System-environment dynamics of GHZ-like states in noninertial frames

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Quantum coherence, quantum entanglement and quantum nonlocality are important resources in quantum information precessing. However, decoherence happens when a quantum system interacts with the external environments. We study the dynamical evolution of the three-qubit GHZ-like states in non-inertial frame when one and/or two qubits undergo decoherence. Under the amplitude damping channel we show that the quantum decoherence and the Unruh effect may have quite different influences on the initial state. Moreover, the genuine tripartite entanglement and the quantum coherence may suffer sudden death during the evolution. The quantum coherence is most resistent to the quantum decoherence and the Unruh effect, then comes the quantum entanglement and the quantum nonlocality which is most fragile among the three. The results provide a new research perspective for relativistic quantum informatics.

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Introduction

The interactions with external environments may result in energy dissipations or relative phase changes of a quantum system, leading to the quantum state degenerating from a coherent superposition state to mixed or single states [1]. On the one hand, such decoherence reduces the quantum advantages of the important resources including coherence [2], quantum entanglement [3] and quantum nonlocality [4] of the system. On the other hand, the decoherence also causes the entanglement between the system and the environment. The dynamics of the system is no longer unitary in this case. It plays a fundamental role in the quantum-to-classical transition [5, 6] and has been successfully applied in cavity QED [7] and ion trap experiments [8].

These quantum resources have been investigated mostly in inertial systems. In 2013 quantum teleportation with a uniformly accelerated partner has been demonstrated based on quantum entanglement degenerated in noninertial system [9]. Soon after, Fuentes et al. [10] observed that entanglement in noninertial frames is characterized by the observer-dependent properties. Since then, progresses have been made to the researches on quantum information theory in noninertial systems [11–39].

Among the noninertial systems, the Dirac field [13, 15, 34, 36, 40, 41] can be described by a superposition of the Unruh monochromatic modes from an inertial perspective [42],

$$\begin{aligned} |0\rangle &= \cos\beta|0\rangle_I^+|0\rangle_{II}^- + \sin\beta|1\rangle_I^+|1\rangle_{II}^-, \\ |1\rangle &= |1\rangle_I^+|0\rangle_{II}^-, \end{aligned} \tag{1}$$

where $|n\rangle_I$ and $|n\rangle_{II}$ are the number states of the particle outside the (physically accessible) region and the antiparticle inside the (physically inaccessible) region of the event horizon, respectively. The superscripts $+$ and $-$ denote particle and antiparticle, respectively. $\cos\beta = (e^{-2\pi\omega c/a})^{-1/2}$, where a is the acceleration of the observer, ω is the frequency of the Dirac particle and c is the speed of light in vacuum. As β increases when a increases, in the following the accelerating parameter β of the Unruh effect is used instead of a.

For two qubit states, the authors in [16] show that the decoherence and the loss of entanglement resulted from the Unruh effect will influence each other remarkably. Sudden death of entanglement may appear for any acceleration when the whole system undergoes decoherence. However, when only one qubit undergoes decoherence, such sudden death may only occur when the acceleration parameter is greater than a critical point. Recently, the genuine tripartite nonlocality (GTN) and the genuine tripartite entanglement (GTE) of Dirac fields in the background of a Schwarzschild black hole for Greenberger-Horne-Zeilinger-like (GHZ-like) states have been studied [34, 36]. It is found that the Hawking radiation [43] degrades both the physically accessible GTN and the physically accessible GTE. The former suffers from sudden death at some critical Hawking temperature, while the latter approaches to a nonzero asymptotic value in the limit of infinite Hawking temperature. Moreover, the Hawking effect cannot generate the physically inaccessible GTN, but can generate the physically inaccessible GTE for fermion fields in curved spacetime. More recently, it is shown that for three-qubit mixed states the Hawking effect can also generate the physically inaccessible GTN in curved spacetime [44].

Recently, the influences of different noisy environments on quantum coherence and entanglement for W state in noninertial frames have been investigated intensively [52–55]. Zeng and Cao [53] studied the evolution and distribution of quantum coherence for multipartite W and GHZ states of Dirac fields under amplitude-damping, phase damping and depolarizing channels in the noninertial frames. Wu et al. [54] investigated the quantum coherence for N-partite W and GHZ states under the local amplitude-damping environment when $N-1$ observers are accelerated. They found that quantum coherence is symmetric with respect to all the observers for GHZ state, but to two accelerating observers only for W state. In Ref. [55], the authors studied quantum coherence and entanglement for W state of Dirac fields under bit flip, phase flip and phase damping channels in noninertial frames.

In this paper we study the system-environment dynamics for three-qubit states of the Dirac fields in a noninertial frame. We consider the most typical amplitude-damping channel [56], which can be modeled by the spontaneous decay of a two-level quantum state in an electromagnetic field [46]. We consider the case that one and two of the observers move (or stay) in the noisy environment and investigate whether or not the quantum decoherence and the loss of the quantumness generated by the Unruh radiation would influence each other, as well as the sudden death [47] of coherence, entanglement and nonlocality.

The outline of the paper is as follows. In Section 2 we simply recall some knowledge about the theory of open quantum systems, amplitude damping channel and the quantization of the quantumness of three-qubit X-type states. In Section 3 we investigate the system-environment dynamics of GHZ-like states in noninertial frames. We summarize and discuss our conclusions in the last section.

Some preliminaries

The evolution of a system state ρ_S of an open quantum systems is governed by [7, 48, 49] $U_{SE}(\rho_S\otimes|0\rangle\langle 0|)U_{SE}^\dagger$, where $|0\rangle\langle 0|$ represents the initial state of the environment, U_{SE} is the evolution operator for the system and environment. By tracing over the environment, one gets the evolution of the system,

$$\begin{aligned} L(\rho_S) &= Tr_E[U_{SE}(\rho_S\otimes|0\rangle\langle 0|)U_{SE}^\dagger] \\ &= \sum_\mu {}_E\langle\mu|U_{SE}|0\rangle_E\ \rho_S\ {}_E\langle 0|U_{SE}^\dagger|\mu\rangle_E, \end{aligned} \quad (2)$$

where $|\mu\rangle_E$ is the orthogonal basis of the environment, and L stands for the evolution of the system. Eq. (2) can also be expressed as

$$L(\rho_S)=\sum_\mu M_\mu\rho_S M_\mu^\dagger, \quad (3)$$

where $M_\mu={}_E\langle\mu|U_{SE}|0\rangle_E$ are the Kraus operators [50, 51]. There are at most d^2 independent Kraus operators when the dimension of the system is d [56, 57].

