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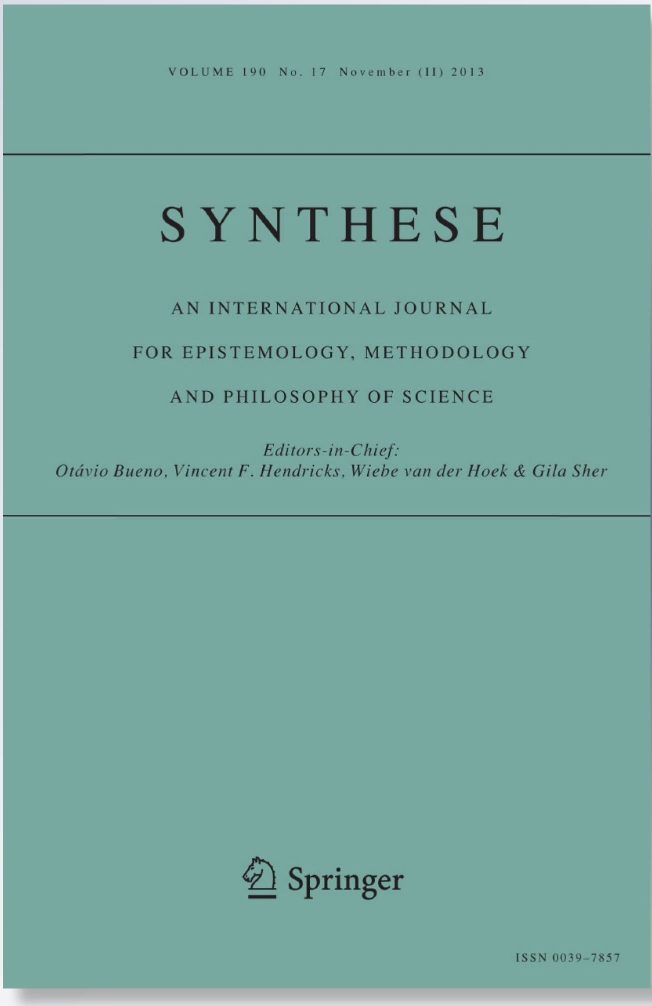

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A modal ontology of properties for quantum mechanics

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Abstract Our purpose in this paper is to delineate an ontology for quantum mechanics that results adequate to the formalism of the theory. We will restrict our aim to the search of an ontology that expresses the conceptual content of the recently proposed modal-Hamiltonian interpretation, according to which the domain referred to by non-relativistic quantum mechanics is an ontology of properties. The usual strategy in the literature has been to focus on only one of the interpretive problems of the theory and to design an interpretation to solve it, leaving aside the remaining difficulties. On the contrary, our aim in the present work is to formulate a "global" solution, according to which different problems can be adequately tackled in terms of a single ontology populated of properties, in which systems are bundles of properties. In particular, we will conceive indistinguishability between bundles as a relation derived from indistinguishability between properties, and we will show that states, when operating on combinations of indistinguishable bundles, act as if they were symmetric with no need of a symmetrization postulate.

Keywords Quantum Mechanics · Modal-Hamiltonian interpretation · Bundles of properties · Contextuality · Indistinguishability

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1 Introduction

From a very general viewpoint, the conceptual challenges of quantum mechanics use to be faced from one of two perspectives. One of them is a sort of instrumentalist stance, which conceives the theory as a mere calculational tool for prediction. The other takes a realist position and tries to elucidate how reality would be if quantum mechanics were true. This paper is clearly framed in this second position: our purpose is to delineate, as precisely as possible for us, an ontology for quantum mechanics that results adequate to the formalism of the theory. Of course, there is not a single possible quantum ontology: the Löwenheim–Skolem theorem teaches us that, if a first order theory has a model, it has a countable model; as a consequence, it cannot have a unique model up to isomorphism. Therefore, we will restrict our aim to the search of an ontology that expresses the conceptual content of the recently proposed modal-Hamiltonian interpretation (Lombardi and Castagnino 2008; Ardenghi et al. 2009; Lombardi et al. 2010; Ardenghi and Lombardi 2011), according to which the domain referred to by non-relativistic quantum mechanics is an ontology of properties.

Although all embodied in the formalism of the theory, the ontological problems posed by quantum mechanics are of different nature: contextuality prevents the simultaneous assignment of determinate values to all the properties of a quantum system; non-separability seems to affect the independent existence of non-interacting systems; indistinguishability challenges certain traditional assumptions about individuals as Leibniz's principle of identity of indiscernibles. The usual strategy in the literature has been to focus on only one of these problems and to design an interpretation to solve it, leaving aside the remaining difficulties. However, one might aspire to formulate a "global" solution, according to which all the problems can be adequately tackled in terms of a single ontology: this is our aim in the present work.

For this purpose, the paper is organized as follows. In Section 2 we will briefly describe the main theses of the modal-Hamiltonian interpretation, stressing its ontological claims on the basis of which the proposed ontology will be introduced in the following sections. Section 3 will be devoted to characterize different kinds of properties as the elemental items of the ontology; here the relation of indistinguishability between properties will be introduced. Section 4 will explain how the features of space and time identify certain properties on the basis of their invariance under space–time transformations. In Section 5 we will introduce the concept of bundle of properties, distinguishing atomic bundles as those composed by the invariant properties identified in the previous section; in this section we will also define indistinguishability between bundles as a relation derived from indistinguishability between properties. In Section 6 we will consider the combinations of bundles, and we will discuss the consequences that the indistinguishability between bundles has on the result of the combination. On this basis, Section 7 analyzes how states, when operating on combinations of indistinguishable bundles, act as if they were symmetric with no need of a symmetrization postulate. Finally, in Section 8 we will review the proposal in order to summarize how this ontology of properties and bundles of properties allows us to face the different ontological challenges of quantum mechanics, and what perspectives are opened up by this new interpretative framework.

2 The modal-Hamiltonian interpretation of quantum mechanics

The main idea behind modal interpretations is that quantum states constrain possibilities rather than actualities: "*the state delimits what can and cannot occur, and how likely it is -it delimits possibility, impossibility, and probability of occurrence- but does not say what actually occurs*" (van Fraassen 1991, p. 279). On the basis of van Fraassen's original idea (1972, 1974), several authors presented realist interpretations that can be viewed as members of the "modal family", since all of them agree on the following points (see Dieks and Vermaas 1998; Dieks 2007; Dieks and Lombardi 2012):

- The interpretation is based on the standard formalism of quantum mechanics.
- The interpretation is realist, that is, it aims at describing how reality would be if quantum mechanics were true.
- The quantum state describes possible properties, with their corresponding probabilities, which evolve unitarily according to the Schrödinger equation.
- A quantum measurement is an ordinary physical interaction: there is no collapse.
- The quantum state refers to a single system, not to an ensemble of systems.

The core feature that distinguishes the different modal interpretations from each other is the rule of actual-value ascription: each member of the family proposes its own rule for selecting the *preferred context*, that is, the set of the observables that acquire an actual value without violating the restrictions imposed by the contextuality of quantum mechanics (Kochen and Specker 1967).

The modal-Hamiltonian interpretation endows the Hamiltonian of the system with a central role, both in the definition of systems and subsystems and in the identification of the preferred context. The first step is, then, to identify the systems referred to by the theory. By adopting an algebraic perspective, a quantum system is defined in the following terms (see Lombardi and Castagnino 2008):

Systems postulate (SP): A *quantum system* $\mathcal{S}$ is represented by a pair $(\mathcal{O}, H)$ such that (i) $\mathcal{O}$ is a space of self-adjoint operators on a Hilbert space $\mathcal{H}$, representing the observables of the system, (ii) $H \in \mathcal{O}$ is the time-independent Hamiltonian of the system $\mathcal{S}$, and (iii) if $\rho_0 \in \mathcal{O}'$ (where $\mathcal{O}'$ is the dual space of $\mathcal{O}$) is the initial state of $\mathcal{S}$, it evolves according to the Schrödinger equation in its von Neumann version.

