A new interval constructed belief rule base with rule reliability

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A new interval constructed belief rule base with rule reliability

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Abstract

Combination rule explosion problem of belief rule base (BRB) is a difficult problem to solve in complex systems and has attracted wide attention at present. Aiming at the problem of combination rule explosion in belief rule base, a new interval constructed belief rule base with rule reliability (IBRB-r) is proposed. On the basis of BRB, IBRB-r innovatively introduced rule reliability and established the belief table in the form of interval. This approach can not only clearly indicate the contribution degree of each rule to the model but also solve the problem of combination rule explosion. Therefore, IBRB-r is more suitable for complex system modeling. In the case study section, the structural safety assessment of liquid launch vehicle is introduced to conduct a concrete example analysis. The experimental results show that the proposed model is effective and accurate.

Keywords: belief rule base, combination rule explosion, rule reliability, complex system, liquid launch vehicle

1. Introduction

The belief rule base (BRB) method based on evidence reasoning was developed on the basis of decision theory, fuzzy theory, traditional IF-THEN rule base and D-S evidence theory[1]. It is a method driven by mixed data and knowledge, which can deal with uncertain information and has good processing performance for small sample data[2]. At present, the BRB has been widely used in fault diagnosis[3], medical diagnosis[4], risk assessment[5] and other fields.

In practical engineering applications, the safety assessment of complex systems can no longer meet the needs of practical systems by subjective judgment alone. The expert knowledge base in the BRB can participate in the security assessment of complex systems well and has a certain validity and accuracy. However, the assessment model based on BRBs still has the following two problems. On the one hand, due to the complex system situation, when the assessment indexes and assessment referential values are too many, the rules of the safety assessment model based on BRB may cause the phenomenon of "combination explosion". When there are too many attributes or referential points, the traditional BRB adopts the Cartesian product form to establish the belief table, which easily produces the phenomenon of "combination explosion". This will increase the complexity of the algorithm and reduce the processing performance of the model. On the other hand, the traditional BRB does not consider rule reliability and cannot clearly indicate the contribution of each rule to the assessment model. In practical engineering applications, rules are not completely reliable due to the limitations of environmental noise and conditions. This will directly reduce the accuracy of the model and indirectly lead to the difficulty of the reduction rules, which is not conducive to human judgment.

There are two solutions to the rule redundancy problem in BRB. First, rule reduction methods are used to reduce rules. In this respect, domestic and foreign scholars have performed much research. Wu et al.[6] introduced information entropy and K-prototypes to remove redundant rules based on fuzzy rough set theory. Yang et al. [7] proposed a rule reduction method based on data envelopment analysis for a belief rule base. Ben Li et al.[8] used Petri nets to solve the problem of combinatorial rule explosion in complex systems. Zhang et al.[9] used the density-based spatial clustering with noise (DBSCAN) algorithm to reduce belief rules and proposed a new training method based on parameter learning. Chang et al.[10] introduced grey target (GT), multi-dimensional scale (MDS), isometric mapping (ISOMAP), principal component analysis (PCA) and other feature extraction methods to carry out rule reduction to screen important attributes. Second, the original BRB is replaced by a hierarchical BRB model. Hierarchical BRB can split a multi-attribute dataset, selecting two attributes at a time and working its way up. This method can effectively reduce the number of attributes to solve the problem of combination rule explosion to a certain extent.

The above two methods have been proven to be effective in reducing rules, but there are still many problems. Using the rule reduction method to reduce rules can easily lead to the loss of precision of BRB, an increase in model complexity and the influence of model representation ability. Specifically, (1) the model cannot guarantee high accuracy after the reduction rule, such as GT and MDS. (2) The accuracy after the reduction rule is ideal, but the complexity of the algorithm is too high to be realized, such as ISOMAP, PCA, DBSCAN, etc. (3) Belief rules based on feature extraction cannot guarantee the integrity and consistency of rule reduction at the same time, such as PCA. With hierarchical BRB modeling, there are two problems: (1) The large structure of the hierarchical BRB model network easily causes the problem of high model complexity. (2) The initial model is constructed in a hierarchical way, and the middle layer cannot be trained. As a result, the results of the middle layer cannot be determined, and the uncertainty of the model increases.

Although these two methods can effectively reduce rules, they need to establish BRB model first and then reduce rules on this basis. In essence, this method of establishing the original model first and then reducing the rules still cannot reduce the rules effectively. In addition, both the rule reduction method and hierarchical BRB method do not consider rule reliability and cannot clearly indicate the contribution degree of each rule to the model.

Since the traditional BRB does not consider reliability, the current research only considers attribute reliability and attribute weight. Feng et al.[11] innovatively introduced premise attribute reliability into the belief rule base and proposed a BRB model with attribute reliability. However, the model without considering the reliability of rules has some disadvantages. On the one hand, the unreliable redundant rules cannot be removed, which increases the difficulty of rule reduction, resulting in a large number of rules. On the other hand, the complexity of the algorithm increases and the performance of the model is reduced.

From the above discussion, there is much of a lack of the method of conventional BRB and the lack of reliability of the rules. Therefore, a new interval constructed belief rule base with rule reliability (IBRB-r) is proposed. On the one hand, the IBRB-r model has abandoned the way that the old BRB builds the belief table in the form of carte accumulation but is based on the addition of the interval. This rule combination greatly reduces the number of rules and reduces the complexity of the model. The initial model is built in the interval form, the original BRB model is established, and the explosive problem of the combination rule is solved fundamentally. The IBRB-r model also introduces rule reliability. It uses the experience of the expert to evaluate the knowledge of the rules, which form the reliability of the rules and the removal of the rules. It is found that the IBRB-r model retains the advantages of the traditional BRB. At the same time, it completely solves the problem of the combination rule of BRB without breaking the model structure, fully considers the reliability of the rules and is more suitable for engineering applications.

The framework of this paper is organized as follows: The first part summarizes the shortcomings of the traditional BRB. To address these shortcomings, the IBRB-r model is proposed. The second part gives a preliminary introduction to BRB and puts forward the challenges of establishing the IBRB-r model. The third part elaborates the overall structure of the IBRB-r model from three aspects: modeling, reasoning and optimization. The fourth part provides case studies to demonstrate the effectiveness and accuracy of the IBRB-r model. The fifth part is the summary of the thesis and the prospect of future work.

2. Preliminary: BRB and existing problems

In 2006, Yang proposed a belief rule base inference method based on evidential inference rules[12]. BRB is essentially a rule-based expert system that can use a mixture of data and knowledge to drive modeling and establish nonlinear mapping between input and output. The belief distribution of a BRB can effectively represent the multi-source information of uncertainty, including probabilistic uncertainty and fuzzy uncertainty[13]. BRB has a set of belief rules, and the $k-th$ rule can be expressed as follows:

$$\begin{aligned}
&\mathrm{R}_k: \\
&\text{If } x_1 \text{ is } A_1^k \wedge x_2 \text{ is } A_2^k \wedge \ \ldots \ \wedge x_M \text{ is } A_M^k \\
&\text{Then result is } \{(D_1, \beta_{1,k}), (D_2, \beta_{2,k}), \ldots, (D_N, \beta_{N,k})\} \\
&\text{with rule weight } \theta_k \\
&\text{and attribute weight } \delta_{1,k}, \delta_{2,k}, \ldots, \ \delta_{M,k} \\
&k \in \{1, 2, \ldots, L\}, \sum_{i=1}^{N} \beta_{i,k} \le 1
\end{aligned} \tag{1}$$

where the k rule of the BRB is denoted as R_k. The M premises attribute is denoted by $x_i\,(i = 1, \ldots, M)$. The set of referential values of M premise attributes in rule k is denoted as $A_i^k\,(i = 1, \cdots, M)$. The N results are denoted as $D_i\,(i = 1, \ldots, N)$, and the corresponding belief degree of each result under the k rule is denoted as $\beta_{i,k}\,(i = 1, \cdots, N)$. The rule weight of rule k is denoted as θ_k. The total number of rules is denoted L.

BRB is mainly composed of knowledge base, inference engine and optimization algorithm. The knowledge base of BRB is expert knowledge acquired through long-term and extensive practice, which is the accumulation of professional knowledge. BRB adopts evidence reasoning (ER), and the reasoning process can be traced and explained. The optimization algorithm of BRB uses projection covariance matrix adaptation evolutionary strategies (P-CMA-ES). This chapter will analyze the original BRB model in detail from the perspectives of modeling, reasoning and optimization and point out the existing problems.

