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Probabilistic Coverage in Mobile Directional Sensor Networks: A Game Theoretical Approach

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Abstract
Directional sensor nodes deployment is indispensable to a large number of applications including Internet of Things applications. Nowadays, with the recent advances in robotic technology, directional sensor nodes mounted on mobile robots can move toward the appropriate locations. Considering the probabilistic sensing model along with the mobility and motility of directional sensor nodes, area coverage in such a network is more complicated than in a static sensor network. In this paper, we investigate the problem of self-deployment and working direction adjustment in directional sensor networks in order to maximize the covered area. Considering the tradeoff between energy consumption and coverage quality, we formulate this problem as a finite strategic game. Then, we present a distributed payoff-based learning algorithm to achieve Nash equilibrium. The simulation results demonstrate the performance of the proposed algorithm and its superiority over previous approaches in terms of increasing the area coverage.

Keywords: *directional sensor networks, mobile sensor networks, area coverage, probabilistic sensing model, payoff-based learning algorithm.*

1. Introduction

Mobile directional sensor networks (MDSNs) consist of directional sensor nodes which can move and rotate on their own and interact with the physical environment. A directional sensor, such as a video sensor, infrared sensor, and ultrasound sensor, is capable of adjusting its working direction and sensing an angular area at each unit of time. Such networks enable a variety of applications in industry and also our daily life, i.e. in IoT, sensor nodes collect information from the environment and send it to the sink through the wireless network [1]. Therefore, area coverage is an important and challenging problem due to the energy constraint of sensor nodes [2-4].

Most of the studies on area coverage problems in DSNs adopted the binary sensing model in which the sensing region is a deterministic sector that is a coarse approximation to sensing region in reality. In this model, an event is detectable by the sensor if and only if it falls into its covered sector. Whereas in reality the probability of event detection decreases as its distance from the sensor increases, so the sensing model of a sensor node is practically probabilistic [5-7]. In [6], the authors have shown through experimental study that the normal distribution is reasonable for modeling the sensing range of sensor nodes. The authors in [7] have considered an exponentially reducible sensing range for sensor nodes. In this model, the sensing capacity is exponentially reduced by increasing the distance between the points and the sensor nodes.

In this paper, we propose two algorithms, namely binary coverage based on game theory (BCGT) and probabilistic coverage based on game theory (PCGT). We model the area coverage problem as a finite strategic game in which the utility function is designed to capture the tradeoff between the amount of covered area and the energy consumption due to movement and rotation. Then, we propose a learning algorithm to solve this problem based on the log-linear learning algorithm proposed in [8]. It is proved that in this algorithm each sensor as a player finally selects the action profile that maximizes the total payoff.

To the best of our knowledge, this is the first work that employs probabilistic sensing model for mobile directional sensor networks. The contributions of this work are as follows:

- We formulate the problem of determining the direction and location of directional sensor nodes in order to maximize coverage for both binary and probabilistic sensing models as a multiplayer repeated game in which each sensor as a player try to maximize its utility function. The utility function is designed to capture the tradeoff between the worth of covered area and the energy consumption due to movement and rotation.
- Then we prove that the proposed game is an exact potential game and its potential function is equivalent to covering the area with the least energy consumption to maximize network lifetime.
- We propose a variant of log-linear algorithm called binary log-linear learning algorithm (BLLL) that converge to pure Nash equilibrium.

  The performance of our proposed algorithm is evaluated via simulations and compared to previous approaches. The simulation results show that our proposed algorithm significantly improves the sensing coverage performance.

The paper is organized as follows: Section 2 briefly reviews some recent research related to solving the sensor coverage problem. In section 3 we introduce some preliminary game theory knowledge. In section 4, first, the proposed approach is formulated for both binary and probabilistic sensing models based on game theory. Then, the binary log-linear learning algorithm is introduced to converge the game into an efficient action profile. In section 5, simulation results are presented through several experiments. Finally, we conclude the paper in section 6.

2. Related Works

In this section, we briefly review the research work on coverage in wireless sensor networks. The coverage problem is usually divided into three categories: area coverage, point coverage, and barrier coverage [9-11]. The purpose of area coverage is to cover the whole area. Next, point coverage is the coverage for Points of Interest (PoI). Finally, the barrier coverage guarantees that every movement that crosses a barrier of sensors will be detected.

Habibi *et al.* [12] proposed a distributed Voroni-based strategy to maximize the sensing coverage in a mobile sensor network. In this algorithm, each sensor moves through a gradient-based nonlinear optimization approach and places inside its Voroni cell.

For the first time, Ai *et al.* [13] studied the problem of covering targets with directional sensors. They formulated the problem as maximum coverage with minimum number of sensors and proved that it is NP-complete. Therefore, several greedy heuristic methods are presented to solve the problem. Here, the main idea is the selection of sensing sectors, which cover the maximum number of targets.

Mohamadi *et al* [14] proposed two Greedy-based algorithms for target coverage in directional sensor networks with adjustable sensing range. They used both scheduling and adjusting sensing range techniques to form cover sets to cover all targets in the network and maximize network lifetime.

In [15], the authors provided a GA-based algorithm to solve the MNLAR (Maximum Network Lifetime with Adjustable Ranges) problem. GA-based algorithm forms cover sets of directional sensors with appropriate sensing ranges.

Yu *et al*. [11] addressed the problem of K-coverage in wireless sensor networks with both centralized and distributed protocols. Protocols introduced a new concept of Coverage Contribution Area (CCA). Based on this concept, a lower sensor spatial density was provided. In addition, the protocols considered the remaining energies of the sensors. Therefore, the proposed protocols prolonged the network lifetime.