Of course, any quantum system can be partitioned in many ways; however, not any partition leads to parts that are, in turn, quantum systems (see Harshman and Wickramasekara 2007). On this basis, a composite system is defined as

Composite systems postulate (CSP): A quantum system represented by $\mathcal{S}$: $(\mathcal{O}, H)$, with initial state $\rho_0 \in \mathcal{O}'$, is *composite* when it can be partitioned into two quantum systems $\mathcal{S}^1$: $(\mathcal{O}^1, H^1)$ and $\mathcal{S}^2$: $(\mathcal{O}^2, H^2)$ such that (i) $\mathcal{O} = \mathcal{O}^1 \otimes \mathcal{O}^2$, and (ii) $H = H^1 \otimes I^2 + I^1 \otimes H^2$, (where I^1 and I^2 are the identity operators in the corresponding tensor product spaces). In this case, the initial states of $\mathcal{S}^1$ and $\mathcal{S}^2$ are obtained as the partial traces $\rho_0^1 = Tr_{(2)}\,\rho_0$ and $\rho_0^2 = Tr_{(1)}\,\rho_0$; we say that $\mathcal{S}^1$ and $\mathcal{S}^2$ are *subsystems* of the composite system, $\mathcal{S} = \mathcal{S}^1 \cup \mathcal{S}^2$. If the system is not composite, it is *elemental*.

As we have said above, the contextuality of quantum mechanics prevents us from consistently assigning actual values to all the observables of a quantum system in a given state. Therefore, the second step is to identify the preferred context by means of a rule of actual-value assignment:

> **Actualization rule (AR):** Given an elemental quantum system represented by $\mathcal{S}$: $(\mathcal{O}, H)$, the actual-valued observables of $\mathcal{S}$ are H and all the observables commuting with H and having, at least, the same symmetries as H.

This actualization rule implies that any observable that has less symmetries than the Hamiltonian cannot acquire a definite actual value, since this actualization would break the symmetry of the system in an arbitrary way.

Before proceeding, let us stress that we should distinguish between the *mathematical language* (self-adjoint operators, functionals, eigenstates and eigenvalues of an operator, etc.) and the *physical language* (observables, states, values of the observables, etc.): each term of the mathematical language represents a physical item whose name belongs to the physical language. It is this physical interpretation of the mathematical formalism what turns it into the formalism of a physical theory. But since this distinction would make the reading long and tedious, we shall follow the usual presentations, where both languages are mixed under the assumption that the reader knows the difference between mathematical and physical terms. Nevertheless, we will pay special attention to the *ontological language*: the ontological meaning of language is a central issue when the task is to understand the picture of reality supplied by the interpretation (see Lombardi and Castagnino 2008):

Mathematics	Physics	Ontology
Self-adjoint operators	Observables	Type-properties
Eigenvalues of a self-adjoint operator	Values of an observable	Case-properties
Probability function	Physical probability	Ontological propensity
Functionals	States	Codification of propensities

The goal will be to describe the quantum ontology in the ontological language; for this purpose we will introduce ontological postulates (Proposition 1, Proposition 2, etc.), definitions (Definition 1, Definition 2, etc.), and corollaries (Corollary 1, Corollary 2, etc.).

3 Elements of the ontology: properties

3.1 Type-properties

The assumption of an ontology of substances and properties is implicit in the quantum physicists' everyday discourse. Anchored in the ordinary language of subjects and predicates, they usually speak about electrons as having a certain momentum or photons as having a certain polarization, as if there existed an underlying "something" to which properties are "stuck". But perhaps ordinary language is not the only factor that favors an ontological picture containing the categories of substance and of property. In the discourse of physics, states "label" quantum systems; observables are "applied" to states and are conceived as representing the properties of the system. In the orthodox formalism of quantum mechanics, states, represented by vectors of the Hilbert space, are fundamental from a logical viewpoint; observables, in turn, are logically posterior because represented by operators acting on those previously defined vectors. When the logical priority of states over observables, embodied in the Hilbert space formalism, is endowed with an ontological content, the assumption of an ontology of substances and properties, with the traditional ontological priority of substances over properties, turns out to be "natural".

The modal-Hamiltonian interpretation, on the contrary, adopts an algebraic approach as its formal starting point. In this formalism, the basic elements of the theory are the observables; states are logically posterior since they are represented by functionals over the set of observables. If this logical priority of observables over states is transferred to the ontological domain, observables turn out to embody the representation of the elemental items of the ontology. In other words, whereas a realm of substances and properties seems to be the natural reference of the theory in the Hilbert space formalism, the algebraic approach favors the adoption of an ontology of properties, in which the ontological category of substance is absent. On this basis, we introduce the first ontological postulate:

Proposition 1 *There exist universal type-properties, symbolized as* $[A]$, $[B]$, $[C]$, *etc., each one of which has countless instances. We will symbolize an instance of a universal type-property* $[A]$ *as* $[A^i]$.

An example of universal type-property is the energy $[H]$, which can be instantiated as the energy $[H^1]$ of *this* particular system, for instance, a free electron.

In physics, each instance $[A^i]$ of the universal type-property $[A]$ is represented by an observable A^i belonging to a certain set of observables $\mathcal{O}^i$ $\left(A^i \in \mathcal{O}^i\right)$. Mathematically, each observable is represented by a self-adjoint operator A^i belonging to a set of self-adjoint operators $\mathcal{O}^i$ $\left(A^i \in \mathcal{O}^i \subseteq \mathcal{H}^i \otimes \mathcal{H}^i\right)$.

3.2 Case-properties

The nature of possibility has been one of the most controversial issues in the history of philosophy. However, two general conceptions can be identified, both of which find their roots in Antiquity. One of them, usually called "*actualism*", is the conception that reduces possibility to actuality. This was the position of Diodorus Cronus; in Cicero's words, "Diodorus defines the possible as that which either is or will be" (cited in Kneale and Kneale (1962), p. 117) The other conception, called "*possibilism*", conceives possibility as an ontologically irreducible feature of reality. From this perspective, the stoic Crissipus defined possible as "that which is not prevented by anything from happening even if it does not happen" (cited in Bunge (1977), p. 172). For actualists, the adjective 'actual' is redundant: non-actual possible items (objects, properties, facts, etc.) do not exist, they are nothing. According to possibilists, on the contrary, not every possible item is an actual item: possible items—*possibilia* constitute a basic ontological category (see Menzel 2007)

As we have seen, according to modal interpretations, the formalism of quantum mechanics does not determine what actually is the case, but rather describes what *may* be the case, that is, possibility. In the particular case of the modal-Hamiltonian interpretation, possibility is conceived from a possibilist, non-actualist perspective: what is possible cannot be reduced to actuality and does not need to become actual to be real. This means that reality spreads out in two realms, the *realm of possibility* and the *realm of actuality*. In Aristotelian terms, being can be said in different ways, as possible being or as actual being, and none of them is reducible to the other. On this basis, we will distinguish between possible and actual case-properties.

Proposition 2 *Any instance* $[A^i]$ *of a universal type-property* $[A]$ *has possible case-properties* $[a^i_j]$.

Following with the previous example, we can talk of the possible case-properties $[\omega^i_j]$ of the energy $[H^1]$ of *this* free electron, where $[H^1]$ is an instance of the universal type-property energy $[H]$.

In physics, the possible case-properties $[a^i_j]$ are represented by the possible values a^i_j of the observable A^i. Mathematically, the possible values a^i_j are represented by the eigenvalues of the self-adjoint operator $A^i : A^i \left|a^i_j\right\rangle = a^i_j \left|a^i_j\right\rangle$.

Although the case-properties of an instance of a universal type-property belong to the realm of possibility, one of them may enter the world of actuality:

Proposition 3 *Given any instance* $[A^i]$ *of a universal type-property* $[A]$*, among all its possible case-properties* $[a^i_j]$*, no more than one of them becomes actual. We will symbolize the actual case-property resulting from the actualization of the possible case-property* $[a^i_k]$ *as* $[\boldsymbol{a^i_k}]$*. When one of the* $[a^i_j]$ *becomes actual, say* $[a^i_k]$*, we will say that the instance* $[A^i]$ *actualizes and acquires the actual value (case-property)* $[\boldsymbol{a^i_k}]$.