Consider a three-qubit state ρ_{ABC} in which one subsystem, say C, undergoes the amplitude damping channel. The action of the amplitude damping channel on the qubit C can be represented by the following phenomenological map [48, 49, 58],

$$|0\rangle_C|0\rangle_{e_C}\rightarrow|0\rangle_C|0\rangle_{e_C}, \quad (4)$$

$$|1\rangle_C|0\rangle_{e_C}\rightarrow\sqrt{1-P}|1\rangle_C|0\rangle_{e_C}+\sqrt{P}|0\rangle_C|1\rangle_{e_C}, \quad (5)$$

where $|0\rangle_C$ ($|1\rangle_C$) stands for the ground (excited) state of the subsystem C, $|0\rangle_{e_C}$ and $|1\rangle_{e_C}$ are the states of the environment with no and one excitation of its modes, respectively. Eq. (4) indicates that the system has no decay and the environment is untouched. Eq. (5) shows that the system remains with probability $1-P$ and the environment exits with probability P. P is time-dependent, $P=(1-e^{-\Gamma t})$, where Γ is called the decoherence rate [56]. In this paper, we consider the same environment to all the qubits, i.e., P is the same for each subsystem.

Eqs. (4) and (5) can also be expressed in the form of Eq. (3) with Kraus operators [16, 50, 51]:

$$M_0=\begin{pmatrix}1 & 0\\ 0 & \sqrt{1-P}\end{pmatrix},\quad M_1=\begin{pmatrix}0 & \sqrt{P}\\ 0 & 0\end{pmatrix}. \quad (6)$$

That is,

$$L(\rho_C)=\sum_{i=0}^{1}M_i\rho_C M_i^\dagger. \quad (7)$$

When two qubits, say B and C, are coupled to the noisy environment independently, the evolution of the reduced state ρ_{BC} is given by [16]

$$L(\rho_{BC})=\sum_{i,j=0}^{1}(M_i\otimes M_j)\rho_{BC}(M_i\otimes M_j)^\dagger. \quad (8)$$

In the computational basis $\{|000\rangle,|001\rangle,\cdots,|111\rangle\}$, the density matrix of a three-qubit X-type state has the following general form,

$$\rho_X=\begin{pmatrix} d_1 & 0 & 0 & 0 & 0 & 0 & 0 & f_1\\ 0 & d_2 & 0 & 0 & 0 & 0 & f_2 & 0\\ 0 & 0 & d_3 & 0 & 0 & f_3 & 0 & 0\\ 0 & 0 & 0 & d_4 & f_4 & 0 & 0 & 0\\ 0 & 0 & 0 & f_4^* & e_4 & 0 & 0 & 0\\ 0 & 0 & f_3^* & 0 & 0 & e_3 & 0 & 0\\ 0 & f_2^* & 0 & 0 & 0 & 0 & e_2 & 0\\ f_1^* & 0 & 0 & 0 & 0 & 0 & 0 & e_1 \end{pmatrix}.$$

The GTN of ρ_X is characterized by [59]

$$S(\rho_X) = \max\{8\sqrt{2}|f_i|, 4|N|\}, \quad (9)$$

where $N = d_1 - d_2 - d_3 + d_4 - e_4 + e_3 + e_2 - e_1$. According to the Svetlichny inequality [60], ρ_X is genuine tripartite nonlocal if $S(\rho_X) > 4$. The GTE of ρ_X is given by [61]

$$E(\rho_X) = 2\max\{0, |f_i| - m_i\}, i = 1, 2, 3, 4, \quad (10)$$

where $m_i = \sum_{j\neq i}^{4} \sqrt{d_j e_j}$. The C_{l_1} quantum coherence is given by [2]

$$C(\rho_X) = C_{l_1}(\rho_X) = \sum_{i\neq j} |\rho_{ij}| = 2\sum_{i=1}^{4} |f_i|. \quad (11)$$

System-environment dynamics for GHZ-like states

In this section, let us consider the GHZ-like states $|GHZ\rangle = \alpha|000\rangle + \sqrt{1-\alpha^2}|111\rangle$ of the Dirac fields shared by Alice, Bob and Charlie.

Case (i): Alice and Bob stay stationary while Charlie moves with uniform acceleration. With respect to the Minkowske modes for Alice and Bob and the Rindler modes for Charlie, by using Eq. (1), the GHZ-like states can be written as $|\psi\rangle_{ABC_IC_{II}} = \alpha\cos\beta|0000\rangle + \alpha\sin\beta|0011\rangle + \sqrt{1-\alpha^2}|1110\rangle$. By tracing over the inaccessible modes C_{II}, we have the following density matrix,

$$\rho_{ABC_I} = \begin{pmatrix} d_1 & 0 & 0 & 0 & 0 & 0 & 0 & f_1 \\ 0 & d_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ f_1 & 0 & 0 & 0 & 0 & 0 & 0 & e_1 \end{pmatrix},$$

where $d_1 = \alpha^2\cos^2\beta$, $d_2 = \alpha^2\sin^2\beta$, $e_1 = 1-\alpha^2$ and $f_1 = \alpha\sqrt{1-\alpha^2}\cos\beta$. Under the bipartition $AB|C_I$ ρ_{ABC_I} can be also expressed as

$$\begin{aligned} &\rho_{ABC_I} \\ = \ &|00\rangle\langle 00| \otimes (d_1|0\rangle\langle 0| + d_2|1\rangle\langle 1|) + |00\rangle\langle 11| \otimes (f_1|0\rangle\langle 1|) \\ &+|11\rangle\langle 00| \otimes (f_1|1\rangle\langle 0|) + |11\rangle\langle 11| \otimes (e_1|1\rangle\langle 1|). \end{aligned}$$

