# Abduction in economics: a conceptual framework and its model

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**Abstract** We discuss in this paper the scope of *abduction* in Economics. The literature on this type of inference shows that it can be interpreted in different ways, according to the role and nature of its outcome. We present a formal model that allows to capture these various meanings in different economic contexts.

**Keywords** Abduction · IBE · Economics

## 1 Introduction

The extensive literature on the methodology of Economics does not usually mention abduction as a step of economic reasoning nor reflects on the origin of economic hypotheses. However, given that abduction is a stage of every scientific development, it is also used in Economics. In effect, as Magnani (2009) states, "abduction is a basic kind of human cognition", and thus cannot be absent in economic reasoning.

The pervasiveness of abduction in economic reasoning stems from the difficulties derived from the complexities of real world economic phenomena. In turn, the complexity of economic events arises from their singularity, from the reflexive character of human affairs and from the variable consequences on future events and the

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<sup>&</sup>lt;sup>1</sup> We do not use the word *complexity* as a technical term, but as it is understood in colloquial speech, i.e., something intricate that calls for thorough and possible inexact assessments to understand it.

human reactions to predictions of these events (Grunberg and Modigliani 1954). These characteristics make extremely difficult to derive generalizations through inductive inference. Hence, in addition to empirical data, background knowledge, metaphors, analogies and intuitions also enter into play in order to provide the building material for the models that become central in economic reasoning. The models obtained from such fragmented pieces of information can be naturally seen as the outcomes of *abduction*. Precisely, the goal of this paper is to dissect the process of abduction in Economics.

As a contribution to the understanding of the scope of abduction in this discipline we introduce a simple formal model, expressive enough to cover all the different meanings attributed in the literature to this form of inference. Several examples exhibit how abduction can be used in economic contexts and how it might yield both explanations, useful tools or even further insights for research.

The plan of the paper is as follows: in Sect. 2 we will introduce different notions of abduction; Sect. 3 provides a general account on how abduction operates in Economics. Section 4 presents a formal framework of abduction while in Sect. 5 several examples of abductive economic reasoning are presented in this setting.

## 2 Abduction and inference to the best explanation

The aim of this Section is to present the notions of abduction considered in the current literature on this topic. We will also introduce a related concept, namely *inference to the best explanation* (IBE), and discuss the role of both notions in Economics.

## 2.1 Explanatory abduction

In this subsection we will consider the first and historically initial conception of abduction, i.e., its explanatory role. Although Aristotle discussed abduction under the name of **apagoge** (in *Posterior Analytics* I, 13), the modern view of abduction was originally formulated by Charles S. Peirce. Two meanings can be discerned in his use of the term. First, he considers abduction as a type of logical inference. While deduction infers a result from a rule and a case, and induction infers a rule from the case and the result, abduction infers the case from the rule and the result. So understood, abduction can be identified with the fallacy of affirming the consequent. Thus, its result can only be conditionally accepted.

In a second sense, Peirce sees abduction as a way of arriving at scientific hypotheses. He formulates it as (Peirce et al. 1931–1958, p. 5.189):

- The surprising fact C is observed.
- But if A were true, C would be a matter of course.
- Hence, there is reason to suspect that A is true.

The conception implied in this second formulation is more general than the previous one. A might be a case or a hypothetical rule (Niiniluoto 1999). However, Peirce states that, given that the fallacy of affirming the consequent remains, this procedure of discovery and postulation of hypotheses is only a first step in scientific research.



That is, abduction in this second sense is a heuristic method assisted by some criteria formulated by Peirce. For him, the hypotheses should be explanatory (Peirce et al. 1931–1958, pp. 5.171, 5.189, 5.197), economical (Peirce et al. 1931–1958, pp. 6.395, 6.529, 8.43) and capable of being tested in experiments (Peirce et al. 1931–1958, pp. 2.96, 2.97, 4.624, 5.597, 5.634, 8.740).

Peirce et al. (1931–1958, pp. 2.756–2.760) distinguishes three forms of induction: (1) crude induction, i.e., "common sense" empirical generalizations; (2) quantitative induction, i.e., statistical induction and, (3) qualitative induction, "the collaborative meshing of abduction and retroduction, of hypothesis conjecture and hypothesis testing" (Rescher 1977, p. 3). This abduction corresponds to its second formulation and "retroduction [to] the process of eliminating hypotheses by experiential/experimental testing" (Rescher 1977, ibid.).

Aliseda (2004) holds that

Abduction is thinking from evidence to explanation, a type of reasoning characteristic of many different situations with incomplete information. Note that the word explanation—which we treat as largely synonymous with abduction—is a noun which denotes either an activity, indicated by its corresponding verb, or the result of that activity. These two uses are closely related (...). The process of explanation produces explanations as its products (...).

Boersema (2003) explores the meaning of explanation for Peirce concluding that we can find elements of today's prevailing models of explanation. It is clear that explanation, in usual examples of abduction, often points to causes: "we want to know the cause" (Peirce et al. 1931–1958, p. 7.198; see also pp. 2.204, 2.212, 2.213, 3.395, 3.690, 7.221). For instance, as described in Aliseda (2006):

You observe that a certain type of clouds (nimbostratus) usually precede rainfall. You see those clouds from your window at night. Next morning you see that the lawn is wet. Therefore, you infer a causal connection between the nimbostratus at night, and the lawn being wet.