This ontological postulate implies that an instance $[A^i]$ of the universal type-property $[A]$ may or may not actualize. This is a consequence of the Kochen-Specker theorem, which establishes one of the central differences between the quantum world and the classical world: in the quantum case, omnimode determination does not hold.

In their ordinary language, physicists use to talk about the value that an observable "effectively" acquires, in general as the result of a measurement. However, quantum mechanics does not account for actualization and, as a consequence, neither in the physical language nor in the mathematical language there is a representation of the actual case-property $[\boldsymbol{a^i_k}]$ as different as the possible case-property $[a^i_k]$.

3.3 Indistinguishability between properties

In their book on identity and individuality in physics, French and Krause (2006) note that the category of individual requires some "principle of individuality" that makes an individual to be that individual and not another. The metaphysical question is, then, what confers individuality to individuals. The answers to this question can be broadly divided into two kinds: (i) those that appeal to a "transcendental individuality", that is, something over and above any set of properties of the individual, like, for instance, substance, and (ii) those that appeal to some subset of the properties of the individual, together with some further principle which ensures that no other individual must possess that subset; in this case, the properties that confer individuality are usually spatio-temporal properties under the assumption of impenetrability, which guarantees that two individuals cannot occupy the same spatial region at the same time.

In the discussions about the ontological commitments of quantum mechanics, several authors have pointed out the serious challenge posed by the theory to the notion of individual. Already in the 60s, Heinz Post (1963) argued that elementary particles cannot be regarded as individuals, but they must be seen as "non-individuals" in some sense. Paul Teller (1998) addresses the problem in terms of "haecceity", that is, what makes an object to be different from all others in some way that transcends all properties. According to this author, quantum mechanics provides good reasons for rejecting any aspect of quantum entities that might be thought to do the job of haecceity: "I suggest that belief in haecceities, if only tacit and unacknowledged, plays a crucial role in the felt puzzles about quantum statistics" (Teller 1998, p. 122). In turn, quantum non-separability leads Tim Maudlin to assert that the world cannot be conceived as just a set of separate and localized objects, externally related only by space and time (Maudlin 1998, p. 60). All these authors stress the fact that the notion of individual does not fit into the structure of quantum mechanics (see also French and Krause (2006), and references therein).

The quantum feature that has given rise to a deep skepticism about the notion of individual is the indistinguishability of "identical particles", which is introduced in the formalism of quantum mechanics as a restriction on the set of states: non-symmetric states are rendered inaccessible. Steven French (1998) considers that such a restriction is consistent with the ontological view of particles as individuals: quantum statistics is recovered by regarding those states as possible but never actually realized. However, the restriction on the non-symmetric states has an unavoidable ad hoc flavor in the context of the theory. In this sense, Redhead and Teller (1992) reject the talk of individuals by claiming that the posit of inaccessible non-symmetric states amounts to the introduction of a surplus structure in the formalism. When, on the other hand, indistinguishability is understood in terms of the identification of the complexions resulting from the permutations of identical particles, the traditional idea of individual runs into troubles. Particles cannot be individuated by labeling them, and individuality becomes a controversial notion in the light of this fact.

As we have seen, the modal-Hamiltonian interpretation, on the basis of the algebraic approach, adopts an ontology of properties, where observables represent the elemental items of the ontology. Therefore, our aim will be to derive the indistinguishability of quantum systems from the indistinguishability of their properties:

Definition 1 Any two instances $[A^1]$ y $[A^2]$ of the universal type-property $[A]$ are *indistinguishable*, that is, only numerically different, if their respective case-properties $[a_j^1]$ and $[a_j^2]$ are represented by the same number: $a_j^1 = a_j^2$. We will symbolize indistinguishability as $[A^1] \triangleq [A^2]$.

Corollary 1 *The indistinguishability between instances of universal type properties is an equivalence relation.*

The indistinguishable instances of a given universal type-property *do not satisfy Leibniz's Principle of Identity of Indiscernibles* (for a discussion about this principle in the context of physics, see French and Krause (2006)). This happens not because the principle is false in this case, but because it does not apply: the principle applies to individuals, whereas here we are considering items belonging to the ontological category of property.

In spite of the fact that physicists use to talk about "indistinguishable particles", quantum mechanics is not endowed with a formal relationship for indistinguishability and, as a consequence, there is no mathematical representation for this relation. Some authors developed logical systems specifically designed to deal with indistinguishability in a formal context: the semiextensional quasisets theory, developed by Newton da Costa and Decio Krause (1994, 1997, 1999) (see also Krause (1992), and da Costa et al. (1992)), and the intensional quasets theory, developed by dalla Chiara and di Francia (1993, 1995), describe collections of objects having cardinality but not order type, that is, objects to which the concept of individual of classical logic does not apply. The strategy to be developed in the present work is different, because based on a complete renunciation of the category of individual object.

4 Elements of the ontology: space and time

4.1 Features of space and time

As it is well known, in non-relativistic quantum mechanics space and time have classical features:

Proposition 4 *Space and time are Galilean, that is, time is homogeneous, space is homogeneous and isotropic, and their properties are represented by the group of the Galilean transformations.*

The Galilean group is defined by the commutation relations between its generators. In absence of external fields, these generators represent the basic magnitudes of the theory: the energy, the three momentum components, the three angular momentum components, and the three boost components. In turn, external fields modify the evolution of the system and, as a consequence, their effect is accounted for by the Hamiltonian: it ceases to be the generator of time-displacements in the commutation relations, and only retains its role as generator of the dynamical evolution (see Lombardi et al. 2010). Nevertheless, the Schrödinger equation is still covariant under the Galilean transformations (for the conditions to be satisfied by external fields in order to preserve the covariance of the Schrödinger equation, see Brown and Holland (1999)).

As stressed by Jean-Marc Lévi-Leblond (1974), although it is usual to read that non-relativistic quantum mechanics is covariant under the Galilean transformations, this issue has been scarcely treated in the standard literature on the theory. For instance, the commutation relations defining the Galilean group are often not even quoted in the textbooks on the matter (an exception is Ballentine (1998)). This situation is very odd to the extent that space–time symmetries endow the formal skeleton of quantum mechanics with the physical flesh and blood that make the theory to be a physical theory: they identify the fundamental physical magnitudes of the theory, like energy, position, momentum, spin, etc. From our ontological perspective, we can say that the features of space and time impose constraints to universal type-properties, since those features rule how their instances behave under time-translations, space-translations, space-rotations and velocity boosts. Moreover,

Corollary 2 *Among all the universal type-properties, the features of space and time distinguish, through the Galilean group, the universal type-properties whose instances are invariant under all the transformations of the group. These universal type-properties are the mass* $[M]$*, the internal energy* $[W]$ *and the squared spin* $[S^2]$*, and all the remaining universal type-properties that can be defined as functions of them.*

Mathematically, the Galilean group is a Lie group. The *Casimir operators* of the group are operators that commute with all the generators of the group and, as a consequence, are invariant under all the transformations of the group. In particular, the universal type-properties mass $[M]$, internal energy $[W]$ and squared spin $[S^2]$ are physically represented by the observables mass M, internal energy W and squared spin S^2, which, in turn, are mathematically represented by Casimir operators of the Galilean group.

4.2 Actualization

As we have stressed in Section 2, the main difference among the diverse modal interpretations rests on the rule that selects the preferred context of the actual-valued observables. In the case of the modal-Hamiltonian interpretation, the actualization rule depends on the Hamiltonian, which is not invariant under all the Galilean transformations (it is not invariant under velocity-boosts). Nevertheless, it can be proved that, when the definition of quantum system is considered, the rule admits a Galilean-invariant formulation (see Ardenghi et al. 2009; Lombardi et al. 2010). The natural way to obtain this invariant formulation is by appealing to the Casimir operators of the Galilean group: if the actualization rule has to select a Galilean-invariant set of actual-valued observables, such a set must depend on those Casimir operators, which are invariant under all the transformations of the Galilean group. Precisely,

Proposition 5 (Actualization rule AR'**)** *The only instances of universal type-properties that actualize are the instances* $[M^i]$ *of the mass* $[M]$*,* $[W^i]$ *of the internal energy* $[W]$*,* $[S^{2i}]$ *of the squared spin* $[S^2]$*, and the instances of the universal type-properties that, in each case, are represented by operators obtained as functions of the Casimir operators of the Galilean group.*

Let us recall that a continuous space–time transformation admits two interpretations. Under the active interpretation, the transformation corresponds to a change from a system to another—transformed—system; under the passive interpretation, the transformation consists in a change of the viewpoint—the reference frame—from which the system is described (see Brading and Castellani 2007). Nevertheless, in both cases the invariance of the fundamental law of a theory under its symmetry group implies that the behavior of the system is not altered by the application of the transformation: in the active interpretation language, the original and the transformed systems are equivalent; in the passive interpretation language, the original and the transformed reference frames are equivalent.

