# Modeling the coevolution between citations and coauthorship of scientific papers

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# Abstract

Collaborations and citations within scientific research grow simultaneously and interact dynamically. Modelling the coevolution between them helps to study many phenomena that can be approached only through combining citation and coauthorship data. A geometric graph for the coevolution is proposed, the mechanism of which synthetically expresses the interactive impacts of authors and papers in a geometrical way. The model is validated against a dataset of papers published on PNAS during 2007-2015. The validation shows the ability to reproduce a range of features observed with citation and coauthorship data combined and separately. Particularly, in the empirical distribution of citations per author there exist two limits, in which the distribution appears as a generalized Poisson and a power-law respectively. Our model successfully reproduces the shape of the distribution, and provides an explanation for how the shape emerges via the decisions of authors. The model also captures the empirically positive correlations between the numbers of authors' papers, citations and collaborators.

### 1 Introduction

Coauthorship and citations are typical relationships in scientific activities, which can be observed from datasets of scientific papers, and expressed as graphs called coauthorship and citation networks respectively. Coauthorship networks, in essential, are hypergraphs, in which nodes represent authors, and the authors of a paper forms a hyperedge. Citation networks are acyclic directed graphs, in which papers are directly linked via citation relations. Those networks provide big pictures of the topological connections between authors and papers respectively. Modelling the mechanisms and temporal dynamics of

those networks (especially, with consideration of the papers' publication time) sheds light on modes of collaborations within science, and helps to discover trends of academic research [1–4].

Matthew effect describes the phenomenon that success breeds success, which is also called preferential attachment or cumulative advantage [5]. The effect has been observed in the behaviors of scientific collaborations and citations in specific disciplines and countries [6–9]. As early as 1965, Price explained the emergence of fat-tails in the distributions of citation networks as a consequence of Matthew effect. His explanation is modelled by a rule, namely the probability of a paper's receiving a new citation is proportional to the citations it has received. The rule successfully predicts the scale-free property [10, 11]. Price's model has been generalized to illustrate other properties of citation networks in various contexts [12–16]. Meanwhile, although coauthorship networks receive far less attention than citation networks [17], there are still a range of models working on the elementary mechanism for the emergences of scale-free, small-world and degree-assortativity [18–25]. Each of those models has addressed Matthew effect.

The behaviors of collaboration and citation are coupled and coevolutionary. Modelling the coevolution has its own meaning, because some phenomena can be studied only by combining citation and coauthorship data, such as the distribution of citations per author, self-/coauthor-/feedback-citation (a researcher cites others, who have previously cited them) [26, 27].

There has been relatively little work on modelling the coevolution of citation and coauthorship networks so as to simulate their simultaneous growth and dynamic interactions. The first model (perhaps the only one according to our present knowledge) is named TARL (topics, aging, and recursive linking), which successfully reproduces a power law distribution of citations per paper [28]. The model novelly introduces "topics", which enables the simulated citation networks to obtain positive and tunable clustering coefficients. The model concentrates on the citation patterns, and has validated the properties of the citation network against a dataset of papers published on the Proceedings of the National Academy of Sciences (PNAS) 1982-2001.

To the ground, the behaviors of collaborations and citations are due to the self-organization decisions made by authors: "yes" or "no". For example, the event that whether a paper cites another paper can be treated as a "yes/no" decision. Meanwhile, Matthew effect "is at the heart of self-organization across social and natural sciences" [5]. A geometric graph is proposed to model those decisions and their Matthew effect. The present model is built on a cluster of concentric circles, where each circle has a

time stamp. The modelled decisions synthetically consider two factors of generating collaborations and citations, namely the homophily (in sense of research interests, topics, etc.) and the academic impacts (with Matthew effect) of authors and papers. The model codes are available upon requests of readers.

The model is validated against a dataset of papers published on PNAS 2007-2015. The reasonability of the model is verified by the respectable model-data matching in a range of topological and statistical features of the empirical citation and coauthorship networks simultaneously, such as the distribution of collaborators/citations per author, and that of citations per paper. Those distributions emerge two limits, namely a generalized Poisson and a power-law in small and large variable regions respectively. There exists a cross-over between the two limits. Our model successfully reproduces the shapes of those distributions, and reveals how the decisions of nodes in networks generate the emerged limits. In addition, the data PNAS 2007-2015 evidences the positive correlation between each two of the three author-indexes, namely numbers of papers, citations and collaborators, which are also captured by our model.

This paper is organized as follows: the model and data are described in Sections 2 and 3 respectively; the distributions of specific indexes per author, the correlation between each two of the three authorindexes are analyzed in Sections 4 and 5 respectively; and the conclusion is drawn in Section 6.

## 2 The model

The presented model adopts the viewpoint from research-teams in focusing on the roles of research-teams performing in the production and dissemination of knowledge. Each author is assigned research-teams, each paper is written by a group of authors called paper-team (which is called article team in Reference [24]), and the members of a paper-team usually come from the same research-team. The model simultaneously generates two graphs interacting to each other, namely a hypergraph and a directed acyclic graph, to simulate the coevolution process of coauthorship and citation networks.

The model is built on a cluster of concentric circles  $S_t^1$ , t = 1, ..., T. For the sake of simplicity, the number of modelled "authors" is supposed to linearly grow over time t. The parameter t can be regarded as the t-th unit of time. Denote the i-th author-node by  $a(\theta_i, t_i)$ , the i-th paper-node by  $p(\theta_i, t_i)$ , where  $(\theta_i, t_i)$  is its spatio-temporal coordinate.

Select a small fraction of author-nodes as "leaders" of research-teams to attach zones to express their academic impacts (called influential zones). For each leader node, its research team is formed by the nodes

within the influential zone of the leader. All paper-nodes are attached influential zones. The influential zones are designed to express the ability of capturing academic communities' response to authors and papers. This design is built on the perception that counting citations can offer a quantitative proxy of eye-catching ability. Note that counting citations is not a measure of the novelty and importance, which are impossible to be measured objectively. The detail of zones is given as follows.

Let constants  $\alpha_l > 0$  and  $\beta_l \in [0.5, 1]$  (l = 1, 2, 3). The zone of a leader  $a(\theta_i, t_i)$  is defined as an interval of angular coordinate with center  $\theta_i$  and arc-length  $\alpha_1 t_i^{-\beta_1} t^{\beta_1 - 1}$ . The definitions of paper-node-zones are the same as those of leaders except parameters  $(\alpha_2, \beta_2)$  for the papers written by leaders as "the first authors" and  $\alpha_3, \beta_3$  for the papers written by non-leaders as "the first authors"). The zonal sizes are all required to be less than  $2\pi$ . The reason of choosing boundless circles is that we do not need to consider spacial boundary effects on influential zones.

