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## Editorial Introduction. Truth Values: Part II

This issue is a continuation of the *Studia Logica* special issue on truth values Part I (see vol. 91, No. 3, 2009) and contains a series of contributions concentrating again on some central issues in the logical and philosophical analysis of truth values. The topics dealt with include Suszko's Thesis (the claim that there are but two logical values, the True and the False), logical many-valuedness, generalizations of bi-lattices of truth values, truth values as idealizations of basic proof-theoretical properties, truth values as entities codifying the relationship between language and the world, and the interpretation of truth values in Basic Fuzzy Logic. The following comments may serve as a brief guide to the papers of this special issue.

*Stefano Aguzzoli, Matteo Bianchi and Vincenzo Marra* in "A Temporal Semantics for Basic Logic" investigate the interpretation of truth values, alias truth degrees, in Petr Hájek's *Basic Fuzzy Logic*. It is shown that formulas in Basic Fuzzy Logic can be interpreted as modal formulas over appropriate flows of time, where the logic of each instant is Łukasiewicz's logic with a finite or infinite number of truth values. This reduction might be seen to contribute to a modal understanding of Basic Fuzzy Logic.

Closely related to questions concerning the nature and understanding of truth values in many-valued logics is the problem of motivating multiple-valued logic and the problem of finding a suitable methodology for it.

In the paper "Multi-valued Semantics: Why and How", *Arnon Avron* considers Roman Suszko's method used to obtain a two-valued semantics for any non-trivial structural Tarskian consequence relation defined on a propositional language. Suszko's reduction method provides the background for discussing the merits of many-valued non-deterministic matrices. The property of analyticity is highlighted as the main advantage of matrices-based semantics over the bivaluation semantics. Intuitively, analyticity means that only formulas which are subformulas of formulas occurring in an inference are relevant to the question whether the conclusion follows from the assumptions. Moreover, it is argued that in the context of many-valued non-deterministic matrices truth values are best be conceived of as information carriers.

Truth values are intimately associated with the general philosophical notions of truth and falsity and thus can play an important role in representing and studying some well-known semantic paradoxes, most notably the Liar Paradox.

The paper by *Roy T. Cook*, “What is a Truth Value And How Many Are There?”, focuses on a phenomenon known as the Revenge Problem in connection with the semantic paradoxes and self reference in general. The consideration of this problem in the paper rests essentially upon explicating truth values as a kind of “place-holders” for various relations between statements of our language and an extra-language reality (“the world”). By taking into account some additional – other than classical – semantic relations, holding between statements and the world, one is led to truth values other than the traditional, classical values the True and the False. The author claims that once truth values are understood in this way, consideration of the Liar paradox and the revenge problem shows that our language is indefinitely extensible, as is the class of truth values that statements of our language can take.

Suszko’s Thesis and the Suszko Reduction may be discussed not only from a practical, methodological perspective, but also from the point of view of non-standard consequence relations to which Suszko’s reduction to a bivalent semantics does not apply.

The point of departure in *Grzegorz Malinowski’s* paper “Beyond Three Inferential Values” is the notion of a  $q$ -consequence relation developed by Malinowski in the 1990’s in order to provide a counterexample to Suszko’s Thesis or, put differently, to replace the reduction to a bivalent semantics by a reduction to a trivalent semantics. The  $q$ -consequence relations to which this new reduction applies are still well-behaved, though not Tarskian. The problem of lifting the reduction method to an  $n > 3$  is raised, and Malinowski presents some partial solutions to this technically as well as conceptually intricate problem.

When Frege introduced truth values into logic, he was strongly motivated by a functional approach to language analysis. Accordingly, many of the known logical systems, classical logic in the first place, can be equipped with a truth-functional semantics, and consequently, are said to be truth-functional.

However, *João Marcos* pinpoints attention upon the well-known fact that some logics fail to have a truth-functional semantics, and raises

the question: “What is a Non-truth-functional Logic?” He provides examples of logics with truth-tabular semantics (genuinely finite-valued or infinite-valued) and logics with no truth-tabular semantics at all, and demonstrates how the corresponding semantically described classes of logics can be adequately characterized from an abstract point of view in terms of single-conclusion consequence relations. He also explores some possible extensions of the intuitive notion of truth-functionality.

The Fregean truth values the True and the False are abstract Platonic objects *par excellence*. However, there is room for truth values not only in the treatment of logics which are associated with a Platonistic ontology. One may even try to develop an understanding of truth values from the anti-realistic point of view of inferentialism.

*Greg Restall's* paper “Truth Values and Proof Theory” presents an account of truth values for classical logic, intuitionistic logic and the modal logic *S5* as arising out of an inferentialist interpretation of sequent calculi. These calculi are seen as supplying a normative constraint on acts of assertion and denial. From this perspective, existing semantic structures can be understood in terms of a more basic notion of logical consequence.

Truth values can be generalized in many ways. One such generalization strategy proposed by Nuel Belnap suggests a reinterpretation of the very notion of a truth value as information received from different sources. The resulting semantic constructions are often presented as combinations of classical truth values.

*Dmitry Zaitsev* in “A Few More Useful 8-valued Logics for Reasoning with Tetralattice *EIGHT*<sub>4</sub>” suggests to take as a basis for a Belnap-style generalization not the set of classical truth values, but the truth values of Kleene's three-valued “logic of uncertainty”. Whereas Belnap's four generalized truth values form a specific algebraic structure called a “bilattice” (with information and logical orderings), the eight truth values obtained by generalizing Kleene's logic constitute a tetralattice, where elements are ordered by information, truth, falsity and uncertainty. Zaitsev explores some features of this algebraic structure and formulates some logics, which correspond to the truth order, the falsity order and their combination.

Like Part I, the second part of this special issue of *Studia Logica* is based on submissions received after a call for papers and on some invited

lectures presented at the International Workshop on Truth Values, held from May 29 – June 1, 2008, at the Institute of Philosophy, Dresden University of Technology, Germany, see (<http://www.truthvalues2008.com/>). The speakers at the workshop were: Arnon Avron (Tel Aviv), Nuel Belnap (Pittsburgh), Jean-Yves Béziau (Neuchâtel), Walter Carnielli (São Paulo), Michael Dunn (Bloomington), Melvin Fitting (New York), Siegfried Gottwald (Leipzig), Petr Hájek (Prague), Norihiro Kamide (Tokyo), Beata Konikowska (Warsaw), Saul Kripke (New York), Grzegorz Malinowski (Łódź), Sergei Odintsov (Novosibirsk), Yaroslav Shramko (Krivoi Rog), and Heinrich Wansing (Dresden).

Altogether we received 21 submissions; the paper by Dmitry Zaitsev has been contributed at a later stage. Each contribution underwent a strict anonymous refereeing, and we would like to thank the referees for their excellent work and assistance. From the invited workshop presentations, the paper by Norihiro Kamide and the joint paper by Beata Konikowska and Arnon Avron have appeared in a regular issue of *Studia Logica* (see vol. 91, No. 2, 2009). The workshop in Dresden was generously supported by The German Research Council (DFG), grant WA 936/6-1, by the German Society for Analytic Philosophy (GAP) and by the Gesellschaft von Freunden und Förderern der Technischen Universität Dresden. We gratefully acknowledge this sponsorship. Moreover, we would like to thank Jacek Malinowski for supporting the idea of a *Studia Logica* special issue devoted to truth values.

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