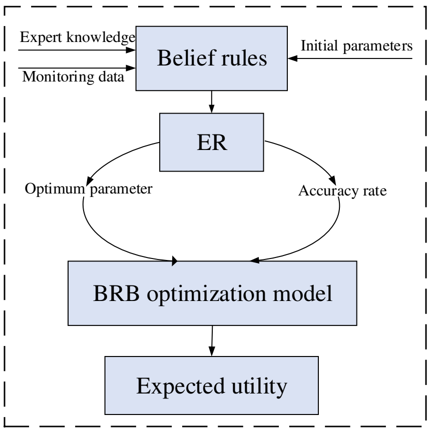

Figure 1. Overall structure of the BRB

2.1 BRB modeling process: Modeling and problems

This section will examine the traditional BRB modeling process in detail and point out its problems. The following will focus on analyzing the modeling process of traditional BRB from three aspects: problem mechanism analysis, referential point setting and belief table construction.

(1) Problem mechanism analysis

The first step in modeling should be to identify the problem the model is intended to solve. It is important to identify the factors that influence the problem and determine the possible outcomes of the problem. The influencing factors of the problem are taken as the prerequisite attributes of the traditional BRB, and the possible results of the problem are taken as the labels of the traditional BRB.

(2) Set referential values and referential points

Referential points should be selected where the attributes have typical significance, usually in most data sets. The referential point contains upper and lower bounds and can represent the range of data values. The number of referential points should be defined according to the actual problem. The more general referential points there are, the higher the model accuracy, but the model complexity will also increase.

For example, if premise attribute 1 has 5 referential points and premise attribute 2 has 6 referential points, 30 belief rules need to be established. However, if you have 5 premise attributes and 20 referential points for each premise attribute you need to establish 20^5 rules. It can be seen that with an increasing number of premise attributes and referential values, the number of rules increases exponentially, and the established rules are prone to the problem of combination rule explosion.

(3) Build belief table

After setting the premise attributes and the result referential points and referential values, the belief table needs to be established. According to the corresponding belief rules, the belief table is constructed. The following is a brief example of how traditional BRB build belief tables.

For example, suppose two premise attributes each have three reference levels, and the result has four reference levels. The referential point and referential value of premise attribute 1 are set in Table 1, and the referential point and referential value of premise attribute 2 are set in Table 2. The referential points and referential values of the results are shown in Table 3.

Table 1 Referential points and values of prerequisite attribute 1

Referential point	A	B	C
Referential value	1	2	3

Table 2 Referential points and values of prerequisite attribute 2

Referential point	I	J	K
Referential value	4	5	6

Table 3 Referential points and referential values for the results

Result	L	M	O	H
Referential value	1	2	3	4

In this case, 9 belief rules need to be constructed, and the corresponding belief table is shown in Table 4.

Table 4 BRB belief table

Number	Attribute 1	Attribute 2	Output belief distribution
1	A	I	{(L,0.9), (M,0.1), (O,0), (H,0)}
2	A	J	{(L,0.8), (M,0.2), (O,0), (H,0)}
3	A	K	{(L,0.7), (M,0.2), (O,0.1), (H,0)}
4	B	I	{(L,0.6), (M,0.3), (O,0.1), (H,0)}
5	B	J	{(L,0.5), (M,0.3), (O,0.2), (H,0)}
6	B	K	{(L,0.4), (M,0.3), (O,0.2), (H,0.1)}
7	C	I	{(L,0.2), (M,0.3), (O,0.2), (H,0.3)}
8	C	J	{(L,0), (M,0.2), (O,0.2), (H,0.6)}
9	C	K	{(L,0), (M,0.1), (O,0.2), (H,0.7)}

The above traditional BRB constructs belief tables in the form of cartesian products. In the above example, the number of attributes is small, and the number of referential values and referential points is small, so the combination of rules of the belief table does not produce a combination explosion. However, when the number of premise attributes and referential values and referential points increase, it is easy to produce the problem of combination rule explosion. This is mainly due to the way the rules are combined because the number of rules that build belief tables as cartesian products increases exponentially. It can be seen that the traditional BRB is only suitable for a simple system with a small number of prerequisite attributes and referential values but not for a complex system with a large number of prerequisite attributes and referential values.

2.2 The reasoning process of BRB: Reasoning and problems

As the inference engine of BRB, ERs are a multi attribute decision-making method formed on the basis of decision theory and D-S evidence theory. Its belief framework has good performance in describing uncertain problems, so it is chosen by BRB for model reasoning. The inference of BRB is mainly divided into four parts: calculation of rule fitness, calculation of rule activation weight, rule synthesis by the ER analytic algorithm, and calculation of expected utility value.

First, the rule fitness is calculated. This step completes the transformation of the input data according to the different properties of the premise attributes. The equivalent transformation based on utility can fully retain the features of the original data and is suitable for the input data transformation of BRB reasoning process. The specific conversion rules are as follows:

$$a_i^k = \begin{cases} \dfrac{A_i^{l+1} - x_i}{A_i^{l+1} - A_i^l} & k = l, A_i^l \le x_i \le A_i^{l+1} \\ 1 - a_i^k & k = l+1 \\ 0 & k = 1 \cdots K, k \ne l, l+1 \end{cases} \tag{2}$$

where the matching degree between rule k and attribute i is denoted as a_i^k. The $i-th$ attribute sample is called x_i. The referential value of the $i-th$ attribute in rule l is denoted as A_i^l.

Then, the rule activation degree is calculated. BRB combines rules in the form of cartesian products, and each combination rule has a different practical meaning. In practical models, not every rule is equally important. The activation degree of each rule is different. The rule activation weight is calculated as follows:

$$\omega_k = \frac{\theta_k \prod_{i=1}^{M} (a_i^k)^{\delta_i}}{\sum_{i=1}^{K} \theta_l \prod_{i=1}^{M} (a_i^l)^{\delta_i}} \tag{3}$$

where the rule activation weight of rule k is denoted as ω_k. The attribute weight of the $i-th$ premise attribute is denoted as $\delta_i (i = 1 \cdots M)$.

Then, the ER analytic approach is used for rule synthesis. In 2007, Yang proposed the ER analytical method[14]. In the process of model reasoning, the ER analytical method is widely used. The ER parsing method is expressed as follows:

$$\beta_n = \frac{\mu \times \left[\prod_{i=1}^{L} \left(\omega_l \beta_{n,l} + 1 - \omega_l \sum_{i=1}^{N} \beta_{i,l} \right) - \prod_{l=1}^{L} \left(1 - \omega_l \sum_{i=1}^{N} \beta_{i,l} \right) \right]}{1 - \mu \times \left[\prod_{l=1}^{L} (1 - \omega_l) \right]} \tag{4}$$

$$\mu = \frac{1}{\sum_{n=1}^{N} \prod_{l=1}^{L} \left(\omega_l \beta_{n,l} + 1 - \omega_l \sum_{i=1}^{N} \beta_{i,l} \right) - (N-1) \prod_{l=1}^{L} \left(1 - \omega_l \sum_{i=1}^{N} \beta_{i,l} \right)} \tag{5}$$

where the corresponding belief degree of the $i-th$ result under rule l is denoted as $\beta_{i,l} (i = 1 \cdots N)$. The utility value is μ.

Finally, the model output results are calculated based on utility theory. After the fusion of all rules is completed, the output set of the BRB inference process can be expressed as follows:

$$y = \{(D_n, \beta_n),\ n = 1, ..., N\} \tag{6}$$

After obtaining the output result set of the BRB inference process, the final output result can be expressed as:

$$Z = \sum_{n=1}^{N} u(D_n) \beta_n \tag{7}$$

where $u(D_n)$ is the utility value of D_n. Z is the final expected utility value of outcome set y and the final output of the BRB model.

The BRB model uses ER reasoning. ER inference integrates multiple sources of information and can show better processing ability for uncertain information. Moreover, the process of ER inference is traceable and explainable, and the process is obvious in the circuit. However, traditional

BRB reasoning does not consider rule reliability, which will have a great impact on model integrity and representation ability. Therefore, it is necessary to introduce rule reliability into the new model to enhance the integrity of the model.

2.3 BRB optimization process: Parameter training model

The initial parameters of BRB are given by experts, including the belief, attribute weight and rule weight. Based on long-term practice, experts can give a general distribution in line with the trend of the actual system, but it may not be the optimal solution of the model. Therefore, it is necessary to design an optimization model to correct the initial distribution given by expert knowledge so that BRB can achieve the optimal effect.