Since the Galilean group of continuous space–time transformations is the symmetry group of quantum mechanics, in the passive interpretation language we have to say that the invariance of the quantum laws amounts to the equivalence between inertial reference frames, that is, reference frames time-displaced, space-displaced, space-rotated or uniformly moving with respect to each other: the application of a Galilean transformation does not introduce a modification in the physical situation, but only expresses a change of the perspective from which the system is described. On the other hand, from a realist viewpoint, the fact that certain observables acquire an actual value is an objective fact in the behavior of the system. Therefore, the set of actual-valued observables selected by a realist interpretation must be also Galilean-invariant, since such a set must not change as the result of a mere change of descriptive viewpoint. But the Galilean-invariant observables are always functions of the Casimir operators of the Galilean group. As a consequence, one is led to the conclusion that any realist interpretation that intends to preserve the objectivity of actualization may not stand very far from the modal-Hamiltonian interpretation.

5 Non-elemental items: bundles

5.1 Bundles of instances of universal type-properties

One of the main areas of controversy in contemporary metaphysics is the problem of the nature of individuals or particular objects: is an individual a substratum supporting properties or a mere "bundle" of properties? (see Loux 1998). The idea of a substratum acting as a bearer of properties and/or as the principle of individuation has pervaded the history of philosophy. For instance, it is present under different forms in Aristotle's "primary substance", in Locke's doctrine of "substance in general" or in Leibniz's monads. Nevertheless, many philosophers belonging to the empiricist tradition, from Hume to Russell, Ayer and Goodman, have considered the posit of a characterless substratum as a metaphysical abuse. As a consequence, they have adopted some version of the "bundle theory", according to which an individual is nothing but a bundle of properties: properties have metaphysical priority over individuals and, therefore, they are the fundamental items of the ontology.

Our idea of an ontology of properties favors the conception of a quantum system as a bundle of properties:

Definition 2 A *bundle* h^i is a collection of instances of universal type-properties: $h^i = \left\{[A^i], [B^i], [C^i], ...\right\}$

The concept of bundle is the ontological correlate of the physical concept of *system*. This means that the bundle h^1 is physically represented by a system $\mathcal{S}^1$, which is identified by a set of observables $\mathcal{O}^1$. In turn, mathematically, a physical system $\mathcal{S}^1$ is represented by the set of operators $\mathcal{O}^1$, or by the Hilbert space $\mathcal{H}^1$ if $\mathcal{O}^1 = \mathcal{H}^1 \otimes \mathcal{H}^1$.

Although this proposal is framed in the general idea underlying bundle theories, it is relevant to stress the difference between this conception of quantum system and the classical notion of individual as a bundle of properties. According to the traditional versions of the bundle theory, an individual is the convergence of certain case-properties, under the assumption that the type-properties corresponding to that individual are all determined in terms of a definite case-property. For instance, a particular billiard ball is the convergence of a definite value of position, a definite shape, say round, a definite color, say white, etc. So, in the debates about the metaphysical nature of individuals, the problem is to decide whether this individual is a substratum in which definite position, roundness and whiteness inhere, or it is the mere bundle of those case-properties. But in both cases the properties taken into account are actual: bundle theories identify individuals with *bundles of actual case-properties*.

The fact that the modal-Hamiltonian interpretation adopts an ontology of properties as the reference of quantum mechanics does not mean that it identifies a quantum system with a bundle of properties in the same sense as in traditional bundle theories, designed under the paradigm of classical individuals. We know that not all the instances of the universal type-properties constituting a quantum system can acquire an actual case-property; only the instances selected by the preferred context actualize. Of course, in each context one could insist on the classical idea of instances of universal type-properties with their definite actual case-properties with no contradiction: the picture of a bundle of actual case-properties that defines an individual could be retained in each context. But as soon as we try to extend this ontological picture to all the contexts by conceiving the individual as a bundle of bundles, the Kochen-Specker theorem imposes an insurmountable barrier: it is not possible to actually ascribe the case-properties corresponding to all the instances of the universal type-properties of the system in a non-contradictory manner. Therefore, the classical idea of an individual as a bundle of bundles of actual case-properties does not work in the quantum framework.

Moreover, according to the indeterministic nature of quantum mechanics, even for the instances of universal type-properties that actualize, the particular possible case-property that becomes actual is not determined. So, also for this reason, the strategy of identifying a system as a bundle of actual case-properties is not adequate in the quantum context. Therefore, it seems reasonable to conceive a quantum system as a bundle of instances of universal type-properties instead of relying on actual case-properties. This reading has the advantage of being immune to the challenge represented by the Kochen-Specker theorem, since this theorem imposes no restriction on type-properties.

It is worth noting that, by contrast with traditional bundle theories, when the quantum system is conceived in this way, the account of its identity over time poses no difficulty: the space of observables remains invariant during the entire "life" of the system. On the other hand, nothing happening in the realm of actuality modifies the identity of the quantum system: it is the same no matter what possible case-properties become actual.

5.2 Atomic bundles

Mathematically speaking, the Galilean group has irreducible representations, in which the Casimir operators are multiples of the identity: $M = mI$, $W = wI$ and $S^2 = s(s+1)I$; as a consequence, each irreducible representation is labeled by a triplet (m, w, s). In physics it is assumed that each irreducible representation of the Galilean group represents a kind of *elemental particle*, characterized by its mass m, internal energy w and its spin s. We will use the symbol $\mathcal{P}^{\alpha}$ to denote a particular quantum system that is an elemental particle, and we will define the concept of atomic bundle as the ontological correlate of the physical concept of elemental particle.

Definition 3 A bundle h^{α} is *atomic* if (i) there is no more than one instance of each universal type-property in it, (ii) the instances $[M^{\alpha}]$ of the mass $[M]$, $[W^{\alpha}]$ of the internal energy $[W]$ and $[S^{2\,\alpha}]$ of the squared spin $[S^2]$ always belong to it, and (iii) these instances are represented by observables M^{α}, W^{α} y $S^{2\,\alpha}$, which, in turn, are mathematically represented by operators that are multiples of the identity.

On the basis of this definition it can be said that an atomic bundle is mathematically represented by the Casimir operators M^{α}, W^{α}, $S^{2\,\alpha}$, which are multiples of the identity ($M^{\alpha} = m^{\alpha}I$, $W^{\alpha} = w^{\alpha}I$, $S^{2\,\alpha} = s^{\alpha}(s^{\alpha}+1)I$ respectively) and, therefore, in each irreducible representation of the Galilean group each one of them has a single eigenvalue (m^{α}, w^{α}, $s^{\alpha}(s^{\alpha}+1)$ respectively). This physically means that, in any kind of elemental particle, each one of the observables M^{α}, W^{α}, $S^{2\,\alpha}$ has a single possible value m^{α}, w^{α}, $s^{\alpha}(s^{\alpha}+1)$. In turn, this ontologically means that each one of the instances $[M^{\alpha}]$, $[W^{\alpha}]$, $[S^{2\,\alpha}]$ of the universal type-properties $[M]$, $[W]$, $[S]$ has a single possible case-property $[m^{\alpha}]$, $[w^{\alpha}]$, $[s^{\alpha}(s^{\alpha}+1)]$ respectively. According to the Actualization Rule (Proposition 5), in the atomic bundle h^{α} the instances $[M^{\alpha}]$, $[W^{\alpha}]$ and $[S^{2\,\alpha}]$ actualize, that is, they acquire an actual case-property; but since all of them have a single possible case-property, this is the possible case-property that actualizes:

