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One-Way Deficit and Holevo Quantity of Generalized n-qubit Werner State

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Abstract Originated from the work extraction in quantum systems coupled to a heat bath, quantum deficit is a kind of significant quantum correlations like quantum entanglement. It links quantum thermodynamics with quantum information. We analytically calculate the one-way deficit of the generalized n-qubit Werner state. We find that the one-way deficit increases as the mixing probability p increases for any n. For fixed p, we observe that the one-way deficit increases as n increases. For any n, the maximum of one-way deficit is attained at $p = 1$. Furthermore, for large n ($2^n \to \infty$), we prove that the curve of one-way deficit versus p approaches to a straight line with slope 1. We also calculate the Holevo quantity for the generalized n-qubit Werner state, and show that it is zero.

Keywords Generalized n-qubit Werner state · one-way deficit · Holevo quantity.

1 Introduction

Similar to quantum entanglement [1] and quantum discord [2,3], quantum deficit [4,5,6] is a kind of important nonclassical correlation, which characterizes the work

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extraction from a correlated system coupled to a heat bath by using nonlocal operations [4]. Oppenheim et al. defined the work deficit [4] to be the difference between the information of the whole system and the localizable information [7]. By means of relative entropy over all local von Neumann measurements on one subsystem, Streltsov et al. [9,10] introduced the one-way deficit (OWD) as a resource of entanglement distribution. OWD is able to characterize quantum phase transitions in the XY model and even topological phase transitions in the extended Ising model [8]. These results enlighten extensive researches of quantum phase transitions from the perspective of quantum information processing and quantum computation. For a bipartite composite quantum system ρ_{AB} associated with subsystems A and B, the one-way deficit with respect to von Neumann measurement $\{\Pi_k\}$ on one subsystem is given by [11]

$$\Delta^{\rightarrow}(\rho_{AB}) = \min_{\{\Pi_k\}} S(\sum_k \Pi_k \rho_{AB} \Pi_k) - S(\rho_{AB}), \tag{1}$$

where $S(\cdot)$ denotes the von Neumann entropy.

The Holevo bound characterizes the capacity of quantum states in classical communication [12,13]. It is a keystone in many applications of quantum information theory[14,15,16,17,18,19,20]. With respect to the Holevo bound, the maximal Holevo quantity referred to weak measurements has been studied in [21]. The Holevo quantity of the SU(2)-invariant states has been investigated in [22]. The Holevo quantity of an ensemble $\{p_i;\rho_{A|\Pi_i}\}$, corresponding to a bipartite quantum state ρ_{AB} with the projective measurements $\{\Pi_i\}$ performed on the subsystem B, is given by [21]

$$\chi\{\rho_{AB}|\{\Pi_i^B\}\} = \chi\{p_i;\rho_{A|\Pi_i}\} \equiv S(\sum_i p_i \rho_{A|\Pi_i}) - \sum_i p_i S(\rho_{A|\Pi_i}), \tag{2}$$

where

$$p_i = \mathrm{tr}_{AB}[(I_A \otimes \Pi_i)\rho_{AB}(I_A \otimes \Pi_i)], \quad \rho_{A|\Pi_i} = \frac{1}{p_i}\mathrm{tr}_B[(I_A \otimes \Pi_i)\rho_{AB}(I_A \otimes \Pi_i)]. \tag{3}$$

It characterizes the A's accessible information about the B's measurement outcome when B projects the subsystem B by the projection operators $\{\Pi_i^B\}$.

In this paper, we consider the generalized n-qubit Werner state given in [23, 24]. The state becomes the Werner state [25] when $n = 2$. We study the OWD and the Holevo quantity under the bipartition of any single qubit (B subsystem) and the remaining $n-1$ qubits (A subsystem) for the generalized n-qubit Werner state. Here we perform a projective measurement on subsystem B. The general projective measurement operators are of the form,

$$\Pi_1 = \mathbb{I}_A \otimes |u\rangle_{BB}\langle u| \qquad \text{and} \qquad \Pi_2 = \mathbb{I}_A \otimes |v\rangle_{BB}\langle v|, \tag{4}$$

where $|u\rangle = \cos(\theta)|0\rangle + e^{i\phi}\sin(\theta)|1\rangle$ and $|v\rangle = \sin(\theta)|0\rangle - e^{i\phi}\cos(\theta)|1\rangle$ with $0 \le \theta \le \pi/2$ and $0 \le \phi \le 2\pi$. In section 2, we analytically calculate OWD between the subsystems A and B, and derive the linear relationship between OWD and the mixing probability p at the thermodynamic limit ($n \to \infty$). The Holevo quantity between the subsystems A and B is investigated in section 3.

