# Quantum and Semi-Quantum Lottery: Strategies and Advantages

Sandeep Mishra<sup>1</sup> and Anirban Pathak<sup>1,\*</sup>

<sup>1</sup>Jaypee Institute of Information Technology, A 10, Sector 62, Noida, UP-201309, India

\*Corresponding author: anirban.pathak@gmail.com

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#### Abstract

Lottery is a game in which multiple players take chances in the hope of getting some rewards in cash or kind. In addition, from the time of the early civilizations, lottery has also been considered as an apposite method to allocate scarce resources. Technically, any scheme for lottery needs to be fair and secure, but none of the classical schemes for lottery are unconditionally secure and fair. As fairness demands complete unpredictability of the outcome of the lottery, it essentially requires perfect randomness. Quantum mechanics not only guarantees the generation of perfect randomness, it can also provide unconditional security. Motivated by these facts, a set of strategies for performing lottery using different type of quantum resources (e.g., single photon states, and entangled states) are proposed here, and it's established that the proposed strategies leads to unconditionally secure and fair lottery schemes. A scheme for semi-quantum lottery that allows some classical users to participate in the lottery involving quantum resources is also proposed and the merits and demerits of all the proposed schemes are critically analysed. Its also established that the level of security is intrinsically related to the type of quantum resources being utilized. Further, its shown that the proposed schemes can be experimentally realized using currently available technology, and that may herald a new era of commercial lottery.

## 1 Introduction

Lottery is a game of chances where multiple players hope for getting the rewards. The use of lotteries has been prevalent since the time of early human civilizations. For example, during the Roman empire, lotteries were used as a form of amusement for giving gifts to the guests [1]. The first commercial application of the lottery was done by Roman Emperor Augustus Caesar as an alternative to increase in taxes for the purpose of funding the infrastructure projects of the city [2]. In the modern history, the first officially recorded state controlled lottery was organized by Queen Elizabeth I in 1556-59 for the purpose of funding a set of projects [3]. In this particular case, 4,00,00 tickets of 0.5 pound each were issued with a reward of 5,000 pounds to the winner. Since then, the

lotteries have globally evolved as a mechanism that states can adopt to collect money (without levying higher taxes) required for various people-centric projects. Thus, the lottery is historically used for socially meaningful purposes, but it has close resemblance with the gambling and it can always be viewed as a kind of gambling. Consequently, nowadays commercial lottery with financial rewards is not considered righteous in many countries.

Despite the above issue, we are interested in the lottery as the use of lottery is not restricted to gambling and collection of funds by the states. In fact, lotteries have many other applications in the diverse fields [4]. In the modern theory of allocation of resources, there are primarily four ways of allocation namely merit, queue, auction and lotteries. In "merit", the persons are allotted points based on many parameters and the entity with maximum points get the first preference. In "queue", the entity which has submitted the application first gets the reward. In "auction", one who is willing to pay the maximum gets the rights. Of all the above, lotteries does a randomization and the entity gets hold of the resource purely by luck. In fact, lottery is the only method which is free from any type of inherent bias. Debates are going on to find an optimum method for the allocation of resources [5], but the lottery is often considered as a better and fairer way of allocating the scarce resources among the large number of applicants [6, 7]. Specially, if the number of indivisible goods (k) is lesser than the number of applicants (n) then lottery is considered as a suitable way for the allocation. Tracing down in history, the lottery has been used by governments for the allotment of lands to the farmers. Even nowadays lotteries are used for the allocation of low cost houses by many governments across the world. Lotteries are prevalent medium in many countries for fair grant of admission to the students in the elementary schools. Lotteries are also used to grant work permits from the pool of eligible applicants. Lotteries are considered as a good way of placing the teams in various groups of major sporting events such as Olympics and the events organized by FIFA, NBA, etc. Even there is a recorded history of use lottery in the legal system where punishment to the accused were delivered via use of lottery in a situation where the act of crime was committed by a mob and it was difficult to trace out the right person who dealt the fatal blow [8]. With respect to technology, lottery based CPU scheduling among the various competing processes has been proposed and used for instantaneous fair CPU allocation [9]. Indeed, recently, lottery ticket hypothesis for graph theory has been proposed for training of the neural networks [10]. Nowadays, serious debate is also going on for the use of lotteries for the funding of the research projects as the prevalent medium of the peer review process has many intrinsic biases [11]. In fact, many funding agencies such as the Health Research Council of New Zealand, Volkswagen Foundation in Germany and the Swiss National Science Foundation are using the lotteries to fund the research projects after the initial screening of the eligible projects [12, 13].