There is a background knowledge that helps in identifying the explanation, and consequently, the cause.<sup>2</sup> The final aim of scientific knowledge according to Peirce is, as Rescher remarks and argues, "the actual truth" (Rescher 1977).<sup>3</sup> Boersema, however, contends that Peirce does not take into account only metaphysical (causal) aspects in his account of explanation but also epistemological and axiological elements, in the context of a broad theory of inquiry.

The need of retroduction as a second step of Peircean qualitative induction means, however, that the second sense of abduction presents a way to suggest hypotheses or possible explanations pointing to true causes, but not sufficient justifications for accepting them. Nevertheless, Niiniluoto (1999) indicates that Pierce considers instances,

<sup>&</sup>lt;sup>3</sup> Peirce's notion of *truth* is another topic the paper will not deal with. For more on this topic see Haack (1977).



<sup>&</sup>lt;sup>2</sup> A subsequent problem would be to clarify what is the meaning of *cause* for Peirce. For example, he criticizes the "grand principle of causation" (Peirce et al. 1931–1958, p. 6.68). However, dealing with this topic is beyond the scope of this paper.

each of which is "an extreme case of abductive inferences", "irresistible or compelling" and comes to us "like a flash" (Peirce et al. 1931–1958, p. 5.181). In these cases, Niiniluoto contends, "for Peirce [abduction] is not only a method of discovery but also a fallible way of *justifying* an explanation" (Niiniluoto 1999, italics in the original). That is, the strength of this flash would produce a change in the epistemic state of the agent.

Niiniluoto thus distinguishes the procedure of suggesting hypotheses embodying a "weak conception" of abduction and the justification of the hypotheses, which reflects a "strong conception". He equates the latter to an IBE: "in the strong interpretation, abduction is not only an inference to a potential explanation but to the *best explanation*" (Niiniluoto 1999, italics in the original). In short, the weak conception is the best way of arriving at hypotheses without providing justifications. The strong conception, in turn, is a fallible way of justifying explanations. This latter conception implies a change of the epistemic state of the agent by which she accepts the hypothesis, acquiring new knowledge (Thagard 1978). Obviously, this acceptance does not mean that the hypothesis is infallible: it is just an accepted hypothesis.

Lipton (2004) considers IBE as a tool of exploration, generation and justification of the hypotheses. Cresto (2002, 2006) propose conceiving IBE as a complex process which proceeds in two steps: the abductive stage and, after testing, the selective stage, in which the epistemic state of the agent changes. Developing ideas in Levi (1984, 2001) Cresto applies expected utility theory to the IBE, considering the epistemic virtues of simplicity (or parsimony), unification power, fertility, testability, economy, and accuracy as essential elements of her proposal. Similarly, Harman (1965) proposes simplicity, plausibility and explanation power as criteria for judging hypotheses while Thagard (1978) considers consilience (how much a theory explains), simplicity and analogy. In turn, Lipton (2004) mentions unification, elegance and simplicity as virtues leading to what he calls the "loveliest explanation". According to him, this "loveliest explanation" finally becomes the "likeliest explanation". In addition to empirical adequacy, which is required but not sufficient, other epistemic virtues come into play in the whole process of IBE. Each context indicates which virtue has more or less weight in the epistemic utility calculus. For example, as Keynes contends, vagueness may be more virtuous than precision when dealing with the complex social realm. For him, elegance and simplicity may be misleading and economy may be a vice instead of a virtue. This is compatible with Peirce's thought: for him "simplicity" does not imply a "simplified" hypothesis, but "the more facile and natural, the one that instinct suggests, that must be preferred" (Peirce et al. 1931–1958, p. 6.477).

The choice of these criteria is one of the key points in the process of postulating hypotheses and deciding among them (and, eventually, of justifying them). In the formal framework to be presented in Sect. 4 some of the salient criteria are **simplicity** (in Peirce's sense), **unification power** (external coherence), **internal coherence** and **testability**.

## 2.2 Other forms of abduction

Gabbay and Woods (2005, pp. 40, 41, 50, 88, 109; Chap. 5) introduce the concepts of *non-explanatory* and *instrumental* abduction. Magnani (2009) develops these notions



further, exploring different kinds of explanatory abduction (*creative* and *selective*) and adding the concept of *manipulative abduction*. In this subsection we will briefly discuss these notions.

In the sciences it is usual to resort to the use of non-explanatory hypotheses or assumptions, just for their instrumental value. That is, when their utility lies in how they help reach research goals and not in how well they help to explain. Magnani (2009, p. 465; see also p. 77) asserts:

Abduction exhibits an instrumental and strategic aspect, for instance, when intertwined with the exquisite epistemological problem of the role of unfalsifiable hypotheses in scientific reasoning. In this case, an abductive hypothesis can be highly implausible from the "propositional" point of view and nevertheless it can be adopted for its instrumental virtues, such as in the Newtonian case of actionat-a-distance. Highly implausible hypotheses from the "propositional" point of view can be conjectured because of their high "instrumental" plausibility, where a different role of characteristicness is at stake. We have to note that in some sense all abductions embed instrumental factors. In the general case, one accepts because doing so enables ones target to be attained, notwithstanding that lacks the relevant epistemic virtue. However, in cases such as Newton's, is selected notwithstanding that it is considered to be epistemically hopeless.