2 OWD for generalized n-qubit Werner state

As the two-qubit Werner state is a special case of the Bell-diagonal states, while the quantum discord coincides with the one-way deficit for Bell-diagonal states [26], the one-way deficit is equal to the quantum discord for two-qubit Werner state [2]. We first study the relation between the non-local correlations and the total number of qubits n. The generalized n-qubit Werner state is given as follows,

$$\rho_{W_{AB}} = p|\phi\rangle_{AB\,AB}\langle\phi| + \frac{(1-p)}{2^n}\mathbb{I}_{AB}, \tag{5}$$

where $0 \leq p \leq 1$, $|\phi\rangle_{AB}$ is the n-qubit GHZ state under bipartition, $|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_A^{\otimes n-1}|0\rangle_B + |1\rangle_A^{\otimes n-1}|1\rangle_B \right)$, $\mathbb{I}_{AB}/2^n$ is the n-qubit maximally mixed state. To calculate the OWD between subsystems A and B for the state $\rho_{W_{AB}}$, we calculate the von Neumann entropy of $\rho_{W_{AB}}$.

Denote $N = 2^n$. The matrix representation of the state $\rho_{W_{AB}}$ has the form,

$$\rho_{W_{AB}} = \begin{bmatrix} a_{11} & 0 & 0 & \dots & a_{1N} \\ 0 & a_{22} & 0 & \dots & 0 \\ 0 & 0 & a_{33} & \dots & 0 \\ \vdots & & & \ddots & \vdots \\ a_{N1} & & & & a_{NN} \end{bmatrix}_{N\times N}, \tag{6}$$

where $a_{11} = a_{NN} = \left(\frac{1-p}{2^n} + \frac{p}{2}\right)$, $a_{1N} = a_{N1} = \frac{p}{2}$ and $a_{22} = a_{33} = a_{44} = \ldots = a_{N-1\;N-1} = \left(\frac{1-p}{2^n}\right)$. From the characteristic equation of the matrix $\rho_{W_{AB}}$, $(a_{22} - \lambda)(a_{33} - \lambda)\cdots(a_{N-1\;N-1} - \lambda)(\lambda^2 - \lambda(a_{11} + a_{NN}) + a_{11}a_{NN} - a_{N1}a_{1N}) = 0$, we have the eigenvalues [28],

$$\lambda_1 = \lambda_2 = \lambda_3 = \ldots = \lambda_{N-2} = \frac{1-p}{2^n} \tag{7}$$

and

$$\begin{aligned} \lambda_{N-1} &= \frac{1}{2}\left\{(a_{11} + a_{NN}) + \sqrt{(a_{11} - a_{NN})^2 + 4a_{1N}a_{N1}}\right\} \\ &= \frac{1 + (2^n - 1)p}{2^n}; \end{aligned} \tag{8}$$

$$\begin{aligned} \lambda_N &= \frac{1}{2}\left\{(a_{11} + a_{NN}) - \sqrt{(a_{11} - a_{NN})^2 + 4a_{1N}a_{N1}}\right\} \\ &= \frac{1-p}{2^n}. \end{aligned} \tag{9}$$

Therefore, we have the entropy $S(\rho_{W_{AB}})$,

$$\begin{aligned} S(\rho_{W_{AB}}) &= (N-2)\left\{-\left(\frac{1-p}{2^n}\right)\log_2\left(\frac{1-p}{2^n}\right)\right\} - \left(\frac{1+(2^n-1)p}{2^n}\right) \\ &\quad\cdot \log_2\left(\frac{1+(2^n-1)p}{2^n}\right) - \left(\frac{1-p}{2^n}\right)\log_2\left(\frac{1-p}{2^n}\right), \end{aligned} \tag{10}$$

$$\begin{aligned} &= -(2^n-1)\left(\frac{1-p}{2^n}\right)\log_2\left(\frac{1-p}{2^n}\right) - \left(\frac{1+(2^n-1)p}{2^n}\right) \\ &\quad\cdot \log_2\left(\frac{1+(2^n-1)p}{2^n}\right). \end{aligned} \tag{11}$$

