which

$$A = \varphi_{k,0} \left( \mathbf{x}_{1}, t \right) \varphi_{k,0} \left( \mathbf{x}_{2}, t \right)$$
  
+  $e^{i\phi_{1}} e^{i\phi_{2}} \varphi_{k,0} \left( \mathbf{x}_{1}, t - \Delta T \right) \varphi_{k,0} \left( \mathbf{x}_{2}, t - \Delta T \right).$  (5)

Whenever  $\Delta T \ll \tau_1$ , the amplitude of detecting a pair of particles at time  $t - \Delta T$  will be approximately equal to the amplitude of detecting a pair of particles at time t, and in fact, they have only a constant phase difference causing [3]

$$\varphi_{k,0} \left( \mathbf{x}_{1}, t \right) \varphi_{k,0} \left( \mathbf{x}_{2}, t \right) = \sum_{k_{1},k_{2}} c_{k_{1}} c_{k_{2}} e^{i(\mathbf{k}_{1} \cdot \mathbf{x}_{1} - \omega_{1}t)} e^{i(\mathbf{k}_{2} \cdot \mathbf{x}_{2} - \omega_{2}t)}, \qquad (6)$$

$$\varphi_{k,0} \left( \mathbf{x}_{1}, t - \Delta T \right) \varphi_{k,0} \left( \mathbf{x}_{2}, t - \Delta T \right) = \sum_{k_{1},k_{2}} c_{k_{1}} c_{k_{2}} e^{i(\omega_{1} + \omega_{2})\Delta T} \times e^{i(\mathbf{k}_{1} \cdot \mathbf{x}_{1} - \omega_{1}t)} e^{i(\mathbf{k}_{2} \cdot \mathbf{x}_{2} - \omega_{2}t)},$$

where  $c_{k_1}, c_{k_2}$  are the expansion coefficients in the Fourier transformation and can be determined by evaluation of system. Energy conservation yields  $\omega_1 + \omega_2 =$  $(E_1 - E_3)/\hbar + \Delta\omega$ , where  $E_1$  and  $E_3$  are the unperturbed energies of initial and final states, respectively.  $\Delta\omega \sim \frac{1}{\tau_1} + \frac{1}{\tau_3}$  (the uncertainty in  $\omega_1 + \omega_2$ ), and its value is much less than the individual uncertainty of  $\omega_i$  since  $\tau_2$  is relatively short [3], and thus

$$\varphi_{k,0} \left( \mathbf{x}_{1}, t - \Delta T \right) \varphi_{k,0} \left( \mathbf{x}_{2}, t - \Delta T \right)$$
(7)  
= e<sup>i(E\_1 - E\_3) \Delta T / \hbar} \varphi\_{k,0} \left( \mathbf{x}\_{1}, t \right) \varphi\_{k,0} \left( \mathbf{x}\_{2}, t \right),</sup>

leading to

$$R_{c} = \frac{1}{16} R_{0} \left[ 1 + e^{-i(\Delta E \Delta T/\hbar + \phi_{1} + \phi_{2})} \right] \times \left[ 1 + e^{i(\Delta E \Delta T/\hbar + \phi_{1} + \phi_{2})} \right], \qquad (8)$$

in which  $R_0 = \langle 0 | \sum_{k_1,k_2} c_{k_1}^{\dagger} c_{k_2}^{\dagger} c_{k_1} c_{k_2} | 0 \rangle$  is the coincidence rate with the half-silvered mirrors removed (shorter length) and  $\Delta E = E_1 - E_3$ . Finally, one finds [3]

$$R_{c} = \frac{1}{4} R_{0} \cos^{2} \left( \frac{\Delta E \Delta T / \hbar + \phi_{1} + \phi_{2}}{2} \right)$$
$$= \frac{1}{4} R_{0} \cos^{2} \left( \phi_{1}' - \phi_{2}' \right), \qquad (9)$$

where

$$\phi_1' = \phi_1/2, \qquad (10)$$
  
$$\phi_2' = -\left(\phi_2 + \Delta E \Delta T/\hbar\right)/2.$$

### III. FRANSON EXPERIMENT IN THE PRESENCE OF MINIMUM LENGTH

In this section, we intend to study the implications of the quantum features of gravity on Eq. (9) and thus QNL using the perturbation theory. Now, suppose that the quantum gravity modifications to the two emitted photons in Franson experiment have been considered. It means that the Hamiltonian of atoms and thus their energy levels are also perturbed by the modifications of the QG scenarios, and thus we have  $\hat{H}_{GUP} = \hat{H} + \beta \hat{H}_p$ , where  $\hat{H}_p$  refers to the perturbed Hamiltonian in the GUP frame, and  $E_{GUP} = E + \beta E_p$  for the energy levels.  $E_p$  can be determined using the perturbation theory (up to the desired level).