As described above, lottery is an integral part of many important processes that are associated with our daily life. However, not all forms of lottery can be considered as fair. To understand this point, we first need to understand the meaning of a fair lottery. A lottery is considered as fair if and only if all the participants have an equal chance of winning. Thus, it requires perfect randomness. Further, once the results are announced then no one should be able to forge the ticket and claim to be the winner. Moreover, every participant should be able to verify the outcome of the process. An important point of concern is that the fairness of the lottery is inherently dependent upon the security of the lottery scheme being used. Thus, randomness and security are the primary

concerns associated with the schemes of the lottery. Currently, the lottery schemes being used depend upon the credibility of the trusted authorities or security based on some mathematical complexities. Security derived in such a way is conditional. In fact, an unconditional security cannot be obtained in the classical word. Further, randomness used in classical schemes is weak compared to the randomness that can be generated quantum mechanically. Naturally, often issues have been raised regarding the fairness of the lottery schemes and such things will come up again and again until and unless we have an unconditionally secure lottery scheme. By unconditionally secure, we mean that any potential adversary even with the unlimited resources would not be able to manipulate the outcome. Such issues were raised for the classical cryptographic schemes, too, but the advent of quantum cryptography [14] provided a new way forward for unconditionally secure cryptography [15, 16, 17]. The use of quantum states is currently being explored for providing unconditional security in various applications such as bit commitment [18, 19, 20, auctions [21, 22], voting [23, 24, 25, 26], multi-party computation [27]. Lottery is inherently related to quantum states as quantum mechanics has intrinsic randomness and lottery demands a complete randomization of the outcome [28].

Quantum strategies for fair and unconditionally secure lottery is a demand of the time. A step in this direction was provided by Sun et al. in Ref. [29], where they proposed the schemes for lottery and auction on the backbone of the quantum blockchain. However, the mentioned protocol was not mature as it was based on quantum bit commitment which still does not provide unconditional security. Further, it used the elementary idea of quantum blockchain where communication between nodes was done via the QKD protocol for which trust between the nodes would have been required at the forefront. However, the blockchain requires consensus on the contents of the decentralized data between between non-trusting parties. Barring this work, the field for use of quantum systems in implementing lottery schemes has remained largely untouched till now. Hence, we try to explore the use of quantum resources towards the development of fair and unconditionally secure lottery schemes which can be implemented via the currently available quantum hardware.

The rest of the paper is structured as follows. In Section 2, we introduce some of the basic ideas and nomenclature required for better understanding of the article with specific attention to the requirements that a good scheme of lottery should satisfy. Subsequently, a set of schemes for quantum and semi-quantum lottery are proposed in Section 3. This is followed by security analysis of the proposed schemes in Section 4. Finally, the paper is concluded in Section 5.

## 2 Basic notations and definitions

The basic requirements to be satisfied by a lottery scheme can be briefly mentioned as follows:

- i) **Eligibility:** Only the registered and legitimate entities can take part in the lottery.
- ii) **Equi-probability:** All the entities have equal probability to win the lottery. Thus, if there are n participants, then the probability of winning for every participant  $(p_i)$  must be the same (complete randomization) and the total probability should

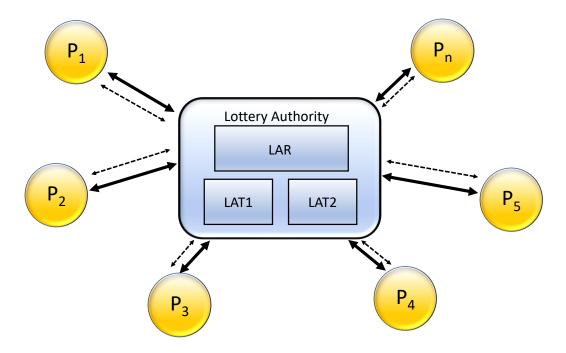


Figure 1: (Color online) Schematic of the quantum lottery scheme with solid lines denoting quantum channels while dashed lines denoting classical channels.

be equal to unity. i.e.,

$$p_1 = p_2 = p_3 = \dots = p_n = \mathcal{P}, \text{ s.t. } \sum_{i=1}^n p_i = 1.$$
 (1)

- iii) Binding: No one can change the lottery ticket after it has been issued.
- iv) Verifiability: Anyone can verify the outcome of the process.
- v) **Secure:** An adversary even with unconditional computational power cannot manipulate the outcome.

## 3 Quantum lottery schemes

The proposed lottery schemes consist of the following stakeholders (see fig 1):

- 1. Lottery Authority: The lottery authority (LA) is responsible for the conduct of lottery. Further, it will consist of multiple personnels but for the sake of simplicity, we can consider it to be consisting of three agents only, namely 'lottery authority for registration' (LAR), 'lottery authority for ticketing 1' (LAT1) and 'lottery authority for ticketing 2' (LAT2). The role of LAR is to register the interested parties and record their details. The role of LAT1 and LAT2 is to generate the lottery tickets for every eligible participant. Further, they cooperate with each other to declare the winning lottery ticket.
- 2. **Participants:** Participants  $(P_i)$  consist of the set of people who are interested to participate in the lottery. It is to be mentioned that the proposed lottery scheme provides a method by which every participant can verify the winning lottery ticket.