To compute $\min_{\{\Pi_k\}} S(\sum_k \Pi_k \rho_{W_{AB}} \Pi_k)$ under measurement (4) on subsystem B, let us consider the following state,

$$\begin{aligned}\rho = & p\cos^2(\theta)|0\rangle^{\otimes n-1}\langle 0|^{\otimes n-1} + pe^{i\phi}\cos(\theta)\sin(\theta)|0\rangle^{\otimes n-1}\langle 1|^{\otimes n-1} \\ & + pe^{-i\phi}\cos(\theta)\sin(\theta)|1\rangle^{\otimes n-1}\langle 0|^{\otimes n-1} + p\sin^2(\theta)|1\rangle^{\otimes n-1}\langle 1|^{\otimes n-1} \\ & + \sum_{i=0}^{2^{n-1}-1}\left(\frac{1-p}{2^{n-1}}\right)|i\rangle\langle i|. \end{aligned} \tag{12}$$

Denote $L = 2^{n-1}$. The density matrix ρ has the form,

$$\rho = \begin{bmatrix} b_{11} & 0 & 0 & \dots & b_{1L} \\ 0 & b_{22} & 0 & \dots & 0 \\ 0 & 0 & b_{33} & \dots & 0 \\ \vdots & & & \ddots & \vdots \\ b_{L1} & 0 & 0 & \dots & b_{LL} \end{bmatrix}_{LL}, \tag{13}$$

where

$$b_{11} = p\cos^2\theta + \frac{1-p}{2^{n-1}}, \qquad b_{LL} = p\sin^2\theta + \frac{1-p}{2^{n-1}}, \tag{14}$$

$$b_{1L} = pe^{i\phi}\cos(\theta)\sin(\theta), \qquad b_{L1} = pe^{-i\phi}\cos(\theta)\sin(\theta) \tag{15}$$

and

$$b_{22} = b_{33} = \dots = b_{L-1\ L-1} = \frac{1-p}{2^{n-1}}. \tag{16}$$

The eigenvalues of ρ are $\beta_1 = \beta_2 = \dots = \beta_{L-2} = \frac{1-p}{2^{n-1}}$, $\beta_{L-1} = \frac{1+(2^{n-1}-1)p}{2^{n-1}}$ and $\beta_L = \frac{1-p}{2^{n-1}}$.

From (4) and (12), it is direct to verify that $\Pi_1\rho_{W_{AB}}\Pi_1 = \frac{\rho}{2}\otimes|u\rangle_{BB}\langle u|$ and $\Pi_2\rho_{W_{AB}}\Pi_2 = \frac{\rho}{2}\otimes|v\rangle_{BB}\langle v|$. Hence,

$$\sum_k \Pi_k\rho_{W_{AB}}\Pi_k = \Pi_1\rho_{W_{AB}}\Pi_1 + \Pi_2\rho_{W_{AB}}\Pi_2 \tag{17}$$

$$= \frac{\rho}{2}\otimes(|u\rangle_{BB}\langle u| + |v\rangle_{BB}\langle v|) \tag{18}$$

$$= \frac{\rho}{2}\otimes\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{19}$$

From the calculation of the eigenvalues of ρ, we have the following eigenvalues of the matrix $\sum_k \Pi_k\rho_{W_{AB}}\Pi_k$, $\alpha_1 = \alpha_2 = \dots = \alpha_{2L-4} = \frac{1-p}{2^n}$, $\alpha_{2L-3} = \alpha_{2L-2} =$