First, there are clear optimization objectives. Mean square error (MSE) is an important indicator to measure the effectiveness of a model [15]. The smaller the numerical gap between the model output and the label value is, the better the model optimization effect. The objective function of BRB optimization can be expressed as:

$$\min\ MSE(\theta, \beta, \delta) \tag{8}$$

Among them, the optimized parameters include rule weight, attribute weight and belief. The MSE is calculated as follows:

$$MSE(\theta, \beta, \delta) = \frac{1}{T_{train}} \sum_{k=1}^{K} (Z^* - Z)^2 \tag{9}$$

where T_{train} is the number of training samples and Z^* and Z are the label value and the output value of the BRB model, respectively. Based on the above analysis and discussion, the complete optimization objective function of the BRB can be expressed as:

$$\begin{aligned}
&min\ MSE(\theta, \beta, \delta) \\
&st. \sum_{n=1}^{N} \beta_{n,k} = 1, k = 1, \cdots, L \\
&0 \le \beta_{n,k} \le 1, n = 1, ..., N, k = 1, ..., L \\
&0 \le \theta_k \le 1, k = 1, .., L \\
&0 \le \delta_i \le 1, i = 1, .., M
\end{aligned} \tag{10}$$

Then, an appropriate optimization algorithm is selected to construct the optimization model. The optimization model continuously modifies the parameters by calculating the MSE value of the label value and the BRB model output value. This process is shown in Figure 2.

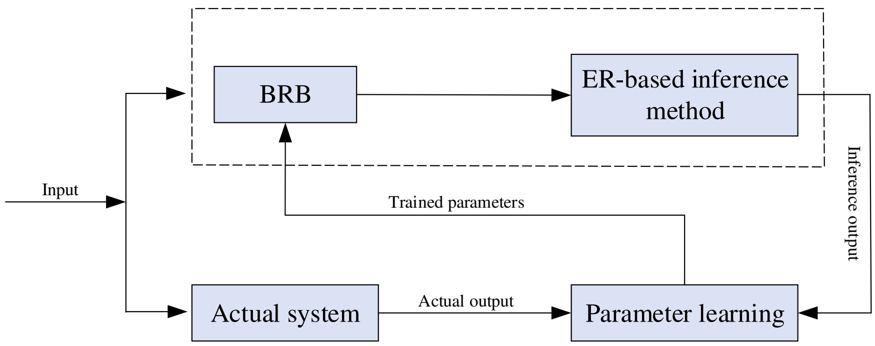

Figure 2. Parameter training model of BRB

2.4 Question

According to the above analysis of the traditional BRB, it can be found that the traditional BRB still has many shortcomings. These deficiencies greatly degrade the performance of the model and must be addressed properly. It can be summarized as follows:

(1) Rule combination explosion in BRB modeling process

The combination explosion of rules is a thorny issue for the BRB. Based on the analysis of the traditional BRB belief table construction method in Section 2.1, it can be found that combining rules in the form of cartesian products easily produces a combination explosion. As the number of premise attributes and referential points increases, the combination of rules increases exponentially. This problem seriously increases the complexity of the model and reduces the efficiency of optimization, so it must be properly solved.

(2) Rule reliability is not fully considered in BRB reasoning

Traditional BRB reasoning uses the ER analytic approach, but the ER analytic approach does not consider rule reliability. This results in an incomplete model that cannot clearly indicate how much each rule contributes to the model.

(3) BRB is not suitable for engineering applications with many attributes and referential values

Traditional BRB modeling methods are not suitable for engineering applications. This is caused by the complex conditions of data noise, a large number of premise attributes, and many referential values and referential points in engineering applications.

Based on the above analysis, a new interval constructed belief rule base with rule reliability (IBRB-r) is proposed. In this model, belief tables are constructed in the form of intervals, and regular reliability is considered. This not only solves the problem of combination rule explosion in the process of traditional BRB modeling but also solves the problem that rule reliability is not considered in the process of traditional BRB reasoning, which can be fully applied to engineering practice.

3. Model description based on IBRB-r

Similar to a traditional BRB, the IBRB-r model is also composed of a series of belief rules. The difference is that the new model has a great change in the modeling method, and rule reliability is introduced into the reasoning method. Assuming that the premise attributes are independent of each other, the IBRB-r model can be described by the IF-THEN statement as:

$$\begin{aligned}
&\mathrm{R}_k: \\
&\text{If } x_1 \in [a_1, b_1] \vee x_2 \in [a_2, b_2] \vee \ldots \vee x_M \in [a_M, b_M] \\
&\text{Then result is } \{(D_1, \beta_{1,k}), (D_2, \beta_{2,k}), \ldots, (D_N, \beta_{N,k})\} \\
&\text{with rule reliability } \gamma_k \\
&\text{and rule weight } \iota_k \\
&k \in \{1, 2, \ldots, L\}, \sum_{i=1}^{N} \beta_{i,k} \le 1
\end{aligned} \tag{11}$$

where the M premise property is denoted as $x_i (i = 1, \ldots, M)$. The reference interval of the M premise attribute can be denoted as $[a_i, b_i]$, where $i = 1, \ldots, M$. The rule reliability of rule k is denoted as γ_k. The rule weight of rule k is denoted as ι_k.

3.1 Problem formulation

In Section 2, the IBRB-r model is proposed based on the lack of traditional BRB analysis. IBRB-r can perfectly solve the problem that the traditional BRB model easily causes the explosion of combination rules and does not consider the reliability of rules. This section will propose how to construct the IBRB-r model from the following three perspectives and formulate the problem.

Problem 1: How to reasonably design the modeling process of IBRB-r model. According to the analysis in Section 2, there are problems in the combination of rules in the modeling process of traditional BRB, which easily causes the explosion of combined rules. Therefore, it is necessary to propose a new rule combination method to reasonably design the modeling process of the new model. The set of parameters after reasonable modeling can be described as:

$$w^0 = \{\beta_1, \ldots, \beta_L, \gamma_1, \ldots, \gamma_L, \iota_1, \ldots, \iota_L\} \tag{12}$$

where L represents the number of rules. $\beta_1, \ldots, \beta_L$ represents the initial belief given by the expert knowledge. $\gamma_1, \ldots, \gamma_L$ represents the initial rule reliability. $\iota_1, \ldots, \iota_L$ represents the initial rule weight. w^0 represents the parameter set consisting of belief, rule weight, and rule reliability.

Problem 2: How to reasonably design the reasoning process of IBRB-r model. The traditional BRB does not consider rule reliability, so it is necessary to add rule reliability in the reasoning process to reasonably design the reasoning process of IBRB-r model. The reasoning process of the new model can be formulated as:

$$Z = inference(w^0, \eta, x) \tag{13}$$

where Z represents the expected utility value of the IBRB-r model. η represents the parameter set in the model inference process. x represents the set of prerequisite attributes of the model. $inference(\cdot)$ represents the formal description function of the inference process.

Question 3: How to reasonably optimize the parameters of IBRB-r model. It is also a problem to select an appropriate optimization algorithm and optimize the parameters of IBRB-r. The optimization process of the new model can be described as:

$$w_{best} = optimize(x, Z, w^0, \zeta) \tag{14}$$

where w_{best} is the optimal parameter set of IBRB-r model. ζ is a set of parameters in the optimization model. $optimize(\cdot)$ is a formal description function of the optimization process.

3.2 IBRB-r modeling process: a new way to set reference intervals and a new way to build belief tables

The modeling method of the IBRB-r model has been greatly changed from that of the traditional BRB. This section will introduce the modeling process of the IBRB-r model in detail from three aspects: problem mechanism analysis, referential point setting and belief table construction.

(1) Mechanism analysis of the problem

In terms of mechanism analysis, IBRB-r is the same as a traditional BRB. They all need to first identify the main influencing factors and possible outcomes of the problem, that is, the premise properties and possible outcomes. After that, data samples of premise attributes and possible outcomes are fed into the model for modeling, reasoning, and optimization.

(2) Setting the new reference interval

Different from the traditional BRB, IBRB-r sets the referential values and referential points of the premise attributes in the form of reference levels and reference intervals. IBRB-r replaces the referential value of the premise attribute with the reference interval and the referential point with the reference level. When the sample data of the premise attribute falls into one of the reference intervals, the corresponding belief rule will be activated. The activated rules are then involved in model inference and optimization.

IBRB-r's new reference interval setting is suitable for engineering applications. In practical engineering applications, monitoring data are easily affected by noise, and data samples have great uncertainty. This method of representing the data reference interval in the form of an interval can better describe the uncertainty of the model to be applied to engineering practice.