## 3.1 BB84 based quantum lottery scheme

It has always been an endeavour since ages to develop a scheme by which secret messages can be sent from one party to another with minimum number of assumptions. classical schemes were basically based on the assumption of trust that the encryption key is available to only authorized people or that some problems are too complex to be solved in polynomial time [30]. However, the advent of quantum cryptography in 1984 (i.e., the introduction of BB84 protocol for quantum key distribution (QKD)) [14] altogether changed the rules of the game by providing a scheme for unconditionally secure distribution of keys. Further, Ekert showed that the unconditional secure distribution of keys can be done via the use of entanglement [31] and the presence of unauthorized interceptor can be detected by checking the correlations between entangled particles. This lead to two different ways of secure key distribution and each has its own advantages and disadvantages. Currently, this field of unconditional secure cryptography is quite mature to be used in practical situations and scenarios [15, 16, 32, 33]. Parallel to the development of quantum key distribution technology, researchers have been working on the development of true random number generators whose outputs are completely non-deterministic as well as private. Since quantum mechanics has intrinsic randomness associated with it [34], so the quantum systems can be used to generate truly random numbers [35]. Currently, the quantum technology is quite mature that we can generate very high quality random numbers at great speeds [36, 37, 38, 39] for a wide variety of applications including secure communication, e-commerce, multi-party computations and lottery. In fact, there are many commercial quantum random number generator (QRNG) devices available in the market such as ID Quantique [40], Toshiba [41], PicoQuant [42], MPD [43] etc.

Taking inspiration from quantum cryptographic protocols, and the availability of the required hardware, we will first propose a lottery strategy based on BB84 states [44] and then briefly elaborate about its physical implementation. BB84 states are one of the most important and widely studied set of states studied in context of quantum key distribution. These states can be experimentally produced in a wide variety of physical systems, but it is more useful when implemented using photonic systems. In photonics, BB84 states are essentially polarization-encoded qubits or equivalently a sequence of photons which are randomly polarized in horizontal, vertical,  $45^{\circ}$  or  $-45^{\circ}$  which respectively correspond to the states  $|0\rangle$ ,  $|1\rangle$ ,  $|+\rangle$  and  $|-\rangle$  [45]. These states can be easily generated by passing the photons from a laser diode source to an attenuator (neutral density filter) or by performing heralding on the output of certain spontaneous parametric down conversion process, and then passing the single photon through the relevant polarizer [46]. The lottery scheme using BB84 states involves the following three stages:

#### 3.1.1 Registration phase

This phase is required for the registration of every participant with the LA and generation of unique digital signatures similar to that proposed by Wallden et al. [47] for every participant  $P_i$ . The steps involved in the process can be enumerated as follows:

- **Step 1** The participant  $P_i$  will first register with the LA by sending the documents and purchasing the lottery ticket with LAR.
- **Step 2** LAR will verify the credentials of the  $P_i$  and then use a QRNG to issue a unique 256 bit participant's identity (PID) for the participant  $P_i$ . Further, LAR keeps a

record of all the allocated PIDs in the database. The purpose of generation of PID for every participant is to maintain the privacy of the participants as from here on the participant will only be using PIDs in all the subsequent steps. Because of the use of QRNG by LAR, the probability of PID collision for two participants will be asymptomatically very small. However, if the PID generated for a new participant collides with the existing set of allocated PIDs, then the QRNG is used again to generate a new PID.

- Step 3 For the generation of digital signatures, the participant  $P_i$  will generate two large identical, but random sequences of BB84 states  $(|0\rangle, |1\rangle, |+\rangle, |-\rangle)$  in accordance with the output generated by QRNG.  $P_i$  will then send the first and second sequence respectively to LAT1 and LAT2 via the use of quantum channel.
- **Step 4** LAT1 (LAT2) will randomly choose to either forward the signature element to LAT2 (LAT1) or keep it with themselves to directly measure it. Further, in either case the position of the elements in the sequence is recorded.
- Step 5 LAT1 (LAT2) measures the states that they have directly received from  $P_i$  or through LAT2 (LAT1) by randomly choosing either the rectilinear basis ( $|0\rangle, |1\rangle$ ) or diagonal basis ( $|+\rangle, |-\rangle$ ). In this way, after the measurements, both LAT1 and LAT2 exclude at least one of the four possible states and generate an eliminated signature for the sequence. e.g., if the measurement result is  $|0\rangle$  then the participant must have never sent  $|1\rangle$ . The eliminated signature will serve as the quantum digital signature for the participant  $P_i$  to be used in the next phase.

Similarly, unconditionally secure digital signatures are generated for every participant  $P_i$ . The registration phase in only meant for the generation of signatures for every participant that will be used in the next phase for the authentication of the participants before the generation of the tickets.