Consider that the Charlie's qubit couples to the noisy environment. From Eqs. (6) and (7), ρ_{ABC_I} evolves into

$$\begin{aligned} &\rho'_{ABC_I} \\ = &\begin{pmatrix} d_1 + Pd_2 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{1-P}f_1 \\ 0 & (1-P)d_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & pe_1 & 0 \\ \sqrt{1-P}f_1 & 0 & 0 & 0 & 0 & 0 & 0 & (1-P)e_1 \end{pmatrix}. \end{aligned}$$

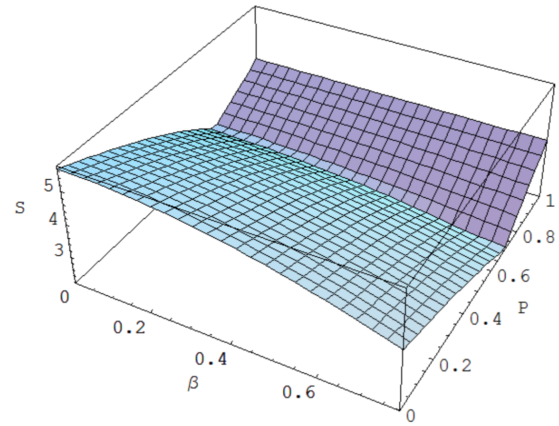


FIG. 1: $S(\rho'_{ABC_I})$ as a function of the acceleration parameter β and decoherence parameter P for the initial GHZ state, when the Charlie's qubit undergoes acceleration and decoherence.

Using Eqs. (9), (10) and ((11)), we obtain

$$\begin{aligned} S(\rho'_{ABC_I}) = \ &\max\{8\sqrt{2}\sqrt{1-P}\alpha\sqrt{1-\alpha^2}\cos\beta, \\ &4[\alpha^2\cos^2\beta + 2P\alpha^2\sin^2\beta - \alpha^2\sin^2\beta \\ &+(2P-1)(1-\alpha^2))]\}, \end{aligned}$$

$$\begin{aligned} E(\rho'_{ABC_I}) = \ &2\max\{0, \sqrt{1-P}\alpha\sqrt{1-\alpha^2}\cos\beta - \\ &\sqrt{(1-P)\alpha^2\sin^2\beta P(1-\alpha^2)}\}, \end{aligned}$$

$$C(\rho'_{ABC_I}) = 2\sqrt{1-P}\alpha\sqrt{1-\alpha^2}\cos\beta.$$

In FIG. 1 we show the behavior of GTN of ρ'_{ABC_I} for the GHZ state with $\alpha = 1/\sqrt{2}$. It is seen that the increase of either the parameter β of the Unruh effect or the decoherence parameter P reduces the GTN. Moreover, the increase of the two parameters will cause the sudden death of GTN. With the increase of the acceleration parameter β, $S(\rho'_{ABC_I})$ is larger than 4 at first and then smaller than 4, but will not tend to zero. However, with the increase of P the GTN tends to zero first and then increases.

FIG. 2 (a) and (b) show the behavior of GTE and quantum coherence of ρ'_{ABC_I} ($\alpha = \frac{1}{\sqrt{2}}$), respectively. We observe that with the increase of β the GTE and quantum coherence decrease slowly. But the increase of P has a stronger influence on GTE and quantum coherence, which makes them tend to 0. And for large β, with the decrease of P the GTE and quantum coherence behavior differently, as GTE is a convex function, while quantum coherence is a concave function.

Similarly, by tracing over the mode C_I we have the reduced state $\rho_{ABC_{II}}$. Correspondingly, using Eqs. (6)

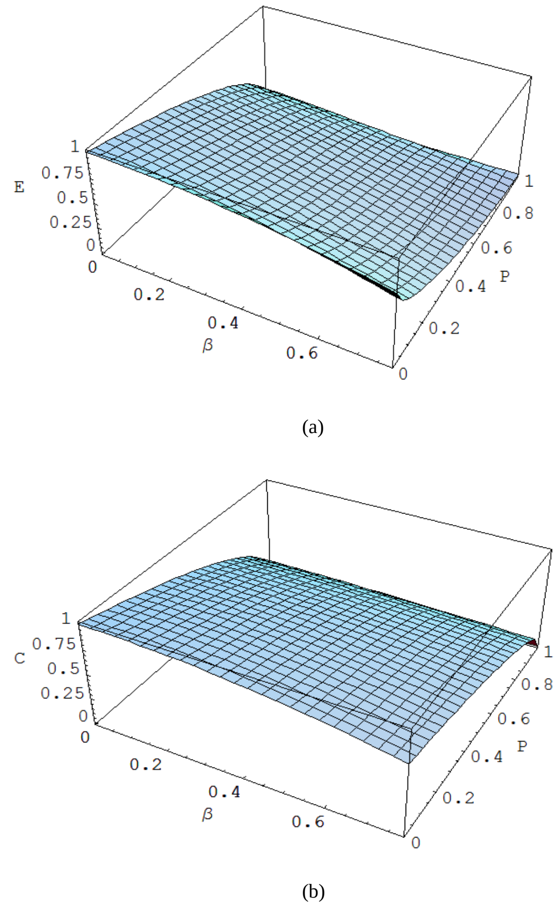


FIG. 2: $E(\rho'_{ABC_I})$ and $C(\rho'_{ABC_I})$ as functions of the acceleration parameter β and the decoherence parameter P for the initial GHZ state with $\alpha = \frac{1}{\sqrt{2}}$, when the Charlie's qubit undergoes acceleration and decoherence.