For example, suppose two premise attributes each have three reference levels, and the result has four reference levels. The following is an example of how to set the new reference level and reference interval. The reference level and reference interval settings of the two premise attributes are shown in Table 5 and Table 6, respectively. The reference levels and reference intervals of the results are shown in Table 7.

Table 5 Reference levels and reference intervals of prerequisite attribute 1

Referential point	A	B	C
Reference interval	$[a_1, b_1]$	$[a_2, b_2]$	$[a_3, b_3]$

Table 6 Reference levels and reference intervals of prerequisite attribute 2

Referential point	I	J	K
Reference interval	$[c_1, d_1]$	$[c_2, d_2]$	$[c_3, d_3]$

Table 7 Reference levels and reference intervals of results

Referential point	L	M	O	H
Reference interval	1	2	3	4

(3) New belief table construction method

The IBRB-r model proposes a new way of constructing a belief table and a new way of rule combination. Its rules are combined in additive form, not in Cartesian product form. This combination of rules results in a dramatic change in the construction of belief tables and has the following advantages. On the one hand, the combination of rules in the form of addition avoids the exponential growth of the number of rules and can effectively reduce the number of rules. This kind of rule combination perfectly solves the problem of combination rule explosion. On the other hand, the method of constructing a belief table in addition to form is more suitable for engineering practice. Due to the large number of premise attributes, referential points and referential values in engineering practice, it is easy to cause large and complex problems in the model. Using an addition form to construct a belief table can not only solve the problem of combination rule explosion but also simplify the model and reduce the complexity of the model.

The following is an example of the new belief table construction. If the value of each attribute is divided into different non-overlapping interval ranges, each interval corresponds to a rule. If you have two premise properties, premise property 1 has three reference intervals, and premise property 2 has four reference intervals, then the number of rules is 3+4=7. However, the traditional BRB requires 3*4=12 rules. It can be seen that in this case, the number of rules in the IBRB-r model is reduced by 42% compared with the traditional BRB. Assuming premise property 1 has 6 reference intervals and premise property 2 has 10 reference intervals, then the number of rules is 6+10=16. However, the traditional BRB requires 6*10=60 rules. It can be seen that in this case, the number of rules in the IBRB-r model is reduced by 73% compared with the traditional BRB. Given that premise property 1 has 30 reference intervals and premise property 2 has 30 reference intervals, the number of rules is 30+30=60. However, the traditional BRB requires 30*30=900 rules. It can be seen that in this case, the number of rules of the IBRB-r model is reduced by 93% compared with the traditional BRB.

As shown in Table 5-7, assume that the reference levels of premise attribute 1 are A, B, and C, and the reference levels of premise attribute 2 are I, J, and K. Figure 3-4 shows the comparison of the two rule combinations.

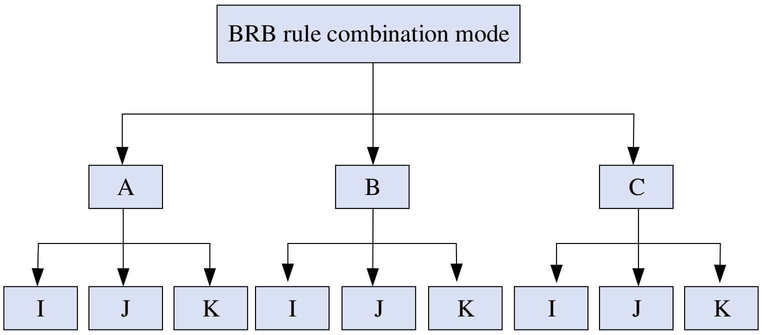

Figure 3. Rule construction in a traditional BRB

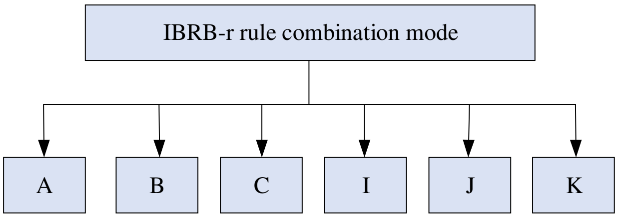

Figure 4. Rule construction of IBRB-r

As a result, the new belief table construction table is shown in Table 5. Compared with the traditional BRB belief table construction method, the belief table constructed by the IBRB-r model greatly reduces the difficulty of rule construction. In addition, the way IBRB-r constructs belief tables completely solves the problem of exploding combination rules.

Table 5 IBRB-r belief degree

Number	Reference interval	Output belief distribution
1	$[a_1, b_1]$	{(L,0.9), (M,0.1), (O,0), (H,0)}
2	$[a_2, b_2]$	{(L,0.8), (M,0.2), (O,0), (H,0)}
3	$[a_3, b_3]$	{(L,0.7), (M,0.2), (O,0.1), (H,0)}
4	$[c_1, d_1]$	{(L,0.6), (M,0.3), (O,0.1), (H,0)}
5	$[c_2, d_2]$	{(L,0.5), (M,0.3), (O,0.2), (H,0)}
6	$[c_3, d_3]$	{(L,0), (M,0.1), (O,0.2), (H,0.7)}

3.3 IBRB-r inference process: New ER rules and a new rule activation mode are introduced

(1) New rule activation mode

After completing the modeling process, the traditional BRB activates the rule when the matching degrees of the prerequisite attributes are not 0. However, the IBRB-r model is different from the traditional BRB. When the data sample of the premise attribute falls in a certain interval, the corresponding rule of this interval is activated. For example, referring to the belief table shown in Table 5, when premise attribute 1 falls in the interval $[a_1, b_1]$ and premise attribute 2 falls in the interval $[c_2, d_2]$, rules 1 and 5 are activated accordingly.

(2) Newly introduced ER rule

The inference process of IBRB-r introduces ER rules, which further considers the rule reliability compared with ER analytic approach. Different ways of obtaining evidence are different, and it is easy to be disturbed by the environment in the process of obtaining evidence. As a result, evidence may not be completely reliable, so ER rule introduces evidence reliability, that is, regular reliability of the IBRB-r model. Different from ER analytic approach, the reasoning process of the ER rule algorithm is mainly shown in Figure 5.

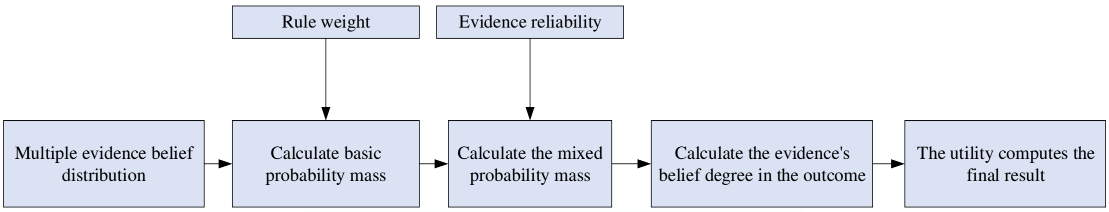

Figure 5. Implementation process of ER rule

The rules in the IBRB-r model modeling process are actually used as evidence in the ER rules. Mark the independent evidence in section L as $e_i(i=1,...,L)$. The identification framework is denoted as Θ, which consists of N assessment level $D_n(n=1,...,N)$. This can be represented as $\Theta=\{D_1,...,D_N\}$. A piece of evidence can then be represented as the following belief distribution:

$$\begin{aligned} & e_i=\{(D_n,\beta_{n,i}),n=1,...,N;(\Theta,\beta_{\Theta,i})\}, \\ & 0\le\beta_{n,i}\le 1,\ \sum_{n=1}^{N}\beta_{n,i}\le 1 \end{aligned} \tag{15}$$

where $\beta_{n,i}$ is expressed as the belief degree that the assessment scheme is evaluated as D_n under the evidence e_i. $\beta_{\Theta,i}$ is expressed as global ignorance, that is, the belief of the $i-th$ attribute with respect to the identification framework Θ.