Corollary 3 *In any atomic bundle h^{α}, the instances $[M^{\alpha}]$, $[W^{\alpha}]$, $[S^{2\,\alpha}]$ of the universal type-properties M^{α}, W^{α}, $S^{2\,\alpha}$ actualize, and they acquire the actual case-properties $[\boldsymbol{m}^{\boldsymbol{\alpha}}]$, $[\boldsymbol{w}^{\boldsymbol{\alpha}}]$, $[\boldsymbol{s}^{\boldsymbol{\alpha}}(\boldsymbol{s}^{\boldsymbol{\alpha}}+\boldsymbol{1})]$ respectively.*

5.3 Indistinguishable atomic bundles

Although physicists, in their ordinary language, use to talk about "indistinguishability among particles", in the ontology proposed here there are not particles as individuals: the ontological correlate of the concept of particle is the concept of atomic bundle, but bundles have no principle of individuality that makes them individuals. So, from our perspective, bundles are not individuals and indistinguishability is not a relationship between individuals. Let us begin by considering the indistinguishability between two atomic bundles, which "inherit" the indistinguishability between their respective instances:

Definition 4 Two atomic bundles h^{α} and h^{β} are *indistinguishable*, symbolized as $h^{\alpha} \triangleq h^{\beta}$, when the respective instances of universal type-properties belonging to them are indistinguishable: $\forall [A_i^{\alpha}] \in h^{\alpha}, \forall [A_i^{\beta}] \in h^{\beta}, [A_i^{\alpha}] \triangleq [A_i^{\beta}] \Rightarrow h^{\alpha} \triangleq h^{\beta}$

Corollary 4 *Indistinguishable atomic bundles are only numerically different.*

Corollary 5 *Given two indistinguishable atomic bundles h^{α} y h^{β}, their respective actual case-properties are represented by the same numbers:*

$$h^{\alpha} \triangleq h^{\beta} \Rightarrow m^{\alpha} = m^{\beta}, w^{\alpha} = w^{\beta}, s^{\alpha}(s^{\alpha} + 1) = s^{\beta}(s^{\beta} + 1).$$

The indistinguishability between two atomic bundles h^{α} y h^{β} expresses the physical indistinguishability between the corresponding elemental particles $\mathcal{P}^{\alpha}$ y $\mathcal{P}^{\beta}$, and agrees with the physical fact that all the elemental particles of the same kind—that is, with the same values of mass, internal energy and spin—are indistinguishable.

6 Combinations of bundles

6.1 Combinations and kinds of combinations

Up to this point we have characterized an ontological concept of indistinguishability that expresses, in ontological terms, the physical relationship between the corresponding elemental particles. Now we have to show that this ontological concept of indistinguishability admits the same formal treatment as that physical indistinguishability receives in the practice of physics. For this purpose, we will begin by defining the combination of bundles.

Definition 5 Given two bundles h^1 and h^2, physically represented by the sets of observables $\mathcal{O}^1$ and $\mathcal{O}^2$ respectively, and mathematically represented by the set of self-adjoint operators $\mathcal{O}^1$ and $\mathcal{O}^2$ respectively, the *combination* between bundles is defined as an operation $h^1 \diamond h^2 = h^{\mathrm{c}}$, where h^{c} is a new bundle physically represented by the set of observables $\mathcal{O}^{\mathrm{c}} = \mathcal{O}^1 \otimes \mathcal{O}^2$ and mathematically represented by the set of self-adjoint operators $\mathcal{O}^{\mathrm{c}} = \mathcal{O}^1 \otimes \mathcal{O}^2$.

Let us stress again that, in the context of this ontological proposal, bundles are not individuals, that is, they do not have a principle of individuality that preserves their identity through change. As a consequence, any combination of bundles is a new bundle, in which the identity of the components is not retained, precisely because they are not individuals.

In the practice of quantum mechanics, certain relationships between the observables of a composite system and the observables of its components are usually assumed. For instance, it is accepted that the observable A^1 of a subsystem S^1 and the observable $A = A^1 \otimes I^2$ of the composite system $S = S^1 \cup S^2$ represent the same property (see discussion in Lombardi and Castagnino (2008), Subsect. 4.1). In order to express this fact in the ontological language, we introduce the following postulate:

Proposition 6 *Given two bundles h^1 and h^2, which combine in the bundle $h^c = h^1 \diamond h^2$, the instance $[A^1] \in h^1$ of the universal type-property $[A]$, represented by the operator $A^1 \in \mathcal{O}^1$, is the same instance as $[A^1 \otimes I^2] \in h^c = h^1 \diamond h^2$ of the same universal type-property $[A]$, represented by the operator $A^1 \otimes I^2 \in \mathcal{O}^1 \otimes \mathcal{O}^2$: $[A^1] = [A^1 \otimes I^2]$.*

It is clear that, when we say that $[A^1] = [A^1 \otimes I^2]$, the symbol '=' represents *logical identity*: 'a = b' means that the terms 'a' and 'b' refer or denote the same item in the ontological realm.

Finally, we will distinguish two forms of combination of bundles, depending on the existence of interaction or not.

Definition 6 Given two bundles h^1 and h^2, which combine in the bundle $h^c = h^1 \diamond h^2$, where the instance $[H^c]$, belonging to h^c, of the universal type-property energy $[H]$ is represented by an observable H^c, represented in turn by an operator $H^c = H^1 \otimes I^2 + I^1 \otimes H^2 + H_{\text{int}} \in \mathcal{O}^c = \mathcal{O}^1 \otimes \mathcal{O}^2$, there are two kinds of combinations between h^1 and h^2:

* when $H_{\text{int}} \neq 0$, the combination is an *interaction*,
* when $H_{\text{int}} = 0$, the combination is an *aggregation*, which will be symbolized as $h^1 * h^2 = h^c$. By extension, we will call the bundle h^c the '*aggregate*' of h^1 and h^2.

6.2 Aggregates of bundles

Here we will only consider aggregates, since we are interested in the statistics of quantum systems. Let us insist that, even in the case of aggregates, in which there is no interaction, the result of the combination is a new bundle where the component bundles, being not individuals, lose their identity. For this reason, it is reasonable to suppose that the aggregation of two bundles is a combination that does not depend on the order of the combination: in the mathematical representation of any instance belonging to the bundle $h^c = h^1 * h^2$, it doesn't matter if the first instance comes from h^1 and the second from h^2, or vice versa.

Proposition 7 *Let us consider the bundles $h^1 = \left\{[A_i^1]\right\}$ and $h^2 = \left\{[A_j^2]\right\}$, physically represented by the sets of observables $\mathcal{O}^1$ and $\mathcal{O}^2$ respectively, and mathematically represented by the sets of self-adjoint operators $\mathcal{O}^1$ y $\mathcal{O}^2$ respectively. The instances $[C^c], [C^{c'}]$ belonging to $h^c = h^1 * h^2$, and represented by the operators $C^c = \sum_{ij} k_{ij} \left(A_i^1 \otimes A_j^2\right) \in \mathcal{O}^c = \mathcal{O}^1 \otimes \mathcal{O}^2$ and $C^{c'} = \sum_{ij} k_{ij} \left(A_j^2 \otimes A_i^1\right) \in \mathcal{O}^c = \mathcal{O}^1 \otimes \mathcal{O}^2$, are such that $[C^c] = [C^{c'}]$ and, therefore, $[A_i^1 \otimes A_j^2] = [A_j^2 \otimes A_i^1]$.*

The mathematical counterpart of this ontological condition, $[A_i^1 \otimes A_j^2] = [A_j^2 \otimes A_i^1]$, is the restriction on the admissible operators representing instances of the composite bundle: the admissible operators are those of the form $C^c = \sum_{ij} k_{ij} \left(A_i^1 \otimes A_j^2\right) \in \mathcal{O}^c \subseteq \mathcal{O}^1 \otimes \mathcal{O}^2$ such that $A_i^1 \otimes A_j^2 = A_j^2 \otimes A_i^1$.