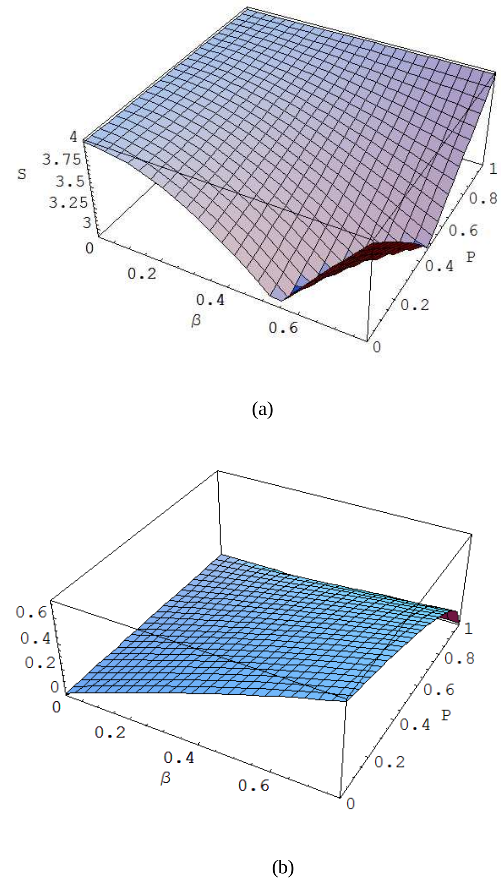


FIG. 3: $S(\rho'_{ABC_{II}})$ and $E(\rho'_{ABC_{II}})$=$C(\rho'_{ABC_{II}})$ as functions of the acceleration parameter β and the decoherence parameter P with respect to the initial GHZ state, when Charlie's qubit undergoes acceleration and decoherence.

and (7) we get

$$\rho'_{ABC_{II}} = \begin{pmatrix} d_1 + Pd_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-P)d_2 & 0 & 0 & 0 & 0 & \sqrt{1-P}f_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{1-P}f_2 & 0 & 0 & 0 & 0 & e_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

where $d_1 = \alpha^2 \cos^2\beta$, $d_2 = \alpha^2 \sin^2\beta$, $e_2 = 1 - \alpha^2$ and $f_2 = \alpha\sqrt{1-\alpha^2}\sin\beta$. From straightforward calculation we have

$$S(\rho'_{ABC_{II}}) = \max\{8\sqrt{2}\sqrt{1-P}\alpha\sqrt{1-\alpha^2}\sin\beta, 4[\alpha^2\cos^2\beta + 2P\alpha^2\sin^2\beta + (1-\alpha^2) - \alpha^2\sin^2\beta]\}$$

and

$$E(\rho'_{ABC_{II}}) = C(\rho'_{ABC_{II}}) = 2\sqrt{1-P}\alpha\sqrt{1-\alpha^2}\sin\beta.$$

We plot the GTN of $\rho'_{ABC_{II}}$ for GHZ the state with $\alpha = \frac{1}{\sqrt{2}}$ in FIG. 3(a). It is shown that the increase of β has a greater impact on $S(\rho'_{ABC_{II}})$, while the increase of P has less impact on $S(\rho'_{ABC_{II}})$. However, the values of GTN are all smaller than 4. The behavior of GTE (quantum coherence) of $\rho'_{ABC_{II}}$ for the GHZ state ($\alpha = \frac{1}{\sqrt{2}}$) is shown in FIG. 3(b). We see that the effect of the increase of β and P on GTE (or quantum coherence) is completely the opposite. The GTE (or quantum coherence) increases with the increase of β, but decreases with the increase of P. Moreover, the quantum coherence of the initial state of GHZ-like states $|GHZ\rangle = \alpha|000\rangle + \sqrt{1-\alpha^2}|111\rangle$ satisfies the following strong nonlinear relationship.

$$C^2(\rho'_{ABC_I}) + C^2(\rho'_{ABC_{II}}) = 4(1-P)\alpha^2(1-\alpha^2), \quad (12)$$

which is given by the decoherence parameter P, and has nothing to do with the Unruh effect.

Case(ii): we now let Alice still stay at an asymptotically flat region, while Bob and Charlie move with uniform acceleration a. Using Eqs. (1), we can rewrite the GHZ-like states as $|\psi\rangle_{AB_IB_{II}C_IC_{II}}$, with the detailed expression given in [34, 36]. By tracing over the freedom in the region II, the density matrix $\rho_{AB_1C_I}$ of the physically accessible part I is of the form [36],

$$\rho_{AB_IC_I}=\begin{pmatrix} d_1 & 0 & 0 & 0 & 0 & 0 & 0 & f_1\\ 0 & d_2 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & d_3 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & d_4 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ f_1 & 0 & 0 & 0 & 0 & 0 & 0 & e_1 \end{pmatrix},$$

where $d_1=\alpha^2\cos^4\beta$, $d_2=d_3=\alpha^2\sin^2\beta\cos^2\beta$, $d_4=\alpha^2\sin^4\beta$, $e_1=1-\alpha^2$ and $f_1=\alpha\sqrt{1-\alpha^2}\cos^2\beta$. It can also be written as in the form of bipartition $A|B_IC_I$,

$$\rho_{AB_IC_I}=|0\rangle\langle 0|\otimes N_1+|0\rangle\langle 1|\otimes N_2+|1\rangle\langle 0|\otimes N_3+|1\rangle\langle 1|\otimes N_4,$$

where $N_1=\begin{pmatrix} d_1 & 0 & 0 & 0\\ 0 & d_2 & 0 & 0\\ 0 & 0 & d_3 & 0\\ 0 & 0 & 0 & d_4\end{pmatrix}$, $N_2=\begin{pmatrix} 0 & 0 & 0 & f_1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$,

$N_3=\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ f_1 & 0 & 0 & 0\end{pmatrix}$, $N_4=\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & e_1\end{pmatrix}$.

Now we consider that both Bob and Charlie's qubits couple to the noisy environment independently. From Eqs. (6) and (8), state $\rho_{AB_1C_I}$ evolves to

$$\rho'_{AB_IC_I}=\begin{pmatrix} d'_1 & 0 & 0 & 0 & 0 & 0 & 0 & f'_1\\ 0 & d'_2 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & d'_3 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & d'_4 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & e'_4 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & e'_3 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ f'_1 & 0 & 0 & 0 & 0 & 0 & 0 & e'_1 \end{pmatrix},$$

where $d'_1=d_1-P(d_2+d_3)+P^2d_4$, $d'_2=(1-P)d_2+P(1-P)d_4$, $d'_3=(1-P)d_3-P(1-P)d_4$, $d'_4=(1-P)^2d_4$, $e'_1=(1-P)^2e_1$, $e'_3=P(1-P)e_1$, $e'_4=Pe_1$ and $f'_1=(1-P)f_1$. We obtain

$$\begin{aligned}&S(\rho'_{AB_IC_I})=\max\{8\sqrt{2}\sqrt{1-P}\alpha\sqrt{1-\alpha^2}\cos\beta,\\ &4[\alpha^2(\cos^4\beta-2\sin^2\beta\cos^2\beta+(1-2P+2P^2))\\ &-(1-2P+2P^2)(1-\alpha^2)]\}\end{aligned}\tag{13}$$

and

$$\begin{aligned}&E(\rho'_{AB_IC_I})\\ &=2\max\{0,[(1-P)\alpha\sqrt{1-\alpha^2}\cos^2\beta\\ &-\alpha\sin\beta\sqrt{(1-P)\cos^2\beta-(1-P)P\sin^2\beta}\\ &-(1-P)\alpha\sin^2\beta\sqrt{P(1-\alpha^2)}]\}.\end{aligned}\tag{14}$$