As we will see in subsequent sections, Economics also uses extensively these kinds of abductions.

Magnani (2009, p. 39ff.) also considers *manipulative* abduction, defined as (pp. 465–466):

Manipulative abduction is a process in which a hypothesis is formed and evaluated resorting to a basically extra-theoretical and extra-sentential behavior that aims at creating communicable accounts of new experiences to integrate them into previously existing systems of experimental and linguistic (theoretical) practices. Manipulative abduction represents a kind of redistribution of the epistemic and cognitive effort to manage objects and information that cannot be immediately represented or found internally. An example of manipulative abduction is the case of the human use of the construction of external diagrams in geometrical reasoning, useful to make observations and "experiments" to transform one cognitive state into another for example to discover new properties and theorems.

As we will argue in Sect. 3, Economics has always used diagrams that do not only represent theory but also help to discover aspects of it. Furthemore, different kind of exploratory data analyses are performed just to detect new properties and conjectures.

Finally, we want to mention a classification of explanatory and manipulative abduction proposed also in Magnani (2009), distinguishing between *creative* and *selective* abduction. Creative abduction, as its name indicates, advances completely new hypotheses. Instead, selective abduction *is the process in which a hypothesis is abductively selected from a pre-stored encyclopedia of "abducibles"* (p. 468).



## 3 Abduction in economic reasoning

We contend that abduction is an essential component of economic analysis, theoretical and practical. Economic theory generally proceeds by constructing models (Morgan and Morrison 1999), that is, mental schemes based on mental experiments (Nersessian 1992). They are often written in mathematical language but, apart from their formal expression, they use metaphors, analogies and pieces of intuition to motivate their assumptions and to give support to their conclusions (Frigg 2012). In dealing with ongoing economic processes, agents and analysts must generally evaluate whether the situation resembles in a relevant way some instances observed or studied in the past, and whether this warrants applying somehow the "lessons" drawn from those experiences. The problem in judging "whether some pasts are good references for the future" becomes particularly severe when the economy is seen to undergo important changes (Crespo et al. 2010). Simplicity in the Peircean sense, explanatory power, coherence and testability are rather unconsciously considered in this abduction of possible explanatory models.

The retroductive phase also involves problems implying abductive-like decisions. Although it sounds rather obvious, it must be recognized that there is a gap between the formulation of a question to be answered through measurement and the actual measurement providing the right answer. The difference arises from the fact that problems are qualitative while data are quantitative. In consequence, rough data (which certainly are the quantitative counterparts of qualitative concepts) must be organized according to the qualitative structure to be tested. That is, a correspondence between theory and data must be sought. So, for example, in economic theory there exists a crucial distinction between ordinal and cardinal magnitudes in the characterization of preferences. But once measurements are involved it is clear that the theoretical relational structure must be assumed to be homomorphic to a numerical structure (Krantz et al. 1971). This implies that if there exists a data base of numerical observations about the behavior of a phenomenon or a system, we might want to infer the properties of the qualitative relational structure to which the numerical structure is homomorphic. Of course, this is impaired by many factors:

- The syntactic representation of the qualitative structure can be somewhat ambiguous (Barwise and Hammer 1994).
- Although the observations fall in a numerical scale, the real world is too noisy, allowing only a statistical approximation.
- The complexity of the phenomena may be exceedingly high. Then, only rough approximations may make sense.

These factors, which preclude a clear cut characterization of the observations, leave ample room for arbitrary differences. In this sense, the intuition and experience of the economist and the econometrician determine the limits of arbitrariness in an abductive-like fashion. As an example, consider the question "Did a specific economy grow in the last year?" To provide an answer, first, one has to define clearly what does

Therefore, it remains clear that abduction in economics is mostly "model-based" rather than "sentential". For the meaning of these notions see Magnani (2009).



it mean that an economy grows and which variables can be used to measure the phenomenon of growth. Economic theory states that economic growth means growth of the national income. But in order to answer the question an economist has to define what real world data will represent national income; i.e. she has to embed the available data into the framework given by the theory. In this case the national product is an available variable which is well-defined and relatively easy to measure and is considered (theoretically) equivalent to the national income. Therefore it is easy to check out whether the economy grew or not. But in the case where the question is something like "Did well-being increase in the last twenty years?" the procedure is far less simple. How do we define well-being and moreover, how do we make the concept operational? This is where the intuition of the economist is called in. Although theoretical concepts may be lacking, a set of alternative models of the notion of wellbeing and its evolution in time should be provided in order to check out which one fits better the real world data. When this question is settled it is possible to consider the development of a theory formalizing the properties satisfied in the chosen model. That is, when the abducting process is completed the theory-building phase can start.