In [16], a Probabilistic coverage preserving protocol (CPP) is designed to achieve energy efficiency and to ensure a certain coverage rate. The purpose of the proposed protocol is to select the minimum number of probabilistic sensors to reduce energy consumption.

A graph model named Cover Adjacent Net (CA-Net) was proposed by Weng *et al* [10] to simplify the problem of k-barrier coverage while reducing the complexity of computation. Based on the developed CA-Net, two distributed algorithms, called BCA and TOBA, were presented for the purpose of energy balance and maximum network lifetime.

Mostafaei *et al.* [9] Proposed a distributed boundary surveillance (DBS) algorithm to cover the boundary and reduce energy consumption of sensors. DBS selects the minimum number of sensors to increase the network lifetime using learning automata.

Li et al. [17] Proposed the Voronoi-based distribution approximation (VDA) algorithm. In the proposed algorithm, in order to maximize the coverage of the desired area, the most Voronoi edges are covered. In

[18], the authors proposed the distributed Voronoi-based self-redeployment algorithm (DVSA), aiming to improve the overall field coverage of mobile directional sensor networks. This paper utilized the geometrical features of Voronoi diagram and the advantages of a distributed algorithm.

Recently, game theoretic approaches have been taken into consideration to solve coverage problem in WSNs [19-22]. In [23], the authors have proposed an algorithm based on game theory for the problem of maximizing coverage and reducing energy consumption. They have shown that the desired solution in this model is a NE strategy profile. In [24] the authors have proposed a game-theoretical complete coverage algorithm. This algorithm is used to ensure whole network coverage mainly through adjusting the covering range of nodes and controlling the network redundancy. The game theory control method has many advantages including robustness to failures and environmental disturbances, reducing communication requirements and improving scalability. The primary goal of game theory-based approaches is to design rules that guarantee the existence and efficiency of a pure Nash equilibrium [25]. Proper utility functions and reinforcement learning methods are designed for the coverage game of WSNs in [26, 27]. In these algorithms each player must have access to the utility values of its alternative actions. In [28] coverage of an unknown environment was investigated by robots. A state-based potential game was designed to control the robots' actions. The reward of sensing the areas and the penalty of energy consumption due to the sensors' movement were considered in the utility function. The sensors updated their action profile using the Binary Log-Linear Learning (BLLL) [29] in which the sensors must know an estimate of the outcome of their future actions. Hence, an estimation algorithm was used to assist the sensors in predicting the probability of targets in unknown areas. An improved EM algorithm was introduced to estimate the number of targets and other probability distribution parameters. In this study, we propose a game theory based algorithm to optimally cover targets and reduce energy consumption.

3. Background in Game Theory

In this section, we consider a brief review of the concepts in game theory. More information about game theory and learning in game theory are mentioned in [30, 31].

A strategic game $G := \langle V. A. U \rangle$ has three components: A set of players, V, an action set $A = A_1 \times \cdots A_n$, where A_i is the finite action set of player i, and the collection of utility functions U, where the action profile models the benefit of player i^{th} over action profiles.

For an action profile $a = (a_1. a_2. \cdots. a_n) \in A$, $a_{-i} = \{a_1. \cdots. a_{i-1}. a_{i+1}. \cdots. a_n\}$ denotes the action profile of all players other than player i. Therefore, the action profile a can be represented as $(a_i. a_{-i})$. Similarly, the utility function $U_i(a)$ is represented by $U_i(a_i. a_{-i})$. The concept of (pure) Nash equilibrium (*NE*) is the most important one in game theory. Consider the strategic game *G*, an action profile $a^* \in A$ is a pure Nash equilibrium if for all players $i \in N$ and for all $a_i \in A_i$ it holds that $u_i(a^*) \geq u_i(a_i. a_{-i}^*)$. Simply speaking, Nash equilibrium is a set of strategies in which each player does not benefit from one-sided change.

The strategic game G is an exact potential game [32] with potential function $\phi: A \rightarrow R$ if for every player $i \in N$, for every $a_{-i} \in A_{-i}$ and for every $a_i. a_i' \in A_i$,

$$\phi(a_i. a_{-i}) - \phi(a_i'. a_{-i}) = u_i(a_i. a_{-i}) - u_i(a_i'. a_{-i}) \tag{1}$$

It is proved that any action profile that maximizes the potential function is a Nash equilibrium [33].

In some cases, the actions available for player *i* is restricted to a subset of A_i, denoted by $F_i(a_i. a_{-i}) \subseteq A_i$, which is the set of feasible actions of player *i* when the action profile is $(a_i. a_{-i})$. The introduction of *F* leads to the notion of restricted strategic game $G_{res} \coloneqq \langle V. A. U. F \rangle$.

An action profile a^* is a restricted NE of the restricted strategic game G_{res} if $\forall i \in V$ and $\forall a_i \in F_i(a_i^*. a_{-i}^*)$, it holds that $u_i(a^*) \geq u_i(a_i. a_{-i}^*)$.

The game G_{res} is a restricted exact potential game with potential function ϕ(a) if for every player *i*, for every $a_{-i} \in A_{-i}$, and for every $a_i \in A_i$,, Eq. (1) holds for every $a_i' \in F_i(a_i. a_{-i})$.

The existence of *NE* in an exact potential game is guaranteed [33]. It then follows that any restricted exact potential game has at least one restricted *NE*.