We plot the GTN of $\rho'_{AB_IC_I}$ for $\alpha=\frac{1}{\sqrt{2}}$ in FIG. 4(a). It is seen that $S(\rho'_{AB_IC_I})$ is larger than 4 at first and then becomes smaller than 4 with the increase of β or P. The increase of the two parameters cause the sudden death of GTN. With the increase of acceleration parameter β, GTN slowly approaches to 3, but never reaches 0. However, with the increase of the decoherence parameter P, the GTN decreases monotonically and tends to 0. FIG. 4(b) shows the behavior of GTE of $\rho'_{AB_IC_I}$ with $\alpha=\frac{1}{\sqrt{2}}$. One sees that when only the parameter β increases, the GTE will not decrease to zero. But when P also exerts influence, the GTE decreases to zero first, and then increases. Moreover, compared with FIG. 1 and FIG. 2, we can find that the GTN and GTE change faster with the change of the parameters. Hence, the decoherence when the two subsystems undergo noisy channel is stronger than that of one system does.

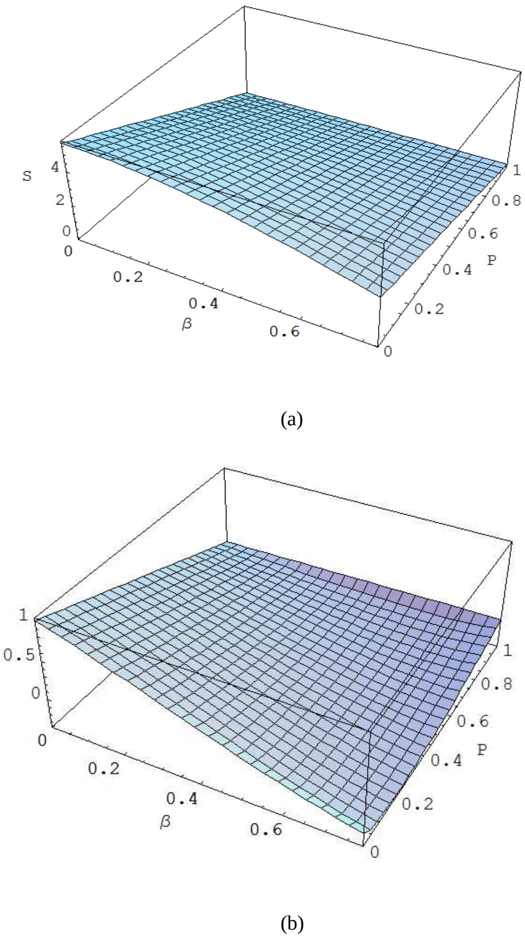


FIG. 4: $S(\rho'_{AB_IC_I})$ and $E(\rho'_{AB_IC_I})$ as functions of the acceleration parameter β and decoherence parameter P for the initial GHZ state, when Bob's and Charlie's qubits undergo acceleration and decoherence.

The quantum dynamics of the rest reduced density matrices can be similarly analyzed. Due to the symmetry between Bob and Charlie, we only need to consider the following three situations.

$$\rho'_{AB_IC_{II}} = \begin{pmatrix} d'_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d'_2 & 0 & 0 & 0 & 0 & f'_2 & 0 \\ 0 & 0 & d'_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d'_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e'_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f'_2 & 0 & 0 & 0 & 0 & e'_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

where $d'_1 = d_1 - P(d_2+d_3) + P^2 d_4$, $d'_2 = (1-P)d_2 + P(1-P)d_4$, $d'_3 = (1-P)d_3 - P(1-P)d_4$, $d'_4 = (1-P)^2 d_4$, $e'_4 = P(1-\alpha^2)$, $e'_2 = (1-P)(1-\alpha^2)$ and $f'_2 = (1-P)\alpha\sqrt{1-\alpha^2}\sin\beta\cos\beta$. According to Eqs. (9) and (10), we have

$$S(\rho'_{AB_IC_{II}}) = \max\{8\sqrt{2}(1-P)\alpha\sqrt{1-\alpha^2}\sin\beta\cos\beta, 4[\alpha^2(\cos^2\beta - \sin^2\beta)^2 + (1-2P)(1-\alpha^2)]\},$$

$$E(\rho'_{AB_IC_{II}}) = 2\max\{0, (1-P)\alpha\sqrt{1-\alpha^2}\sin\beta\cos\beta - (1-P)\alpha\sin^2\beta\sqrt{P(1-\alpha^2)}\}.$$

FIG. 5(a) and (b) show the behavior of GTN and GTE of $\rho'_{AB_IC_{II}}$ when $\alpha = \frac{1}{\sqrt{2}}$, respectively.