It is interesting to notice that, in the case of aggregates of (indistinguishable or not) atomic bundles, for all the instances of universal type-properties that actualize, the mathematical counterpart of Proposition 7 is satisfied as a direct consequence of the nature of those instances. In fact, given two atomic bundles

$$h^{\alpha} = \left\{[H^{\alpha}], [M^{\alpha}], [W^{\alpha}], [S^{2\,\alpha}], \ldots\right\} \tag{1}$$

$$h^{\beta} = \left\{[H^{\beta}], [M^{\beta}], [W^{\beta}], [S^{2\,\beta}], \ldots\right\} \tag{2}$$

which combine as $h^{\alpha} {*} h^{\beta} = h^{c}$:

(a) Let us consider the instances $[M^{\alpha}] \in h^{\alpha}$ and $[M^{\beta}] \in h^{\beta}$ of the universal type-property mass $[M]$. On the basis of Proposition 6 we know that

$$[M^{\alpha}] = [M^{\alpha} \otimes I^{\beta}] \quad \text{and} \quad [M^{\beta}] = [M^{\beta} \otimes I^{\alpha}] \tag{3}$$

The instances $[M^{\alpha} \otimes I^{\beta}], [M^{\beta} \otimes I^{\alpha}] \in h^{c}$ are mathematically represented by the operators $M^{\alpha} \otimes I^{\beta}, M^{\beta} \otimes I^{\alpha} \in \mathcal{O}^{c} = \mathcal{O}^{\alpha} \otimes \mathcal{O}^{\beta}$ respectively. Therefore, in the mathematical domain,

$$\begin{aligned} M^{\alpha} \otimes I^{\beta} &= m^{\alpha} I^{\alpha} \otimes I^{\beta} = m^{\alpha} \left(I^{\alpha} \otimes I^{\beta}\right) = m^{\alpha} \left(I^{\beta} \otimes I^{\alpha}\right) \\ &= I^{\beta} \otimes m^{\alpha} I^{\alpha} = I^{\beta} \otimes M^{\alpha} \end{aligned} \tag{4}$$

$$\begin{aligned} M^{\beta} \otimes I^{\alpha} &= m^{\beta} I^{\beta} \otimes I^{\alpha} = m^{\beta} \left(I^{\beta} \otimes I^{\alpha}\right) = m^{\beta} \left(I^{\alpha} \otimes I^{\beta}\right) \\ &= I^{\alpha} \otimes m^{\beta} I^{\beta} = I^{\alpha} \otimes M^{\beta} \end{aligned} \tag{5}$$

The fact that $M^{\alpha} \otimes I^{\beta} = I^{\beta} \otimes M^{\alpha}$ ($M^{\beta} \otimes I^{\alpha} = I^{\alpha} \otimes M^{\beta}$) in the mathematical domain expresses the ontological fact that $[M^{\alpha} \otimes I^{\beta}]$ and $[I^{\beta} \otimes M^{\alpha}]$ ($[M^{\beta} \otimes I^{\alpha}]$ and $[I^{\alpha} \otimes M^{\beta}]$) are different names of the same instance of the universal type-property mass belonging to the aggregate. This is reasonable when it is admitted that the result of the aggregation of two bundles must not depend on the order in which the component bundles are combined. In other words, the fact that the tensor product commutes in this particular case expresses mathematically the ontological claim of Proposition 7, according to which

$$[M^{\alpha} \otimes I^{\beta}] = [I^{\beta} \otimes M^{\alpha}] \quad \text{and} \quad [M^{\beta} \otimes I^{\alpha}] = [I^{\alpha} \otimes M^{\beta}] \tag{6}$$

(b) Let us now consider the instance $[M^{\alpha} \otimes I^{\beta} + I^{\alpha} \otimes M^{\beta}] \in h^{c}$, represented by the operator $M^{\alpha} \otimes I^{\beta} + I^{\alpha} \otimes M^{\beta} \in \mathcal{O}^{c} \subseteq \mathcal{O}^{\alpha} \otimes \mathcal{O}^{\beta}$. On the basis of Eqs. (4) and (5), we know that:

$$M^{\alpha} \otimes I^{\beta} + I^{\alpha} \otimes M^{\beta} = I^{\beta} \otimes M^{\alpha} + M^{\beta} \otimes I^{\alpha} \tag{7}$$

And due to the commutativity of the sum,

$$I^{\beta} \otimes M^{\alpha} + M^{\beta} \otimes I^{\alpha} = M^{\beta} \otimes I^{\alpha} + I^{\beta} \otimes M^{\alpha} \tag{8}$$

Therefore, from Eqs. (7) and (8),

$$M^{\alpha} \otimes I^{\beta} + I^{\alpha} \otimes M^{\beta} = M^{\beta} \otimes I^{\alpha} + I^{\beta} \otimes M^{\alpha} \tag{9}$$

Equation (9) expresses, in the mathematical domain, the ontological fact that $[M^{\alpha} \otimes I^{\beta} + I^{\alpha} \otimes M^{\beta}]$ and $[M^{\beta} \otimes I^{\alpha} + I^{\beta} \otimes M^{\alpha}]$ are different names of the same instance of the universal type-property mass belonging to the aggregate. This ontological fact is a particular case of the Proposition 7, according to which

$$[M^{\alpha} \otimes I^{\beta} + I^{\alpha} \otimes M^{\beta}] = [M^{\beta} \otimes I^{\alpha} + I^{\beta} \otimes M^{\alpha}] \tag{10}$$

(c) Analogous conclusions can be drawn for all the instances represented by the Casimir operators of the Galilean group, since they are multiples of the identity, and for all the instances represented by functions of those Casimir operators that are also multiples of the identity.

Summing up, Proposition 7 restricts the operation of aggregation to the particular cases in which the tensor product commutes: $A_i^1 \otimes A_j^2 = A_j^2 \otimes A_i^1$. Nevertheless, this is not yet equivalent to the commutativity of the operation of aggregation itself, which would imply that $A_i^1 \otimes A_j^2 = A_i^2 \otimes A_j^1$. As we will see, the commutativity of aggregation is related with indistinguishability.

6.3 Aggregates of indistinguishable atomic bundles

In an aggregate of indistinguishable bundles, it can be expected that the instances belonging to the aggregate do not distinguish between the component bundles. In the atomic case,

Proposition 8 *In the case of the aggregate of indistinguishable atomic bundles h^{α} and h^{β}, $h^{\alpha} \triangleq h^{\beta}$, the operation of aggregation is commutative: $h^{\alpha} * h^{\beta} = h^{\beta} * h^{\alpha} = h^{c}$*

Proposition 8 adds an additional restriction to the instances belonging to the bundle resulting from the aggregation. Mathematically, it requires that the operators $C^{c} = \sum_{ij} k_{ij} \left(A_i^{\alpha} \otimes A_j^{\beta}\right) \in \mathcal{O}^{c} = \mathcal{O}^{\alpha} \otimes \mathcal{O}^{\beta}$ representing the instances belonging to $h^{c} = h^{\alpha} * h^{\beta}$, when $h^{\alpha} \triangleq h^{\beta}$, are such that $A_i^{\alpha} \otimes A_j^{\beta} = A_i^{\beta} \otimes A_j^{\alpha}$. In other words, the instances $[C^{c}] \in h^{c}$ are represented by observables *symmetric* with respect to the permutation between h^{α} and h^{β}. Taking the simpler case $C^{c} = A^{\alpha} \otimes B^{\beta}$ and considering the components of the operators, if $A^{\alpha} = [a_{ij}]$ and $B^{\beta} = [b_{mn}]$, then $C^{c} = [c_{ijmn}] = [a_{ij} b_{mn}] = [a_{mn} b_{ij}] = [c_{mnij}]$.

Summarizing up to this point: the indistinguishability between instances of universal type-properties leads to the indistinguishability between bundles. Due to their indistinguishability, when two indistinguishable bundles combine in an aggregate, the operation of aggregation commutes. And it is this commutation what leads to the symmetry of the bundle resulting from the aggregation: the operators representing the instances belonging to the new bundle are symmetric with respect to the permutation of the indices corresponding to the two component operators.