The power-law factors  $t_i^{-\beta_l}$  (l=1,2,3) in influential zones induce the scale-free property of synthetic networks (see Appendix Eq. 2). The parameters of zones are used to tune synthetic data to fit the empirical distributions of collaborators per author and those of citations per paper. The formula of zones is dependent on t. If running the model with different initial time (which is equal to choose different  $\alpha_l$ ), the result will be different.

In the empirical data, the distributions of paper-team-sizes and references per paper appear two common features, namely hook heads and fat tails, which can be sufficiently fitted by generalized Poisson and power-law distributions respectively (which can be seen in following Fig. 2a, Fig. 4i). For such kind of distributions, we denote their probability density function (PDF) by f(x),  $x \in \mathbb{Z}^+$ .

After above preparations, we introduce the model as follows, where the constants  $N_1$ ,  $N_2$ ,  $N_3$ ,  $T \in \mathbb{Z}^+$ ,  $p \in [0,1]$ , and the paper-team-sizes and reference-lengths of paper-nodes are drawn from specific PDFs with form f(x).

#### 1. Generate a coauthorship network

For time t = 1, 2, ..., T do:

1.a. Sprinkle  $N_1$  author-nodes as potential authors uniformly and randomly on a circle  $S_t^1$ , and select  $N_2$  nodes randomly from the new nodes as leaders.

1.b. For each new node i, search the existing leaders whose zones cover i, and join those leaders' research team. For each such leader j, under probability p (p = 1 if i also is a leader), generate a

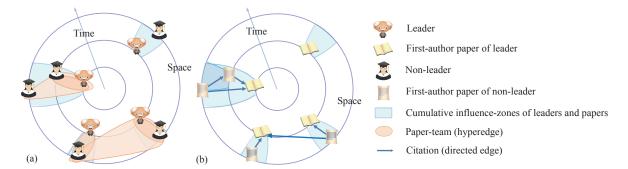


Figure 1. Illustration of the model. Leaders and papers have influential zones, the sizes of which are all required to be less than  $2\pi$ . The rules of modelled authors publishing and citing papers are outlined in the model. Those behaviors happen with in zones locally in most cases, and across zones in few cases. The coordinates of papers are the same as those of their first authors.

paper-team with size m by grouping together i, j and m-2 research-team-members of j nearest to i, where m is a random variable of a given f(x) or the research team size of j plus one if the former is larger than the latter.

1.c. Select  $N_3$  nodes with non-zero degree randomly from all existing nodes, and generate a paper-team with size m for each selected node l by grouping together l and m-1 randomly selected nodes with the same degree of l, where m is a random variable of a given f(x) or the number of nodes with the degree of l if the former is larger than the latter.

#### 2. Generate a citation network

For time t = 1, 2, ..., T do:

- 2.a. Each paper-team publishes a paper, where "the first author" is the new author-node in Step 1.b or the author-node selected in Step 1.c. The spatio-temporal coordinates of paper-nodes are those of their first authors.
- 2.b. Each new paper cites the existing papers that have zones covering it, and then cites other existing papers uniformly at random (with no replacement) until the length of its references (out-degrees) equals a random variable of a given f(x).

Note that in the publication version of this work (Scientometrics, 2017, 112: 483-507), there is expression error in Step 1.b, where "neighbor" should be replaced as "research-team-member".

The homophily of citations and coauthorship based on topics and research interests is a reason for the clear community structures in empirical data. In fact, with the development of sciences, researchers cannot

understand all of the sciences, but focus on their special fields, collaborate with some of the researchers in the same research-team, and cite some of the papers in their fields. This part of connections is imitated by Step 1.b and the first half of Step 2.b, which contributes to the main part of connections in our model.

The parameters of f(k) in Steps 1.b, 1.c and 2.b are not required to be the same. The f(k) in Steps 1.b and 1.c tunes the distribution of paper-team sizes. The f(k) in Step 2.b tunes the distribution of references per paper. Parameters  $\alpha_1$  and  $\beta_1$  tune the distribution of collaborators per author. Parameters  $\alpha_l$  and  $\beta_l$  (l=2,3) tune the distribution of citations per paper. There is no parameter directly tunes the distribution of citations per author. By far, we tune those parameters manually to fit empirical data. Firstly, we tune the parameters in Step 1 to fit the features of coauthorship data, then tune the parameters in Step 2 to fit the features of citation data. Lastly, we fine tune the chosen parameters with an optimal trade-off between model-date fits for the features of citation and coauthorship data separately and combined (e. g. distribution of citations per author, self-citation rate). In the future, we will try to find an effective way to automatically learn parameters from empirical data.

If the authors only collaborate with the authors with the same research interests, and papers only cite the papers with the same topics, the networks of citations and coauthorship are highly clustered, but would not have giant components and the small-world property. However, it is against the empirical data, a possible reason of which is the existence of a small fraction of cross-disciplinary studies [29]. Authors collaborating across research-teams would result in the connections of papers from different topics even disciplines via citations. Step 1.c and the second half of Step 2.b are designed to imitate the cross-disciplines phenomenon, which make the modeled networks have the small-world property and giant components. In addition, researchers can belong to different research-teams at different time, which is also equivalently imitated by Step 1.c to some extents.

The model is partly based on our previous results. The first part of the model is a generalization of the coauthorship model in Reference [30]. The parameter p in Step 1.b is newly introduced to tune the average number of papers per author. The second part of the model comes from the citation models in References [31,32], but with modifications, such as the increment of nodes, the connection rules, etc.

The innovation of the model here is mapping the coordinates of paper-nodes to those of their first authors, which connects papers to authors and makes the networks of them grow simultaneously and interactively (Fig. 1). The combination makes the new provided model not only reproduce a range of statistical features of empirical coauthorship and citation networks (which have captured by our previous

models [30,32]), but also predict some features generated by the interactions between coauthorship and citations, e.g. the distribution of citations per author, self-citation, etc.

There are some oversimple assumptions in our model. (a) The linear growth of authors does not hold in reality. If changing it, the formula of zone sizes should be changed to capture features of empirical data. (b) To express degree assortativity (authors prefer to collaborate with other authors with similar degrees), nodes with the same degree (the number of collaborators) are grouped in Step 1.c. Actually, no author does that surely when choosing coauthors. (c) The authorship order in Step 2.a does not consider heterogeneous conventions on different scientific fields.

