Ancilla-Assisted Protection of Information: Application to Atom-Cavity Systems

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One of the major obstacles faced by quantum-enabled technology is the environmental noise that causes decoherence in the quantum system, thereby destroying much of its quantum aspects and introducing errors while the system undergoes quantum operations and processing. A number of techniques have been invented to mitigate the environmental effects, and many of these techniques are specific to the environment and the quantum tasks at hand. Here, we propose a protocol that makes arbitrary environments effectively noise-free or transparent using an ancilla, which, in particular, is well suited to protect information stored in atoms. The ancilla, which is the photons, is allowed to undergo restricted but a wide class of noisy operations. The protocol transfers the information of the system onto the decoherence-free subspace and later retrieves it back to the system. Consequently, it enables full protection of quantum information and entanglement in the atomic system from decoherence. We propose experimental schemes to implement this protocol on atomic systems in an optical cavity.

I. INTRODUCTION

Quantum-enabled technologies involve well-controlled systems to store information and processing, such as atomic and solid state devices, and systems that are suitable for networking and communication, such as photonic systems. For information processing, it is often necessary to protect coherence in the state of a quantum system for a long enough duration. However, a quantum system cannot be fully isolated from its environment, and the latter induces decoherence in the system. These effects thereby destroy much of its quantum aspects, e.g., quantum coherence, entanglement, etc., and introduces uncontrolled errors in information processing [1–4]. Taking over environmental noise is thus one of the major challenges in quantum-enabled technologies today.

Some of the well-known techniques to eliminate environmental effects include dynamical decoupling [5-7], weak measurements, Zeno effect [8] and coherent feedback control [9] to suppress the decoherence, quantum error-correction [4, 10–13], error-mitigating methods [14–19], and use of decoherence-free subspace [20–23] for quantum computation and simulations.

Dynamical decoupling is an open-loop quantum control technique. It is implemented by a periodic sequence of instantaneous control pulses [5] and achieves decoherence suppression without increasing the Hilbert space dimension. Still, it cannot be applied to non-Markovian processes [24, 25] (e.g., decoherence due to amplitude damping). In the decoherence-free subspace approach, quantum information is encoded in a particular quantum state that does not experience a specific type of decoherence [26, 27]. Both the quantum error correction and the decoherence-free subspace-based schemes use the Hilbert space dimension larger than that of the system dimension. The quantum Zeno effect is also exploited to protect a quantum system [8]. There are other interesting methods to protect a quantum state and entanglement distribution using weak measurement and their reversals [28–40] and decoherence suppression via quantum measurement reversal [41]. In these schemes, the quantum state is firstly transferred to more robust states by a weak measurement to resist decoherence. After that, another weak measurement is performed that reverses the state back to the original state. Due to the failure rates of the weak measurements, however, these schemes have limited success probabilities. Weak force sensing is also a technique to protect the quantum state, and this is based on coherent quantum noise cancellation in a non-linear hybrid optomechanical system [42]. The optomechanical cavity contains a movable mechanical mirror, a fixed semitransparent mirror, an ensemble of ultra-cold atoms, and an optical parametric amplifier (OPA). Using the coherent quantum noise cancellation (CQNC) process, one can eliminate the back action noise at all frequencies. Also, by tuning the OPA parameters, one can suppress the quantum shot noise at lower frequencies than the resonant frequency. There are also techniques to suppress noise in an atomic system using a field in a squeezed coherent state [43]. The interaction of a quantized electromagnetic field with a medium of three-level Λ atoms has been studied adequately in the last several years [44–46]. The interaction of a quantized electromagnetic field in a squeezed coherent state with a three-level Λ atom is studied numerically by the quantum Monte Carlo method and analytically by the Heisenberg-Langevin method in the regime of electromagnetically induced transparency (EIT) [47].

Here we introduce a generic protocol that is particularly suitable for storing quantum information in an atomic system placed inside an optical cavity. It makes arbitrary noisy environment noise-free for the atom using externally supplied photons. The

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Figure 1. Quantum circuit for QCT: The protocol uses two ancilla *A* and *B* in the initial state $|++\rangle^{AB}$. In the pre-processing phase, the nonlocal unitary U_{ABS} is applied on the system *S* and the ancilla *AB*, which couples the three qubits before they are exposed to the environments. Due to the environment, the system *S* undergoes an arbitrary noisy operation Λ^S , while the ancilla *AB* may be exposed to a wide class of noise given by the map Φ^{AB} in Eq. (3). In the post-processing phase, we undo the operation U_{ABS} , i.e., apply U_{ABS}^{\dagger} , followed by the non-local unitary operation V_{ABS} , which neutralizes the effect of environmental noise on *S*.

protocol enables full protection of quantum information and entanglement from decoherence and environmental noise. We also propose an experimentally realizable scheme for that.

To mitigate noise from a system, the protocol requires an ancilla. For instance, the system may be an atom, and the ancilla may be two photons, as we shall consider later. The protocol exploits non-local evolutions on the system and ancilla. In the process, the information stored in the system is transferred onto the decoherence-free subspace [21, 22] of the ancilla before the system is exposed to environmental noise and retrieved back to the system at the end. The action of an arbitrary channel Λ^S on a state ρ^S of a *d*-dimensional system (qudit) *S* can be expressed as

$$\Lambda^{S}(\rho^{S}) = \sum_{m} F_{m}^{S} \rho^{S} F_{m}^{S\dagger}, \tag{1}$$

where F_m^S are the Kraus operators satisfying $\sum_m F_m^{S\dagger} F_m^S = 1$. Here, 1 is the $d \times d$ identity matrix. The protocol implements the transformation of the operation on the system part from arbitrary noisy operation Λ to identity operation as,

$$\Lambda^S \to \mathbb{I}^S.$$

The protocol achieves a transformation in the system's operation, transforming it from an arbitrary noisy operation Λ^S to the identity operation \mathbb{I}^S , where \mathbb{I}^S signifies the identity channel acting on the system. The goal of this paper is to introduce a protocol implementing such a transformation on the system's operations. The protocol relies on two ancilla and unitary operations on the composite (see Fig. 1). We also introduce an experimentally implementable model based on an atom-cavity system. We discuss all these in the following sections.