As in the case of Proposition 7, in the case of aggregates of indistinguishable atomic bundles, for all the instances of universal type-properties that actualize, the mathematical counterpart of Proposition 8 is satisfied as a direct consequence of the nature of those instances. In fact, given two indistinguishable atomic bundles $h^{\alpha} \triangleq h^{\beta}$,

$$h^{\alpha} = \left\{[H^{\alpha}], [M^{\alpha}], [W^{\alpha}], [S^{2\,\alpha}], \ldots\right\} \tag{11}$$

$$h^{\beta} = \left\{[H^{\beta}], [M^{\beta}], [W^{\beta}], [S^{2\,\beta}], \ldots\right\} \tag{12}$$

which combine as $h^{\alpha} * h^{\beta} = h^{c}$:

a) Let us consider the instances $[M^{\alpha}] \in h^{\alpha}$ and $[M^{\beta}] \in h^{\beta}$ of the universal type-property mass $[M]$. On the basis of the definition of indistinguishability between atomic bundles (Definition 4), we know that

$$h^{\alpha} \triangleq h^{\beta} \Rightarrow [M^{\alpha}] \triangleq [M^{\beta}] \tag{13}$$

Moreover, from Proposition 6 we know that

$$[M^{\alpha}] = [M^{\alpha} \otimes I^{\beta}] \quad \text{and} \quad [M^{\beta}] = [M^{\beta} \otimes I^{\alpha}] \tag{14}$$

Therefore, from Eqs. (13) and (14), we can conclude that $[M^{\alpha} \otimes I^{\beta}] \in h^{c}$ and $[M^{\beta} \otimes I^{\alpha}] \in h^{c}$ are both instances of the universal type-property mass $[M]$ and that they are indistinguishable:

$$[M^{\alpha} \otimes I^{\beta}] \triangleq [M^{\beta} \otimes I^{\alpha}] \tag{15}$$

Mathematically, the instances $[M^{\alpha} \otimes I^{\beta}], [M^{\beta} \otimes I^{\alpha}] \in h^{c}$ are represented by the operators $M^{\alpha} \otimes I^{\beta}, M^{\beta} \otimes I^{\alpha} \in \mathcal{O}^{c} = \mathcal{O}^{\alpha} \otimes \mathcal{O}^{\beta}$ respectively. Therefore, in the mathematical domain:

$$M^{\alpha} \otimes I^{\beta} = m^{\alpha} I^{\alpha} \otimes I^{\beta} = m^{\alpha}\left(I^{\alpha} \otimes I^{\beta}\right) = m^{\alpha}\left(I^{\beta} \otimes I^{\alpha}\right) \tag{16}$$

From the definition of indistinguishability between instances of universal type-properties (Definition 1),

$$[M^{\alpha}] \triangleq [M^{\beta}] \Rightarrow m^{\alpha} = m^{\beta} \tag{17}$$

Replacing Eq. (17) into Eq. (16),

$$m^{\alpha}\left(I^{\beta} \otimes I^{\alpha}\right) = m^{\beta}\left(I^{\beta} \otimes I^{\alpha}\right) = m^{\beta} I^{\beta} \otimes I^{\alpha} = M^{\beta} \otimes I^{\alpha} \tag{18}$$

Therefore, from Eqs. (16) and (18),

$$M^{\alpha} \otimes I^{\beta} = M^{\beta} \otimes I^{\alpha} \tag{19}$$

This shows that, in the bundle h^c resulting from the aggregation of indistinguishable atomic bundles h^α and h^β, the two indistinguishable instances $[M^\alpha \otimes I^\beta] \triangleq [M^\beta \otimes I^\alpha]$ are effectively indistinguishable even mathematically, since they are represented by *the same symmetric operator* $M^\alpha \otimes I^\beta = M^\beta \otimes I^\alpha \in \mathcal{O}^c = \mathcal{O}^\alpha \otimes \mathcal{O}^\beta$. In physical terms, once the particles have entered in the aggregate, their masses are also properties of the aggregate but they are completely indistinguishable.

b) As proved above (see Eq. (10)), $[M^\alpha \otimes I^\beta + I^\alpha \otimes M^\beta] \in h^c$ and $[M^\beta \otimes I^\alpha + I^\beta \otimes M^\alpha] \in h^c$ are different names of the same instance of the universal type-property mass $[M]$ belonging to the aggregate: $[M^\alpha \otimes I^\beta + I^\alpha \otimes M^\beta] = [M^\beta \otimes I^\alpha + I^\beta \otimes M^\alpha]$. We have also proved that this ontological identity is mathematically represented by the equality between operators (see Eq. (9)): $M^\alpha \otimes I^\beta + I^\alpha \otimes M^\beta = M^\beta \otimes I^\alpha + I^\beta \otimes M^\alpha$. Therefore, the instance $[M^\alpha \otimes I^\beta + I^\alpha \otimes M^\beta] = [M^\beta \otimes I^\alpha + I^\beta \otimes M^\alpha]$, physically interpreted as the total mass of the aggregate of the two particles, is also represented by a symmetric operator.

c) Again, analogous conclusions can be drawn for all the instances represented by the Casimir operators of the Galilean group, since they are multiples of the identity, and for all the instances represented by functions of those Casimir operators that are also multiples of the identity.

7 States

7.1 States as codification of propensities

As we have seen in Section 3, the modal-Hamiltonian interpretation adopts an algebraic approach as its formal starting point: observables are the basic elements of the theory, represented by self-adjoint operators, and states are represented by functionals over the set of those operators. This formal priority of observables expresses the ontological priority of properties. Moreover, we have seen that any instance $[A^i]$ of a universal type-property $[A]$ has *possible case-properties* $[a^i_j]$, where possibility is interpreted from a possibilist, non-actualist perspective: each possible case-property may become actual or not. In this interpretative context, states measure an ontological propensity to actualization (see Lombardi and Castagnino (2008), Subsect. 4.2).

Proposition 9 *Given a bundle h^1, its state e codifies the measure of the propensity to actualization for all the possible case-properties of all the instances of universal type-properties belonging to the bundle.*

Mathematically, if the bundle h^1 is represented by the set of operators $\mathcal{O}^1$, the state e is represented by an operator $\rho \in \mathcal{O}^{1'}$, where $\mathcal{O}^{1'}$ is the dual of $\mathcal{O}^1$. Without loss of generality, here we will consider the expectation values, mathematically computed as $Tr(\rho A^1)$: the measures of the propensities to actualization for the possible case-properties $[a^1_i]$ of the instance $[A^1]$ can be obtained by replacing the operator A^1 by the corresponding eigenprojector $|a^1_i\rangle\langle a^1_i|$ in the trace.

7.2 States of aggregates of indistinguishable bundles

In the discussions about "identical particles", the arguments are usually tied to the Hilbert space formalism, whose vectors are the basic mathematical entities representing states that, in turn, are assumed to be applied to particles. In general, the problem is posed in terms of considering the distribution of two particles, 1 and 2, over two states $|a\rangle$ and $|b\rangle$, and the question is: how many combinations (complexions) are possible to obtain the state of the composite system? The classical answer is given by the Maxwell–Boltzmann statistics, according to which there are four possible combinations: the principle of individuality, no matter which one, makes particle 1 in $|a\rangle$ and particle 2 in $|b\rangle$ a different combination than particle 1 in $|b\rangle$ and particle 2 in $|a\rangle$. When the situation is conceived in these terms, the problem consists in explaining why a permutation of the particles does not lead to a different complexion in quantum statistics. The modal-Hamiltonian conception of quantum systems as bundles of instances of universal type-properties, based on the algebraic formalism, leads to a different reading of the problem from the very beginning.