### 3 The data

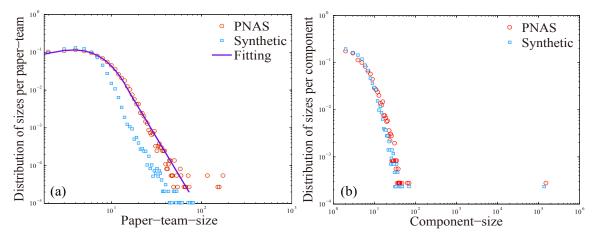
The information of citations and collaborations in 36,732 papers published on PNAS during 2007–2015 is utilized to validate the model in terms of major statistical properties. The empirical data are gathered from the Web of Science (http://www.webofscience.com). Authors are identified by their names on their papers. For example, the author named "Michael R. Strand" on his paper is represented by the name per se. Another way of identifying authors is using surnames and all initials of authors (e. g. represent "Michael R. Strand" by "Strand, MR"). Compared with using surnames and all initials, using names on papers is a reliable way to distinguish authors from one another in most cases. However, it mistakes one author as two if the author changes his/her name in different papers, and two authors as one if they have the same name. Those deficiencies will cause the inaccurate of expressing real situations, such as the number of authors, the number of components, clustering coefficient, etc.

Several references say that the inaccurate (caused by using surnames and all initials) does not much affect certain research findings, such as the distribution type of collaborators per author, and that type of papers per author [17,27,36–38]. Reference [39] analyzes the papers in PNAS 2012, and shows the difference between the distribution of collaborators per the author identified by using surnames together with all initials and that by a proxy of ground-truth. Those distributions have the same type approximately. Here we mainly focus on those distributions per se and some properties based on them. We use names on papers to identify authors, because it seems more reliable (Table 1). In elsewhere, we will show the same mathematical laws (i.e. types) underlying the considered distributions for the authors identified by both methods respectively, and analyze why this happens.

Table 1. Top ten authors (identified by using surnames together with all initials and by using names on papers respectively) according to the number of papers/collaborators.

Papers	Zhang Y 137, Wang J 122, Wang Y 117, Liu Y 112, Li Y 104, Liu J 95, Zhang J 90,
	Kim J 89, Chen J 80, Li J 80
Collaborators	Wang Y 1313, Zhang Y 1297, Wang J 1165, Liu Y 984, Liu J 856, Li Y 816,
	Zhang J 762, Chen Y 720, Li J 695, Kim J 688.
Papers	Nair Prashant 62, Croce Carlo M 61, Wang Wei 52, Mak Tak W 49, Onuchic Jose N 45,
	Schultz Peter G 44, Ayala Francisco J 41, Langer Robert 41, Weissman Irving L 41,
	Flavell Richard A 40
Collaborators	Wang Jun 527, Croce Carlo M 482, Wang Wei 383, Langer Robert 355, Schultz Peter G 352,
	Henrissat Bernard 350, Wang Tao 331, Wang Ying 329, Mak Tak W 328, Zhang Yan 323

Those data come from PNAS 2007-2015.



**Figure 2.** The distributions of sizes per paper-team, and those per component. Panels (a, b) show those distributions for the empirical data PNAS 2007-2015 (red circles) and the synthetic data generated by our model with the parameters in Table 2 (blue squares). The fitting is a mixture of generalized Poisson and power-law distributions, the parameters of which are listed in Table 5.

Synthetic data are generated by the model to compare with the empirical data. The model parameters are listed in Table 2. Denote the PDFs of generalized Poisson and power-law by  $f_1(x) = a(a+bx)^{x-1}e^{-a-bx}/x!$  and  $f_2(x) = cx^{-d}$  respectively, where  $a, b, c, d \in \mathbb{R}^+$  and  $x \in \mathbb{Z}^+$ . Generate random variables of a distribution f(x) with head  $f_1(x)$  and tail  $f_2(x)$  by sampling random variables of  $f_1(x)$  and  $f_2(x)$  with probability q and 1-q respectively. In order to make f(x) smooth and capture the empirical features at certain levels (Fig. 2a), we have tried many times to find proper q and  $f_i(x)$ 's domain  $I_i$ , i = 1, 2 for each step.

In PNAS 2007-2015, the maximum collaborators per author and maximum citations per paper are not very large. So the values of  $\alpha_l N_1$  (l = 1, 2, 3) should be also not very large (Table 2), which leads the probability of zone-overlapping is small. The paper-teams within and between research-teams are

modelled by Steps 2.b and 2.c respectively. In reality, the number of paper-teams within a research-team is far more than that between research-teams. So  $N_1$  is set to be far larger than  $N_3$ .

Table 2. The parameters of the synthetic data.

Network sizes	T = 4,500	$N_1 = 100$	$N_2 = 20$	$N_3 = 1$	p = 0.13		
Influential zones	$\alpha_1 = 0.133$	$\alpha_2 = 0.082$	$\alpha_3 = 0.02$	$\beta_1 = 0.42$	$\beta_2 = 0.44$	$\beta_3 = 0.44$	
f(k) in Step 1.b	a = 6	b = 0	c = 15,507	d = -3.70	q = 0.968	$I_1 = [2, 250]$	$I_2 = [11, 150]$
f(k) in Step 1.c	a = 6	b = 0			q = 1	$I_1 = [2, 250]$	
f(k) in Step 2.b	a = 1.35	b = 0.45	c = 19,424	d = -3.95	q = 0.9999	$I_1 = [0, 65]$	$I_2 = [20, 65]$

In order to make the synthetic data capture the empirical indicators listed in Table 3 at certain levels, we have attempted many times to find above parameters. Since the model is stochastic, we run 20 times with the same parameters, and find that the model is robust on those indicators (Table 4).

Table 3. Typical statistic indicators of the analyzed data.

Network	NN	NE	GCC	AP	MO	PG	NG	AC	SC	SC2
PNAS-Citation	36,732	42,932	0.263	12.586	0.922	0.651	9,145	0.287	0.312	0.399
Synthetic-Citation	95,993	133,318	0.136	8.034	0.713	0.807	16,128	0.280	0.231	0.388
PNAS-Coauthorship	162,531	1,074,836	0.896	6.662	0.909	0.844	5,013	0.554		
Synthetic-Coauthorship	175,699	886,140	0.825	6.808	0.948	0.864	3,578	0.164		

The indicators are the numbers of nodes (NN) and edges (NE), global clustering coefficient (GCC), average shortest path length (AP), modularity (MO), the node proportion of the giant component (PG), the number of components (NG), the assortativity coefficient of degrees for coauthorship networks and that of in-degrees for citation networks (AC), the proportion of self-citations (SC) and that of citations by collaborators (SC2). The AP of the last two networks is calculated by sampling 15,000 pairs of nodes respectively.