II. PROTECTION OF INFORMATION AGAINST NOISE

Let us denote two qudits A and B as ancilla. The system is in an arbitrary state ρ^{S} . A quantum circuit scheme to realize the protocol is given in figure 1.

The noise mitigation protocol – Without loss of generality, we consider a qubit (d = 2) system, which is exposed to arbitrary qubit noisy channels. The protocol can be extended to an arbitrary *d*-dimensional atom (see Appendix). For d = 2, the Kraus operators in Eq. (1) can be expressed in terms of superposition in the orthonormal Hilbert-Schmidt bases, as $F_m^S = \sum_{i=0,x,y,z} c_{mi} \sigma_i^S$, where $c_{mi} \in \mathbb{C}$, $\sigma_0 = 1$, and σ_x^S , σ_y^S , and σ_z^S are the Pauli *x*, *y*, and *z* matrices, respectively. Throughout the text we denote $|0\rangle$ and $|1\rangle$ as the eigenstates of σ_z^S operator. The step-by-step description of the protocol is given below (see Appendix for more details).

Step 1: The ancilla AB is initiated in the state $|++\rangle$ where $|+\rangle^{A/B} = (|0\rangle^{A/B} + |1\rangle^{A/B})/\sqrt{2}$.

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Step 2: The composite ABS is evolved by a global unitary operation U_{ABS} , given by

$$U_{ABS} = |00\rangle\langle 00|^{AB} \otimes \mathbb{1}^{S} + |01\rangle\langle 01|^{AB} \otimes \sigma_{z}^{S} + |10\rangle\langle 10|^{AB} \otimes \sigma_{x}^{S} - i|11\rangle\langle 11|^{AB} \otimes \sigma_{y}^{S}.$$

$$(2)$$

Step 3: The system *S* undergoes an arbitrary (possibly unknown) qubit channel Λ^S , as a result of its interaction with its environment. Similarly, the ancilla may be allowed to experience environmental noise resulting in a specific class of channels Φ^{AB} . The form of Kraus operators of the channels $\Phi^{AB} = \sum_{\mu} E^{AB}_{\mu} \rho_{AB} (E^{AB}_{\mu})^{\dagger}$ are given by

$$E_{\mu}^{AB} = q_{\mu 0} \mathbb{1} \otimes \mathbb{1} + q_{\mu 1} \sigma_x \otimes \mathbb{1} + q_{\mu 2} \sigma_y \otimes \sigma_x + q_{\mu 3} \sigma_z \otimes \sigma_x, \tag{3}$$

where $q_{\mu i} \in \mathbb{C}$ and $\sum_{\mu} E_{\mu}^{AB\dagger} E_{\mu}^{AB} = \mathbb{1}^{AB}$. The form the Kraus operators depend on the idea of decoherence-free subspace which we shall discuss later and obvious example of the operation Φ^{AB} is the identity operation.

Step 4: The composite ABS is evolved with the unitary U_{ABS}^{\dagger} followed by another unitary V_{ABS} given by

$$V_{ABS} = |++\chi++|^{AB} \otimes \mathbb{1}^{S} + |+-\chi+-|^{AB} \otimes \sigma_{x}^{S} + i |--\chi--|^{AB} \otimes \sigma_{y}^{S} + |-+\chi-+|^{AB} \otimes \sigma_{z}^{S}.$$
(4)

On the level of channel, the Steps 1-4 implement a transformation that leads to

$$\Phi^{AB} \otimes \Lambda^{S} \to \Psi^{AB} \otimes \mathbb{I}^{S}, \tag{5}$$

for an arbitrary noisy channel Λ^S on *S*. Note that, in place of applying the unitary V_{ABS} in Step 4, we may also perform a non-unitary operation with the Kraus operators $\{| + +\chi + + |^{AB} \otimes \mathbb{1}^S, | + -\chi + - |^{AB} \otimes \sigma_x^S, | - -\chi - - |^{AB} \otimes \sigma_y^S, | - +\chi - + |^{AB} \otimes \sigma_z^S\}$ to make the channel Λ^S transparent.

Role of decoherence-free subspace – The protocol transfers the information of the initial system state to the ancilla after Step 2, i.e., before the system is exposed to arbitrary local noise. And the information transferred to the ancilla does not alter as it is encoded onto the decoherence-free subspace [21, 22] corresponding to the noise given by the Kraus operators (3). In addition, arbitrary local noise on the system does not degrade the information stored in ancilla either. Step 4 only retrieves the information from the ancilla to the system. Thus, the system recovers its initial state at the end of the protocol.

For a given set of channels Φ on some system with the Kraus operators E_{μ} satisfying $\sum_{\mu} E_{\mu}^{\dagger} E_{\mu} = 1$, we define a decoherence-free subspace \mathcal{H}_d if

$$\rho = \Phi(\rho) = \sum_{\mu} E_{\mu} \rho E_{\mu}^{\dagger}, \tag{6}$$

where ρ is state that live in \mathcal{H}_d . Note, this alternatively requires that $[\rho, E_\mu] = 0$, $\forall \mu$. This notion of the decoherence-free subspace is similar to the one introduced in [21, 22] except that it is defined for a quantum channel without having an explicit semi-group structure.

Now, we discuss how the protocol transfers the information about the initial state of the system S to the ancilla AB. The global state after Step 2 (see above) is

$$\rho_2^{ABS} = U_{ABS} \rho_1^{ABS} U_{ABS}^{\dagger},$$

where $\rho_1^{ABS} = |++\rangle + |^{AB} \otimes \rho^S$, and ρ^S is an arbitrary state of S. Now the reduced density matrix of AB is given by

$$\rho_{AB} = \frac{1}{4} \bigg[(\mathbb{1} \otimes \mathbb{1}) + r_z (\mathbb{1} \otimes \sigma_x) - r_y (\sigma_x \otimes \sigma_y) + r_x (\sigma_x \otimes \sigma_z) \bigg], \tag{7}$$

where $r_i = tr(\rho^S \sigma_i)$ with i = x, y, z. Note, all the information about ρ^S is transferred to AB in terms of $\{r_x, r_y, r_z\}$ and no information remains in S as its reduced state becomes maximally mixed after the evolution.