#### 3.1.2 Ticketing phase:

In this phase, lottery ticket numbers are generated by every participant. The steps involved in this phase are as follows:

- **Step 1** Participant  $P_i$  will first send the PID to both LAT1 and LAT2. After that,  $P_i$  will reveal the classical information corresponding to the BB84 sequence used in the registration stage.
- Step 2 Both LAT1 and LAT2 will then match the set of states revealed by  $P_i$  with the corresponding eliminated signatures for every position. If the number of mismatches as recorded by either of LAT1 and LAT2 is greater than a threshold limit, then the participant is not allowed to take part further.  $P_i$  is allowed to participate only if he is authenticated by both LAT1 and LAT2.
- **Step 3**  $P_i$  will use any of the experimentally available BB84 based QKD protocol to generate two keys, namely  $K_{P_i}^{LAT1}$  and  $K_{P_i}^{LAT2}$  corresponding to LAT1 and LAT2.
- **Step 4**  $P_i$  will then generate a random 256-bit unique ticket  $(TID_i)$  via use of QRNG. These TIDs will be used for the draw of lots. Also, the hash value of the  $TID_i$

will be generated and publicly announced. The generation of  $TID_i$  by the participant will prevent any kind of manipulation by the lottery authority during ticket allocation phase. Further, the public announcement of hash of  $TID_i$  precludes any forging of the lottery ticket after the reward have been announced. Here, it is to be mentioned that any adversary can use the dictionary attack in which the publicly announced hash value of  $TID_i$  can be compared with the pre-calculated hash value of all possible 256 bit  $TID_i$ s. But, such an attack is computationally impossible.

Step 5  $P_i$  sends the  $TID_i$  to both LAT1 and LAT2 using the key  $K_{P_i} = K_{P_i}^{LAT1} \oplus K_{P_i}^{LAT2}$ . In this way, the TID sent by  $P_i$  can be opened only if LAT1 and LAT2 cooperate with each other. As an alternative,  $P_i$  can use any other experimentally available quantum secret sharing protocol using BB84 states [48, 49] to send the  $TID_i$  to LAT1 and LAT2.

The same procedure will be repeated by every participant  $P_i$  to send the correspondingly generated  $TID_i$  to the LAT1 and LAT2. No lottery tickets will be accepted after the closing of the phase.

### 3.1.3 Rewards phase:

The steps involved in this phase are as follows:

- **Step 1** LAT1 and LAT2 cooperate with each other to open the tickets  $TID_i$ .
- **Step 2** The winning ticket is announced as the bit wise XOR of all the received TIDs. i.e.,  $T^W = \oplus TID_i$ .
- Step 3 The reward for every participant is calculated as proportional to the Hamming distance of the participants' ticket (TIDi) from the winning ticket (TW). Hamming distance between two strings basically corresponds to the number of positions in which two strings are different. For bit strings, it corresponds to the number of 1's present in the XOR of the two strings. In the proposed scheme, there is finite possibility that two or more participants have the same TID. For such an event, the rewards can be distributed equally to all participants having the winning ticket, but for most of the cases the specific scheme for the calculation of rewards will depend upon the particular application of the lottery scheme.

In this phase, our main focus is to just propose a method for generation of the winning ticket rather than commenting on a particular method for distribution of the rewards. The allocation of rewards will depend upon the application of lottery scheme and is thus left open to the potential users to develop their own methods. Further, such methods only constitute as a part of the post-processing techniques.

Before, moving to the next lottery scheme, let us just briefly describe the physical implementation of the above mentioned scheme. Figure 2 shows the schematic of the resource requirements for the participants and the lottery authority to implement the BB84 based lottery scheme. We can see that the resource requirements at both the ends are symmetric in nature. Both of them require a QRNG to perform certain steps in the protocol and more so with respect to generation of TIDs and PIDs. As mentioned before, currently there are a wide variety of commercial QRNGs in the market, though the cost is a bit on the higher end. To reduce cost, the participants may use pseudo random number

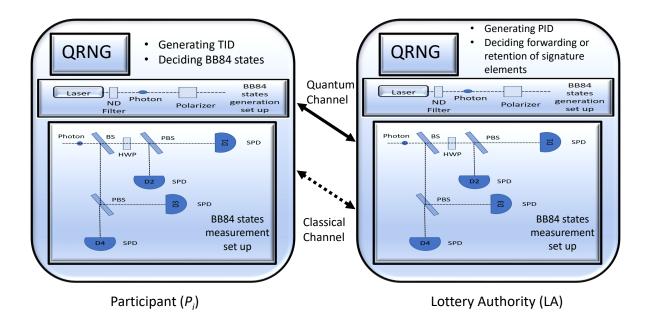


Figure 2: (Color online) Schematic of the resource requirements for BB84 based lottery scheme. All the participants as well as lottery authority require a QRNG, a set up for generation of BB84 states and a set up for measurement of BB84 states.

generators (PRNGs) which pass NIST test [50] but that may lead to a compromise with the security aspects. Other than QRNG, both of them require set-up for the generation and the measurement of BB84 states. The current optical technology is quite mature enough to generate and measure BB84 states with very high fidelity. For photonics based implementation, the qubits are encoded in the polarization states of the single photon with horizontal, vertical,  $45^{\circ}$  or  $-45^{\circ}$  respectively corresponding to the states  $|0\rangle$ ,  $|1\rangle$ ,  $|+\rangle$  and  $|-\rangle$  [45]. These BB84 states are used for both the authentication of the participants as well as sending of the TIDs from the participants to the lottery authority. Further, the participants and lottery authority are connected to each other via bi-directional quantum as well as the classical authenticated channel. Current technology now allows us to send qubits from one party to another through free space communication as well as via use of optical fibres. So, the proposed lottery scheme seems feasible to be implemented via the use of currently available technology.