4. The Proposed Algorithm

in this section, we propose a new game theory-based algorithm for both binary and probabilistic sensing models in mobile directional sensor networks in order to maximize the area coverage. We prove that the proposed method is a potential game and converges to Nash equilibrium using a distributed learning algorithm.

4.1. Problem Formulation

Suppose that *N* mobile directional sensor nodes are randomly deployed in a two-dimensional mission space. Figure 1 shows the binary sensing area of a directional sensor denoted by $(S. R. \theta_f. \theta_s. \vec{v})$. *S* is the location of the sensor node on a two-dimensional plane. *R* represents the maximum sensing radius. The horizontal orientation of the sensor and angle of view are indicated by $\theta_s \in (0.2\pi]$ and θ_f, respectively. $\vec{v}$ is a unit vector that defines the orientation of the directional sensor. Let *q* be a point in the area. Point *q* is covered by sensor *S* if:

$$\left\|\vec{d}\right\|_2 \leq R \tag{2}$$

$$\varphi = \cos^{-1}\frac{\vec{d} \cdot \vec{v}}{\left\|\vec{d}\right\|_2} \leq \frac{\theta_f}{2} . \quad 0 \leq \varphi \leq \pi \tag{3}$$

where $\vec{d}$ is the distance vector from sensor S to q. The first condition indicates whether q is within the sensing range of S, and the second one examines whether q is in the sensor's angle of view.

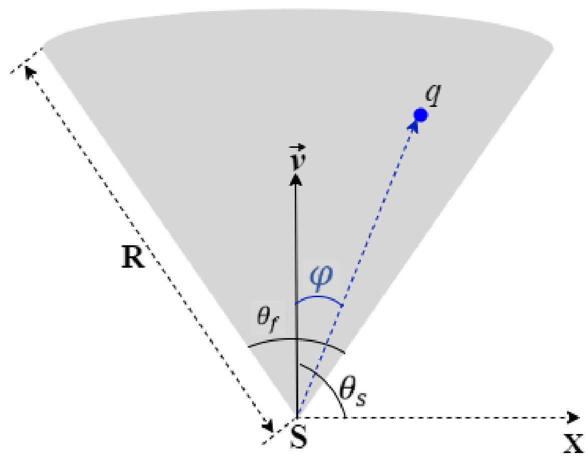

Figure 1. a Binary sensing model

We assume that communication range of each sensor (R^c) is at least twice the sensing range ($R^c \geq 2R$). Thus, each sensor can transmit its state information to its neighbors.

The two-dimensional mission space is discretized into a squared lattice. Each square of the lattice is 1×1 and is represented by the coordinate of its center $q = (q_x . q_y)$. The coordinates of the center of all squares are determined by Q. The location of sensor i^{th} is denoted by $l_i \in Q,\ \ i = 1. \dots . N$. Sensors can move and rotate in the mission space. The direction of sensor i^{th} is indicated by $\theta_i \in (0.2\pi]$. The motion of each sensor is limited to adjacent square lattices in four directions. The full area coverage is provided if the center of all the squares is covered by sensors.

Under these assumptions, the problem is to find the appropriate location and orientation of each sensor to maximize the area coverage and minimize energy consumption. We model this problem as an optimization problem. For this purpose, we define several notations as follows:

- n_q: the number of directional sensors that cover $q \in Q$.
- $x_{ni\theta_s}$: a binary variable that indicates whether sensor n is placed at location i with orientation θ_s.
- $y_{i\theta_s q}$: a binary variable that indicates whether a sensor at location i with orientation θ_s covers control point q.
- E_j^{mov}: the energy consumed due to movement.
- E_j^{rotate}: the energy consumed due to rotation.

The goal is to maximize the coverage and reduce energy consumption due to the movement and rotation of sensor nodes. To this end, we define the objective function as follows:

$$Maximize \sum_{\substack{q \in Q \\ n_q \neq 0}} \sum_{j=1}^{n_q} \frac{1}{j} - \sum_{i=1}^{N} (E_i^{move} + E_i^{rotate}) \tag{4}$$

$$subject\ to: \sum_i \sum_{\theta_s} x_{ni\theta_s} = 1.\ \ \forall n \tag{5}$$

$$n_q = \sum_n \sum_i \sum_{\theta_s} x_{ni\theta_s} y_{i\theta_s q}.\ \ \forall q \tag{6}$$

$$x_{ni\theta_s}.\, y_{i\theta_s q} \in \{0.1\} \tag{7}$$

Equation (5) ensures that only one working direction is assigned to each sensor node. n_q is calculated by (6).

In the following subsections, we formulate the utility function for both binary and probabilistic sensor nodes.

4.2. Coverage Problem as an Exact Potential Game

In our coverage problem, we are concerned with devising motion and orientation laws for repositioning of a finite number of mobile directional sensor nodes so that their converged positions in the limit correspond to a deployment with desirable coverage performance. In this section, we present our formulation of this problem in terms of a restricted exact potential game $G_{mc} := \langle V, A, U_{mc}, F_{mc} \rangle$. In the following, we describe the game components in more detail:

- Player set *V*: The set of players consists of the *N* sensors in the mission space, denoted by $V = \{s_1. s_2. \dots s_N\}$
- Action set *A*: The action of a player *i* is shown by $a_i = (l_i. \theta_i) \in A_i$, where A_i is the available action set for sensor s_i. The joint action set across all players is $\boldsymbol{A} = \prod_{i=1}^{N} A_i$.
- Feasible action set F_{mc}: The feasible action for each agent is determined based on a motion vector in any four cardinal directions; more formally, we have $F_{mc} = \prod_{i=1}^{N} \bigcup_{a_i \in A} F(a_i)$, where $F(a_i)$ is the set of feasible locations that a sensor in location a_i can move to, with any direction θ_i.