Concerning the systems A and $B_{II}C_{II}$ we have

$$\rho'_{AB_{II}C_{II}} = \begin{pmatrix} d'_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d'_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & d'_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d'_4 & f'_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & f'_4 & e'_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

where $d'_1 = d_1 - P(d_2+d_3) + P^2 d_4$, $d'_2 = (1-P)d_2 + P(1-P)d_4$, $d'_3 = (1-P)d_3 - P(1-P)d_4$, $d'_4 = (1-P)^2 d_4$, $e'_4 = 1-\alpha^2$ and $f'_4 = (1-P)\alpha\sqrt{1-\alpha^2}\sin^2\beta$. According to Eqs. (9) and (10), we obtain

$$S(\rho'_{AB_{II}C_{II}}) = \max\{8\sqrt{2}(1-P)\alpha\sqrt{1-\alpha^2}\sin^2\beta, 4[\alpha^2(\cos^2\beta - \sin^2\beta)^2 - (1-\alpha^2)]\}$$

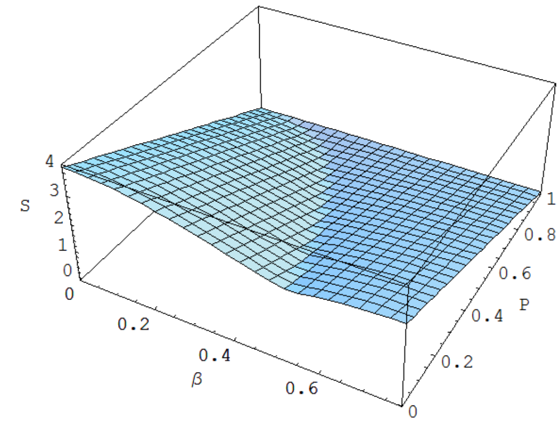
and

$$E(\rho'_{AB_{II}C_{II}}) = 2\max\{0, (1-P)\alpha\sqrt{1-\alpha^2}\sin^2\beta\}.$$

In FIG. 6(a) and (b), we plot the GTN and GTE of $\rho'_{AB_{II}C_{II}}$ as functions of β and P when $\alpha = \frac{1}{\sqrt{2}}$, respectively.

With respect to the systems A, B_I and B_{II}, we have

$$\rho'_{AB_IB_{II}} = \begin{pmatrix} d'_1 & 0 & 0 & 0 & 0 & 0 & 0 & f'_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e'_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e'_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e'_2 & 0 \\ f'_1 & 0 & 0 & 0 & 0 & 0 & 0 & e'_1 \end{pmatrix},$$



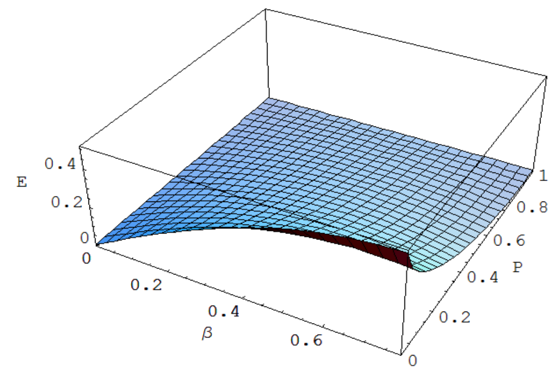


FIG. 5: $S(\rho'_{AB_IC_{II}})$ and $E(\rho'_{AB_IC_{II}})$ as functions of the acceleration parameter β and decoherence parameter P for the initial GHZ state, when Bob's and Charlie's qubits undergo acceleration and decoherence.

where $d'_1 = \alpha^2\cos^2\beta$, $e'_1 = (1-P)^2\alpha^2\sin^2\beta$, $e'_2 = P(1-P)\alpha^2\sin^2\beta + (1-P)(1-\alpha^2)$, $e'_3 = P(1-P)\alpha^2\sin^2\beta$, $e'_4 = P^2\alpha^2\sin^2\beta + P(1-\alpha^2)$ and $f'_1 = (1-p)\alpha^2\sin\beta\cos\beta$. From Eqs. (9) and (10) we get

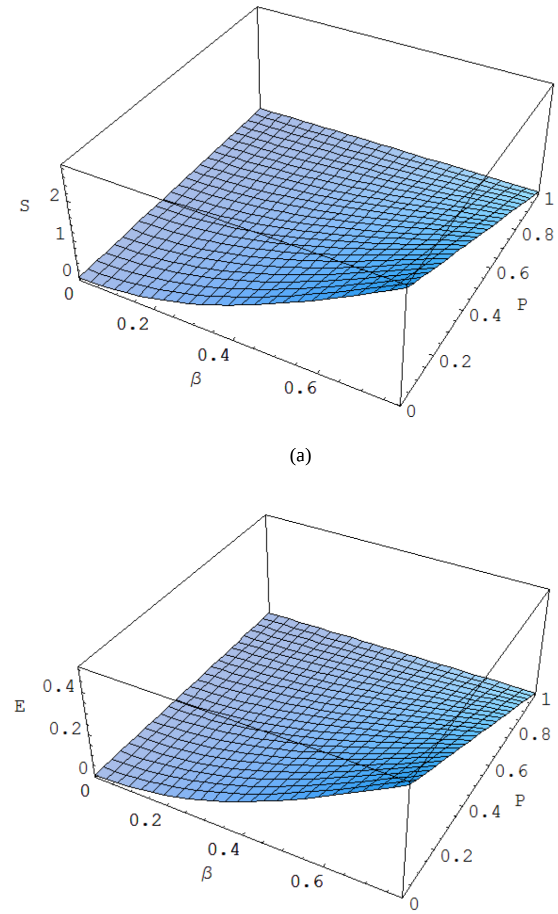
$$S(\rho'_{AB_IB_{II}}) = \max\{8\sqrt{2}(1-P)\alpha^2\sin\beta\cos\beta, 4[\alpha^2(\cos^2\beta + (2P+2P^2-1)\sin^2\beta) + (1-P)(1-\alpha^2)]\}$$

and

$$E(\rho'_{AB_IB_{II}}) = 2\max\{0, (1-P)\alpha^2\sin\beta\cos\beta\}.$$

In FIG.7 (a) and (b) we plot the behavior of GTN and GTE of $\rho'_{AB_IB_{II}}$ for GHZ state ($\alpha = \frac{1}{\sqrt{2}}$), respectively.

From the figures above, we observe that (1) the influence of the quantum decoherence and the Unruh effect on the dynamical evolution of the systems are not always in the same rhythms. (2) For the most cases, the quantum decoherence has a stronger influence than Unruh effect, as it may result in the phenomena of sudden death. (3) The decoherence of two subsystems is stronger than that of only one subsystem.



(a)

(b)

FIG. 6: $S(\rho'_{AB_{II}C_{II}})$ and $E(\rho'_{AB_{II}C_{II}})$ as functions of the acceleration parameter β and decoherence parameter P for the initial quantum state is GHZ state, when Bob's and Charlie's qubit undergoes acceleration and decoherence.