## 3.2 Entanglement based quantum lottery scheme

Till now, we have proposed a lottery scheme based on BB84 states, but this is not the only quantum system that can used. In fact, we already know that for quantum cryptography, there are many entanglement based protocols [31, 33, 32]. The use of entanglement brings altogether new features such as device independence [51, 52, 27]. i.e. we need not trust the devices used for the implementation. Similar to the use of entanglement in cryptography, here we will propose a lottery scheme by exploiting the feature of entanglement that can be found only in quantum systems. It is to be mentioned that quantum entanglement is a very costly resource which is very difficult to maintain. So, we will try to minimize the use of quantum entanglement in the proposed scheme. We will keep the registration

phase and rewards phase same as that used in the already discussed scheme while using the entanglement only in the ticketing phase using schemes similar to that of quantum secret sharing [53, 54, 55]. This is done with the motivation that ticketing is the most important phase where tickets are generated by the participants and are sent to the lottery authority. The proposed scheme makes use of the Bell state  $|\psi^-\rangle = \frac{1}{\sqrt{2}}\{|01\rangle - |10\rangle\}$  and single qubit local unitary operators, namely  $U_0 = |0\rangle\langle 0| + |1\rangle\langle 1|$ ,  $U_1 = |0\rangle\langle 0| - |1\rangle\langle 1|$ ,  $U_2 = |1\rangle\langle 0| + |0\rangle\langle 1|$ ,  $U_3 = |0\rangle\langle 1| - |1\rangle\langle 0|$ . In principle, the scheme can use any of the four Bell states  $(|\psi^{\pm}\rangle, |\phi^{\pm}\rangle)^{-1}$  but we will use only  $|\psi^-\rangle$  in the scheme. Further, the scheme will make use of the entanglement swapping for two Bell states [56, 57]. The steps involved in the ticketing phase while implementing entanglement based lottery scheme are as follows:

Step 1 same as that of 3.1.2

Step 2 same as that of 3.1.2

- Step 3  $P_i$  will prepare 256 pair of Bell states  $|\psi^-\rangle$  and stores the first qubit of all 256 pairs with him. The sequence of second qubits of one set of 256 Bell states is sent to LAT1 while the sequence of second qubits of the other set of 256 Bell states is sent to LAT2. So,  $P_i$  shares 256 Bell states with LAT1 (Set I) and another 256 Bell states with LAT2 (Set II). The combined state of  $P_i$ , LAT1 and LAT2 can be written as  $\{|\psi^-\rangle^1\otimes|\psi^-\rangle^2\otimes\cdots\otimes|\psi^-\rangle^{256}\}_{LAT1}^{P_i}\bigotimes\{|\psi^-\rangle^1\otimes|\psi^-\rangle^2\otimes\cdots\otimes|\psi^-\rangle^{256}\}_{LAT2}^{P_i}$ .
- Step 4  $P_i$  randomly picks half of his qubits, then choose to measure them either in the computational basis  $(|0\rangle, |1\rangle)$  or Haddamard basis  $(|+\rangle, |-\rangle)$  and publicly announce the basis used for measurement of his qubits. LAT1 and LAT2 will use the same basis as announced by  $P_i$  to measure their corresponding qubits. If there is no adversary during the transmission phase, then the measurement outcomes of  $P_i$  will be opposite to that of LAT1. e.g. If  $P_i$  gets  $|0\rangle$  ( $|1\rangle$ ) then LAT1 will get  $|1\rangle$  ( $|0\rangle$ ) while if  $P_i$  gets  $|+\rangle$  ( $|-\rangle$ ) then LAT1 will get  $|-\rangle$  ( $|+\rangle$ ). Similar is the case for measurement outcomes of  $P_i$  and LAT2. If the error is below the threshold limit, then they proceed to next step else they abort the protocol. Further, they will rearrange the qubits to have the combined state as  $\{|\psi^-\rangle^1 \otimes |\psi^-\rangle^2 \otimes \cdots \otimes |\psi^-\rangle^{128}\}_{LAT1}^{P_i} \otimes \{|\psi^-\rangle^1 \otimes |\psi^-\rangle^2 \otimes \cdots \otimes |\psi^-\rangle^{128}\}_{LAT2}^{P_i}$ .
- Step 5  $P_i$  will choose one qubit from Set I and another qubit from Set II. Further,  $P_i$  will randomly choose one of the above qubits to apply any one of the operators  $U_0, U_1, U_2, U_3$  to encode the bits 00, 01, 10, 11 respectively.  $P_i$  will then perform a Bell measurement on the two qubits and then publicly announce the outcome. The same operation is performed for all the qubits of  $P_i$  by taking one qubit from Set I while other from Set II. In this way,  $P_i$  will send the encoded 256 bit TID to LAT1 and LAT2. Further, the hash of the TID is publicly announced.
- **Step 6** LAT1 and LAT2 can cooperate with each other to get the TID sent by  $P_i$  after performing the necessary joint Bell measurements of their respective qubits and the properties of entanglement swapping.