4.2.1 Utility Function Using Binary Sensing Model

In this section, the directional sensors are considered as binary sensing models. As depicted in figure 1, the binary sensing model for sensor s_i is defined as the following:

$$C_q(a_i) = \begin{cases} 0. & (d > R) \vee (\varphi > \dfrac{\theta_f}{2}) \\ 1. & (d \leq R) \wedge (\varphi \leq \dfrac{\theta_f}{2}) \end{cases} \tag{8}$$

Let $D(a_i)$ be a set of points that s_i can cover. For each $q \in Q$, $n_q(a)$ represents the number of sensors that observe the point q. We define a utility function for each sensor s_i as the following:

$$\sum_{q \in D(a_i)} \frac{1}{n_q(a)} \tag{9}$$

Due to energy constraints in sensor networks, we consider energy consumption in the design of the utility function. The energy consumption of sensor s_i due to movement is defined as follows:

$$E_i^{move}(a_i) = K_i(|l_i - l_i'|) \tag{10}$$

Where $K_i > 0$ is a coefficient, l_i and l_i' refer to the present and previous sensor locations, respectively. The energy consumption of sensor s_i due to rotation is defined as follows:

$$E_i^{rotate}(a_i) = K'_i(\left|\frac{\theta_i - \theta_i'}{2\pi}\right|) \tag{11}$$

Where $K'_i > 0$ is a coefficient, θ_i and θ_i' refer to the present and previous sensor orientations, respectively. Therefore, the utility function of the sensor s_i indicates the contribution of that sensor to the area coverage and energy consumption due to movement and rotation. We consider the utility function for sensor s_i as the following:

$$u_i(a) = \sum_{q \in F(a_i)} \frac{1}{n_q(a)} - E_i^{move}(a_i) - E_i^{rotate}(a_i) \tag{12}$$

The following lemma shows that the defined game is a potential game.

Lemma 1. The strategic game $G_{mc} := \langle V, A, U_{mc}, F_{mc} \rangle$ is an exact potential game with the following potential function:

$$\phi(a) = \sum_{\substack{q \in Q \\ n_q \neq 0}} \sum_{j=1}^{n_q(a)} \frac{1}{j} - \sum_{i=1}^{N} (E_i^{move}(a_i) + E_i^{rotate}(a_i)) \tag{13}$$

Proof:

As shown in [32], a potential game has to satisfy the following condition.

For any agent $i = 1. \dots . N$ and two consecutive action profiles $a_i = (l_i . \theta_i)$ and $a'_i = (l'_i . \theta'_i)$, equation (1) is established. Define $\eta_1 = D(a_i) \backslash D(a'_i)$ and $\eta_2 = D(a'_i) \backslash D(a_i)$. Since for each $q \in \eta_1$, $n_q(a) = n_q(a') + 1$ and for each $q \in \eta_2$, $n_q(a') = n_q(a) + 1$. Thus we have:

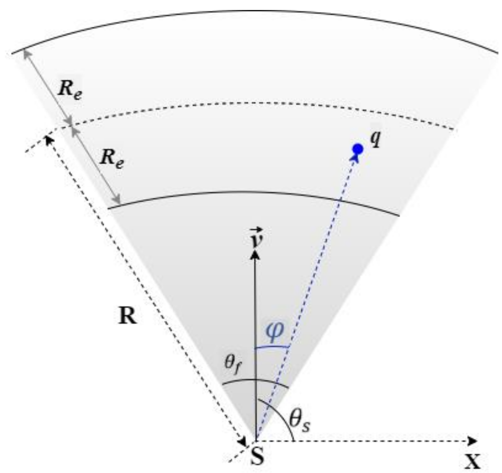

Figure 2. a Probabilistic sensing model

$$\phi(a_i.a_{-i}) - \phi(a'_i.a_{-i}) \tag{14}$$
$$= \sum_{q\in\eta_1}(\sum_{j=1}^{n_q(a)}\frac{1}{j} - \sum_{j=1}^{n_q(a')}\frac{1}{j}) + \sum_{q\in\eta_2}(-\sum_{j=1}^{n_q(a)}\frac{1}{j} + \sum_{j=1}^{n_q(a')}\frac{1}{j})$$
$$-\left(E_i^{move}(a_i) + E_i^{rotate}(a_i)\right) + (E_i^{move}(a'_i) + E_i^{rotate}(a'_i))$$
$$= \sum_{q\in\eta_1}\frac{1}{n_q(a)} - \sum_{q\in\eta_2}\frac{1}{n_q(a')} - \left(E_i^{move}(a_i) + E_i^{rotate}(a_i)\right) + (E_i^{move}(a'_i)$$
$$+ E_i^{rotate}(a'_i)) = u_i(a_i.a_{-i}) - u_i(a'_i.a_{-i})$$

4.2.2. Utility Function Using Probabilistic Sensing Model

In probabilistic sensing models, the probability of point detection is a reduction function of the sensing distance. As shown in figure 2, the probabilistic sensing area in DSNs can be denoted by $(S.R.R_e.\theta_f.\theta_s.\vec{v})$. Similarly, S is the location coordinate on a two-dimensional plane, R_e indicates the uncertain sensing range, and $R - R_e$ specifies the maximum certain sensing range. The point q is probabilistically covered if the Euclidean distance d between q and S is in the range $(R - R_e.R + R_e)$. The horizontal orientation of the sensor and angle of view are indicated by $\theta_s \in (0.2\pi]$ and θ_f, respectively. $\vec{v}$ is a unit vector and defines the orientation of the directional sensor. In DSNs, the probabilistic sensing model for sensor s_i is described as follows:

$$C_q(a_i) = \begin{cases} 0. & (d > R + R_e) \vee (\varphi > \frac{\theta_f}{2}) \\ e^{-\lambda[d-(R-R_e)]^\beta} & (R - R_e < d \le R + R_e) \wedge (\varphi \le \frac{\theta_f}{2}) \\ 1. & (d \le R - R_e) \wedge (\varphi \le \frac{\theta_f}{2}) \end{cases} \quad (15)$$

Where λ and β are parameters that measure the probability of point detection and vary in different types of sensors.