In Step 3, the ancilla AB is exposed to the noise channels Φ^{AB} with the Kraus operators $E^{AB}_{\mu} = \sum_{ij=0}^{1} q_{\mu ij} \sigma_z^i \sigma_x^j \otimes \sigma_x^{2-i}$. But the interesting fact is that the reduced state ρ^{AB} does not alter by this evolution, i.e.,

$$\rho^{AB} = \Phi^{AB}(\rho^{AB}) = \sum_{\mu} E^{AB}_{\mu} \rho^{AB} E^{AB\dagger}_{\mu}.$$
(8)

It implies that the information about *S*, in terms of $\{r_x, r_y, r_z\}$, is stored in a subspace of *AB* that is decoherence-free for the class of noisy channels given by Φ^{AB} . Further, an arbitrary local noisy evolution (Λ^S) on the system *S* cannot destroy or degrade the information about *S* being stored in *AB*. After Step 4, the information about *S* is retrieved, which is how it is protected from arbitrary local noises.

Clearly, arbitrary noisy channels on the system (S) can be made transparent using the above protocol if the ancilla (AB) is restricted to experiencing a particular class of noises. Not only that, but for a multipartite system, it can protect entanglement while the subsystems are exposed to local environments. Beyond these restricted noises on ancilla, the protocol does not always protect quantum information in the system in general. However, there are realistic cases where the ancilla (AB) does not experience the same noise as the system does. For instance, the atoms and photons undergo different noisy evolutions when they are exposed to the 'same' environment. Below we explore such a situation where an atom in a cavity is considered as the system undergoing arbitrary noisy evolution, and photons act as the ancilla.

III. PROTECTING ATOMS IN OPTICAL CAVITIES

In quantum information processing tasks, it is often necessary to protect coherence in the state of a quantum system for a long enough duration. But, the inevitable interaction with the environment makes the quantum coherence in a state to decay. The environmental effects can be nullified using the noise mitigation protocol introduced above hence protecting the state and the quantum information indefinitely. To demonstrate that, we present an experimental scheme in Figure 2, where the state of an atom is protected for an indefinite time.

In this scheme, we consider a three-level atom with a two-fold degenerate ground state and an excited state trapped inside an optical cavity. The system qubit (*S*) consists of the state space spanned by the low-energy states $|\pm 1\rangle$ of the atom, whereas the ancilla qubits (*A* and *B*) consist of two single-photons.

To implement the operation U_{ABS} we first notice that this operator can be decomposed as a product of two controlled two-qubit operators as follows:

$$U_{ABS} = (\mathbb{1}_B \otimes C_x^{AS})(\mathbb{1}_A \otimes C_z^{BS}), \tag{9}$$

where $C_x^{AS} = |0\rangle\langle 0|^A \otimes \mathbb{1}^S + |1\rangle\langle 1|^A \otimes \sigma_x^S$ and $C_z^{BS} = |0\rangle\langle 0|^B \otimes \mathbb{1}^S + |1\rangle\langle 1|^B \otimes \sigma_z^S$ are respectively the control-NOT and control-Phase gates acting on one photon and the atom.

The control-Phase and the control-NOT operations between a photon and the atom can be performed using the technique given in Ref. [48–51]. For this technique to work, the three-level atom should have the Λ -transition, where the transition is allowed only between $|\pm 1\rangle \leftrightarrow |0\rangle$ and forbidden between $|+1\rangle \leftrightarrow |-1\rangle$ levels. Due to the conservation of angular momentum, the right-circular polarized light interacts with $|+1\rangle \leftrightarrow |0\rangle$ transition and the left-circular with $|-1\rangle \leftrightarrow |0\rangle$.

For an atom trapped inside a cavity in the strong coupling regime (Purcell regime), in the steady state limit and at resonance, the relation between the input a_{in} and output a_{out} light modes of the cavity can be written as

$$a_{\rm out} = \frac{-\kappa\gamma + 4g^2}{\kappa\gamma + 4g^2} a_{\rm in},\tag{10}$$

where κ is the decay rate of the cavity, g characterizes the coupling strength between the cavity and the atom and γ is the atomic decay rate such that $\kappa \gg g \gg \gamma$. Therefore, if $g^2 \gg \kappa \gamma$ then $a_{out} \sim a_{in}$ whereas if $g^2 \ll \kappa \gamma$ then $a_{out} \sim -a_{in}$.

If we choose the right-circular polarization as the normal mode of the optical cavity, then the transition $|-1\rangle \leftrightarrow |0\rangle$ is always decoupled, i.e, g = 0. Therefore, if the atom is prepared in the state $|-1\rangle$ then it will reflect the photon with a π -phase, irrespective of its polarization. However, if the atom is in the state $|+1\rangle$ then the photon will experience a σ_z operation on its polarization states, i.e., $|L\rangle \rightarrow -|L\rangle$ and $|R\rangle \rightarrow |R\rangle$. The general transformation can be written as

$$|L\rangle \otimes |\pm 1\rangle \to -|L\rangle \otimes |\pm 1\rangle, \quad |R\rangle \otimes |\pm 1\rangle \to \pm |R\rangle \otimes |\pm 1\rangle, \tag{11}$$

which is exactly $-C_z$ operation. The C_z operation can be converted into the C_x operation by applying the Hadamard operation on the target qubit. It can be done efficiently on the atomic system by the optimized-STIRAP techniques where the excited state $|0\rangle$ is adiabatically eliminated [57, 58]. Optimized-STIRAP has duration around ~ 100ns [59] and the typical errors are less than 10^{-4} [60, 61]. To perform controlled operations, we use a single atom (A-system) trapped in a cavity. This A-system is made up of three levels: $|0\rangle$, $|+1\rangle$, and $|-1\rangle$. When transitioning from $|0\rangle$ to $|\pm1\rangle$, the coherence time is typically around $(\gamma/2)^{-1}$, approximately ~ 1µs. As discussed above, our qubit $(|+1\rangle, |-1\rangle)$ is created by eliminating the excited state $|0\rangle$, making the influence of decoherence (from γ) negligible on the overall operation time. To conduct controlled operations, we need light pulses with widths much larger than κ/g^2 . Therefore, boosting the coupling strength g can speed up the execution of controlled operations. However, in modern experiments with atoms in cavities, pulses lasting ~ 1µs to ~ 2µs are commonly used [62]. As for U_{ABS} , it involves two reflections separated by an atomic rotation, and with current atom-cavity settings, the estimated time is