Suppose that the weight of evidence is $\iota_i(i=1,...,L)$, and $\iota_i\in[0,1]$. The evidence reliability is $\gamma_i(i=1,...,L)$, and $\gamma_i\in[0,1]$. Then, the belief distribution of evidence mixed weighting with reliability can be expressed as:

$$m_i=\{(D_n,\tilde{m}_{n,i}),\forall D_n\subseteq\Theta;(\beta(\Theta),\tilde{m}_{\beta(\Theta),i})\} \tag{16}$$

where the power set is denoted by $\beta(\Theta)$. The mixed probability mass of the $i-th$ attribute in the hierarchy D_n is denoted as $\tilde{m}_{n,i}$, which can be obtained by the following formula:

$$\tilde{m}_{n,i}=\begin{cases}0, & D_n=\varnothing\\ c_{\gamma w,i}m_{n,i}, & D_n\subseteq\Theta, D_n\neq\varnothing\\ c_{\gamma w,i}(1-\gamma_i), & D_n=\beta(\Theta)\end{cases} \tag{17}$$

where the normalized coefficient is denoted by $c_{\gamma w,i}=1/(1+\omega_i-\gamma_i)$, which satisfies $\sum_{n=1}^{N}\tilde{m}_{n,i}+\tilde{m}_{\beta(\Theta),i}=1$. The joint support of any two pieces of evidence $\beta_{n,e(2)}$ is calculated as follows:

$$\hat{m}_{n,e(2)}=\left[(1-\gamma_i)m_{n,j}+(1-\gamma_j)m_{n,i}\right]+\sum\nolimits_{\substack{A\cap B=D_n\\ A,B\subseteq\Theta}}m_{A,i}m_{B,j} \tag{18}$$

$$\beta_{n,e(2)}=\begin{cases}0, & D_n=\varnothing\\ \dfrac{\hat{m}_{n,e(2)}}{\sum\nolimits_{A\subseteq\Theta}\hat{m}_{A,e(2)}}, & D_n\subseteq\Theta, D_n\neq\varnothing\end{cases} \tag{19}$$

Then, the joint support of L independent evidence $\beta_{n,e(L)}$ can be generalized to be computed in the following way：

$$\forall D_n\in\Theta, \hat{m}_{n,e(k)}=\left[(1-\gamma_k)m_{n,e(k-1)}+m_{\beta(\Theta),e(k-1)}m_{n,k}\right]+\sum\nolimits_{A\cap B=D_n}m_{A,e(k-1)}m_{B,k} \tag{20}$$

$$\hat{m}_{\beta(\Theta),e(k)}=(1-\gamma_k)m_{\beta(\Theta),e(k-1)} \tag{21}$$

$$m_{n,e(k)}=\begin{cases}0, & D_n=\varnothing\\ \dfrac{\hat{m}_{n,e(k)}}{\sum\nolimits_{A\subseteq\Theta}\hat{m}_{A,e(k)}+\hat{m}_{\beta(\Theta),e(k)}}, & D_n\neq\varnothing\end{cases} \tag{22}$$

$$\beta_{n,e(k)}=\begin{cases}0, & D_n=\varnothing\\ \dfrac{\hat{m}_{n,e(k)}}{\sum\nolimits_{A\subseteq\Theta}\hat{m}_{A,e(k)}}, & D_n\subseteq\Theta, D_n\neq\varnothing\end{cases} \tag{23}$$

where $k=3,4,...,L$ 。 $\beta_{n,e(k)}$ is the belief degree of the former k attributes with respect to the level D_n after fusion, and $m_{n,e(1)}=m_{n,1}$, $m_{\beta(\Theta),e(1)}=m_{\beta(\Theta),1}$. Through the above formula calculation, the comprehensive assessment results can be obtained as follows:

$$e(L)=\{(D_n,\beta_{n,e(L)}), n=1,...,N,(\Theta,\beta_{\Theta,e(L)})\} \tag{24}$$

The utility at level D_n is denoted as $u(D_n)$. The final output is Z , the expected utility. The final expected utility is calculated as follows:

$$Z=\sum_{n=1}^{N}u(D_n)\beta_{n,e(L)}+u(\Theta)\beta_{\Theta,e(L)} \tag{25}$$

3.4 IBRB-r parameter optimization process: P-CMA-ES algorithm

For the BRB parameter optimization problem, the main optimization methods include sequential quadratic programming (SQP) and constrained particle swarm optimization (PSO), P-

CMA-ES, and the differential evolution algorithm (DE). Zhou et al.[16] used PSO, SQP, P-CMA-ES and other algorithms to optimize the hidden power set BRB and found that the model using the P-CMA-ES algorithm had higher accuracy. Cao et al.[17] compared the optimization effects of DE and P-CMA-ES on interpretable BRBs and found that P-CMA-ES has high accuracy and interpretability.

Due to the superiority of P-CMA-ES in the BRB optimization algorithm, the P-CMA-ES algorithm was used to optimize the initial parameters in subsequent experiments IBRB-r and BRB. The optimization process of the P-CMA-ES algorithm is mainly divided into six parts: parameter initialization, sampling, projection, updating the next generation mean, updating the covariance matrix, and recursive execution[18].

First, the parameters are initialized. Initialize the parameters of the BRB that need to be optimized, including the belief degree, rule weights, and attribute weights. The initialization parameter can be expressed as:

$$w^0 = \sigma^0 \tag{26}$$

$$\sigma^0 = \{\beta_{1,1},...,\beta_{N,L},\gamma_1,...,\gamma_L,\iota_1,...,\iota_L\} \tag{27}$$

Then, the sample. The parameters of each generation are obtained through a sampling operation, which can be expressed as:

$$\sigma_i^{s+1} \sim w^s + \varepsilon^s N(0, C^s), i = 1,...,\lambda \tag{28}$$

where σ_i^{s+1} is used to represent the solution i in generation $s+1$. w^s is used to represent the mean of the s generation search distribution. ε^s is used to represent the step size of generation s. C^s is used to represent the $s-th$ covariance matrix. $N(\cdot)$ is used to represent the normal distribution. λ is used to represent the number of offspring.

Then, the projection. To satisfy the constraints, the solution is projected onto the feasible hyperplane, which can be expressed as:

$$\begin{aligned} &\sigma_{\mathrm{i}}^{s+1}(1+n_e\times(\tau-1):n_e\times\tau) \\ &=\sigma_{\mathrm{i}}^{s+1}(1+n_e\times(\tau-1):n_e\times\tau)-\mathbf{Q}^T\times(\mathbf{Q}\times\mathbf{Q}^T)^{-1} \\ &\times\sigma_{\mathrm{i}}^{s+1}(1+n_e\times(\tau-1):n_e\times\tau)\times\mathbf{Q} \end{aligned} \tag{29}$$

where the parameter vector is denoted as $\boldsymbol{Q} = [1,...,1]_{1\times N}$. The number of variables with constraints is denoted $n_e = 1,...,N$. The number of equality constraints is denoted as $\tau = 1,..,N+1$.

Then, the mean of the next generation search distribution is updated[19]. Denote the weight coefficient as h_i and the offspring population size as ν . The $i-th$ solution of λ solutions of the $s+1$ generation search distribution is denoted as $\sigma_{i:\lambda}^s$. The operation to update the mean can be expressed as:

$$w^{s+1} = \sum_{i=1}^{\nu} h_i \sigma_{i:\lambda}^{s+1} \tag{30}$$

Then, the covariance matrix is updated[20]. Update the covariance matrix using the following equation:

$$C^{s+1} = (1-e_1-e_2)C^s + e_1 P_e^{s+1}(P_e^{s+1})^T + e_2 \sum_{i=1}^{v} h_i (\frac{K_{i:\lambda}^{s+1} - \psi^s}{\eta^s}) \times (\frac{K_{i:\lambda}^{s+1} - \psi^s}{\eta^s})^T \tag{31}$$

where the step of generation s is denoted as η^s . The learning rate is denoted as e_1 and e_2 . The evolutionary path of generation $s+1$ is denoted as P_e^{s+1}. The offspring population of generation s is denoted as ψ^s . The $i-th$ parameter vector in the λ vector of the $s+1$ generation is denoted as $K_{i:\lambda}^{s+1}$.

After the modeling, inference and optimization of the IBRB-r model, the complete IBRB-r model can be obtained. Compared with the traditional BRB model, the IBRB-r model has undergone great changes in the modeling process and reasoning process. Among them, IBRB-r designed a new reference interval and a new confidence table construction method in the modeling process, and a new rule activation method and a new ER rule were designed in the reasoning process. The specific IBRB-r model structure is shown in Figure 6.