Therefore, in the language of quantum field theory and due to the existence of minimal length, the field operator of each photon is modified as  $\varphi_k^{GUP}(x) = \varphi_k(x) + \beta \varphi_k^p(x)$ , where the index p denotes the correction terms in the GUP framework [16, 24]. The time evolution of  $\varphi_k^{GUP}(x)$ is obtained by

$$\varphi_{k}^{GUP}(x,t) = e^{i\hat{H}_{GUP}t/\hbar}\varphi_{k}^{GUP}(x)e^{-i\hat{H}_{GUP}t/\hbar} 
= \varphi_{k}(x,t) - i\beta\Gamma_{k}(x,t) + \beta\varphi_{k}^{p}(x,t) 
+ \mathcal{O}(\beta^{2}),$$
(11)

where  $\Gamma_k(x,t) = [\varphi_k(x,t), \hat{H}_p]t/\hbar$ .

For the counterparts of Eqs. (1) and (2), similar to the above argument, and by following the Franson approach, the field corresponding to the *i*th photon, at the detector  $D_i$  can be written as

$$\varphi_{k}^{GUP}(\mathbf{x}_{i},t) = \frac{1}{2}\varphi_{k,0}^{GUP}(\mathbf{x}_{i},t) + \frac{1}{2}e^{\mathrm{i}\phi_{1}}\varphi_{k,0}^{\mathrm{GUP}}(\mathbf{x}_{i},t-\Delta T). \quad (12)$$

Finally, the corresponding coincidence rate  $R_c^{QG}$  is achieved by

$$R_{c}^{QG} = \eta_{1}'\eta_{2}'\langle 0|\varphi_{k}^{GUP,\dagger}(\mathbf{x}_{1},t)\varphi_{k}^{GUP,\dagger}(\mathbf{x}_{2},t) \\ \times \varphi_{k}^{GUP}(\mathbf{x}_{2},t)\varphi_{k}^{GUP}(\mathbf{x}_{1},t)|0\rangle,$$
(13)

summarized into

$$R_c^{QG} = \eta_1' \eta_2' \langle 0 | B^{\dagger} B | 0 \rangle, \qquad (14)$$

in which

$$B = \varphi_{k}(x_{2}, t)\varphi_{k}(x_{1}, t)$$

$$+ \frac{1}{2}\beta \bigg[ \varphi_{k,0}^{p}(x_{2}, t)\varphi_{k}(x_{1}, t) + e^{i\phi_{2}}\varphi_{k,0}^{p}(x_{2}, t - \Delta T)\varphi_{k,0}(x_{1}, t) \bigg]$$

$$- i\Gamma_{k,0}(x_{2}, t)\varphi_{k,0}(x_{1}, t) - ie^{i\phi_{2}}\Gamma_{k,0}(x_{2}, t - \Delta T)\varphi_{k,0}(x_{1}, t)$$

$$+ \varphi_{k}(x_{2}, t)\varphi_{k,0}^{p}(x_{1}, t) + e^{i\phi_{1}}\varphi_{k}(x_{2}, t)\varphi_{k,0}^{p}(x_{1}, t - \Delta T)$$

$$- i\varphi_{k}(x_{2}, t)\Gamma_{k,0}(x_{1}, t) - ie^{i\phi_{1}}\varphi_{k,0}(x_{2}, t)\Gamma_{k,0}(x_{1}, t - \Delta T) \bigg].$$

$$(15)$$

Using the Fourier expansion, one can write

$$\varphi_{k,0}^{p}\left(\mathbf{x}_{1},t\right)\varphi_{k,0}^{p}\left(\mathbf{x}_{2},t\right) = \sum_{k_{1},k_{2}}c_{k_{1}}^{\prime}c_{k_{2}}^{\prime}\mathrm{e}^{\mathrm{i}\left(\mathbf{k}_{1}\cdot\mathbf{x}_{1}-\omega_{1}\mathrm{t}\right)}\mathrm{e}^{\mathrm{i}\left(\mathbf{k}_{2}\cdot\mathbf{x}_{2}-\omega_{2}\mathrm{t}\right)},\qquad(16)$$