The inferences that allow economists and econometricians to detect patterns in reams of data cannot be called statistical inductions. They are more a result of a detective-like approach to scarce and unorganized information, where the goal is to get clues out of data bases of observations and to disclose hidden explanations that make them meaningful. In other words: it is a matter of making guesses, which later can be put in a deductive framework and tested by statistical procedures. So far, it seems that it is just an "artistic" feat, which can only be performed by experts.<sup>5</sup>

As indicated by Autor (2012), summarizing the experience of editing a scholarly journal, the process of economic reasoning can be described as:

Economic research often begins with a big interesting question, which also tends to be sprawling and unmanageable. So the researcher breaks down the question into chunks, carefully examining assumptions and interpretations along the way, diving deeply into analysis. Papers in the refereed literature result from such deep dives. But as these papers are discussed and digested, their lessons are brought back from the deep where they can be more broadly appreciated. This process is as indispensable for scholars as it is for end users. Academics master and ultimately digest frontier scholarship by distilling its insights down to a few big facts, simple models, and reliable predictive relationships.

Economists have a background of general rules. When a surprising or abnormal fact appears, the first step is to try to come up with an explanation according to those rules. By a surprising or abnormal event we mean one that creates an "irritation of doubt" (Peirce and Carolyn 1987, pp. 261 and 263, quoted by Magnani 2009, p. 3). As Gabbay and Woods (2005) assert, abduction is triggered by the irritation of ignorance. This irritation may be weak or strong. It is weak when we can suspect that the event could be explained using our previous knowledge (rules, theories). Instead, it is strong when we do not find in our previous knowledge any possible explanation. Weak ignorance

<sup>&</sup>lt;sup>5</sup> This might be a reason for why formal logicians, until recently, did not intensively study abduction in contrast to the other forms of inference.



or abnormal/surprising events often lead to selective abduction while strong ignorance to creative abduction.

The best explanation obtains by delimiting the possible hypotheses until only one of them remains. In this process the economist uses information about similar situations as well as the features of the specific case to capture simple and coherent hypotheses and models.

Let us give a sketchy description of the reasoning process in Economics. Economists have a background of general rules. When a surprising fact appears the first step is to try to come up with an explanation according to those rules. The best explanation obtains by delimiting the possible hypotheses until only one of them remains. In this process the economist uses information about similar situations as well as the features of the specific case to capture simple and coherent hypotheses and models.

We may distinguish the following steps in this process:

- 1. An abnormal/surprising/ignorance irritating event (*query event*) is detected, requiring an explanation.
- 2. The event is carefully described.
- 3. Some stylized facts are extracted from the description.
- 4. Situations sharing the same stilized facts are given particular attention.
- 5. Possible explanations based on a theory, on a modified theory, on a combination of theories (sometimes deciding on possibly competing theories) or on an entirely new theory are conceived.
- 6. Formal expressions, capturing the relations deemed essential in the explanation of the relevant stylized facts, according to the previous step, are formulated.
- 7. Only those combinations of deductive chains and inductive plausibility that are both externally and internally coherent are chosen, discarding other possibilities.
- 8. This provides an original coherent explanation (or set of explanations) of the event.
- 9. The conclusions are tested.

Abduction is hidden, particularly in steps 3, 4, 5 and 7. Steps 6 and 7 are mostly deductive. Step 9 is also inductive and retroductive. The whole process is a Peircean qualitative inductive process in the already mentioned sense defined by Rescher (1977), but almost always also uses instrumental assumptions. The so-called *as if* arguments are pervasive in Economics.

Good economists have a guess instinct (Peirce et al. 1931–1958, pp. 6.476–6.477) present in their scientific processes. This is not a mysterious miracle but an intellectual intuition, stemming from a theoretical framework or background knowledge, of experience, of hard work with theories, models and data. This leads good economists to foresee a set of probably successful models. Combining this gift with hard empirical work economists often overcome the problems of under-determination of theories by formulating local or context-dependent theories. Context-dependence is a characteristic feature of IBE (Cresto 2006; Day and Kincaid 1994).

However, the economists always try to improve their models. This is because, given the fluctuating ontological condition of the economic material, a close relation with real situations is needed. The analogies sometimes work and sometimes not. Old or conventional theories may be misleading. Thus, economists need that special "gift for using vigilant observation to choose good models" (Keynes 1973, p. 297).



This improvement, however, has a limit. On one hand, the frequent urgency of decisions that cannot wait for further investigation, and the economy of research (Rescher 1977, p. 65ff., extensively quoting Peirce), actually lead to accept conclusions that are fallible but reasonable inferences to the best explanation. On the other hand, the problems of quantification—be they of conceptual, institutional, accuracy of data, calculation and even presentation nature—also lead the researcher to accept a sufficiently examined fallible conclusion as a good one.

Economists, as has already been mentioned in the previous section, also use manipulative abduction. This is a category that might include very different ways to proceed: drawing diagrams representing economic relations, running laboratory experiments, finding natural experiments, applying specific economic policies are based on economic hypotheses and theories, exploring data in different ways, etc. However, they also serve as hypotheses that might generate new knowledge.