Figure 2. Protecting quantum information in an atomic system: The atomic state space is spanned by the low-energy states $|\pm 1\rangle$ and the excited state $|0\rangle$. The ancilla consists of two single-photons (*A* and *B*) in circularly polarization basis $\{|R\rangle, |L\rangle\}$. The photons are initiated in the states $(|R\rangle + |L\rangle)/\sqrt{2}$. In order to implement U_{ABS} operation, we make two photons *A* and *B* interact with the atom *S* inside the cavity. Let the two photons be τ time apart where the photon *B* comes first. The interaction of the photon *B* with the atom results in the C_z^{BS} operation. Then applying Hadamard operation on the atom using STIRAP technique with the help of a classical laser pulses (Ω) followed by the interaction of the photon *A* yields C_x^{AS} operation. Both operations together result in U_{ABS} . Afterward, the two photons can be stored in an optical quantum memory [52–54] to be used subsequently to implement U_{ABS}^{\dagger} and V_{ABS} operations. Circulators [49] are used to direct the photons towards the cavity and optical quantum memory. The electro-optic-modulator (EOM) [55, 56] is used to perform Hadamard operation on the photons in order to convert U_{ABS} .

around ~ $5\mu s$. Additionally, atomic rotations are performed using STIRAP, a process that takes about ~ 100ns [59]. It's crucial to highlight that the extended time required for controlled operations is mainly due to the input pulse width. Nevertheless, this duration can be significantly shortened by increasing the coupling strength, bringing the total time to around a few ~ 100ns [59]. Controlled operations between atoms and a cavity exhibit low fidelities. For instance, a CNOT operation is executed with an 86% fidelity [49]. This reduction in fidelity primarily arises from frequency fluctuations and scattering losses associated with the cavity mirrors rather than cavity decay rates. Hence, by tuning the parameters (g, κ, γ), an atom-cavity system can perform unitary operations with minimal losses, primarily due to photon loss via scattering from mirrors.

Therefore, the unitary U_{ABS} can be implemented by sequentially interacting the two photons with the atom-cavity system. Similarly, we can implement the V_{ABS} operator between the atom and the photons by introducing U_{ABS} along with the Hadamard operation on the polarization states of the photons using HWP and the atomic states using STIRAP technique.

As mentioned earlier, one of the advantages of photonic systems is that they experience less environmental noise than for atomic systems. In fact, the major source of photonic noise is associated with the loss of the photon itself. However, recent experiments demonstrate that, in atom-cavity systems, the photon loss can be restricted to less than 5% [63–65]. This, in turn, implies there is a 0.05 probability with which a photon is lost. Now we shall study the impact of photon loss in the atom-cavity system discussed above. There are several possibilities. For instance, photon-A may get lost in different stages of the protocol, i.e., in step-2 or in step-4, and similarly for photon-B. Further, both the photons A and B may get lost in different stages.

Consider the photon-*A* is lost before step 2 takes place. Because of that, the effect of environmental noise on the atom cannot be eliminated. Say, the atom undergoes an evolution given by the channel Φ_2^S . This channel will be different for different environmental noises the atom is exposed to. For the cases where the photon is being lost with probability *p*, the overall evolution of the atom is $\mathcal{E}^S = (1 - p)\mathbb{I}^S + p\Phi_2^S$, where \mathbb{I}^S represents the identity operation when there is no photon loss (see Eq. (5)). Now, it can be easily seen that the error incurred in the noise mitigation due to the photon loss for any given initial state of the atom is

$$\|\mathbb{I}^{S} - \mathcal{E}^{S}\|_{\diamond} = p\|\mathbb{I}^{S} - \Phi_{2}^{S}\|_{\diamond} \le p,$$

$$(12)$$

where $\|\cdot\|_{\diamond}$ is the diamond norm [66]. Similarly, for the case where a photon is lost in step 4 with a probability *p*, the error in noise mitigation is bounded from above by *p*. This is also true when the photon-*B* is lost in either step 2 or 4. In the case where

both the photon A and B are lost with probabilities p and q, respectively, the overall error in noise mitigation is upper bounded by pq irrespective of the steps in which the photons are lost.

IV. DISCUSSION

Unlike the conventional noise-mitigating protocols that are, in general, specific to the system, and the nature of the environment, our protocol can be applicable to an arbitrary channel on any system. Although, it requires the ancilla to undergo a restricted class of noisy channels. If one allows arbitrary noise on the ancilla, the proposed protocol can only mitigate noise in some specific class noises on the system and its initial states. The methods to improve coherence time using dynamical decoupling and feedback control are particular to the nature of noise in the system. While these protect local coherence to an extent, they are not suitable for preserving non-local coherence, such as quantum entanglement. In contrast, our protocol can protect both local and global coherence for an indefinite time, irrespective of the nature of local noises on the systems. There are techniques to protect quantum entanglement by creating it in a decoherence-free subspace. However, these are specific to a small set of entangled states and do not work for arbitrary states. Instead, in our protocol, a decoherence-free subspace of the ancilla is utilized to eliminate arbitrary local noise in the system to the ancilla by swapping their states. However, this requires the ancilla to be noise-free at all times. On the contrary, our protocol works even if the ancilla is allowed to interact with a large but restricted class of noisy environments.