and

$$\Gamma_{k,0} (\mathbf{x}_{1}, t) \Gamma_{k,0} (\mathbf{x}_{2}, t) = \sum_{k_{1},k_{2}} c_{k_{1}}' c_{k_{2}}' e^{i(\mathbf{k}_{1} \cdot \mathbf{x}_{1} - \omega_{1}t)} e^{i(\mathbf{k}_{2} \cdot \mathbf{x}_{2} - \omega_{2}t)}, \qquad (17)$$

leading to

$$\varphi_{k,0}^{p}\left(\mathbf{x}_{1},t-\Delta T\right)\varphi_{k,0}^{p}\left(\mathbf{x}_{2},t-\Delta T\right) = (18)$$
$$\sum_{k_{1},k_{2}}c_{k_{1}}'c_{k_{2}}'\mathrm{e}^{\mathrm{i}(\omega_{1}+\omega_{2})\Delta T}\mathrm{e}^{\mathrm{i}(\mathbf{k}_{1}\cdot\mathbf{x}_{1}-\omega_{1}\mathrm{t})}\mathrm{e}^{\mathrm{i}(\mathbf{k}_{2}\cdot\mathbf{x}_{2}-\omega_{2}\mathrm{t})}.$$

Here,  $c_{k_1}^\prime,c_{k_2}^\prime$  and  $c_{k_1}^{\prime\prime},c_{k_2}^{\prime\prime}$  are the corresponding coefficients in the Fourier expansion.

In this manner, the corresponding energy conservation leads to

$$\omega_1 + \omega_2 = \Delta E/\hbar + \beta \Delta E_p/\hbar, \tag{19}$$

where  $\Delta E = E_3 - E_1$ , and  $\Delta E_p = E_{3,p} - E_{1,p}$  which yields

$$R_{c}^{QG} = \frac{1}{16} \left( R_{0} + 2\beta (R_{1}' + R_{2}') \right)$$

$$\times \left[ 1 + e^{-i(\Delta E \Delta T/\hbar + \beta \Delta E_{p} \Delta T/\hbar + \phi_{1} + \phi_{2})} \right]$$

$$\times \left[ 1 + e^{i(\Delta E \Delta T/\hbar + \beta \Delta E_{p} \Delta T/\hbar + \phi_{1} + \phi_{2})} \right], \quad (20)$$

and thus

$$R_c^{QG} = \frac{1}{4} R_0^{GUP} \cos^2 \left( \frac{\Delta E \Delta T/\hbar + \beta \Delta E_p \Delta T/\hbar + \phi_1 + \phi_2}{2} \right)$$
$$= \frac{1}{4} R_0^{GUP} \cos^2 \left( \Phi_1' - \Phi_2' \right), \qquad (21)$$

where  $R_0^{GUP} = R_0 + 2\beta(R'_1 + R'_2)$  is the coincidence rate of the shorter length near to the Planck scale, and

$$R'_{1} = \langle 0 | \sum_{k_{1},k_{2}} c^{\dagger}_{k_{1}} c^{\dagger}_{k_{2}} c_{k_{1}} c^{\dagger}_{k_{2}} | 0 \rangle,$$

$$R'_{2} = \langle 0 | \sum_{k_{1},k_{2}} c^{\dagger}_{k_{1}} c^{\dagger}_{k_{2}} c^{\dagger}_{k_{1}} c_{k_{2}} | 0 \rangle,$$

$$\Phi'_{1} = \phi_{1}/2,$$

$$\Phi'_{2} = -(\phi_{2} + \Delta E \Delta T/\hbar + \beta \Delta E_{p} \Delta T/\hbar) / 2.$$
(22)

It is obvious that, at the limit of  $\beta \longrightarrow 0$ , the desired results obtained in quantum mechanics are recovered. Therefore, the coincidence rate stores the effects of QG, and indeed, the existence of minimum length affects the spectrum of the coincidence rate and thus the spectrum of the expectation value of Bell's operator.

#### IV. SUMMARY

It seems that the existence of a non-zero minimal length is unavoidable leading to GUP (or equally, modified commutators algebra) [16, 17]. Consequently, motivated by the deep connection between HUP and the EPR paradox leading to the emergence of QNL, and also Ref. [29], showing that the square of Bell's operators formed by angular momentum operators, changes when HUP is replaced by GUP, we tried to clarify the relation between QNL and GUP. In order to achieve this goal, we resorted to the Franson experiment in which the uncertainty principle plays a crucial role in describing the results, and obtained that GUP affects the coincidence rate spectrum.

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