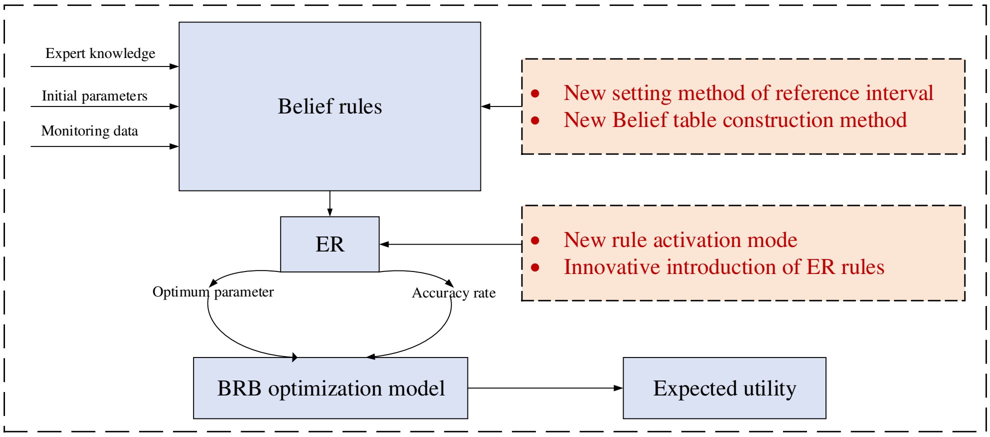

Figure 6. Structure diagram of the IBRB-r model

4. Case study

This experiment takes the result safety assessment of a liquid launch vehicle as the main case to prove the effectiveness and accuracy of the IBRB-r model. At the same time, the effectiveness and accuracy of the IBRB-r model under different data sets are compared. The data "Rocket" for the structural safety of the liquid launch vehicle was collected from a laboratory platform. The monitoring indicators of the experimental platform of a liquid launch vehicle include shaking, inclining, ambient temperature, ambient humidity, etc.[21]. Only the shaking and inclining are used to evaluate the structural safety of the liquid launch vehicle. This is because the ambient temperature and humidity are basically unchanged during the experiment, so the influence of these two factors is not considered temporarily. There are 515 liquid launch vehicle monitoring data, 445 experimental training samples and 70 test samples. The other datasets of the experimental control group are from the public UCI platform dataset.

4.1 Establishment of a structural safety assessment model of a liquid launch vehicle based on IBRB-r

Based on the IBRB-r model, combined with shaking and inclining indexes and assessment results, the safety assessment rules of a liquid launch vehicle structure can be described as IF-THEN rules as follows:

$$
\begin{aligned}
&R_k: \\
&\text{If } Shaking \in [a_1, b_1] \vee Inclining \in [a_2, b_2] \\
&\text{Then safety state is } \{(D_1, \beta_{1,k}), (D_2, \beta_{2,k}), (D_3, \beta_{3,k})\} \\
&\text{with rule reliability } \gamma_k \\
&\text{and rule weight } \iota_k \\
&k \in \{1, 2, ..., L\}, \sum_{i=1}^{3} \beta_{i,k} \leq 1
\end{aligned} \tag{32}
$$

Among them, *Shaking* and *Inclining* of the liquid launch vehicle structure are the prerequisite attributes of IBRB-r. The experiment set 20 reference intervals for each attribute. γ_k is the rule reliability of rule k, and ι_k is the rule weight of rule k. D_1, D_2 and D_3, as the three assessment grades of liquid launch vehicle structure safety, represent normal, medium and low, respectively. Through the analysis of data samples, the reference levels and reference intervals of *Shaking* and *Inclining* are set as shown in Table 8. The larger the sample value of the two attributes, the higher the assessment level. The reference level and reference interval of the assessment result are set as shown in Table 9, which reflects the safe failure probability of the liquid launch vehicle result and thus reflects its safe state.

Table 8 Reference grades and reference intervals for *Shaking* and *Inclining*

No.	Reference degree	*Shaking*	*Inclining*
1	A	[3.1, 3.5]	[0.020, 0.025]
2	B	[3.5, 4.0]	[0.025, 0.028]
3	C	[4.0, 5.0]	[0.028, 0.030]
4	D	[5.0, 5.5]	[0.030, 0.035]
5	E	[5.5, 6.0]	[0.035, 0.038]
6	F	[6.0, 6.5]	[0.038, 0.040]
7	G	[6.5, 7.0]	[0.040, 0.045]
8	H	[7.0, 7.5]	[0.045, 0.048]
9	I	[7.5, 8.0]	[0.048, 0.050]
10	J	[8.0, 8.5]	[0.050, 0.053]
11	K	[8.5, 9.0]	[0.053, 0.055]
12	L	[9.0, 9.5]	[0.055, 0.060]
13	M	[9.5 50]	[0.060, 0.063]

14	N	[50, 55]	[0.063, 0.065]
15	O	[55, 56]	[0.065, 0.070]
16	P	[56, 58]	[0.070, 0.075]
17	Q	[58, 60]	[0.075, 0.078]
18	R	[60, 62]	[0.078, 0.080]
19	S	[62, 64]	[0.080, 0.085]
20	T	[64, 66]	[0.085, 0.090]

Table 9 Liquid launch vehicle safety grades and referential values

Reference degree	D_1	D_2	D_3
Referential value	1.0	0.5	0

After setting the reference level and interval for shaking and inclining, the initial IBRB-r liquid launch vehicle safety assessment model can be obtained. The initial model specifically includes the initial confidence, rule reliability and rule weight, as shown in Table 10. Table 10 not only shows the initial parameters but also clearly shows the method of constructing the confidence table of the IBRB-r model. When the two prerequisite properties are shaking and incline, each with 20 reference intervals, the traditional BRB requires 21 referential points. At this time, the traditional BRB has 21*21=441 rules, while the IBRB-r model only needs 20+20=40 rules, which reduces the number of rules by nearly 91%. This greatly reduces the number of rules and makes the model much less complex.

Table 10 Initial model for safety assessment of liquid launch vehicles

No.	Reference Interval	reliability	weight	Output	No.	Reference Interval	reliability	weight	Output
1	[3.1, 3.5]	1	1	{1,0,0}	21	[0.020, 0.025]	1	1	{0,0,1}
2	[3.5, 4.0]	1	1	{0,1,0}	22	[0.025, 0.028]	1	1	{1,0,0}
3	[4.0, 5.0]	1	1	{0,0,1}	23	[0.028, 0.030]	1	1	{0,1,0}
4	[5.0, 5.5]	1	1	{1,0,0}	24	[0.030, 0.035]	1	1	{0,0,1}
5	[5.5, 6.0]	1	1	{0,1,0}	25	[0.035, 0.038]	1	1	{1,0,0}
6	[6.0, 6.5]	1	1	{0,0,1}	26	[0.038, 0.040]	1	1	{0,1,0}
7	[6.5, 7.0]	1	1	{1,0,0}	27	[0.040, 0.045]	1	1	{0,0,1}
8	[7.0, 7.5]	1	1	{0,1,0}	28	[0.045, 0.048]	1	1	{1,0,0}
9	[7.5, 8.0]	1	1	{0,0,1}	29	[0.048, 0.050]	1	1	{0,1,0}
10	[8.0, 8.5]	1	1	{1,0,0}	30	[0.050, 0.053]	1	1	{0,0,1}
11	[8.5, 9.0]	1	1	{0,1,0}	31	[0.053, 0.055]	1	1	{1,0,0}

12	[9.0, 9.5]	1	1	{0,0,1}	32	[0.055, 0.060]	1	1	{0,1,0}
13	[9.5 50]	1	1	{1,0,0}	33	[0.060, 0.063]	1	1	{0,0,1}
14	[50, 55]	1	1	{0,1,0}	34	[0.063, 0.065]	1	1	{1,0,0}
15	[55, 56]	1	1	{0,0,1}	35	[0.065, 0.070]	1	1	{0,1,0}
16	[56, 58]	1	1	{1,0,0}	36	[0.070, 0.075]	1	1	{0,0,1}
17	[58, 60]	1	1	{0,1,0}	37	[0.075, 0.078]	1	1	{1,0,0}
18	[60, 62]	1	1	{0,0,1}	38	[0.078, 0.080]	1	1	{0,1,0}
19	[62, 64]	1	1	{1,0,0}	39	[0.080, 0.085]	1	1	{0,0,1}
20	[64, 66]	1	1	{0,1,0}	40	[0.085, 0.090]	1	1	{1,0,0}

4.2 Inference and optimization of the structural safety assessment model of a liquid launch vehicle based on IBRB-r

After model establishment, the output belief degree, rule reliability and rule weight will be obtained after model reasoning and parameter optimization. The parameter values after inference and optimization are shown in Table 11.