Our protocol is particularly suitable for various practical situations where different kinds of systems (or degrees of freedom) experience different noises while in the same physical environment. For instance, a photon experiences different noises than an atom. We exploit this advantage to construct an experimental proposal where an atom in a cavity is made noise-free with the use of photons.

In summary, we have introduced a protocol that makes an arbitrary quantum channel transparent for an arbitrary dimensional quantum system. Specifically, we have given the implementation scheme to protect quantum information in an atomic system inside a cavity. The protocol may open new avenues to protect quantum information and correlations against environmental noise.

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Appendix A: The protocol for qubit channels

Detailed derivation of the protocol for arbitrary qubit channels. The protocol is implemented in five steps, to make arbitrary qubit channel Λ^S transparent, i.e., $\Lambda^S \to \mathbb{1}^S$, where $\mathbb{1}^S$ is an identity channel. The protocol uses two qubit ancilla *AB* with is also allowed to undergo a wide class of noisy operation during the protocol, as discussed below.

Step 1: To start with we attach the ancilla AB with the system S in an initial state

$$\rho_1^{ABS} = |++\rangle\!\!\!/++|^{AB} \otimes \rho^S, \tag{A1}$$

where $|\pm\rangle^X = \frac{1}{\sqrt{2}} (|0\rangle^X \pm |1\rangle^X)$ with X = A, B, and ρ^S is any state of S.

Step 2: Initial state evolve with a global unitary (acausal operation) U_{ABS} .

$$\rho_2^{ABS} = U_{ABS} \rho_1^{ABS} U_{ABS}^{\dagger}, \tag{A2}$$

where $U_{ABS} = |00\rangle\langle00|^{AB} \otimes \mathbb{1}^{S} + |01\rangle\langle01|^{AB} \otimes \sigma_{z}^{S} + |10\rangle\langle10|^{AB} \otimes \sigma_{x}^{S} - i|11\rangle\langle11|^{AB} \otimes \sigma_{y}^{S}$. Here $\{\sigma_{x}, \sigma_{y}, \sigma_{z}\}$ are the Pauli matrices.

Step 3: System *S* passes through a noisy channel implementing an arbitrary quantum noisy channel (CPTP map) Λ^S that we want to eliminate, given by $\Lambda^S(\rho_S) = \sum_m F_m^S \rho_S F_m^{S^{\dagger}}$, with Kraus operators $F_m^S = \sum_{i=0}^3 c_{mi}\sigma_i$ where $c_{mi} \in \mathbb{C}$ can take arbitrary values satisfying $\sum_m F_m^S F_m^S = \mathbb{1}^S$. Here we assume $\sigma_0 = \mathbb{1}$. The ancilla *AB* may experience a class of environmental noise, given by the channels $\Phi^{AB}(\rho_{AB}) = \sum_\mu E_\mu^{AB} \rho_{AB} E_\mu^{AB^{\dagger}}$, where the Karus operators are $E_\mu^{AB} = \sum_{j,k=0}^1 q_{\mu jk} \sigma_z^j \sigma_x^k \otimes \sigma_x^{2-j} = \sum_{j=0}^{\infty} c_{mi}\sigma_j^j \sigma_z^k \otimes \sigma_z^{2-j}$.

 $q_{\mu 00} \mathbb{1} \otimes \mathbb{1} + q_{\mu 01} \sigma_x \otimes \mathbb{1} + q_{\mu 10} \sigma_z \otimes \sigma_x + q_{\mu 11} \sigma_z \sigma_x \otimes \sigma_x$, satisfying $\sum_{\mu} E_{\mu}^{AB^{\dagger}} E_{\mu}^{AB} = \mathbb{1}^{AB}$. As a consequence, the overall state evolves as

$$\rho_3^{ABS} = \Phi^{AB} \otimes \Lambda^S \left(\rho_2^{ABS} \right), \tag{A3}$$

The we choose E^{AB}_{μ} , because of decoherence free subspace as discussed in the main text.

Step 4: Now, ρ_3^{ABS} is evolved with hermitian conjugate on the global unitary (acausal operation) U_{ABS}^{\dagger} . Thus,

$$\rho_4^{ABS} = U_{ABS}^{\dagger} \rho_3^{ABS} U_{ABS}, \tag{A4}$$

where $U_{ABS}^{\dagger} = |00\rangle\langle00|^{AB} \otimes \mathbb{1}^{S} + |01\rangle\langle01|^{AB} \otimes \sigma_{z}^{S} + |10\rangle\langle10|^{AB} \otimes \sigma_{x}^{S} + i|11\rangle\langle11|^{AB} \otimes \sigma_{y}^{S}$. The state ρ_{4}^{ABS} can explicitly be expressed as,

$$\rho_4^{ABS} = \sum_m \sum_\mu \left[\left[U_{ABS}^{\dagger} \left(E_{\mu}^{AB} \otimes F_m^S \right) U_{ABS} \right] \rho_1^{ABS} \left[U_{ABS}^{\dagger} \left(E_{\mu}^{AB} \otimes F_m^S \right) U_{ABS} \right]^{\dagger} \right].$$
(A5)

Let's consider a $K_{\mu m}^{ABS} = U_{ABS}^{\dagger} (E_{\mu}^{AB} \otimes F_{m}^{S}) U_{ABS}$, which is the kraus operator applied on the initial state (ρ_{1}^{ABS}) . Then, we can re-write the state ρ_{4}^{ABS} as,

$$\rho_4^{ABS} = \sum_{\mu m} K_{\mu m}^{ABS} \rho_1^{ABS} K_{\mu m}^{ABS^{\dagger}}.$$
 (A6)

The explicit form of the kraus operator $K_{\mu m}^{ABS}$ is,

$$K_{\mu m}^{ABS} = U_{ABS}^{\dagger} \left(\sum_{j,k=0}^{1} q_{\mu j k} \left(\sigma_z^j \sigma_x^k \right)^A \otimes \left(\sigma_x^{2-j} \right)^B \otimes \sum_{i=0}^{3} c_{m i} \sigma_i^S \right) U_{ABS}.$$
(A7)