Definition (Probabilistic Coverage): The desired area is covered by *n* sensors with probability P_c, if for each point *q* in the area, the following equation is established.

$$P(q) = 1 - \prod_{i=1}^{n} (1 - C_q(a_i)) \ge P_c \quad (16)$$

According to Equation (15), $C_q(a_i)$ is the probability of detecting the point *q* by the sensor s_i. $(1 - C_q(a_i))$ is the probability that the point *q* is not covered by the sensor s_i. Since the probabilistic coverage of a point by a sensor node is independent of other sensors, the term $\prod_{i=1}^{n}(1 - C_q(a_i))$ is the probability that the point *q* is not be covered by any of the sensors. Hence, the expression $1 - \prod_{i=1}^{n}(1 - C_q(a_i))$ is the probability that the point *q* be covered by at least one sensor.

In this problem, the goal is to move and rotate the directional sensor nodes in a way that the coverage probability of each point $q \in Q$ is greater than or equal to P_c. We define the utility function of player *i* in the probabilistic sensing model as follows:

$$u_i(a) = \sum_{q \in F(a_i)} w_q(a_i) - E_i^{move}(a_i) - E_i^{rotate}(a_i) \quad (17)$$

Where $w_q(a_i)$ is the contribution of directional sensor s_i in detecting the point *q,* which is defined as follows:

$$w_q(a_i) = \begin{cases} \frac{C_q(a_i)}{\sum_{k=1}^{n_q(a)} C_q(a_k)}. & if\ P(q) \ge P_c \\ 0. & otherwise \end{cases} \quad (18)$$

$E_i^{move}(a_i)$ and $E_i^{rotate}(a_i)$ are defined in (10) and (11). The following lemma shows that our defined game is a potential game.

Lemma 2. The strategic game $G_{mc} := \langle V, A, U_{mc}, F_{mc} \rangle$ is an exact potential game with the following potential function:

$$\phi(a) = \sum_{\substack{q\in Q \\ P(q)\geq P_c}} \sum_{j=1}^{n_q(a)} \frac{C_q(a_j)}{\sum_{l=1}^{j} C_q(a_l)} - \sum_{i=1}^{N}(E_i^{move}(a_i) + E_i^{rotate}(a_i)) \tag{19}$$

Proof:

For any agent $i = 1. \dots . N$ and two consecutive action profiles $a_i = (l_i . \theta_i)$ and $a'_i = (l'_i . \theta'_i)$, (1) is established. Define $\eta_1 = D(a_i) \backslash D(a'_i)$ and $\eta_2 = D(a'_i) \backslash D(a_i)$.). Since for each $q \in \eta_1$, $n_q(a) = n_q(a') + 1$ and for each $q \in \eta_2$, $n_q(a') = n_q(a) + 1$. Thus we have:

$$\begin{aligned}
&\phi(a_i . a_{-i}) - \phi(a'_i . a_{-i}) \\
&= \sum_{\substack{q\in \eta_1 \\ P(q)\geq P_c}} (\sum_{j=1}^{n_q(a)} \frac{C_q(a_j)}{\sum_{l=1}^{j} C_q(a_l)} - \sum_{j=1}^{n_q(a')} \frac{C_q(a_j)}{\sum_{l=1}^{j} C_q(a_l)}) \\
&+ \sum_{\substack{q\in \eta_2 \\ P(q)\geq P_c}} (-\sum_{j=1}^{n_q(a)} \frac{C_q(a_j)}{\sum_{l=1}^{j} C_q(a_l)} + \sum_{j=1}^{n_q(a')} \frac{C_q(a_j)}{\sum_{l=1}^{j} C_q(a_l)}) \\
&- \left(E_i^{move}(a_i) + E_i^{rotate}(a_i)\right) + (E_i^{move}(a'_i) + E_i^{rotate}(a'_i)) \\
&= \sum_{\substack{q\in \eta_1 \\ P(q)\geq P_c}} \frac{C_q(a_i)}{\sum_{l=1}^{n_q(a)} C_q(a_l)} - \sum_{\substack{q\in \eta_2 \\ P(q)\geq P_c}} \frac{C_q(a'_i)}{\sum_{l=1}^{n_q(a')} C_q(a_l)} - \left(E_i^{move}(a_i) + E_i^{rotate}(a_i)\right) \\
&+ (E_i^{move}(a'_i) + E_i^{rotate}(a'_i)) = u_i(a_i . a_{-i}) - u_i(a'_i . a_{-i})
\end{aligned} \tag{20}$$

4.3. Distributed Learning Algorithm

In the game theoretical formulation, the sensor nodes play the coverage game G repeatedly starting from a desired initial configuration. At each time step $t \in \{0,1,2. \dots\}$, one senor s_i is randomly selected and plays an action $a_i(t)$ While other sensors repeat their actions, i.e. $a_{-i}(t) = a_{-i}(t-1)$. The role of the learning algorithm is to provide an action update rule so that the sensor actions converge to a Nash equilibrium.