In this way, all the  $P_i$ s will send their TIDs to the LA and the TIDs can be opened only if both the LAT1 and LAT2 cooperate with each other. So, entanglement based

 $<sup>|1|\</sup>psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\{|01\rangle \pm |10\rangle\} \text{ and } |\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\{|00\rangle \pm |11\rangle\}$ 

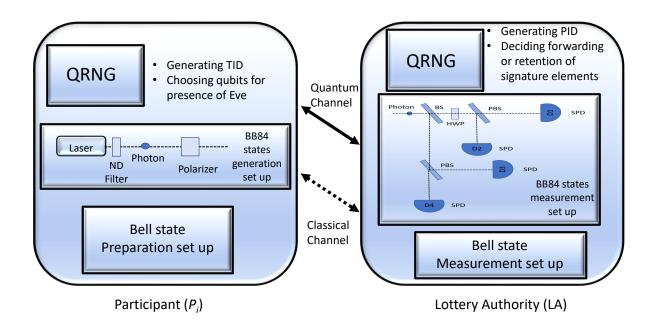


Figure 3: (Color online) Schematic of entanglement based lottery scheme. Participants require a BB84 state generation set up while LA require a BB84 state measurement set up for registration phase while ticketing phase requires Bell state preparation set up with participants and Bell state measurement set up with LA. Other than this, participants and LA also require QRNG.

lottery scheme differs from that of BB84 based scheme only in the method adopted to send the ticketing numbers (TIDs) from participants to the lottery authority. The advantage of using entanglement is just to provide an additional level of security.

Let us now briefly describe the physical implementation of the proposed scheme. Figure 3 shows the schematic of the resource requirements for participants and the lottery authority to implement the entanglement based lottery scheme. In contrast to the BB84 based scheme, the resource requirements here are not symmetric with respect to the participants and the lottery authority except the use of QRNGs at both the ends. As mentioned before, in this scheme too authentication of the participants is done via the use of BB84 based digital signature scheme. But for its implementation, only the BB84 state generation set-up is required at the participants end while the BB84 state measurement set-up is required by the lottery authority. Further, the TIDs are sent from participants to the lottery authority via encoding the TIDs into the Bell states. So, the Bell state generation set up is required by the participants while Bell state measurement set up is required by lottery authority. Similar to the case of BB84 based scheme, participants and the lottery authority are connected to each other via a quantum channel and classical authenticated channel. As far as current technology is concerned, the proposed scheme can be physically implemented, but certainly the use of fragile resources such as entanglement will come at a very heavy cost. In the next section, we will propose a scheme to minimize the cost by allowing all participants to use classical resources, while only lottery authority having access to quantum resources.

## 3.3 Semi-quantum lottery scheme

In the previous two proposed schemes, all the participants need to have the quantum capabilities. However, in the realistic situations the quantum resources are extremely costly and difficult to maintain. In fact, entanglement is a very fragile resource too. In order to overcome these limitations, Boyer et al. [58] in 2007 proposed a semi-quantum scheme of quantum key distribution, in which only one party has full quantum abilities but the other party is classical. The classical party can either reflect back the qubits or can measure the incoming qubits in the computational basis  $(|0\rangle, |1\rangle)$  only. Since then, various semi-quantum protocols have been proposed in quantum cryptography [59, 60, 61] and related areas. As far a current scenario is concerned, infrastructure for classical communication systems is very well developed and is available at a very reasonable cost. But in contrast, the quantum resources such as creation and manipulation of quantum states, quantum entanglement are too costly and difficult to handle. So, the current situation demands the development of schemes in which only few nodes have full quantum capabilities while the rest of the nodes can make of use only classical resources. Such schemes are known as semi-quantum schemes, and those schemes are relevant as they can exploit advantages of the currently available classical infrastructure. Taking the above motivation, we will now present a semi-quantum lottery scheme in which lottery authority will have full quantum capability, but all the participants will have only classical abilities. The proposed semi-quantum lottery scheme is further shown to be equally good in comparison with their quantum counterparts. Let us now describe in detail the semiquantum scheme for lottery.