The explicit form of $K_{\mu m}^{ABS}$ is,

$$\begin{split} K^{ABS}_{\mu m} &= \left[q_{\mu 00} \Big(\mathbbm{1}^A \otimes \mathbbm{1}^B \otimes c_{m0} \mathbbm{1}^S + \mathbbm{1}^A \otimes \sigma_z^B \otimes c_{m1} \sigma_x^S + \sigma_z^A \otimes \sigma_z^B \otimes c_{m2} \sigma_y^S + \sigma_z^A \otimes \mathbbm{1}^B \otimes c_{m3} \sigma_z^S \Big) + \\ q_{\mu 01} \Big(\sigma_x^A \otimes \sigma_z^B \otimes c_{m0} \sigma_x^S + \sigma_x^A \otimes \mathbbm{1}^B \otimes c_{m1} \mathbbm{1}^S + \sigma_y^A \otimes \mathbbm{1}^B \otimes c_{m2} \sigma_z^S - \sigma_y^A \otimes \sigma_z^B \otimes c_{m3} \sigma_y^S \Big) + \\ q_{\mu 10} \Big(\sigma_z^A \otimes \sigma_x^B \otimes c_{m0} \sigma_z^S + \sigma_z^A \otimes \sigma_y^B \otimes c_{m1} \sigma_y^S - \mathbbm{1}^A \otimes \sigma_y^B \otimes c_{m2} \sigma_x^S + \mathbbm{1}^A \otimes \sigma_x^B \otimes c_{m3} \mathbbm{1}^S \Big) + \\ q_{\mu 11} \Big(\sigma_y^A \otimes \sigma_y^B \otimes c_{m0} \sigma_y^S + \sigma_y^A \otimes \sigma_x^B \otimes c_{m1} \sigma_z^S - \sigma_x^A \otimes \sigma_x^B \otimes c_{m2} \mathbbm{1}^S - \sigma_x^A \otimes \sigma_y^B \otimes c_{m3} \sigma_x^S \Big) \Big] \end{split}$$

Without loss of generality, we may consider the initial system state to be a pure state $\rho^{S} = |\psi \chi \psi|^{S}$. Then the overall transformation becomes

$$\rho_{ABS}^{4} = \sum_{\mu m} \left(K_{\mu m}^{ABS} \right) |++\rangle + |^{AB} \otimes |\psi\rangle \langle\psi|^{S} \left(K_{\mu m}^{ABS} \right)^{\dagger}$$
(A8)

To simply understand, we write the effect of each Kraus operator on the global initial state as,

$$\begin{split} K^{ABS}_{\mu m} \left(|++\rangle^{AB} \otimes |\psi^{S}\rangle \right) &= \left(q_{\mu 00} c_{m0} + q_{\mu 01} c_{m1} + q_{\mu 10} c_{m3} - q_{\mu 11} c_{m2} \right) |++\rangle \otimes \mathbb{1} |\psi_{s}\rangle + \\ \left(q_{\mu 00} c_{m1} + q_{\mu 01} c_{m0} + i q_{\mu 10} c_{m2} + i q_{\mu 11} c_{m3} \right) |+-\rangle \otimes \sigma_{x} |\psi_{s}\rangle + \\ \left(q_{\mu 00} c_{m3} - i q_{\mu 01} c_{m2} + q_{\mu 10} c_{m0} - i q_{\mu 11} c_{m1} \right) |-+\rangle \otimes \sigma_{z} |\psi_{s}\rangle + \\ \left(q_{\mu 00} c_{m2} + i q_{\mu 01} c_{m3} - i q_{\mu 10} c_{m1} - q_{\mu 11} c_{m0} \right) |--\rangle \otimes \sigma_{y} |\psi_{s}\rangle . \end{split}$$

Step 5: Finally the state ρ_{ABS}^4 is evolved with the acausal unitary V_{ABS} given by

$$V_{ABS} = H_A \otimes H_B \otimes H_S \left(U_{ABS} \right) H_A^{\dagger} \otimes H_B^{\dagger} \otimes H_S^{\dagger}$$
(A9)

$$= |++\chi++|^{AB} \otimes \mathbb{1}^{S} + |+-\chi+-|^{AB} \otimes \sigma_{x}^{S} + |-+\chi-+|^{AB} \otimes \sigma_{z}^{S} + i|--\chi--|^{AB} \otimes \sigma_{y}^{S},$$
(A10)

where H_X with X = A,B,S is the Hadamard unitary. As a result, the final state becomes

$$\rho_{ABS}^5 = V_{ABS} \ \rho_{ABS}^4 \ V_{ABS}^\dagger = \Psi^{AB} \Big(\left| + + \chi + + \right|^{AB} \Big) \otimes \rho^S, \tag{A11}$$

where $\Psi^{AB}(|++\chi++|^{AB}) = \sum_{\mu} F^{AB}_{\mu m} |++\chi++|^{AB} F^{AB\dagger}_{\mu m}$ is a channel implemented on the ancilla *AB*, with the Kraus operators

$$F_{\mu m}^{AB} = q_{\mu 00} (c_{m0} \mathbb{1} \otimes \mathbb{1} + c_{m1} \mathbb{1} \otimes \sigma_z + c_{m2} \sigma_z \otimes \sigma_z + c_{m3} \sigma_z \otimes \mathbb{1}) + q_{\mu 01} (c_{m0} \sigma_x \otimes \sigma_z + c_{m1} \sigma_x \otimes \mathbb{1} + c_{m2} \sigma_y \otimes \mathbb{1} - c_{m3} \sigma_y \otimes \sigma_z) + q_{\mu 10} (c_{m0} \sigma_z \otimes \sigma_x + c_{m1} \sigma_z \otimes \sigma_y - c_{m2} \mathbb{1} \otimes \sigma_y + c_{m3} \mathbb{1} \otimes \sigma_x) + q_{\mu 11} (c_{m0} \sigma_y \otimes \sigma_y + c_{m1} \sigma_y \otimes \sigma_x - c_{m2} \sigma_x \otimes \sigma_x - c_{m3} \sigma_x \otimes \sigma_y).$$
(A12)

Thus, the arbitrary system state is fully recovered, as if the system is passed through a transparent environment. On the level of operation the transformation leads to, after all the steps of protocol.

$$\Phi^{AB} \otimes \Lambda^{S} \to \Psi^{AB} \otimes \mathbb{1}^{S}. \tag{A13}$$

The same transparency can be attained by applying a non-unitary (semi-causal) CPTP operation on *ABS* with the Kraus operators $\{| + +\chi + + |^{AB} \otimes \mathbb{1}^{S}, | + -\chi + - |^{AB} \otimes \sigma_{x}^{S}, | - -\chi - - |^{AB} \otimes \sigma_{y}^{S}, | - +\chi - + |^{AB} \otimes \sigma_{z}^{S}\}$ in Step 5, instead of V_{ABS} .