In order to maximize potential function and achieve Nash equilibrium, log-linear learning is presented in [34], where only one player updates its action at each iteration. In log-linear learning, sensors can select suboptimal actions with low probability. Therefore, sensors are allowed to explore, and this plays an important role for sensors in finding optimal actions and achieving Nash equilibrium. Log-linear learning assumes that players have a constant action set. In general, convergence to the potential maximizer is not guaranteed when the practical actions available to a player depend on the player's state, i.e. each player is allowed to choose its next action $a_i(t+1)$ from the set of actions $A_i^c(a_i(t))$ that depends on its current action $a_i(t)$. A modified version of log-linear learning called binary log-linear learning was introduced for the problem of constrained action set in [29]. Binary log-linear learning can be used to converge to a set of potential maximizer action profiles if the constrained action sets meet the following two properties.

Property 1 (Feasibility) For any player $s_i \in S$ and any action pair $a_i(0).\, a_i(k) \in A_i$, there exists a sequence of actions $\{a_i(0). \dots .\, a_i(k)\}$ such that $a_i(t) \in A_i^c(a_i(t-1))$ for all $t \in \{1.2. \dots .\, k\}$.

Property 2 (Reversibility) For any agent $s_i \in S$ and any action pair $a_i.\, a'_i \in A_i$, $a'_i \in A_i^c(a_i) \leftrightarrow a_i \in A_i^c(a'_i)$.

We can easily show that the above properties are met according to the problem settings. In binary log-linear learning, only one sensor is randomly selected at each time step. The selected sensor, assuming the other sensors are stationary, selects a trial action randomly in its constrained action set. The sensor receives a hypothetical utility by playing the trial action and updates its action depending on the current utility and hypothetical utility. The general binary log-linear learning algorithm presented in [29] is as follows:

BLLL Algorithm

Initialization: $t = 0.\, T \epsilon \mathrm{R}^{+}\ small.\, a(0) \epsilon A$

while (1)

Pick a random $p_i \epsilon P$.

Pick a random $a'_i \in A_i^c(a_i)$.

$a_j(t+1) = a_j(t)\ \ for\ all\ p_j \neq p_i$.

$\alpha = e^{U_i(a(t))/T}$.

$\beta = e^{U_i(a'_i.a_{-i}(t))/T}$.

$a_i(t+1) = \begin{cases} a_i(t) & w.p.\ \frac{\alpha}{\alpha+\beta}. \\ a'_i & othewise. \end{cases}$

$t = t + 1$.

end while

5. Simulation Results

In this section we present the simulation results of the proposed algorithm. To evaluate the performance of the proposed algorithms, several experiments have been performed in MATLAB. The simulation results are compared with the results of VDA [17], DVSA [18] and RND algorithms. RND means the initial value after the random deployment of sensors (with random position and random direction). The algorithms are compared with respect to coverage. We consider the fraction of the area which is covered by the deployed sensors as the coverage criterion.

Experiment 1. In the first experiment, we consider an example of applying BCGT in a mobile directional sensor network. We consider a 15×15 square in which 20 directional sensors are located in the middle of the area. Sensing range of each sensor is 3, and angle of view of each sensor is considered as 90 degrees. We have chosen $T = 0 \cdot 1$ in the learning algorithm. Figure 3 shows the position and orientation of sensor nodes at iteration 2000.

The evaluation of the potential function in each iteration is shown in figure 4. This figure shows that sensor nodes try to increase their potential function, which corresponds to better location and orientation exploration. It is now necessary to show that maximizing the potential function leads to maximizing the coverage of the whole area. Figure 5 displays that the area coverage is increasing during the time.