## 3.3.1 Registration phase:

- Step 1 same as that of 3.1.1
- Step 2 same as that of 3.1.1
- Step 3 For the generation of digital signatures, LAR will prepare n qubits in the state  $|+\rangle$  and send it to  $P_i$  one by one. The participant  $P_i$  can either measure the incoming qubit in the computational basis  $(|0\rangle, |1\rangle)$  or let it go as it is to LAT1.
- Step 4 LAT1 will randomly choose to measure the qubit in either computational basis  $(|0\rangle, |1\rangle)$  or Hadamard basis  $(|+\rangle, |-\rangle)$  and note the outcome. After performing the measurement, resultant qubit is sent to the participant  $P_i$ .
- Step 5 The participant  $P_i$  will perform exactly the same operation as done by him in Step 3 (i.e., either pass the qubit or measure in computational basis) on incoming qubit from LAT1 and send it to LAT2.
- **Step 6** Similar to LAT1, LAT2 will also randomly choose either computational basis  $(|0\rangle, |1\rangle)$  or Hadamard basis  $(|+\rangle, |-\rangle)$  to measure the qubit and note the outcome.
- Step 7 LAT1 and LAT2 will announce the basis used by them to measure each of the n qubits received by them. Further, they will keep only those outcomes in which they have used the same basis and discard the rest. These outcomes will be used by LAT1 and LAT2 to verify the participant in the ticketing phase.

### 3.3.2 Ticketing phase:

- Step 1 Participant  $P_i$  will first send the PID to LAT1 and LAT2. After that,  $P_i$  will reveal the information on all n qubits, whether he has allowed the qubit to pass to LAT1 and LAT2 or measured in the computational basis before sending them to LAT1 and LAT2.
- Step 2 Both LAT1 and LAT2 will then match the outcomes of their measurement for the cases in which participant  $P_i$  has not measured the qubit before passing it to them. For such cases, the outcome recorded by LAT1 and LAT2 will match with each other. If the number of mismatches is greater than a threshold limit, then the participant is not allowed to take part further.  $P_i$  is allowed to participate only if he is authenticated by LAT1 and LAT2.
- **Step 3**  $P_i$  will use the semi-quantum QKD scheme using BB84 protocol given by Boyer et al. [58] or any other semi-quantum QKD protocol to generate two keys namely  $K_{P_i}^{LAT1}$  and  $K_{P_i}^{LAT2}$  corresponding to LAT1 and LAT2.
- Step 4 same as that of 3.1.2
- Step 5  $P_i$  sends the  $TID_i$  to both LAT1 and LAT2 using the key  $K_{P_i} = K_{P_i}^{LAT1} \oplus K_{P_i}^{LAT2}$  or via use of any other semi-quantum quantum secret sharing protocol. In this way, the TID sent by  $P_i$  can be opened only if LAT1 and LAT2 cooperate with each other.

#### 3.3.3 Rewards phase:

same as that of 3.1.3

So, we can see that the lottery scheme can be implemented using the lesser resources in comparison to that of full quantum lottery schemes. It is to be mentioned here that the above lottery scheme is also a BB84 state based scheme, but this allows the users to have only classical resources with only lottery authority having full quantum capabilities. Figure 4 describes the schematic of the resource requirements for participants and lottery authority to implement the protocol. We can clearly see that, except for the QRNG (required for performing certain steps), participants just need to have access to quantum channel coming out from the lottery authority. After receiving the qubits from the lottery authority, the participants just need a set up for measurement of the incoming qubits in the computational basis  $(|0\rangle, |1\rangle)$ . Also, they can allow the qubits to be returned back to lottery authority as it is without any modification. Further, to reduce the costs the participants may use PRNGs which passes the NIST tests, but at the cost of security. Looking at the lottery authority, they require quantum resources such as QRNG, BB84 states generation as well as measurement set-up. With the current advancements of technology, the proposed protocol can be implemented with the currently available hardware.

So far we have proposed three protocols for lottery, which can be realized using different amount (and type) of quantum resources. Further, we have shown that the proposed theoretical schemes can be implemented using the currently available hardware. However, the security of the protocols is not discussed in detail until now. Consequently, it will be apt to perform security analysis of the proposed lottery schemes in the next section.

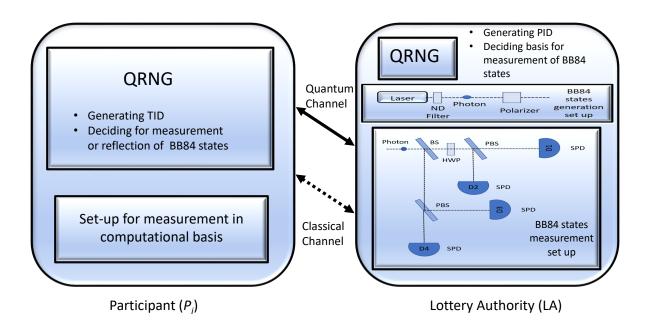


Figure 4: (Color online) Schematic of semi-quantum based lottery scheme. Participants require a QRNG and a set up for measurement of BB84 states in computational basis. LA is fully quantum and requires a QRNG, a set up for BB84 state generation and a set up for measurement of BB84 states.