Appendix B: The protocol for qudit channels

The protocol for qudit channels $(d \ge 3)$ – The protocol for a *d*-dimensional system (*S*) follows similar to the qubit systems, where an arbitrary qudit channel Λ_d^S is made transparent with the help of two *d*-dimensional ancillary systems *A* and *B*. In Step 1, the ancilla state is prepared in $|\psi_0\rangle^A \otimes |\psi_0\rangle^B$ where $|\psi_0\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k\rangle$. In Step 2, the tri-partite unitary operation $U_{ABS}^{(d)} = (\mathbb{1}_B \otimes C_X^{AS})(\mathbb{1}_A \otimes C_Z^{BS})$ is applied. Here the control operations C_X^{AS} and C_Z^{BS} are defined as

$$C_{Z}^{BS} = \sum_{k=0}^{d-1} |k \rangle \langle k|^{B} \otimes Z^{k}, \quad C_{X}^{AS} = \sum_{k=0}^{d-1} |k \rangle \langle k|^{A} \otimes X^{k}, \tag{B1}$$

where $Z = \sum_{k=0}^{d-1} e^{(i2k\pi/d)} |k| |k|$ and $X = \sum_{k=0}^{d-1} |(k+1) \mod d\rangle \langle k|$. In Step 3, the system is exposed to an arbitrary environment and undergoes a noisy operation Λ_d^S . The ancilla *AB* may also undergo noisy operation Φ_d^{AB} with the corresponding Kraus operators

$$E^{AB}_{\mu} = \sum_{i,j} c_{\mu ij} Z^i X^j \otimes X^{d-i}.$$
 (B2)

In Step 4, the ABS composite is evolved with the unitary $U_{ABS}^{(d)\dagger}$ and followed by $V_{ABS}^{(d)}$, where

$$V_{ABS}^{(d)} = \sum_{m,n} |\psi_m \rangle \langle \psi_m |^A \otimes |\psi_n \rangle \langle \psi_n |^B \otimes (Z^m X^n)^{\dagger},$$
(B3)

with $|\psi_m\rangle^A = Z^m |\psi_0\rangle^A$ and $|\psi_n\rangle^B = Z^{[n(d-1) \mod d]} |\psi_0\rangle^B$. The Steps 1-4 result in the overall transformation on the level of channel as

$$\Phi_d^{AB} \otimes \Lambda_d^S \to \Psi_d^{AB} \otimes \mathbb{1}^S, \tag{B4}$$

where the local channel on the system S becomes transparent (see Appendix for more details).

Suppose, an arbitrary noisy quantum channel applied on a *d*-dimensional quantum system *S* in the state η^S , given by $\Lambda_d^S(\eta^S) = \sum_i F_i(\eta^S)F_i^{\dagger}$. Now, we extend the protocol to that makes the channel transparent, i.e., $\Lambda_d^S \to \mathbb{1}^S$. Note, any Karus operator can be expressed as $F_i = \sum_{m,n} c_{mni}S_{mn}$, where S_{mn} are the complete set of Schwinger unitary operators, given by $S_{mn} = Z^m X^n$ with $Z = \sum_{k=0}^{d-1} \xi^k |k \rangle \langle k|$, $X = \sum_{k=0}^{d-1} |(k+1) \mod d \rangle \langle k|$. Here $\{|k\rangle\}$ represents a complete set orthonormal bases in the system Hilbert space.

For the protocol, we attach two d-dimensional anilla AB with the system S, and the ancilla allowed to interact with a wide class of environment. Similar to the qubit case, the protocol is implemented in five steps, as follows.

Step 1: Initially, we attach ancilla AB with system state S.

$$\eta_1^{ABS} = |\psi_0\rangle\langle\psi_0|^A \otimes |\psi_0\rangle\langle\psi_0|^B \otimes \eta^s, \tag{B5}$$

where $|\psi_0\rangle_{A/B} = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k\rangle_{A/B}$ and η^S can be arbitrary state for the system. Here $\{|k\rangle_{A/B}\}$ is a complete set of orthonormal bases on the Hilbert space of A/B.

Step 2: The Initial state η_1^{ABS} is evolved with global (acausal) unitary U_{ABS} , given by

$$U_{ABS} = U_{AS}^{X} U_{BS}^{Z} = \sum_{kk'}^{d^{2}-1} |kk'\rangle kk'| \otimes X^{k} Z^{k'},$$
(B6)

where where $U_{AS}^X = \sum_{k=0}^{d-1} |k \rangle \langle k|^B \otimes X^k$, $U_{BS}^Z = \sum_{k'=0}^{d-1} |k' \rangle \langle k'|^A \otimes Z^{k'}$. As a consequence, the initial state is transformed to $\eta_2^{ABS} = U_{ABS} \eta_1^{ABS} U_{ABS}^{\dagger}$.