Let us consider the aggregate of two indistinguishable atomic bundles $h^{\alpha} \triangleq h^{\beta}$: $h^{\alpha} * h^{\beta} = h^{\mathrm{c}}$. In the previous section we have seen that, due to ontological reasons, the operators C^{c} representing the instances of the aggregate are operators symmetric with respect to the permutation of the indices coming from h^{α} and h^{β}: $C^{\mathrm{c}} = [c_{ijmn}] = [c_{mnij}]$. In turn, we know that any operator can be decomposed into a symmetric part and an antisymmetric part. In the particular case of the operator ρ, which mathematically represents the state e, it can be expressed as

$$\rho = \rho^{\mathrm{S}} + \rho^{\mathrm{A}} \tag{20}$$

where the symmetric part $\rho^{S} = [\rho^{S}_{ijmn}]$ is such that $\rho^{S}_{ijmn} = \rho^{S}_{mnij}$ and the antisymmetric part $\rho^{A} = [\rho^{A}_{ijmn}]$ is such that $\rho^{A}_{ijmn} = -\rho^{A}_{mnij}$. Therefore, the application of the operator ρ to the symmetric operator C^{c} results:

$$\rho C^{\mathrm{c}} = \left(\rho^{S} + \rho^{A}\right) C^{\mathrm{c}} = \rho^{S} C^{\mathrm{c}} + \rho^{A} C^{\mathrm{c}} = \rho^{S} C^{\mathrm{c}} + 0 = \rho^{S} C^{\mathrm{c}} \tag{21}$$

This means that the antisymmetric part of the state has no effect in its application onto symmetric observables and, therefore, it is superfluous: only the symmetric part has physical and ontological meaning. This means that the states of aggregates of indistinguishable atomic bundles behave as if they were represented by symmetric operators. Moreover, if the state e is represented by the operator $\rho = \rho^{\mathrm{S}} + \rho^{\mathrm{A}} \in \mathcal{O}^{1'}$ and the state e^{S} is represented by the symmetric operator $\rho^{S} \in \mathcal{O}^{1'}$, both e and e^{S} codify the same propensities on the case-properties of the instances of the aggregate.

In the particular case of pure states, the symmetric $\rho^{S} = |\varphi\rangle\langle\varphi|$ may be expressed in terms of a symmetric state vector, $|\varphi\rangle = \left|\varphi^{\mathrm{S}}\right\rangle = 1/2\left(\left|\varphi^{1}\right\rangle \otimes \left|\varphi^{2}\right\rangle + \left|\varphi^{2}\right\rangle \otimes \left|\varphi^{1}\right\rangle\right)$, or in terms of an antisymmetric state vector, $|\varphi\rangle = \left|\varphi^{\mathrm{A}}\right\rangle = 1/2\left(\left|\varphi^{1}\right\rangle \otimes \left|\varphi^{2}\right\rangle - \left|\varphi^{2}\right\rangle \otimes \left|\varphi^{1}\right\rangle\right)$. Therefore, the symmetry or antisymmetry of the vectors representing physical pure states of aggregates of "elemental particles" are not the result of an ad hoc symmetrization or antisymmetrization, but are due to ontological reasons: those symmetry properties of the states are a consequence of the symmetry of the observables of the aggregate, and this symmetry is, in turn, a consequence of the ontological picture supplied by the interpretation.

Summing up, from the perspective given by the modal-Hamiltonian interpretation, the problem of the statistics of indistinguishable "particles" can no longer be interpreted in the same terms as those in which classical statistics is understood, that is, by asking how two particles can be distributed over two states. From the new perspective, after the application of the operation of aggregation, there are not two bundles anymore, but *a single bundle with an internal symmetry* manifested in the symmetry of its elements.

8 Conclusions and perspectives

In the context of the modal-Hamiltonian interpretation, the talk of individual entities, as electrons or photons, and their interactions can be retained only in a metaphorical sense. In fact, in the quantum framework even the number of particles is represented by an observable N, which is subject to the same theoretical constraints as any other observable of the system; this leads, especially in quantum field theory, to the possibility of states that are superpositions of different particle numbers (see discussion in Butterfield (1993)). This fact, puzzling from an ontology populated by individuals, is not surprising when viewed from the modal-Hamiltonian ontological perspective, according to which a quantum system is not an individual but a bundle of instances of universal type-properties. The particle picture, with a definite number of particles, is only a contextual picture, whose validity is restricted to the cases in which the observable N belongs to the preferred context. In these cases, we could metaphorically retain the idea of a composite system composed of individual particles. But in the remaining situations, this idea proves to be completely inadequate, even in a metaphorical sense.

On this basis, the modal-Hamiltonian position moves away from the usual arguments involved in the debate about "identical particles" in a relevant sense. In the proposal of a structure for the ontology referred to by quantum mechanics, the starting point is not the particular problem of the indistinguishability between two or more systems ("particles"), but the purpose of supplying an interpretation compatible with the constraints imposed by the Kochen-Specker theorem: the problem of contextuality resulting from this theorem, since arising in a *single system*, is *logically prior* to any problem invoking *more than one system*. For this reason, we consider that the solution to the problem of indistinguishability should derive from an adequate ontological answer to the problem of contextuality, as one of its consequences.

As we have seen, the problem of contextuality is what led us to discard the idea of a bundle of actual case-properties and to conceive the quantum system as a bundle of instances of universal type-properties. This view has the advantage of being immune to the challenge represented by the Kochen-Specker theorem, since this theorem imposes no constraint on type-properties. But when we restrict our attention to the domain of type-properties, it is difficult to see what subset of the bundle may confer individuality to the quantum system: whereas, for instance, impenetrability can be argued for in the domain of actual case-properties, there is no obstacle to two bundles having the same instance of the universal type-property position. For this reason, instead of insisting on the hard search for some principle of individuality applicable to the universal realm, we prefer to endorse the idea that quantum systems are not individuals: they are strictly bundles, and there is no principle that permits them to be subsumed under the ontological category of individual. Therefore, Leibniz's Principle of Identity of Indiscernibles is not applicable to them: two quantum systems conceived as bundles may agree in all their properties and, nevertheless, they may still be two systems, only numerically different.

In the context of this ontological picture, indistinguishability does not arise as the consequence of a restriction on quantum states (symmetrization or antisymmetrization), nor due to the adoption of a set theory that defines a relation of indistinguishability—different than identity—between individuals. Indistinguishability becomes a consequence of the structure of the ontology, as a result of an internal symmetry of the bundle resulting from the aggregate of indistinguishable atomic bundles. Furthermore, the picture of an ontology inhabited by bundles also modifies the understanding of the problem of non-separability. Once we accept that the original bundle-systems lose their identity in the composite system after interaction, non-separability can no longer be conceived as the consequence of the correlations between two individual particles in different spatial positions, but must be understood in terms of the correlations between the case-properties of a single non-individual bundle.

The features of the modal-Hamiltonian proposal make us to consider whether its actualization rule, expressed in terms of the Casimir operators of the Galilean group in non-relativistic quantum mechanics, can be transferred to quantum field theory by changing accordingly the symmetry group: the definite-valued observables of a system in quantum field theory would be those represented by the Casimir operators of the Poincaré group, and the observables commuting with them and having, at least, the same symmetries. Since M and S^2 are the only Casimir operators of the Poincaré group, they would always be actual-valued observables. This conclusion would stand in agreement with a usual physical assumption in quantum field theory: "elemental particles" always have definite values of mass and spin, and those values are precisely what define the different kinds of elemental particles of the theory. Nevertheless, from our ontological viewpoint, strictly speaking those "elemental particles" are atomic bundles constituted by instances of the universal type-properties mass and squared spin.

Of course, this proposal is not closed at all, but opens up a field of new logico-ontological questions. We know that any system of logic implies an ontology. In fact, our ontological picture does not seem to be adequately captured by any formal theory whose elemental symbols are individual variables referring to objects, whether countable or not. This makes us recall Brower's view that logic is subordinate to mathematics: from the intuitionistic perspective, mathematics is fundamental since it arises out of the intuition of succession in time; logic depends on mathematics to the extent that it is a codification of the constructive activity of mathematicians. The situation in intuitionism may serve to understand, by analogy, the case of the ontology proposed here: the modal-Hamiltonian ontology is fundamental, it does not depend on a logical system but, on the contrary, the appropriate logic must be selected a posteriori, on the basis of its ability to express the structure of this specific ontology. In particular, an ontology populated by bundles of instances of universal type-properties cries for a "logic of predicates" in the spirit of the "calculus of relations" proposed by Tarski (1941), where individual variables are absent. Of course, the development of such a system of logic is far beyond the scope of the present paper, but the task deserves to be considered in a future work.

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