Table 4. Specific indicators' mean and standard deviation (SD).

Synthetic	NN	NE	AC	GCC	PG	NG	MO	AP	SC	SC2
Citation										
Mean	9.6E + 04	1.3E + 05	2.7E - 01	1.3E - 01	8.1E - 01	6.8E + 00	7.1E - 01	8.2E + 00	2.3E - 01	3.9E - 01
SD	3.4E + 02	5.3E + 02	8.9E - 03	1.1E - 03	1.8E - 03	1.8E + 02	1.4E - 03	1.2E - 01	1.2E - 03	2.7E - 03
Coauthorship										
Mean	1.8E + 05	8.7E + 05	1.7E - 01	8.2E - 01	8.6E - 01	3.5E + 03	9.5E - 01	6.8E + 00		
SD	8.0E + 02	8.7E + 03	$2.7\mathrm{E}{-02}$	8.1E - 04	2.7E - 03	4.5E + 01	6.5E - 04	$2.1E{-01}$		

The meanings of headers are shown in the notes of Table 3.

Besides the statistical features in next sections, our model captures three topological features of the empirical data (Table 3), namely giant component (Fig. 2b), clear community structure (high modularity) and small world (average shortest path length  $\sim$  log(the number of nodes), positive global clustering coefficient). In the model, the nodes in the same research-team probably belong to the same community. Setting  $N_1 \gg N_3$  makes edges within research-teams are significantly more than those between research-teams, and consequently makes the synthetic networks have a clear community structure. The positive

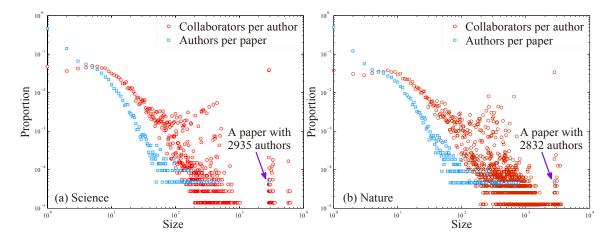


Figure 3. The distributions of collaborators per author and those of authors per paper-team. The data in Panels (a, b) are extracted from the papers published on Science and Nature during 2007-2015 respectively, which are gathered from Web of Science. The authors are identified by their names on papers.

global clustering coefficient due to the homophily in the sense of geometric distances. The small average shortest path length is caused by the random connections generated in Step 1.c and in the second half of Step 2.b.

More than 80% papers of PNAS 2007-2015 belong to biological sciences. In Appendix, in order to show the model's flexibility, we fit another dataset in physical sciences. The model is unsuitable for the datasets having very large paper-team-sizes, e. g. the papers of Nature and Science published during 2002-2015. The papers with very many authors cause that many authors have many collaborators, and consequently the distributions of collaborators per author have no power-law tail (Fig. 3).

# 4 Modeling the distributions of specific indexes per author

The distribution of collaborators per author  $P_{AA}(k)$ , and the distribution of citations per author  $P_{PA}^-(k)$ /
per paper  $P_{PP}^-(k)$  (k > 0) share a common characteristic: a generalized Poisson head, a power-law
tail and a cross-over between them (the purple lines in Fig. 4). Note that the cases of k = 0 are
not considered, because solitary nodes do not affect network topology. The presented geometric model
provides a respectable model-data fit (the blue squares in Fig. 4). With some modifications, the analysis
and calculations in References [25, 30–33] can be employed to show how the model works, which are
described in Appendix. Here an intuitionistic explanation is given as follows.

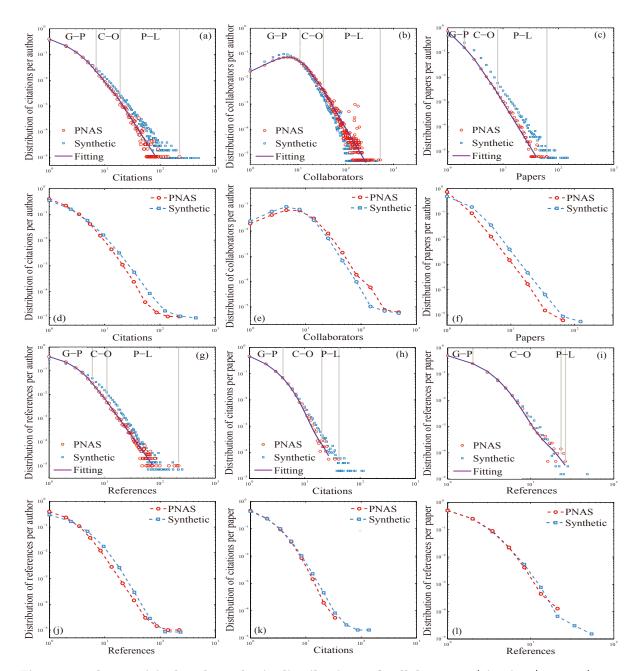


Figure 4. The empirical and synthetic distributions of collaborators/citation/papers/references per author, and those of citations/references per paper. The citations coming from or citing the papers (which are not contained in the empirical data) are not counted. In Panels (a-c, g-i), the regions "G-P", "C-O", "P-L" stand for generalized Poisson, cross-over and power-law respectively. The parameters and goodness of fittings are listed in Table 5. In Panels (d-f, j-l), the data are binned on abscissa axes to extract the trends hiding in noise tails.

Distribution	a	b	c	d	s	B	E	G	P	p-value	$\operatorname{Sum}$
Citations per author	0.810	0.465	27.88	3.337	1.747	7	19	6	30	0.548	0.998
References per author	0.960	0.384	12.29	3.245	1.584	6	11	10	15	0.395	1.003
Collaborators per author	4.916	0.463	294.1	3.213	0.858	11	34	12	45	0.082	1.004
Papers per author	0.703	0.387	21.79	3.821	1.968	2	8	10	25	0.038	0.995
Citations per paper	0.703	0.387	31.89	3.942	1.968	4	20	10	20	0.994	0.999
References per paper	0.768	0.276	23.48	4.131	1.856	2	23	12	15	0.873	1.005
Sizes per paper-team	3.657	0.402	532.4	3.989	1.194	15	16	10	20	0.188	1.000

Table 5. The parameters and goodness of fitting functions.