Step 3: Now, the system S is passed through an arbitrary noisy environment implementing a CPTP map Λ_d^S . The ancilla may also undergo a wide class of noisy channel Φ_d^{AB} due to interaction with its environment. As a result,

$$\eta_3^{ABS} = \Phi_d^{AB} \otimes \Lambda_d^S(\eta_2^{ABS}),\tag{B7}$$

where the kraus operators of the maps Φ_d^{AB} and Λ_d^S are $E_{\mu}^{AB} = \sum_{\alpha,\beta} p_{\mu\alpha\beta} Z^{\alpha} X^{\beta} \otimes X^{d-\alpha}$ and $F_i^S = \sum_{mn} c_{imn} Z^m X^n$ respectively, with $p_{\mu\alpha\beta} \in \mathbb{C}$ and $c_{imn} \in \mathbb{C}$. Note, the Kraus operators satisfy the trace preserving condition $\sum_{\mu} E_{\mu}^{AB^{\dagger}} E_{\mu}^{AB} = \mathbb{1}^{AB}$ and similarly $\sum_i F_i^{S^{\dagger}} F_i^S = \mathbb{1}^S$.

Step 4: The global unitary U_{ABS}^{\dagger} is applied on η_3^{ABS} , to give

$$\eta_4^{ABS} = U_{ABS}^{\dagger} \eta_3^{ABS} U_{ABS}. \tag{B8}$$

Note, it is equivalent to write

$$\eta_4^{ABS} = \sum_{\mu i} \left[U_{ABS}^{\dagger} \left(E_{\mu}^{AB} \otimes F_i^S \right) U_{ABS} \right] \eta_1^{ABS} \left[U_{ABS}^{\dagger} \left(E_{\mu}^{AB} \otimes F_i^S \right) U_{ABS} \right]^{\dagger} = \sum_{\mu i} K_{\mu i}^{ABS} \eta_1 K_{\mu i}^{ABS}^{ABS}^{\dagger}$$
(B9)

where Kraus operators $K_{\mu i}^{ABS}$ are applied on the initial state η_1^{ABS} . The Kraus operators are

$$K_{\mu i}^{ABS} = U_{ABS}^{\dagger} \left(\sum_{\alpha,\beta}^{d-1} p_{\mu\alpha\beta} \left(Z^{\alpha} X^{\beta} \right)^{A} \otimes \left(X^{d-\alpha} \right)^{B} \otimes \sum_{m,n}^{d-1} c_{imn} S_{mn} \right) U_{ABS}$$
(B10)

One can simplify $K_{\mu i}^{ABS}$ by expressing the U_{ABS} and S_{mn} in terms of Z and X. For simplicity, we consider that the ancilla goes through a identity channel ($\Phi_d^{AB} = 1$). In that case, $K_{\mu i}^{ABS} \equiv K_i^{ABS}$. However, the protocol still works even after the ancilla still undergoes a wide class if noisy channels, as mentioned above. Now,

$$K_{i}^{ABS} = \sum_{k,k'=0}^{d^{2}-1} \sum_{m,n=0}^{d-1} c_{mni} |kk'|^{AB} \otimes Z^{k'^{\dagger}} X^{k^{\dagger}} Z^{m} X^{n} X^{k} Z^{k'} = \sum_{k,k'=0}^{d^{2}-1} \sum_{m,n=0}^{d-1} \left(c_{mni} |kk'|^{AB} \otimes e^{i2\pi (km-k'n)/d} S_{mn} \right).$$
(B11)

Here we have used the relations

$$Z^{k'}X^{k} = e^{i2\pi kk'/d}X^{k}Z^{k'}, \quad Z^{k'^{\dagger}}X^{k^{\dagger}} = e^{i2\pi kk'/d}X^{k^{\dagger}}Z^{k'^{\dagger}}, \quad X^{\alpha^{\dagger}}Z^{\beta} = e^{i2\pi \alpha\beta/d}Z^{\beta}X^{\alpha^{\dagger}}, \quad Z^{\beta^{\dagger}}X^{\alpha} = e^{-i2\pi\alpha\beta/d}X^{\alpha}Z^{\beta^{\dagger}}.$$
(B12)

Upon rearrangements, the K_i^{ABS} further reduces to

$$K_{i}^{ABS} = \sum_{m,n=0}^{d-1} c_{mni} \Big[Z^{m} \otimes Z^{n(d-1)mod \ d} \otimes S_{m,n} \Big].$$
(B13)

Now to understand the action of these Kraus operators, we assume that the system S is initially in an arbitrary pure state $|\phi\rangle^{S}$. Note, any mixed state can purified in a larger Hilbert space. Then,

$$|\Psi_4\rangle^{ABS} = K_i^{ABS} \left(|\psi_0\rangle^A \otimes |\psi_0\rangle^B \otimes |\phi\rangle^S \right) = \sum_{m,n=0}^{d-1} c_{imn} |\psi_m\rangle^A \otimes |\psi_n\rangle^B \otimes S_{mn} |\psi\rangle^S ,$$

where $\langle \psi_i | \psi_j \rangle_{A/B} = \delta_{ij}$ and $|\psi_m\rangle^A = Z^m |\psi_0\rangle^A$ and $|\psi_n\rangle^B = Z^{n(d-1)mod \ d} |\psi_0\rangle^B$.

Step 5: In the last step, we apply a global unitary $V^{(d)}_{ABS}$, given by

$$V_{ABS}^{(d)} = \sum_{m,n} |\psi_m \chi \psi_m| \otimes |\psi_n \chi \psi_n| \otimes S_{mn}^{\dagger},$$
(B14)

and, as a result, the overall transformation becomes

$$V_{ABS}^{(d)} \eta_{4}^{ABS} V_{ABS}^{(d)\dagger} = \Psi_{d}^{AB} \Big(|\psi_{0} \chi \psi_{0}|^{A} \otimes |\psi_{0} \chi \psi_{0}|^{B} \Big) \otimes \eta^{S},$$
(B15)

where the Kraus operators corresponding to the CPTP map Ψ_d^{AB} can easily found. Thus on the level quantum channels, the protocol implements the transformation

$$\Phi_d^{AB} \otimes \Lambda_d^S \to \Psi_d^{AB} \otimes \mathbb{1}, \tag{B16}$$

for arbitrary quantum channel of system S and a wide class of noisy channel on the ancilla AB.