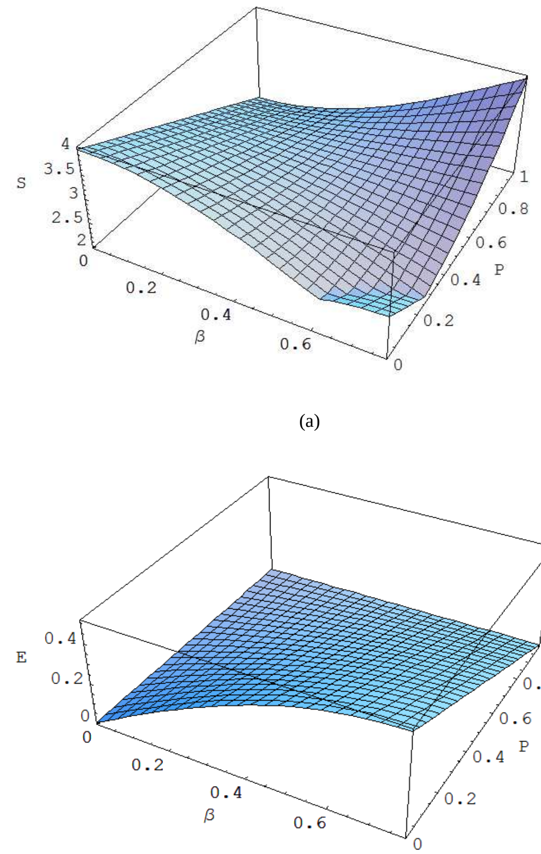
In the case (ii), the influence of P is exerted under the influence of acceleration parameter β. We find that throughout the process of decay the quantum coherence satisfies strictly a beautiful relation. For example, the following relations always hold,

$$C(\rho'_{AB_IC_I}) + C(\rho'_{AB_{II}C_{II}}) = 2(1-P)\alpha\sqrt{1-\alpha^2}, \quad (15)$$

$$\begin{aligned} &C^2(\rho'_{AB_IC_I}) + C^2(\rho'_{AB_{II}C_{II}}) + C^2(\rho'_{AB_IC_{II}}) + \\ &C^2(\rho'_{AB_{II}C_I}) = 4(1-P)^2\alpha^2(1-\alpha^2), \end{aligned} \quad (16)$$

$$\begin{aligned} &C^2(\rho'_{AB_IC_I}) + C^2(\rho'_{AB_{II}C_{II}}) + (1-\alpha^2)[C^2(\rho'_{AB_IB_{II}}) \\ &+C^2(\rho'_{AC_IC_{II}})] = 4(1-P)^2\alpha^2(1-\alpha^2). \end{aligned} \quad (17)$$

Interestingly, when $P = 0$ the right hands of the above equations are just the quantum coherence of the initial GHZ-like state. When the decoherence appears, that is, the value of P increases, the quantum coherence of the single qubit state decreases at a rate of about $(1-P)$. Therefore, the larger the P is, the faster the quantum coherence decreases. When $P \to 1$, the quantum coherence of a single reduced density matrix tends to 0. This phenomena is independent of the parameter β of the Hawking effect.



(a)

(b)

FIG. 7: $S(\rho'_{AB_IB_{II}})$ and $E(\rho'_{AB_IB_{II}})$ as functions of the acceleration parameter β and decoherence parameter P when Bob's and Charlie's qubits undergo acceleration and decoherence.

It is noted that without decoherence ($P = 0$), the entanglement of all the above three-body reduced states is equal to quantum coherence, namely, Eqs (15), (16) and (17) hold both for entanglement and quantum coherence [34]. Nevertheless, after decoherence they are valid only for coherence, because the entanglement of $\rho'_{AB_IC_I}$ and $\rho'_{AB_IC_{II}}$ are slightly reduced due to decoherence. Therefore, the quantum entanglement is more fragile than quantum coherence during the decoherence.

Conclusions and discussions

We have studied the dynamical evolution of the three-qubit GHZ-like states in non-inertial frame when one and two qubits undergo decoherence. Under the amplitude damping channel the influences of the quantum decoherence and the Unruh effect on the initial states has been investigated. It is shown that the GTE and quantum coherence may suffer sudden death. The results can be applied to the cases in which Alice moves along a geodesic while Bob and/or Charlie hover near the event horizon with an uniform acceleration. Our results may also inspire the study on the dynamics of quantum states in the framework of relativity. In addition, one can also consider the dynamical behavior under the influence of amplitude damping or phase damping for other initial quantum states such as W-state or mixed states.