### 4 A formal framework

The ideas presented in the previous sections can be easily formalized. The idea is to see hypotheses and conjectures as *structures* which will be chosen as answers to questions if they satisfy some *methodological* criteria. Other kinds of criteria can be added as extra constraints in the process of answering queries. This description of abduction emphasizes on the existence of events that require an explanation. While it is rather easy to resort to usual tools of the trade, in most of the academic activity in the field and even in some of its practical applications open questions are pervasive. No Ph.D. thesis, no scholarly journal article and no research report involves a mechanical application of time revered methods, but at a certain point requires a formulation of alternative explanations and to weigh pros and cons of them, plus the use of different criteria for their assessment. We contend that any such open question, puzzling observation or mere doubts on the validity of some principle amount to different incarnations of queries that may trigger the process of abduction.

In order to explain how abduction helps in economic model building, we need some previous definitions:<sup>6</sup>

**Definition 1** Given a first order language  $\mathcal{L}$  a structure is  $\Delta = \langle \mathbf{N}, \gamma, \mathcal{F}, \Pi \rangle$ , where  $\mathbf{N}$  is a set of individuals;  $\gamma$  is a function that assigns an individual to each constant of  $\mathcal{L}$ ,  $\mathcal{F}$  is a family of endomorphic functions on  $\mathbf{N}$ , while  $\Pi$  is a set of predicates on  $\mathbf{N}$ . An interpretation of any consistent set of well formed formulas of  $\mathcal{L}$ ,  $\mathcal{T}(\mathcal{L})$  obtains through a correspondence of constants, function symbols and predicate symbols to  $\Delta$ . A model of  $\mathcal{T}(\mathcal{L})$  is an interpretation where every interpreted formula is true.

A structure can be thought of as a database plus the relations and functions that are, implicit or explicitly, true in it. An interpretation is a structure associated to a certain set of well-formed formulas (when deductively closed this set is called a *theory*). If, when replacing the constants by elements in the interpretation and the propositional functions by relations in the structure, all the formulas are made true



<sup>&</sup>lt;sup>6</sup> For a precise characterization of these notions see Shoenfield (1967).

in the interpretation, this structure is called a model. To say that abduction helps in model building means that it is a process that embeds the real-world information in a certain structure that is assumed to be the model of a theory or at least of a coherent part of one.

Our approach, unlike more traditional epistemological analyses in Economics, is based on the distinction "syntactic vs. semantic" as it is understood in Logic. The syntactic approach consists in postulating a formal language, a class of well-formed formulas, a subset of the former called axioms and applying on them some inference rules that just take some formulas and yield others. The set of formulas obtained up from the axioms is called a theory. The semantic approach instead, starts from a class of objects and relations among them. A theory is, again, a family of formulas in a formal language that are interpreted in terms of such objects and relations. Soundness and completeness are properties that relate the theory to its domain. A large corpus of work has been devoted to studying the syntactic/semantic connections and some interesting results (that matter for our approach) were obtained when purely syntactic theories were given semantic counterparts. The most salient example of this is modal logic, studied in pure syntactic terms until Saul Kripke introduced possible worlds semantics (Kripke 1959). This case is relevant for Economics, particularly in the areas of Game and Decision Theories where knowledge and belief can be either defined syntactically or by means of the semantics of state spaces (Dekel and Gul 1997).

In Economics, instead, it is usual to find that there is not a clear distinction between what is meant by "theory" and by "model". One reason is that for most applications, it is excessive to demand a theory to be deductively closed, which means that all its consequences should be immediately available. In the usual practice, statements are far from being deduced in a single stroke. On the other hand -and this clearly explains the confusion between theory and model- most scientific theories have an intended meaning more or less clear in its statements. This does not preclude the formulation of general and abstract theories, but their confrontation with data are always mediated by an intended model (Stigum 1990).

A concern that may arise from our approach is whether any economically meaningful assertion can be embedded in a first-order language. The point is that most theories
of sets, Zermelo-Frenkel and others, intended to provide a comprehensive foundation
for mathematics, are first order (Devlin 1993). Since most of the economic statements
can be expressed as set-theoretic expressions, it seems that the previous definition of
a structure is enough for our purposes. The approach followed here pays tribute to
the fact, pointed out in the work of authors like Hausman (1992), Hands (1985) and
Vilks (1992), that work in Economics is of *semantic* nature, instead as *syntactic*, since
mathematical structures, statistical data, etc., belong to the former realm. <sup>7</sup> While
the label "syntactic" is sometimes used in Economics to refer to axiomatizations of
some phenomena of interest, the truth is that those axioms are sentences about entities in (usually) a real space. Therefore, the syntax already presupposes a semantic
realm. Furthermore, the theorems are usually drawn not by applying formal inference
rules but by using the topological or functional properties of the real space on which

<sup>&</sup>lt;sup>7</sup> For more on this see Aumann (1999a,b), in which the semantic (mathematical and statistical) approach, which is more natural for economists, is shown to be quite distinct from the syntactic (first-order logic) one.



the theory is predicated (eg. Arrow-Debreu's general equilibrium theory). As noted in Hands (2001, p. 311), perhaps the easiest way to understand this version of the semantic view is to contrast it with the standard, **statement** view of scientific theories. Anyway, what economists call a "model" can be seen as a structure and the statements that are true in it.