The ranges of generalized Poisson  $f_1(x)$ , cross-over, and power-law  $f_2(x)$  are [1, E], [B, E], and  $[B, \max(x)]$  respectively. The fitting function is  $f(x) = q(x)sf_1(x) + (1 - q(x))f_2(x)$ , where  $q(x) = e^{-(x-B)/(E-x)}$ . The fitting processes are: Obverse proper G and P; Calculate parameters of  $sf_1(x)$  (i.e. a, b, s) and  $f_2(x)$  (i.e. c, d) through regressing the empirical distribution in [1, G] and  $[P, \max(x)]$  respectively; Find B and E through exhaustion to make f(x) pass Kolmogorov-Smirnov test (p-value> 0.01). The p-value measures the goodness-of-fit. The sum of f(x) over  $[1, \max(x)]$  near-unity shows that f(x) can be regarded as a probability density function.

Behaviors of collaborations and citations are dependent on the choices of authors, the attractiveness of authors and papers. The choices can be simplified by "yes/no" decisions. Take citation behavior as an example. Treat the event that whether a paper cites another paper as a "yes/no" decision. Then the number of a paper's citations is the number of successes in a sequence of n decisions, where n is the number of the papers having willing to cite that paper. Approximate the probability p of "yes" by its expected value  $\hat{p}$ , and suppose those "yes/no" experiments are independent. Then, the number of a paper's citations will follow a binomial distribution  $B(n,\hat{p})$ . When n is large and  $\hat{p}$  is small,  $B(n,\hat{p})$  can be approximated by a Poisson distribution with mean  $n\hat{p}$ . An author could publish several papers. So the number of an author's citations is the sum of several random variables drawn from Poisson, which is still drawn from Poisson. Note that the result holds under above simplifying assumptions.

The "yes/no" experiments could be affected by previous occurrences, e. g. two papers written by the same authors highly probably share some references. Meanwhile, the values of p and n are not constant due to the diversity of attractive abilities of papers and authors. Hence it is reasonable to think the number of citations received by lowly cited authors and papers are drawn from generalized Poisson distributions, which relax the restrictions of Poisson distribution by allowing the probability of occurrence of a single event to affect by previous occurrences [35]. For highly cited papers (with large np) and authors, the numbers of citations are large enough to suppose the "yes/no" decisions are independent. So the number of citations could be considered as random variables drawn from a range of Poisson distributions with

sufficiently large expected values.

The diversity of attractive abilities of papers gives the possibility of existing papers with highly attractive abilities, and then guarantees the relative commonness for the existence of papers getting citations that greatly exceed the average. The commonness appears as the emergence of citation distributions' fat tails. The diversity of attractive abilities induces various probabilities of "yes" p and various numbers of potential citations n. In the model, those are generated by the power-law factor in the formula of influential zones' size, namely  $t_i^{-\beta_l}$ , where l=1,2,3, and i refers to specific node. The factor gives a sufficient diversity for the attractive abilities of nodes, and consequently derives the scale-free property of modelled networks. Eq. 4 in Appendix shows how it works, which illustrates the derivation of power-law by averaging a range of Poisson distributions with expected values proportional to  $s^{-\beta}$  (s=1,...,T,  $T \in \mathbb{Z}^+$ ). Actually, in systems science, diversity is often regarded to be a reason for complexity, and scale-free is one of the symbols of complexity [34].

Note that there exists a difference between BA model [18] and our model. In BA model, nodes make decisions to link previous nodes based on understanding of all nodes' degrees. In our model, most decisions made by nodes are local behaviors, which are restricted by geometric locations (Fig. 1). Meanwhile, a few decisions are made randomly, which are modelled by Step 1.c and the second half of Step 2.b. In reality, authors make decisions based on their knowledge. So the decisions are affected by the locality of authors' knowledge. Uncertainty is also in authors' decisions. In our model, the connection mechanism addresses the locality as well as the uncertainty.

In statistics, mixture distributions, e. g.  $P_{PA}^-(k)$ , mean samples come from different populations. In reality, the main part of the authors in scientific papers is composed of the teachers and students in institutes or universities, which can be treated as two populations. Research modes of students and teachers are different. Many students only write a few papers, and do not write after graduations, but their teachers could continuously write papers collaborating with new students or researchers, and so persistently receive citations. Meanwhile, due to the aging of papers, the citations received by students cannot persistently increase on average.

In our model, the leaders are designed to play the role as teachers, and other team members as students. Note that the distribution  $P_{PA}^-(k)$  of the empirical data is not perfectly fitted by that of the synthetic data. However, the respectable model-data fit after data binning is still impressive to us because the model involves no true free parameters to tune  $P_{PA}^-(k)$ . It also confirms the reasonability of the above

analysis. In addition, a similar analysis has been applied to  $P_{AA}(k)$  [30].

The smoothness of  $P_{PP}^-(k)$  and  $P_{PA}^-(k)$  does not appear in  $P_{AA}(k)$ . In reality, the papers of an author and the citations of a paper increase smoothly over time. However, a paper with very many authors can make those authors' collaborators increase rapidly.

# 5 Modeling the Matthew effect in academic societies

From the social viewpoint, Matthew effect (the rich get richer) naturally exists in academic fields. Authors with many citations and papers can improve their chances of attracting collaborators, especially outstanding students. Consequently, those authors may write more high-quality papers and increase their chances of receiving citations. So the indexes of authors, namely numbers of collaborators, citations and papers, improve mutually and form a positive feedback loop, which is evidenced partially by the positive correlations between those indexes (Fig. 5). Funding plays a part as an activator in promoting the feedback loop and Matthew effect [28]. The authors with voluminous papers and citations easily obtain funding, which in return enables those authors to obtain more collaborators (especially postdocs and visiting scholars), citations, papers and hence more funding.

Matthew effect is often regarded as an explanation for the scale-free properties of many real networks. Here we reverse the question: Does there exist any Matthew effect in the developments of the authors with a kind of the aforementioned indexes following power-laws? With the information of papers' publication time, such Matthew effect can be observed directly as follows.

For each kind of indexes, split authors into two non-overlapped parts by the algorithm in Reference [30] (Table 6 in Appendix), namely a generalized Poisson part and a power-law part. The indexes' cumulative values (over time) of the authors in the two parts follow generalized Poisson and power-law distributions respectively, which are proved by the Kolmogorov-Smirnov tests for the fittings in Fig. 4 (a-c). The indexes of the authors in the generalized Poisson part grow quite slowly on average, meanwhile, those in the power-law part grow fast, even in accelerated ways for citations (Fig. 6). It evidences that the phenomena "the rich getting richer" has existed in academic societies.