$\frac{1+(2^{n-1}-1)p}{2^n}$ and $\alpha_{2L-1} = \alpha_{2L} = \frac{1-p}{2^n}$. The entropy of $\sum\limits_k \Pi_k \rho_{W_{AB}} \Pi_k$ is given by

$$\begin{aligned}
S(\sum_k \Pi_k \rho_{W_{AB}} \Pi_k) &= -(2L-4)\left(\frac{1-p}{2^n}\right)\log_2\left(\frac{1-p}{2^n}\right) - 2\left(\frac{1+(2^{n-1}-1)p}{2^n}\right) \\
&\cdot \log_2\left(\frac{1+(2^{n-1}-1)p}{2^n}\right) - 2\left(\frac{1-p}{2^n}\right)\log_2\left(\frac{1-p}{2^n}\right) \\
&= -(2^{n-1}-1)\left(\frac{1-p}{2^{n-1}}\right)\log_2\left(\frac{1-p}{2^n}\right) - \left(\frac{1+(2^{n-1}-1)p}{2^{n-1}}\right) \\
&\cdot \log_2\left(\frac{1+(2^{n-1}-1)p}{2^n}\right). \qquad (20)
\end{aligned}$$

Note that $S(\sum\limits_k \Pi_k \rho_{W_{AB}} \Pi_k)$ is independent of the parameters θ and ϕ in the measurement operators given in (4). Therefore, the minimization of $S(\sum\limits_k \Pi_k \rho_{W_{AB}} \Pi_k)$ over the measurements is not required. Using Eqs. (1), (11) and (20), we have the OWD of state $\rho_{W_{AB}}$,

$$\begin{aligned}
\Delta^{\rightarrow}(\rho_{W_{AB}}) &= \min_{\{\Pi_k\}} S(\sum_k \Pi_k \rho_{W_{AB}} \Pi_k) - S(\rho_{AB}) \\
&= -(2^{n-1}-1)\left(\frac{1-p}{2^{n-1}}\right)\log_2\left(\frac{1-p}{2^n}\right) - \left(\frac{1+(2^{n-1}-1)p}{2^{n-1}}\right) \\
&\cdot \log_2\left(\frac{1+(2^{n-1}-1)p}{2^n}\right) + (2^n-1)\left(\frac{1-p}{2^n}\right)\log_2\left(\frac{1-p}{2^n}\right) \\
&+ \left(\frac{1+(2^n-1)p}{2^n}\right)\log_2\left(\frac{1+(2^n-1)p}{2^n}\right). \qquad (21)
\end{aligned}$$

In Fig. 1, We plot the OWD as a function of p for different number of qubits n. We observe that the OWD increases as p increases for any n. As n increases, the OWD increases for a given p, which indicates that the nonclassical correlations are dependent upon n. The maximum of OWD is attained at $p = 1$ for any n.

We next study the one-way deficit at thermodynamic limit ($n \to \infty$). The OWD (21) can be rewritten as,

$$\begin{aligned}
\Delta^{\rightarrow}(\rho_{W_{AB}}) &= -\left(1-p-\frac{1-p}{2^{n-1}}\right)\log_2\left(\frac{1-p}{2^n}\right) - \left(\frac{1}{2^{n-1}}+p-\frac{p}{2^{n-1}}\right) \\
&\cdot \log_2\left(\frac{1}{2^n}+\frac{p}{2}-\frac{p}{2^n}\right) + \left(1-p-\frac{1-p}{2^n}\right)\log_2\left(\frac{1-p}{2^n}\right) \\
&+ \left(\frac{1}{2^n}+p-\frac{p}{2^n}\right)\log_2\left(\frac{1}{2^n}+p-\frac{p}{2^n}\right). \qquad (22)
\end{aligned}$$

When $2^n \to \infty$, one obtains

$$\begin{aligned}
\lim_{2^n\to\infty} \Delta^{\rightarrow}(\rho_{W_{AB}}) &= -(1-p)\log_2\left(\frac{1-p}{2^n}\right) - p\log_2\left(\frac{p}{2}\right) \\
&+ (1-p)\log_2\left(\frac{1-p}{2^n}\right) + p\log_2(p) \\
&= p.
\end{aligned}$$