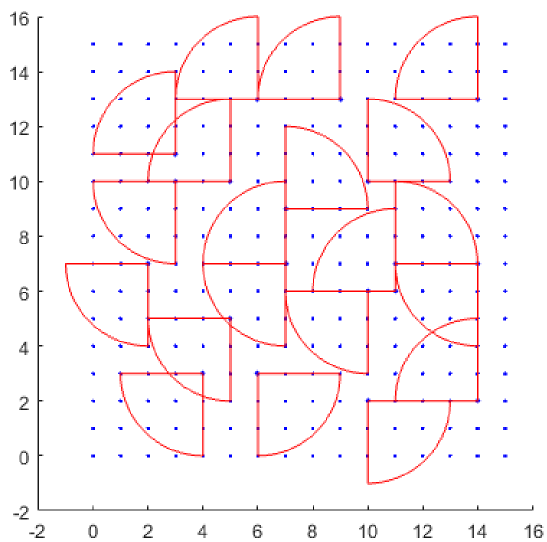

Figure 3. Final configuration of the network at iteration 2000 of BCGT algorithm

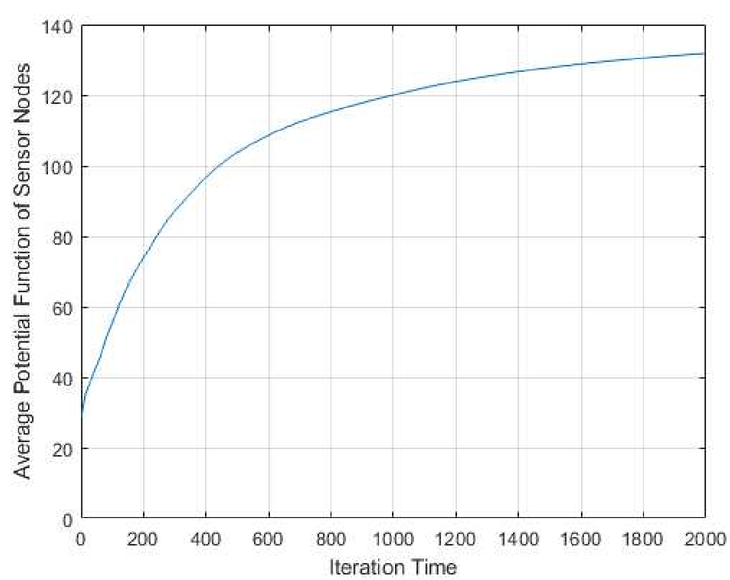

Figure 4. Average potential function of sensor nodes during the time

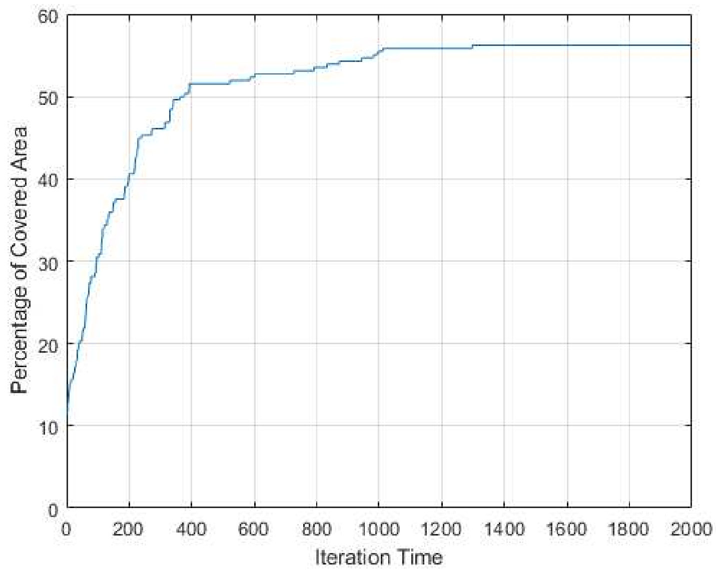

Figure 5. The percentage of covered area during the time

Experiment 2. In this experiment, we compare the performance of the proposed algorithm with RND, VDA and DVSA algorithms in terms of coverage criteria. The directional sensor nodes are randomly placed in the 500×500 square areas. The experiment is performed for $N = 100.200.300.400\ and\ 500$ sensors. The sensing range and angle of view of the sensors are fixed and equal to 50 and 120°, respectively. The comparison of the proposed algorithm BCGT with the existing deployment algorithms is shown in figure 6. As shown in figure 6, BCGT performance is better than RND, VDA and DVSA in terms of coverage criteria. Figure 7 compares the behavior of BCGT with existing algorithms in terms of coverage criteria for setting parameters as $N = 200. R = 50\ and\ \theta_f = 60^{\circ}.90^{\circ}.120^{\circ}.180^{\circ}\ and\ 240^{\circ}$. According to Fig. 7, the BCGT again performs better than existing algorithms. From the comparisons, we conclude that the proposed BCGT performs very well under different number of sensors and angles of view.

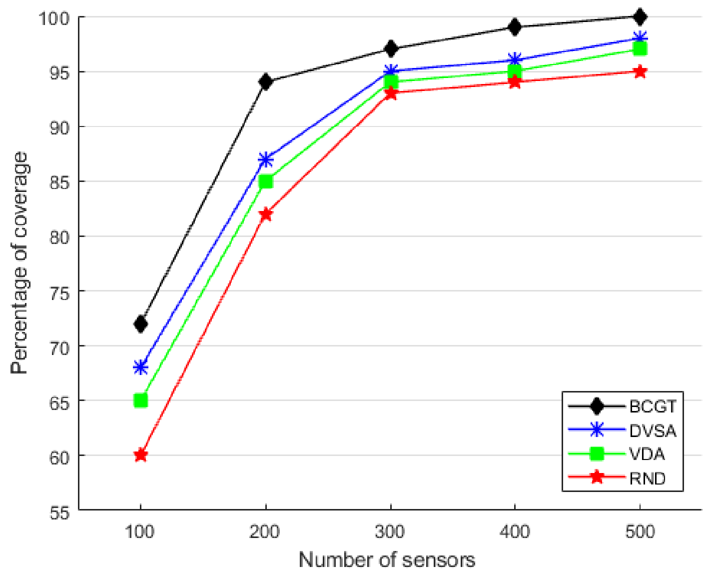

Figure 6. Comparison of BCGT with existing deployment algorithms in terms of coverage ($N = 100 \sim 500 . r = 50\ and\ \theta_f = 120^0$)