The Matthew effects about the aforementioned indexes are coupled due to the positive feedback loop and positive correlations between those indexes. Our model gives a geometrical expression of those coupling Matthew effects. Older leaders have a larger influential zone, so more easily capture collaborators,

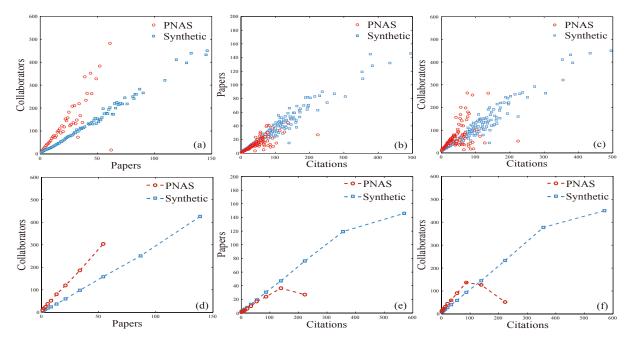


Figure 5. The positive correlations between three author-indexes: numbers of papers, citations and collaborators. In Panels (a-c), each point (x, y) means an author published x papers has y collaborators on average. In Panels (d-f), the data are binned on abscissa axes to improve visibility for the positive slopes.

and then publish more papers to receive more citations. Therefore, our model successfully reproduces the power-law tails in the distributions of those indexes (Fig. 4a-c), and positive correlations between those indexes (the positive slops in Fig. 5). In essentially, the Matthew effect in our model is a cumulative advantage on time.

Matthew effect will lead to "the strongest taking over" in academic societies, which has emerged in the empirical data (Fig. 7). There exist 5.99% (9,742/162,532) authors (called hub-authors), whose numbers of collaborators (more than 32) follow a power-law distribution, published 42.6% papers and received 49.3% citations. The hub-authors and their collaborators obtain nearly 80% papers and citations. In addition, the topological relationships between hub-authors are very close. The proportion of a hub-author having coauthored with another hub-author is 99.5% (9742/9697).

For each hub-author, we calculate the gap between the first appearance time of him/her and each of his/her hub-author-collaborators, and record the maximum gap. The mean of those maximum gaps is 4.180 years. Replace the condition that the number of collaborators is more than 32 by that the number of citations is more than 28 (the number of papers is more than 14). Then the proportion of

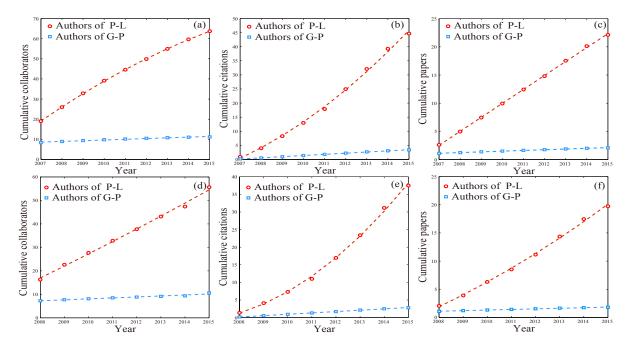


Figure 6. The Matthew effects about three author-indexes: numbers of papers, citations and collaborators. Authors of G-P and P-L are the subsets of the authors, whose indexes follow a generalized Poisson and power-law distribution respectively. The points (33/29/15) splitting the distributions of collaborators/citations/papers per author into G-P and P-L are detected by the algorithm in Table 6. The data in panels (a-c) and panels (e-f) are about the authors first appearing in 2007 and 2008 respectively. The fitting curves are quadratics.

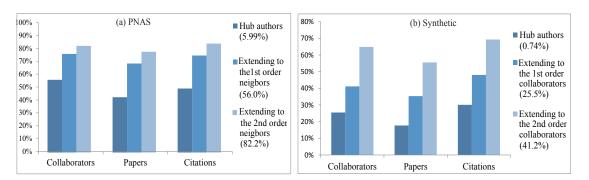


Figure 7. The proportions of papers, citations, collaborators obtained by hub-authors and their collaborators. Hub-authors are those with collaborators more than the cut-off points detected by the algorithm in Table 6 (55 for PNAS and 56 for the synthetic data).

an author satisfying the condition having coauthored with another author also satisfying the condition is 352/401 = 87.8% (243/285 = 85.3%), and the mean of the maximum time-gaps is 1.104 (1.203) years. Those evidence that Matthew effect has been inherited, and works on research-teams not just on individuals.

From the viewpoint of systems science, "the strongest taking over" will suppress system diversity, and consequently harms system flexibility. Hence there exist two critical strategic problems for research administrators: How to reform the existing academic evaluation methods and funding mechanisms, which are mainly oriented by specific indexes with Matthew effect, e. g. the number of citations; How to design a reasonable inspiriting mechanism of scientific research to protect academic diversity.

Some literatures discussed the reason and consequences of Matthew effects in academic society. Reference [40] says Matthew effects are expected if scientific community competes for the same attention device, e. g. a top journal. However, Matthew effects could probably erode the competition, so have negative implications on innovation [41]. For example, the most competitive journals of medical sciences rejected some important articles. As Reference [41] says, "tolerating early failure and focusing on long-term success could be a fine way to guarantee high level innovations".

## 6 Conclusion

The presented model adopts a viewpoint in focusing on the coevolution of collaborations and citations within scientific research, and particular in reproducing statistical and topological features generated by the interactions between collaborations and citations. The good model-data fitting shows the reasonability of the model. Especially, the model provides a respectable prediction of the empirical distribution of citations per author. The model potentially paves a geometric way to figure out the interactional modes of collaborations and citations. For example, it explains the emergence of generalized Poisson and Power law in the distribution of citations per author by the different collaboration modes of teachers and students.

Some shortcomings of the model are indicative of the need for further research: The increment of new nodes should not be fixed; More reasonable expressions are needed for the academic communications between research-teams. We are especially interested in the self-similarity emerged in the empirical data, such as the distribution of citations per paper, and that of collaborators per author. We also should

discuss the role of the coevolution in the formation and evolution of academic communities.

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# 7 Appendix

#### 7.1 Detecting boundary for probability density functions.

The boundary detection algorithm for probability density functions (PDF) is listed in Table 6, which comes from Reference [30].

Table 6. A boundary detection algorithm for PDF

Input: Observations  $D_s$ , s=1,...,n, rescaling function  $g(\cdot)$ , fitting model  $h(\cdot)$ .

For k from 1 to  $\max(D_1,...,D_n)$  do:

Fit  $h(\cdot)$  to the PDF  $h_0(\cdot)$  of  $\{D_s,s=1,...,n|D_s\leq k\}$  by maximum-likelihood estimation;

Do Kolmogorov-Smirnov (KS) test for two data g(h(t)) and  $g(h_0(t))$ , t=1,...,k with the null hypothesis they coming from the same continuous distribution;

Break if the test rejects the null hypothesis at significance level 5%.