Table 11 Optimized parameters

No.	Reference Interval	Rule reliability	Rule weight	Output
1	[3.1, 3.5]	0.99	0.71	{0.00,0.02,0.98}
2	[3.5, 4.0]	0.11	0.37	{0.40,0.07,0.53}
3	[4.0, 5.0]	0.49	0.36	{0.21,0.38,0.41}
4	[5.0, 5.5]	0.51	0.80	{0.50,0.25,0.25}
5	[5.5, 6.0]	0.21	0.97	{0.22,0.73,0.05}
6	[6.0, 6.5]	0.97	0.87	{0.00,0.05,0.95}
7	[6.5, 7.0]	0.32	0.68	{0.54,0.13,0.33}
8	[7.0, 7.5]	0.34	0.71	{0.53,0.35,0.12}
9	[7.5, 8.0]	0.55	0.44	{0.17,0.64,0.19}
10	[8.0, 8.5]	0.46	0.55	{0.60,0.40,0.00}
11	[8.5, 9.0]	0.91	0.39	{0.39,0.10,0.51}
12	[9.0, 9.5]	0.97	0.47	{0.00,0.00,1.00}
13	[9.5 50]	1.00	0.82	{0.97,0.03,0.00}
14	[50, 55]	0.60	0.73	{0.31,0.61,0.08}
15	[55, 56]	0.82	0.84	{0.32,0.52,0.16}
16	[56, 58]	0.99	0.33	{0.93,0.01,0.06}
17	[58, 60]	1.00	0.66	{0.97,0.02,0.01}
18	[60, 62]	0.58	0.35	{0.32,0.47,0.21}

19	[62, 64]	0.99	0.71	{0.95,0.05,0.00}
20	[64, 66]	0.22	0.16	{0.69,0.23,0.08}
21	[0.020, 0.025]	0.87	0.85	{0.72,0.07,0.21}
22	[0.025, 0.028]	0.13	0.10	{0.18,0.48,0.34}
23	[0.028, 0.030]	0.91	0.10	{0.34,0.20,0.46}
24	[0.030, 0.035]	0.57	0.19	{0.70,0.10,0.20}
25	[0.035, 0.038]	0.12	0.06	{0.83,0.07,0.10}
26	[0.038, 0.040]	0.74	0.00	{0.70,0.12,0.18}
27	[0.040, 0.045]	0.25	0.08	{0.09,0.11,0.80}
28	[0.045, 0.048]	0.23	0.33	{0.16,0.51,0.33}
29	[0.048, 0.050]	0.09	0.18	{0.05,0.56,0.39}
30	[0.050, 0.053]	0.10	0.15	{0.22,0.30,0.48}
31	[0.053, 0.055]	0.32	0.80	{0.70,0.10,0.20}
32	[0.055, 0.060]	0.77	0.90	{0.53,0.29,0.18}
33	[0.060, 0.063]	0.18	0.74	{0.62,0.25,0.13}
34	[0.063, 0.065]	0.34	0.45	{0.41,0.18,0.41}
35	[0.065, 0.070]	0.84	0.30	{0.23,0.56,0.21}
36	[0.070, 0.075]	0.69	0.87	{0.62,0.26,0.12}
37	[0.075, 0.078]	0.21	0.36	{0.65,0.08,0.27}
38	[0.078, 0.080]	0.57	0.09	{0.75,0.07,0.18}
39	[0.080, 0.085]	0.68	0.18	{0.72,0.06,0.22}
40	[0.085, 0.090]	0.81	0.33	{0.83,0.04,0.13}

4.3 Analysis and comparison of experimental results of structural safety assessment model of a liquid launch vehicle based on IBRB-r

(1) Curve comparison between the model output value and real value

After modeling, reasoning and optimization of IBRB-r, the expected utility value output by IBRB-r model can be obtained. The comparison of the expected utility value and result label of IBRB-r-based liquid launch vehicle structure safety assessment model is shown in Figure 7. In this experiment, the IBRB-r model is also compared with the traditional BRB model by numerical fitting. The comparison between the output expected utility value and the real value of the BRB is shown in Figure 8.

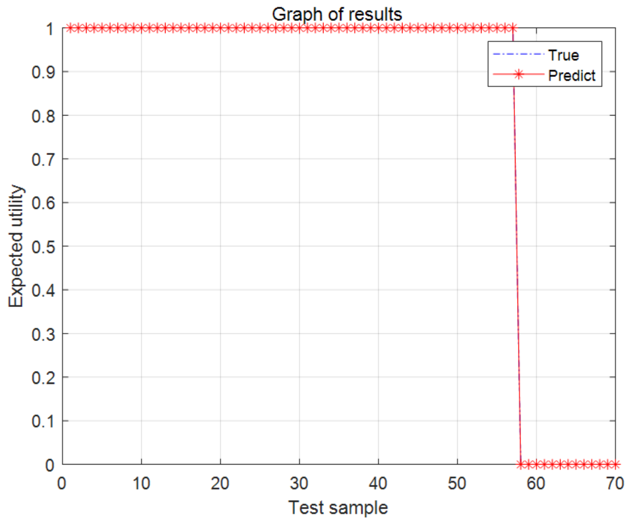

Figure 7 Comparison between the output value of IBRB-r and the real value

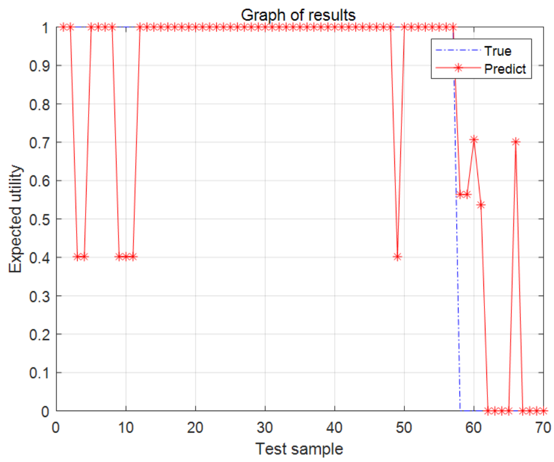

Figure 8 Comparison between the output value of BRB and the real value

In addition, this experiment also combines the IBRB-r model with a backpropagation neural network (BPNN)[22-[23]24], an extreme learning machine (ELM) [25-27]and a radial basis function neural network (RBF)[28], and the comparison between the obtained model output value and the real value is shown in Figures 9-11.

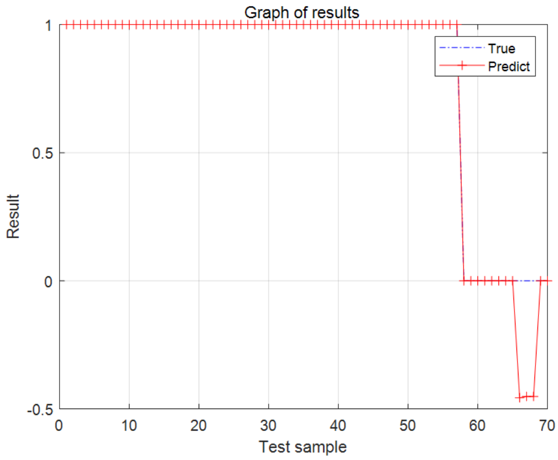

Figure 9. Comparison between the output value of BPNN and the real value

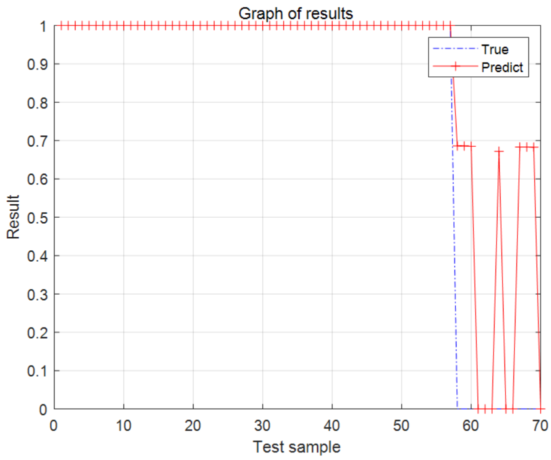

Figure 10. Comparison between the output value of ELM and the real value

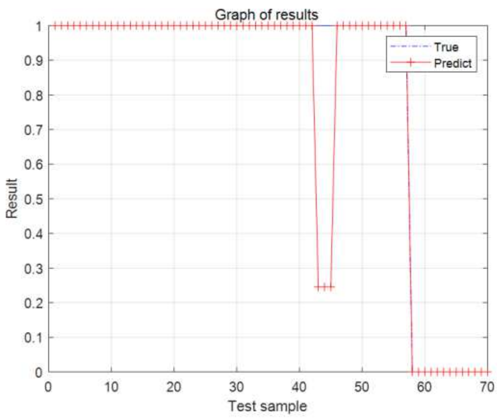

Figure 11. Comparison between the RBF output value and real value

As seen from Figures 7-11, the comparison curve fitting between the output value of the IBRB-r model and the real value is obviously better than that of BRB, BPNN, ELM and RBF. This shows the superiority of IBRB-r model in terms of model accuracy.