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[1] W.H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
[2] T. Baumgratz, M. Cramer, M.B. Plenio, Phys. Rev. Lett. **113**, 140401 (2014);
A. Streltsov, G. Adesso, M.B. Plenio, Rev. Mod. Phys. **89**, 041003 (2017);
M.L. Hu, X. Hu, J.C. Wang, Y. Peng, Y.R. Zhang, H. Fan, Phys. Rep. **762**, 1 (2018).
[3] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[4] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani and S. Wehner, Rev. Mod. Phys. **86**, 419 (2014).
[5] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.O. Stamatescu, H.D. Zeh, Decoherence and the Appearance of a Classical World in Quantum Theory, Springer, 1996.
[6] M.A. Schlosshauer, Decoherence and the Quantum-To-Classical Transition, Springer, 2007.
[7] M. Brune, E. Hagley, J. Dreyer, X. Maitre, A. Maali, C. Wunderlich, J.M. Raimond, S. Haroche, Phys. Rev. Lett. **77**, 4887 (1996).
[8] C.J. Myatt, B.E. King, Q.A. Turchette, C.A. Sackett, D. Kielpinski, W.M. Itano, C. Monroe, D.J. Wineland, Nature **403**, 269 (2000).
[9] P.M. Alsing, G.J. Milburn, Phys. Rev. Lett. **91** 180404 (2003).
[10] I. Fuentes-Schuller, R.B. Mann, Phys. Rev. Lett. **95**, 120404 (2005).
[11] P.M. Alsing, I. Fuentes-Schuller, R.B. Mann, T.E. Tessier, Phys. Rev. A **74**, 032326 (2006).
[12] J. Wang, Q. Pan, S. Chen, J. Jing, Phys. Lett. B **677**, 186 (2009).
[13] E. Martín-Martínez, L.J. Garay, J. León, Phys. Rev. D **82**, 064006 (2010).
[14] J. Wang, Q. Pan, J. Jing, Phys. Lett. B **692**, 202 (2010).
[15] E. Martín-Martínez, L.J. Garay, J. León, Phys. Rev. D **82**, 064028 (2010).
[16] J. Wang, J. Jing, Phys. Rev. A **82**, 032324 (2010).
[17] S. Khan, M.K. Khan, J. Phys. A Math. Theor. **44**, 045305 (2011).
[18] J. Wang, J. Jing, Phys. Rev. A **83**, 022314 (2011).
[19] B. Nasr Esfahani, M. Shamirzaie, M. Soltani, Phys. Rev. D **84**, 025024 (2011).
[20] R.B. Mann, T.C. Ralph, Class. Quantum Gravity **29**, 220301(2012).
[21] Z. Tian, J. Jing, J. High Energy Phys. **04**, 109 (2013).
[22] S. Xu, X.K. Song, J.D. Shi, L. Ye, Phys. Rev. D **89**, 065022 (2014).
[23] Z. Tian, J. Jing, J. High Energy Phys. **07**, 089 (2014).
[24] S. Kanno, J.P. Shock, J. Soda, Phys. Rev. D **94**, 125014 (2016).
[25] Y. Dai, Z. Shen, Y. Shi, Phys. Rev. D **94**, 025012 (2016).
[26] N. Friis, New J. Phys. **18**, 033014 (2016).
[27] X. Liu, Z. Tian, J. Wang, J. Jing, Phys. Rev. D **97**, 105030 (2018).
[28] W.C. Qiang, G.H. Sun, Q. Dong, and S.H. Dong, Phys. Rev. A **98**, 022320 (2018).
[29] A.J. Torres-Arenas, Q. Dong, G.H. Sun, W.C. Qiang, S.H. Dong, Phys. Lett. B **789**, 93 (2019).
[30] S.M. Wu, H.S. Zeng, Class. Quantum Gravity **37**, 115003 (2020).
[31] S.M. Wu, Z.C. Li and H.S. Zeng, EPL, **129**, 40002 (2020).
[32] J. Wang, L. Zhang, S. Chen, J. Jing, Phys. Lett. B **802**, 135239 (2020).
[33] Q. Liu, C. Wen, Z. Tian, J. Jing, J. Wang, Phys. Rev. A **105**, 062428 (2022).
[34] S.M. Wu, H.S. Zeng, Eur. Phys. J. C **82**, 4 (2022).
[35] L. Xiao, C. Wen, J. Jing, J. Wang, Eur. Phys. J. C **82**,684 (2022).
[36] L.J. Li, F. Ming, X.K. Song, L. Ye, D. Wang, Eur. Phys. J. C **82**, 726 (2022).
[37] S.M. Wu, H.S. Zeng and T. Liu, New J. Phys. **24**, 073004 (2022).
[38] J. Jing, S. Long, W. Deng, M. Wang, J. Wang, Sci. China, Phys. Mech. Astron. **65**, 100411, (2022).
[39] J. A. Szypulski, P. T. Grochowski, K. Debski and A. Dragan, arXiv:2112.07250(2021)
[40] M. Aspachs, G. Adesso, I. Fuentes, Phys. Rev. Lett **105**, 151301 (2010).
[41] D.E. Bruschi, J. Louko, E. Martn-Martnez, A. Dragan, I. Fuentes, Phys. Rev. A **82**, 042332 (2010).
[42] W.G. Unruh, Phys. Rev. D **14** 870 (1976).
[43] S.W. Hawking, Nature **248**, 30 (1974).
[44] T. Zhang, X. Wang, S.M. Fei, arXiv:2212.02245 (2022).
[45] A. Salles, F. de Melo, M. P. Almeida, M. Hor-Meyll, S. P. Walborn, P. H. SoutoRibeiro, and L. Davidovich, Phys. Rev. A **78**, 022322 (2008).
[46] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001).
[47] T. Yu and J. H. Eberly, Phys. Rev. Lett. **97**, 140403 (2006).
[48] H. P. Breuer and F. Petruccione, The Theory of Open Quantum Systems, Oxford University Press, Oxford, (2002).
[49] H. Carmichael, An Open Systems Approach to Quantum Optics, Springer, Berlin, (1993).

[50] K. Kraus, States, Effects and Operations: Fundamental Notions of Quantum Theory, Springer, Berlin, (1983).
[51] M.D. Choi, Linear Algebr. Appl. **10**, 285 (1975).
[52] W. Zhang and J. Jing, Multipartite entanglement for open system in noninertial frames. arXiv:1103.4903v1 (2011).
[53] H. S. Zeng and H. M. Cao, Ann. Phys. (Berlin) **533**, 2000606 (2021).
[54] S.M. Wu, Z.C. Li and H.S. Zeng, Quant. Inf. Process. **20**, 277 (2021).
[55] K.I. Kim, M.C. Pak, O.S. An, U.G. Ri, M.C. Ko and N.C. Kim, Phys. Scr. **97**, 075101 (2022).
[56] A. Salles, F. de Melo1, M.P. Almeida1, M. Hor-Meyll, S.P. Walborn, P.H. SoutoRibeiro, L. Davidovich, Phys. Rev. A **78**, 022322 (2008).
[57] D.W. Leung, J. Math. Phys. **44**, 528 (2003).
[58] J. Maziero, T. Werlang, F.F. Fanchini, L.C. Celeri, R.M. Serra, Phys. Rev. A **81**, 022116, (2010).
[59] K. Wang, Z.J. Zheng, Sci. Rep. **10**, 6621 (2020);
K. Wang, Y. Liang, Z.J. Zheng, Quantum Inf. Process. **19**, 140 (2020).
[60] G. Svetlichny, Phys. Rev. D **35**, 3066 (1987).
[61] Z.H. Ma, Z.H. Chen, J.L. Chen, C. Spengler, A. Gabriel,M. Huber, Phys. Rev. A **83**, 062325 (2011);
S.M. Hashemi Rafsanjani, M. Huber, C.J. Broadbent, J.H. Eberly, Phys. Rev. A **86**, 062303 (2012).