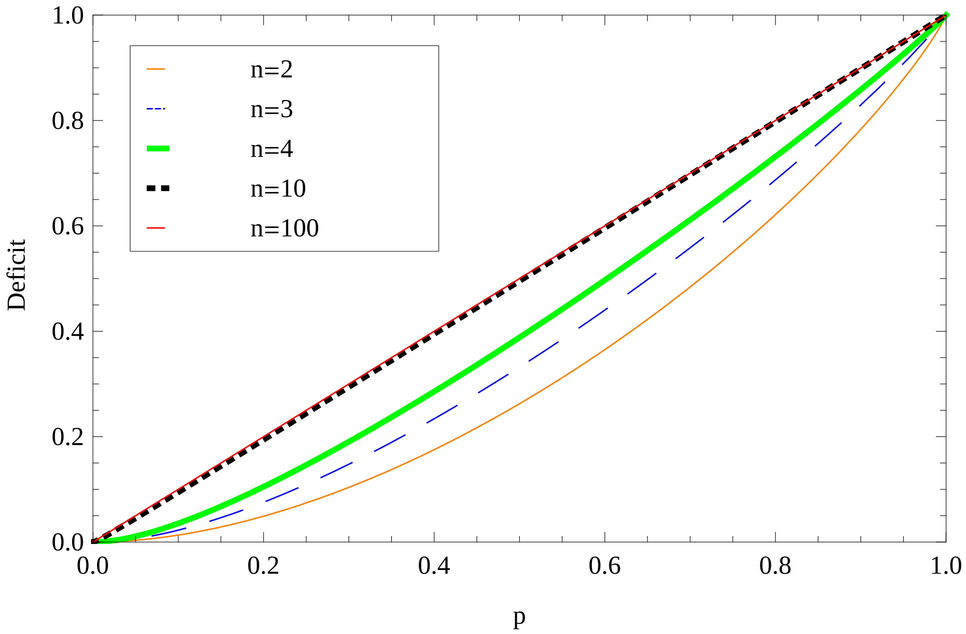

Fig. 1 (Color online) One-way deficit of the generalized n-qubit Werner State as a function of the mixing probability p for different number of qubits n.

Interestingly, the change between OWD and p saturates at thermodynamic limit. The curve in Fig. 1 approaches to a straight line with slope 1, a phenomenon we call it "saturation of one-way deficit".

3 Holevo Quantity for generalized n-qubit Werner state

In this section, we calculate the Holevo quantity of the generalized n-qubit Werner state $\rho_{W_{AB}}$. Denote p_i the probability with respect to the measurement outcome of Π_i, $i = 1, 2$. From (3) we have

$$\begin{aligned} p_1 &= tr(\Pi_1 \rho_{W_{AB}} \Pi_1) \\ &= tr(\frac{\rho}{2} \otimes |u\rangle_{BB}\langle u|) = \frac{1}{2}. \end{aligned}$$

The post measurement state of the subsystem A is

$$\begin{aligned} \rho_{A|\Pi_1} &= \frac{1}{p_1} tr_B(\Pi_1 \rho_{W_{AB}} \Pi_1) \\ &= \frac{1}{p_1} tr_B(\frac{\rho}{2} \otimes |u\rangle_{BB}\langle u|) = \rho. \end{aligned}$$

Similarly, we have $p_2 = \frac{1}{2}$ and $\rho_{A|\Pi_2} = \rho$.

The Holevo quantity of the generalized n-qubit Werner state is then given by

$$\begin{aligned}\chi\{\rho_{AB}|\{\Pi_i\}\} &= S\left(\sum_i p_i \rho_{A|\Pi_k}\right) - \sum_i p_i S\left(\rho_{A|\Pi_k}\right) \\ &= S\left(\frac{1}{2}\rho + \frac{1}{2}\rho\right) - \left(\frac{1}{2}S(\rho) + \frac{1}{2}S(\rho)\right) \\ &= 0.\end{aligned}$$

4 Conclusion

We have analytically calculated the one-way deficit of the generalized n-qubit Werner state, with the projective measurements performed on one-qubit subsystem. We have found that the OWD increases as p increases for any n. When n increases, the OWD increases for any fixed p. For any n, the maximum of OWD is attained at $p = 1$. Furthermore, for large n ($2^n \to \infty$), by analytical calculation we have proved that this curve OWD versus p approaches to a straight line with slope 1. We have also shown that the Holevo quantity of the generalized n-qubit Werner state is 0.

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Conflict of Interest Statements The authors declare no competing interests.

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