## 4 Security analysis

The proposed schemes conform to the requirements of a good lottery scheme as described in section 2. Also, in all the proposed schemes, we can see that the tickets are generated by the participants and the lottery authority only collects the generated TIDs which are then further used for deciding the rewards. In this section, we will first look at the security aspects of the BB84 or entanglement based schemes (henceforth referred to as type I scheme) and then look at the security of semi-quantum schemes (henceforth referred to as type II scheme). For the type I scheme, we can see that after the registration every participant generates a digital signature with lottery authority by the use of unconditionally secure BB84 based digital signature scheme. These digital signatures are used for the authentication of the eligible participants and only eligible participants are allowed to generate the tickets and take part further in the process. So, the eligibility condition is satisfied. Now, since the winning ticket is announced by taking the bit wise XOR of all the valid TIDs, so every TID has an equal probability to win. Further, no one will be able to predict the winning ticket beforehand until and unless one gets hold of all the TIDs. So, the equi-probability condition is satisfied.

The TIDs are sent by the participants to the lottery authority using the unconditionally secure experimentally feasible BB84 protocol. So, it is practically impossible for any adversary to manipulate or change the ticket ID. Further, the TIDs can be opened only if LAT1 and LAT2 cooperate with each other. Even in the entanglement based scheme, the TIDs are sent to the lottery authority via the use of Quantum Secret Sharing (QSS) protocol. In this way, the tickets are sent from participants to lottery authority in an unconditionally secure way. Further, every participant publicly announces the hash value

of their TID before being sent to the lottery authority. This is done to prevent the participant to claim a different TID once the rewards have been announced. In this, the type I lottery schemes are unconditionally secure while sending of the ticket Id but the binding property of the TID via hash function makes it only computationally secure as an adversary can perform the dictionary attack. In this attack, an adversary can compute the hash of all the TIDs before hand and then via the mapping of the TID with their hash value the adversary can get hold of all the transmitted TID. But since one is using 256 bit TID so it is computationally not feasible for the current set of computers to do a mapping of all possible TIDs with their hash value. Now the type I scheme is verifiable as after the announcement of the winning ticket, every participant can announce their TID which can be used to verify the outcome of the lottery. Further, any malicious participant can not change his ticket after the announcement of the winning ticket as every participant has announced the hash value of their TID before being sent to the lottery authority. So, to conclude, the proposed type I lottery schemes satisfy all the requirements to be considered as good schemes.

Let us now look at the security aspects of semi-quantum lottery scheme. The semi-quantum lottery scheme (type II) differs from the type I schemes in terms of resources as in the type II scheme only lottery authority needs quantum resources while the participants can be classical only. Due to this, the registration phase in which digital signatures for participants are generated is different from that of type I scheme. As can be seen from the scheme, the participants signature is encoded in the form of whether the participant has measured the incoming qubit in the computational basis or let it be passed without any modification. Lottery authority can authenticate the participant when he reveals his choices and LAT1 and LAT2 announce the measurement results. For all the qubits in which participant has passed the qubits, the measurement outcomes of LAT1 and LAT2 will be same provided they use the same basis. In this, only authenticated participants will be allowed to proceed in the ticketing phase. Further, in the ticketing phase the TIDs are sent by participants to lottery authority via the use of semi-quantum QKD protocols which have already been proved to be unconditionally secure. So, the semi-quantum lottery scheme also satisfies the requirements of a good lottery scheme.

## 5 Conclusion

The Nature is quantum mechanical and the quantum mechanical world is probabilistic in nature. In short, quantum mechanics is a probabilistic theory and in our daily life we often come across situations that can be best realized within the framework of a probabilistic theory (not essentially quantum mechanics). Lottery is one such phenomenon which can be appropriately realized only in the framework of a probabilistic theory. It's possible to design schemes of lottery in any non-classical probabilistic theory. However, without going into the details of the generalized probability theory (GPT)[62, 63] and the specific toy theories which can support secure lottery schemes, here we have restricted ourselves to quantum mechanics and have provided three specific schemes for lottery. These schemes require different type of quantum resources. To be precise, in contrast to the entanglement based scheme proposed above, the other two schemes, i.e., BB84 state based scheme and semi-quantum schemes can be realized using separable states or more appropriately using single photon states. In fact, these two single-photon based (or equivalently BB84 state based) schemes do not require entanglement and non-locality, and thus such schemes can

also be realized in a non-classical toy theory [64] which has only the feature of uncertainty relations between incompatible observables. This is similar to the availability of a wide variety of practical QKD systems [32, 33] and quantum random number generators [36, 38] with different levels of security aspects. The unconditional security can be derived from the use of only 'incompatibility and uncertainty' feature of quantum mechanics, but the device independence security can be derived only through the use of features such as 'entanglement' and 'non-locality'.

In this work, we have highlighted the importance of lottery in many important works of life and have noted that despite its existence from the days of early civilization there is no fair and secure scheme for lottery. In fact, current implementations of the schemes for lottery are not fully secure. Further, we identified the requirements for a scheme to be considered as a good scheme for lottery. The gap is addressed here by designing a set of secure schemes for lottery and establishing their security. We analysed the resource requirements of the three proposed scheme, and have shown that these schemes, can be implemented using the available devices. We hope that this study will help in providing physical insights towards the development of commercial quantum lottery solutions.

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