Output: The current k as the boundary point.

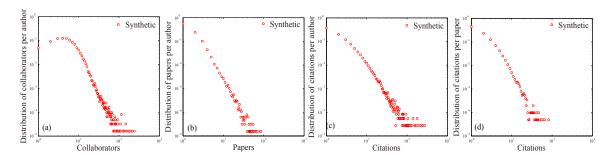


Figure 8. The synthetic distribution of collaborators/citations/papers per author, and that of citations per paper. The model parameters are listed in Table 7.

#### 7.2 Simplifying the model

An obvious weakness of the provided model is that it has a lot of parameters. If ignoring the fitting of the distribution of references per paper and that of paper-team sizes, we can reduce the model's parameters as those in Table 7. The reduction does not affect the synthetic distribution type of collaborators/papers per author, and that of citations per paper/per author (Fig. 8).

Table 7. The parameters of the synthetic data.

Network sizes	T = 4,500	$N_1 = 100$	$N_2 = 20$	$N_3 = 1$	p = 0.13	
Influential zones	$\alpha_1 = 0.133$	$\alpha_2 = 0.082$	$\alpha_3 = 0.02$	$\beta_1 = 0.42$	$\beta_2 = 0.44$	$\beta_3 = 0.44$
f(k) in Step 1.b	a = 5	b = 0				
f(k) in Step 1.c	a = 5	b = 0				
f(k) in Step 2.b	a = 1.35	b = 0.45				

#### 7.3 The underlying formula for the distribution of citations per paper

We only analyze the underlying formula for the distribution type of synthetic "citations per paper" (indegrees), which is similar to that in References [32,33]. The analysis of the formula for the distribution type of synthetic "collaborators per author" is the same as that in Reference [30]. As shown in Fig. 4c, the synthetic in-degree distribution type is a mixture of generalized Poisson and power-law, hence the formula is analyzed piecewise. The formula for the head and that for the tail of the type are deduced respectively. The cross-over can be well fitted by the formula in the notes of Table 5.

The in-degrees contributed by the second half of Step 2.b are due to a random selection. Together with the preset small domain of f(x) in this step, the effect of the second half on in-degree distribution

is small enough to be ignored, when compared with that contributed by the first half.

The first half of Step 2.b makes the expected in-degree of a node generated at time t to be  $k^-(t) \approx \alpha_l \delta p T^{\beta_l} t^{-\beta_l}/\beta_l - 1$ , where l = 2, 3 and  $\delta = N_1/2\pi$ . If t is large enough (suppose larger than a big number  $T_1$ ),  $k^-(t)$  is small enough, and changes slowly over t. Hence the formula for the head is

$$P_S(k) = \frac{1}{T - T_1 + 1} \sum_{t=T_1}^{T} \frac{k^-(t)^k}{k!} e^{-k^-(t)},$$
(1)

which is a mixture Poisson distribution. A generalized Poisson distribution can be well fitted by a mixture Poisson distribution, which can be verified numerically.

The formula for the tail is deduced as follows, where the calculations are inspired by some of the same general ideas as explored in the cosmological networks [42]:

$$P_{L}(k) = \frac{1}{T_{1}} \int_{1}^{T_{1}+1} \frac{k^{-}(t)^{k}}{k!} e^{-k^{-}(t)} dt \propto \frac{1}{k!} \int_{\delta\alpha_{l}(T_{1}+1)^{-\beta_{l}}pT^{\beta_{l}}/\beta_{l}}^{\delta\alpha_{l}pT^{\beta_{l}}/\beta_{l}} \tau^{k-1-\frac{1}{\beta_{l}}} e^{-\tau} d\tau$$

$$\approx \frac{1}{k!} \left( \frac{k-1-\frac{1}{\beta_{l}}}{e} \right)^{k-1-\frac{1}{\beta_{l}}} \int_{\delta\alpha_{l}(T_{1}+1)^{-\beta_{l}}pT^{\beta_{l}}/\beta_{l}}^{\delta\alpha_{l}pT^{\beta_{l}}/\beta_{l}} e^{-\frac{(\tau-k+1+\frac{1}{\beta_{l}})^{2}}{2(k-1-\frac{1}{\beta_{l}})}} d\tau$$

$$\approx \frac{\Gamma(k-\frac{1}{\beta_{l}})}{\Gamma(k+1)} \int_{\delta\alpha_{l}(T_{1}+1)^{-\beta_{l}}pT^{\beta_{l}}/\beta_{l}}^{\delta\alpha_{l}pT^{\beta_{l}}/\beta_{l}} \frac{e^{-\frac{(\tau-k+1+\frac{1}{\beta_{l}})^{2}}{2(k-1-\frac{1}{\beta_{l}})}}}{\sqrt{2\pi(k-1-\frac{1}{\beta_{l}})}} d\tau. \tag{2}$$

Here Laplace approximation is used in the third step, and Stirling's formula is used in the fourth step. When  $k \gg 0$ , the integration part in Eq. 2 is free of k approximatively, which can be verified as follows:

$$\frac{d}{dk} \int_{L_1}^{L_2} \frac{e^{-\frac{(\tau - k + \rho)^2}{2(k - \rho)}}}{\sqrt{2\pi(k - \rho)}} d\tau = \frac{e^{-\frac{(L_1 - k + \rho)^2}{2(k - \rho)}}}{2\sqrt{2\pi(k - \rho)}} \left(1 + \frac{L_1}{k - \rho}\right) - \frac{e^{-\frac{(L_2 - k + \rho)^2}{2(k - s)}}}{2\sqrt{2\pi(k - \rho)}} \left(1 + \frac{L_2}{k - \rho}\right), \tag{3}$$

where  $L_1 = \delta \alpha_l (T_1 + 1)^{-\beta_l} p T^{\beta_l} / \beta_l$ ,  $L_2 = \delta \alpha_l p T^{\beta_l} / \beta_l$ , and  $\rho = 1 + 1/\beta_l$ . This derivative is approximately equal to 0 for  $k \gg 0$ . Hence

$$P_L(k) \propto \frac{\Gamma(k - \frac{1}{\beta_l})}{\Gamma(k+1)} \approx \frac{1}{k^{1 + \frac{1}{\beta_l}}} \sqrt{\frac{k - \frac{1}{\beta_l} - 1}{k}} \left(1 - \frac{\frac{1}{\beta_l} + 1}{k}\right)^{k - \frac{1}{\beta_l} - 1} e^{\frac{1}{\beta_l} + 1} \approx \frac{1}{k^{1 + \frac{1}{\beta_l}}}.$$
 (4)

Stirling's formula is used in the first approximation. The second approximation holds for  $k \gg 0$ . Hence  $P_L(k)$  is approximately a power-law distribution with exponent  $1 + 1/\beta_l$ . So we obtain that the in-

degree distribution tail of the network generated in Step 2.b is a mixture of power-law distributions with exponents  $1 + 1/\beta_1$  and  $1 + 1/\beta_2$  respectively. Note that in Eq. 1, the condition  $k \gg 0$  does not hold, so the power-law does not emerge in the head of the distribution.