In our approach abduction consists in finding a theory, understood as a class of sentences about a field of interest, obtained by choosing the appropriate objects from that field and the relations among them. What we need is to translate the observations into a formal structure, be it of quantitative or qualitative nature (Tohmé et al. 2011) such that (Levesque 1984):

- Each element of interest has a symbolic representation.
- For each (simple) relationship, there must be a connection among the elements in the representation.
- There exist one-to-one correspondences between relationships and connections, and between elements in the data and in the representation.

This representation of the real world information,  $\Lambda$ , facilitates the abduction, by means of its comparison with alternative structures. The result of the abduction will determine an implicit representation of data, as we see if we consider the following definition:

**Definition 2** Given a set of structures  $\{\Delta_i^{\mathcal{C}}\}_{i\in I}$  where I is a set of indexes, selected for satisfying a set of criteria  $\mathcal{C}$ , an abduction is the choice of one of them, say  $\Delta^*$ , by comparison with  $\Lambda$ .

In words, given a class of criteria, there might exist several (although we assume only a finite number) possible structures that may explain the data in  $\Lambda$ . To *abduce*  $\Lambda$ , is to choose one of them.

Up to this point, we have presented a formal reconstruction of the road map presented in Sect. 3, i.e. steps 1 to 9 in the construction of an economic model. The *query event* is detected and described in stylized form by means of  $\Lambda$ , in which other, related, stylized facts are included. Then, candidate explanations are represented by  $\{\Delta_i^{\mathcal{C}}\}_{i\in I}$  and the choice of  $\Delta^*$  follows a process of evaluation in which only coherent pieces of information (internally, in terms of the concordance with the structures and externally, in accord with the criteria) are kept. The key to the process, nevertheless, resides in the coherence with the criteria, which might include also testability.

More generally, the criteria represent all the features that the economist wants to find incorporated into the chosen structure. We will distinguish two main classes of criteria, *methodological* and *substantial*. The former involve all the considerations of *a priori* epistemological nature, that discard possible explanations for the data on the basis of the formal aspects of the structures independently of their factual content. Substantial criteria, instead, focus on how structures are related to other pieces of knowledge. So, for instance, Occam's Razor is a methodological criterion while for (Peirce's favorite example of abduction) Kepler's discovery of the elliptical shape of the planetary trajectories around the sun the idea that trajectories had to be conic curves was a substantial criterion (Marostica 1993).

Both classes of criteria can be put together in a single set  $\mathcal{C}$  such that the set of structures that satisfy them is defined as follows:



**Definition 3** A criterion  $c_j$  defines a set of structures in which it is satisfied,  $\{\Delta_i\}_{i \in I_j}$  (where  $I_j$  is a set of indexes corresponding to this criterion). Then,  $\mathcal{C} = \{c_j\}_{j \in J}$  defines a set of structures  $\{\Delta_i^{\mathcal{C}}\}_{i \in I} = \cap_{j \in J} \{\Delta_i\}_{i \in I_j}$ .

In general, the number of criteria is reduced in order to ensure that the set of possible structures is not empty. The comparison of the structures with the data determines an order on  $\{\Delta_i^{\mathcal{C}}\}_{i\in I}$ :

**Definition 4** Given  $\Lambda$ , and two possible structures  $\Delta_j$ ,  $\Delta_l$  we say that  $\Delta_j \leq \Delta_l$  if and only if  $\mathbf{WFF}(\Delta_j) \bar{\cap} \Lambda \subseteq \mathbf{WFF}(\Delta_l) \bar{\cap} \Lambda$ , where  $\mathbf{WFF}(\cdot)$  is the set of well-formed formulas corresponding to a given structure and  $\bar{\cap}$  is a *satisfaction* operator.

To complete this definition, we have to provide a characterization of the satisfaction operator  $\bar{\cap}$ . Notice that if we had used only the set-theoretic intersection  $\cap$  we would have missed the point of comparing  $\Lambda$  with the potential structures. Since  $\Lambda$  may just consist of a data base of numerical observations, a qualitative structure may not yield even a single one of those observations and still be meaningful. In order to address this question, we have to consider each relation  $\mathcal R$  implicit in  $\Lambda$ . Then consider the collection of sets of observations in  $\Lambda$ , denoted  $2^{\Lambda}$ . Then, an application of the Axiom of Choice for finite sets yields that:

**Definition 5** A proposition  $\lambda_{\mathcal{R}}$  *satisfies*  $\Lambda$  if and only if for every finite subfamily sets in  $\Lambda$  there exists a choice  $\mathcal{S}$  such that for every  $a_1, \ldots, a_n \in \mathcal{S} \subseteq 2^{\Lambda}$ ,  $\mathcal{R}(a_1, \ldots, a_n)$ .