(2) Comparison of IBRB-r model before and after improvement

The improved IBRB-r model is compared with the traditional BRB before improvement in the following dimensions, such as the number of parameters, the number of rules, and the complexity, and the experimental results are shown in Table 12.

Table 12 Comparison before and after model improvement

Method	Parameter quantity	Rule quantity	Time	Accuracy
IBRB-r	200	40	6.2070	100%
BRB	1766	441	23.8170	95.71%

According to Table 12, IBRB-r model comprehensively outperforms the traditional BRB model in terms of the total number of parameters, number of rules, complexity and accuracy. The specific summary is as follows:

1) In terms of the total number of parameters, the traditional BRB has 1766 parameters to be trained and optimized. However, the IBRB-r model only requires 400 parameters to train and optimize to surpass the performance of the traditional BRB. Compared with a conventional BRB, IBRB-r reduces the total number of parameters by 89%.

2) In terms of the number of rules, the traditional BRB has 441 rules, while IBRB-r has only 40 rules. Compared to the traditional BRB, the number of rules is reduced by nearly 91%. The complexity of the model is greatly reduced, the number of rules is greatly reduced, and the problem of combination rule explosion is completely solved.

3) In terms of the time required to complete an experiment, IBRB-r takes 6.2070 seconds, while traditional BRB takes 23.8170 seconds. It can be seen that the traditional BRB has high complexity and high time cost.

4) In terms of experimental accuracy, the new IBRB-r model is obviously superior to the traditional BRB. IBRB-r model achieves better results than the traditional BRB with fewer parameters, fewer rules and less time. This result fully proves the feasibility and superiority of the IBRB-r model.

(3) Accuracy comparison of different methods

In the structural safety assessment model of liquid launch vehicle structures, the accuracy of different methods under this dataset is compared in addition to the comparison before and after the model improvement. The experimental results are shown in Table 13. As seen from Table 13, the IBRB-r model is superior to the traditional BRB and data-driven BPNN, ELM and RBF methods in accuracy.

Table 13 Comparison of different methods for structural safety assessment of liquid launch vehicles

Method	IBRB-r	BRB	BPNN	ELM	RBF
Accuracy	100%	95.71%	81.43%	90%	95.71%

4.4 Comparison of different methods in different data sets

In addition to model comparison before and after improvement, this experiment also compares the methods in different data sets. The data set used is from the UCI platform, and the dataset information is shown in Table 14. A total of 5210 training samples and 90 test samples were set for the Banana dataset. In the Haberman dataset, 276 training samples and 30 test samples were set. There are 110 training samples and 40 test samples in the Iris dataset. The Thyroid dataset set 170 training samples and 45 test samples. In the Bupa dataset, 310 training samples and 35 test samples were set. Appendicitis set 86 training samples and 20 test samples.

Table 14 Dataset description

Datasets	Number of attributes	Number of classes	Total number of data
Banana	2	2	5300
Haberman	3	2	306
Iris	4	3	150
Thyroid	5	3	215
Bupa	6	2	345
Appendicitis	7	2	106

The experiment compares the effect of the model before and after improvement in different data sets. Tables 15 shows the comparison of models in each dataset before and after improvement.

Table 15 Comparison of the model before and after improvement under different datasets

Dataset	Method	Parameter quantity	Rule quantity	Time	Accuracy
Banana	IBRB-r	200	40	51.2500	85.23%
	BRB	1766	441	1170.1000	73.86%
Haberman	IBRB-r	140	28	5.3270	93%
	BRB	522	130	35.9240	80%
Iris	IBRB-r	400	80	21.5090	95%
	BRB	1766	441	124.4500	60%
Thyroid	IBRB-r	500	100	14.8560	97.67%
	BRB	1766	441	20.4170	74.42%
Bupa	IBRB-r	600	120	16.5220	91.43%
	BRB	1766	441	40.0660	88.57%

Appendicitis	IBRB-r	700	140	14.6450	85%
	BRB	1766	441	60.7180	80%

Table 15 shows that the proposed IBRB-r model is universal. With multiple data sets, it outperforms the traditional BRB in many dimensions, such as the number of parameters, the number of rules, the complexity of the model and the accuracy. Compared with the traditional BRB, the number of rules and the number of parameters to be trained and optimized in the IBRB-r model are greatly reduced, which perfectly solves the problem of combination rule explosion in the traditional BRB. The IBRB-r model also achieves the same or even higher accuracy than the traditional BRB in a short time. All these results fully illustrate the universality, accuracy and superiority of the IBRB-r model. In addition, the experiment also compares the accuracy of different methods in different data sets, and the results are shown in Table 16.

Table16 Comparison of different accuracies under different datasets

Dataset / Method	Rocket	Banana	Haberman	Iris	Thyroid	Bupa	Appendicitis
IBRB-r	100%	85.23%	93%	95%	97.67%	91.43%	85%
BRB	95.71%	73.86%	80%	60%	74.42%	88.57%	80%
BPNN	81.43%	63.33%	76.67%	82.50%	72.09%	71.43%	70%
ELM	90%	76.67%	83.33%	85%	86.05%	71.43%	70%
RBF	95.71%	71.11%	80%	77.50%	74.42%	80%	65%

(4) Analysis and summary of experimental results

As an improved model, the IBRB-r model is superior to the traditional BRB model in all aspects while retaining the advantages of the traditional BRB model. According to Tables 12-16, compared with the traditional BRB model based on rule modeling, IBRB-r has the following characteristics: 1) the IBRB-r model completely solves the problem of combination rule explosion. When the number of premise attributes and referential points is large, the number of rules in the traditional BRB increases exponentially. This easily leads to the explosion of combination rules. However, IBRB-r constructs confidence tables in the form of interval addition, which completely solves the problem of combination rule explosion. 2) IBRB-r introduces ER rules and considers the reliability of rules, which makes the model more complete. 3) The IBRB-r model uses simpler models to achieve higher model accuracy than the traditional BRB. According to Tables 15-16, the IBRB-r model with different data sets takes less time but has higher accuracy. This fully shows that IBRB-r can reduce the complexity of the model and have high precision. 4) The IBRB-r model has universality and is more suitable for engineering applications. Because the traditional BRB can only use a hierarchical method to deal with multi-attribute and multi-reference problems, the model network structure is large. As a result, it is difficult to apply it to engineering practice. However, IBRB-r can deal with the problem of multiple attributes and multiple referential points. As shown in Tables 15-16, IBRB-r has better performance on different datasets and is suitable for practical engineering applications.

Compared with the data-driven BPNN, ELM and RBF, IBRB-r has the following characteristics: 1) IBRB-r has low dependency on samples and better processing ability for small sample data. It uses a hybrid data-knowledge driven approach and can obtain high-precision results even when the sample data size is small. However, the model accuracy of BPNN, ELM and RBF based on data-driven mode depends on data samples, and the model accuracy is low in the case of small samples. 2) The inference engine of the IBRB-r model is transparent, and the inference process can be traced. However, the internal structure of the data-driven BPNN, ELM and RBF models is not visible. 3) IBRB-r can better process qualitative and quantitative information. IBRB-r can make full use of expert knowledge for qualitative analysis and data for quantitative analysis. This semi-quantitative modeling method can ensure the high accuracy of the experimental results.

5. Conclusion

The traditional BRB rule combination easily causes the combination rule explosion, and it is not suitable to deal with engineering application problems with multiple attributes and multiple referential points. To solve this problem, the IBRB-r model is improved on the traditional BRB. Its main contributions are as follows: (1) IBRB-r innovatively combines rules in the form of interval addition in the modeling process. This method greatly reduces the complexity of the model and completely solves the problem of combination rule explosion. (2) IBRB-r innovatively introduces ER rules in the reasoning process and considers the reliability of rules.

The improved IBRB-r model can deal with multi-attribute and multi-reference problems well and is suitable for engineering applications. In the experimental part, the effectiveness and accuracy of the IBRB-r model are fully verified by analyzing the safety evaluation case of a liquid rocket body structure. At the same time, the experimental results of IBRB-r and other methods in different data sets are compared, which proves the universality of the IBRB-r model.

In future research, more attention can be paid to the following aspects. (1) The application of the IBRB-r model in engineering practice. (2) The method of new reduction rules for BRB. (3) Combination of IBRB-R and fuzzy fault tree.

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Availability of data and materials The datasets used in the comparison trial section can be downloaded at the following link: https://aistudio.baidu.com/aistudio/datasetdetail/172847

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