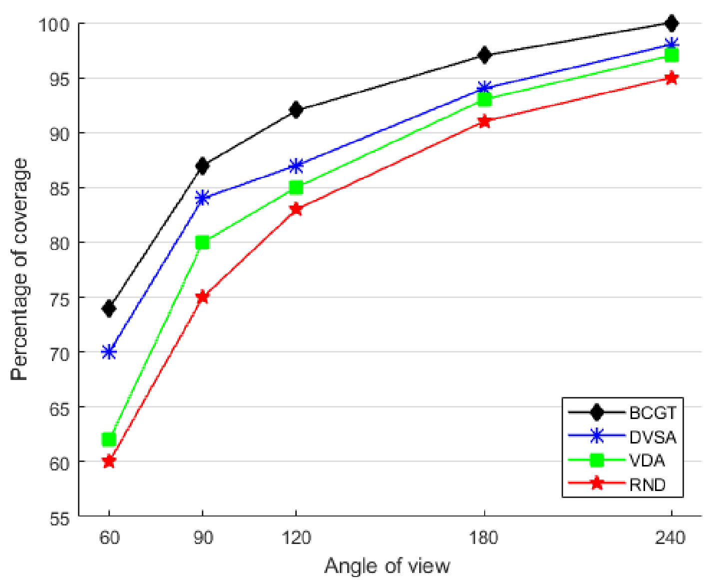

Figure 7. Comparison of BCGT with existing deployment algorithms in terms of coverage ($N = 200 . r = 50\ and\ \theta_f = 60^0 \sim 240^0$)

Experiment 3. In order to establish the probabilistic coverage using the proposed PCGT algorithm, we consider a 500 × 500 area. The sensors are probabilistic with parameters $(R. R_e. \lambda. \beta)$, (50, 15, 0.9, 0.1) and 90 ° viewing angle. We consider the confidence probabilities P_c to be 80%, 85%, 90% and 95%. The sensors are randomly placed in the area. Figure 8 shows the simulation results of the PCGT algorithm for $N =$

100~600 sensors and different confidence probabilities. The results show that at higher confidence probability P_c, more sensor nodes are needed to fully cover the area.

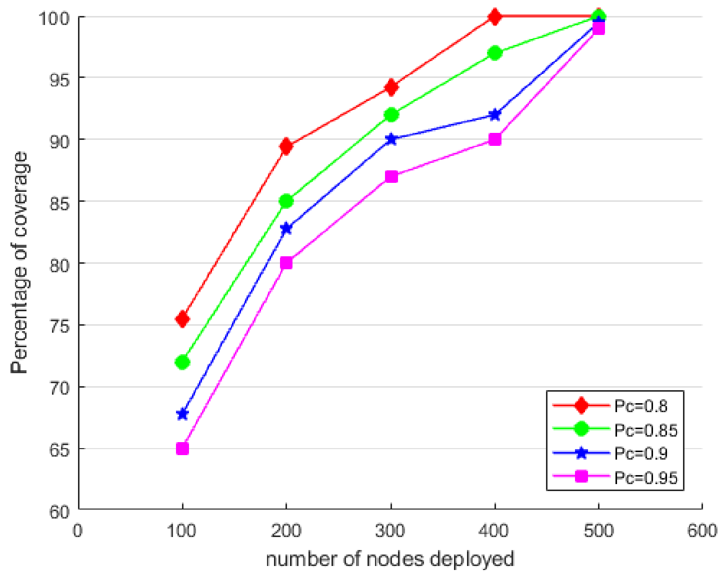

Figure 8. Coverage rate vs. node density

Experiment 4. In the proposed PCGT algorithm, λ and β are two important parameters in the utility function and determining the action of sensor nodes. Therefore, we select two sets of parameters to examine their effect on coverage and compare them with the BCGT algorithm. With the simulation results shown in figure 9, it can be concluded that with increasing the number of sensors, the coverage percentage of the area increases. Similarly, with increasing λ and decreasing β, the probability of sensor coverage and consequently the coverage percentage of the area increases.

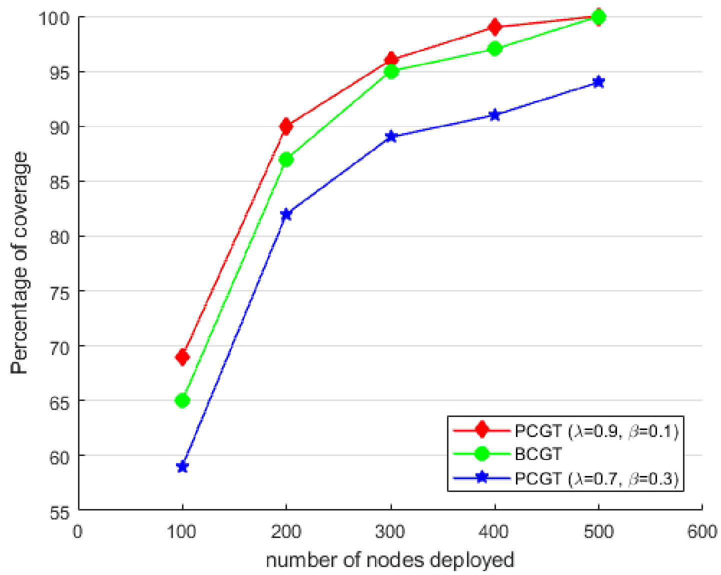

Figure 9. Comparison of BCGT with PCGT for different parameters in terms of coverage

6. Conclusion

In this paper, we proposed a game theory-based algorithm for deploying and orienting a number of mobile directional sensor nodes for both binary and probabilistic sensing models to maximize area coverage. An appropriate utility function for each player is designed to improve coverage quality and reduce energy consumption. Then we proved that the designed game is a potential game and in order to converge the game and achieve Nash equilibrium, we used the binary log-linear learning algorithm. The simulation results showed the performance of our proposed algorithm over previous approaches in terms of coverage rate.

Declarations

Ethical Approval

Not applicable.

Competing interests

The authors declare that they have no conflict of interest.

Authors' contributions

Both of authors wrote the main manuscript text. Dr Elham Golrasan has done the simulations, and Dr Marzieh Varposhti reviewed the manuscript.

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