#### 7.4 Flexibility of the model

The provided model has the flexibility of fitting empirical data from different sciences. We have shown that the model can capture specific features of the empirical data PNAS 2007-2015, the papers of which mainly belong to biological sciences. Here we consider the data from physical sciences: the papers of Physical review E published during 2007-2016 (PRE 2007-2016). The data are gathered from the Web of Science. Authors are identified by their names on papers.

Synthetic data are generated through the provided model to capture specific features of PRE 2007-2016. The parameters of the synthetic data are listed in Table 8. Comparisons on statistic indicators and distributions are shown in Table 9 and Fig. 10 respectively.

Table 8. The parameters of the synthetic data.

Network sizes	T = 5,000	$N_1 = 15$	$N_2 = 3$	$N_3 = 1$	p = 0.3		
Influential zones	$\alpha_1 = 0.2$	$\alpha_2 = 0.082$	$\alpha_3 = 0.02$	$\beta_1 = 0.43$	$\beta_2 = 0.44$	$\beta_3 = 0.44$	
f(k) in Step 1.b	a = 2	b = 0	c = 1,855	d = -3.70	q = 0.995	$I_1 = [2, 250]$	$I_2 = [11, 150]$
f(k) in Step 1.c	a = 2	b = 0			q = 1	$I_1 = [2, 250]$	
f(k) in Step 2.b	a = 1.45	b = 0.45	c = 19,424	d = -3.95	q = 0.9999	$I_1 = [0, 65]$	$I_2 = [20, 65]$

Table 9. Typical statistic indicators of the data.

Network	NN	NE	GCC	AP	MO	PG	NG	AC	SC	SC2
PRE-Citation	24,079	46,061	0.301	8.714	0.892	0.746	4,623	0.272	0.317	0.391
Synthetic-Citation	$65,\!805$	89,787	0.053	7.225	0.676	0.795	12,169	0.094	0.135	0.290
PRE-Coauthorship	37,528	90,711	0.838	6.060	0.950	0.583	4,209	0.394		
Synthetic-Coauthorship	48,956	92,870	0.614	6.680	0.885	0.837	2,183	0.100		

The meanings of headers are the same as those in Table 3.

#### 7.5 TARL model

Constructively suggested by a reviewer, the result of TARL model is compared with that of the proposed model. The pseudo code of TARL model in Reference [28] is repeated as follows.

Initialization

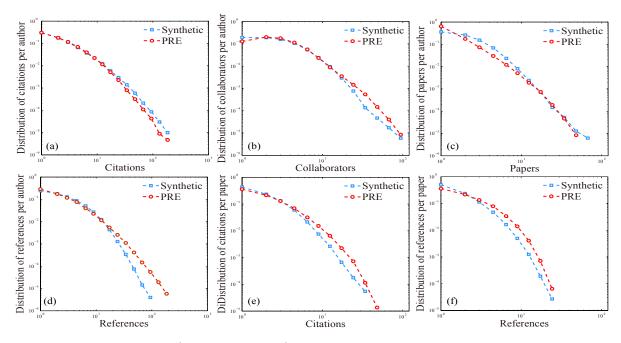


Figure 9. The empirical (PRE 2007-2016) and synthetic distributions of collaborators/citation/papers/ references per author, and those of citations/references per paper. The data are binned on abscissa axes to show the trends hiding in noise tails.

Generate m "papers" and n "authors" with randomly assigned "topics";

Randomly assign l "authors" to the "papers" within the same "topic".

For time t = 1, 2, ..., T do:

Add s new "authors" with randomly assigned "topics";

Deactivate the "authors" older than h;

For each "topic" do:

Randomly partition the "authors" within the "topic" into groups with size l;

For each group do:

Randomly read g "papers" from existing "papers" within the "topic";

"Select a time-slice form (1 to t-1) with probability given in aging-function" [28];

Generate a new "paper" and randomly cite k papers (published or cited in this time-slice) from the read "papers" and their references up to w-th level.

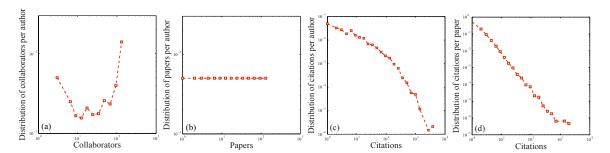


Figure 10. Synthetic distribution of collaborators/citations/papers per author, and that of citations per paper. Those distributions are generated through TARL model.

The generated connections are restricted to the "papers" and "authors" within the same "topic". If no aging-function is given, then all "papers" can be "read" equally. We set the number of "topics" to be 4, and no aging-function (so no time-slice). We let T=200, g=1, h=T, k=2, w=2, m=n=l=s=4. The generated distribution of "collaborators"/"papers" per "author" and that of "citations" per "paper"/per "author" are shown in Fig. 10.

TARL model can generate a "coauthorship" network and a "citation" network, which grow simultaneously. The "citation" network is scale-free (caused by recursive linking), and has a positive clustering coefficient (caused by citing the "papers" within the same "topic"). Our model harmoniously express the citation factors considered in TARL model (i. e. topics, aging and recursive follow-up of citation references) by the connection mechanism induced through the influential zones of "papers". The aging of papers is expressed by decreasing the sizes of influential zones over t. In TARL model, "papers" and "authors" are assigned specific "topics" directly. In our model, we use a continuous way: expressing nodes' "topic" by nodes' spacial coordinate. So the circles could be regarded as "topic spaces". Note that it is not a real topic space, which is a high dimensional space representing textual contents of papers. In our model, "papers" can incompletely "copy" the references of the "papers" it cited, which is induced through the overlapping of influential zones.

TARL model neither consider the Matthew effect on the number of authors' collaborators nor that on papers. In addition, the above instantiation assumes that the number of "papers" per "author" is a constant. Hence, the generated distribution of "papers" per "author" and that of "collaborators" per "author" (Fig. 10) have no power-law tails, which emerge in the corresponding distributions from real data (Fig. 4, Fig. 9). Our model expresses those Matthew effects geometrically: older leaders having a larger influential zone to obtain more "collaborators" and "papers". Therefore, our model can reproduce

those power-law tails.