Consider then, the family of the propositions  $\lambda_{\mathcal{R}}$  for all relations  $\mathcal{R}$  defined over  $\Lambda$ . These relations may represent the closeness of numerical values, or the fact that they belong to a given interval or, closer to Peirce's aim, a hierarchy of observations, ones deemed more relevant than the others. In any case each of these formulas abstract away from the data base. But then:

**Definition 6** Given a structure  $\Delta$ , **WFF** $(\Delta) \cap \Lambda = {\lambda_{\mathcal{R}} : \Delta \models \lambda_{\mathcal{R}}}$ , where  $\models$  is the classical relation of semantical consequence.

That is,  $\mathbf{WFF}(\Delta) \bar{\cap} \Lambda$  consists of those  $\lambda_{\mathcal{R}}$  that are satisfied by  $\Delta$ , and can be seen as well-formed formulas shared by the data base and the theory for which  $\Delta$  is a model. Finally, the relation among structures  $\leq$  simply yields for every pair of structures,  $\Delta_l$ ,  $\Delta_j$  a preference for the structure, say  $\Delta_l$ , that satisfies not only the same formulas of the data base as  $\Delta_j$  but also some more. Notice that the class of the wffs  $\lambda_{\mathcal{R}}$  determine, as much as the candidate structures, the resulting order  $\leq$ .

Even if this description is sound, in practice there exist serious difficulties associated with the detection of patterns and relations, particularly in numerical databases. This fact is well known by statisticians: an approximate generalization is, according to any statistical test, indistinguishable from the form of a wrong generalization. Even if statistical inferences may preclude hasty generalizations, the fact is that qualitative data may not correspond directly to quantitative forms that can be statistically supported.

<sup>&</sup>lt;sup>9</sup> See Simon (1968).



<sup>&</sup>lt;sup>8</sup> This follows in a logic defined over a hypergraph in which the observations constitute the nodes and sets of observations under the relation  $\mathcal{R}$  the hyperedges (Kolany 1993).

Other (non-statistical) methods lead to similar problems. Computational intelligence only provides rough approximations to the task of theory or model building. Systems like BACON (in any of its numerous incarnations) despite their claimed successes are only able to provide *phenomenological* laws (Simon 1984). That is, they are unable to do more than yield generalizations that involve only observable variables and constants. No deeper explanations can be expected to ensue from their use.

In the process of inquiry carried out by economists, the human side has a crucial task, not yet fully elucidated in the literature: the formation of concepts and the elicitation of qualitative relations. In fact, experts excel in detecting patterns and relations in disordered and noisy data. Of course, as it is well known in Combinatorics, more precisely in Ramsey Theory (Graham et al. 1990), with enough elements a regular pattern will exists, be it meaningful or not. In any case, an expert uses the patterns and relations he finds or imposes over the database and this is represented above by the procedure of selection  $\mathcal{S}$ .

Based on this possibility of finding expressions that "refine" the crude information in  $\Lambda$  we have the following result:

**Proposition 1** There exists a maximal structure  $\Delta^*$  in the set  $\{\Delta_i^{\mathcal{C}}\}_{i\in I}$  ordered under  $\prec$ .

A trivial case of a maximal structure  $\Delta^*$  arises when  $\Delta^* \models \Lambda$ . That is, when all the observations are satisfied in the structure. But, as said, this is not only difficult to be found, but also undesirable in the case of numerical data, since it might involve noisy and otherwise imprecise observations.

So far, many structures may be chosen. Sufficient conditions for uniqueness can be achieved if certain methodological criteria are included in C:

**Definition 7** -  $\mathbf{c}^{min}$  (**Minimality**): given two structures  $\Delta_i$ ,  $\Delta_j$ , such that  $\mathbf{WFF}(\Delta_i) \subseteq \mathbf{WFF}(\Delta_j)$  and  $\mathbf{WFF}(\Delta_j) \not\subset \mathbf{WFF}(\Delta_i)$ , select  $\Delta_i$ .

- $\mathbf{c}^{comp}$  (Completeness w.r.t.  $\Lambda$ ): given two structures  $\Delta_i, \Delta_j$ , where  $\Lambda \subseteq \mathbf{WFF}(\Delta_i)$  but  $\Lambda \not\subset \mathbf{WFF}(\Delta_j)$ , select  $\Delta_i$ .
- $\mathbf{c}^{conc}$  (concordance w.r.t. Λ): a given structure  $\Delta$  is selected if for every  $\lambda_{\mathcal{R}}$  derived from Λ, either  $\lambda_{\mathcal{R}}$  or  $\neg \lambda_{\mathcal{R}}$  belongs to WFF( $\Delta$ ).

Then we have the following result:

**Proposition 2** If  $\{\mathbf{c}^{min}, \mathbf{c}^{com}\} \subseteq \mathcal{C}$  and the set of possible structures is otherwise unrestricted,  $\Delta^*$  is unique.

Similarly:

**Proposition 3** If  $\{\mathbf{c}^{min}, \mathbf{c}^{conc}\} \subseteq \mathcal{C}$  and the set of possible structures is unrestricted,  $\Delta^*$  is unique.

These results show that a unique structure can be selected if the restrictions on possible structures obey to methodological criteria like minimality, completeness or concordance. This is not without a cost: if the only true claims in the chosen structure are the ones drawn from the database it is not possible to provide more than a description (data fitting) of the available information. This means in turn that if only methodological criteria are to be used, the result of the inference is the generation of a



prototype, i.e. only a statistical inference is performed. In Economics these criteria are usually violated since sometimes inferences are drawn from partial samples from a bigger database (violation of  $\mathbf{c}^{comp}$ ), some observations are discarded as outliers (violation of  $\mathbf{c}^{min}$ ), or some information is not used (violation of  $\mathbf{c}^{conc}$ ). Nevertheless they represent extreme cases of very desirable properties: minimality involves **simplicity** while completeness and concordance approximate **unification power** (i.e. external coherence). On the other hand, the fact that the abduction yields a structure implies **internal coherence**. A final requirement, **testability**, is satisfied when the structure yields observable outcomes not found in  $\Lambda$ , that have to be checked out in the real world.

The chosen relational structure  $\Delta^*$  and the statements true in it,  $WFF(\Delta^*)$  constitute, in economic parlance, the actual "model" sought for.

To see how this works in practice we will devote the next sections to the discussion of examples drawn from different fields of economic analysis under both methodological and substantial criteria.

# 5 Abductions in practice

We will present now several examples of abduction in order to show how the formal framework of the previous section helps to clarify how they proceed. They are also intended to exhibit many of the different features adscribed to abduction in the literature.

## 5.1 A case of creative abduction: explaining crises

The 2008 global financial crisis (with all its consequences) poses many interesting questions that economists struggle to answer. Maybe the most important is how this happened. This acts already as the query event that triggers the search of explanations. Various alternative hypotheses can be formulated, each of which constitutes a structure as characterized above, involving components, some of which are intended as counterparts of real world entities, and postulating relations among them (Crespo et al. 2010):<sup>10</sup>

- Δ<sub>1</sub>: A situation modeled through a generic dynamic stochastic general equilibrium model (DSGE) in which agents choose solutions of dynamic programming problems in stochastic environments where the main impulses driving the system are random, exogenous, shocks to the aggregate productivity of the economy, or shifts in monetary policy which distort labor supply and demand decisions, and where the expectations of the agents are rational. The severity of the macroeconomic swings is determined by the extraordinary magnitude of the shocks hitting the system (Caballero et al. 2008; Cochrane 2010).
- $\Delta_2$ : A initially lax monetary policy followed by tightening, associated with a strong credit contraction (Taylor 2008; Obstfeld 2009; O'Driscoll 2009).

We dispense here with the detailed expression of the structures.



-  $\Delta_3$ : A great swindle in which a group of economic agents gains to the detriment of others (Hart and Zingales 2009; Kane 2009).

These structural descriptions are not necessarily incompatible but may generate quite distinct policy implications. We will consider these structures in the light of the following family of criteria:

- $-\mathbf{c}^A$ : The hypotheses should satisfy the onset of the crisis.
- $-\mathbf{c}^B$ : The explanations should be internally consistent.
- $-\mathbf{c}^C$ : The border conditions of each hypothesis should be observable.

The cogency of these criteria is evident.  $\mathbf{c}^A$  just captures the idea that, if we seek an explanation for a crisis, it should describe its process. On the other hand  $\mathbf{c}^B$  eliminates inconsistent structures while  $\mathbf{c}^C$  indicates that any reference to external variables should be testable.

To proceed, let us note that in principle, all three hypotheses satisfy  $\mathbf{c}^A$ . But the DSGE model with rational expectations ( $\Delta_1$ ) begs the question about the impulse that would shock the system and the ex-ante probability that agents may have assigned to a large disruption of their plans. That is,  $\Delta_1$  may fail to satisfy  $\mathbf{c}^C$ .

On the other hand, for  $(\Delta_2)$ , if economic actors anticipate correctly an interest rate fluctuation and its consequences, it will not induce defaults or disturb the wealth perceptions of agents. A similar argument holds for malincentives and fraud  $(\Delta_3)$ . There seems to be no adequate alternative to establishing a close association between expectations, even not fully rational, and swings. That is,  $\Delta_2$  and  $\Delta_3$  do not satisfy  $\mathbf{c}^B$ .

Even if  $\Delta_1$  could, as assumed by its most vocal proponents, satisfy  $\mathbf{c}^C$ , a creative abduction can be performed taking into account the shortcomings of the three hypotheses. That is, a new hypothesis can be postulated and shown more adequate that the former structural descriptions:

-  $\Delta_4$ : A crisis is the result of a swing in beliefs and expectations of the economic agents (Leijonhufvud 2009; Heymann 2008; Schiller 2008).

Since only  $\Delta_1$  and  $\Delta_4$  might satisfy the three criteria for abduction. Criteria like  $\mathbf{c}^{min}$  or  $\mathbf{c}^{com}$  are not strictly satisfied by either candidate, so the only remaining possibility is to establish an order  $\leq$  between  $\Delta_1$  and  $\Delta_4$ . In this sense, we have that  $\Delta_1 \leq \Delta_4$ , since  $\Delta_4$  may yield an explanation even in the absence of external shocks.

That is, the abduction is *creative*, *explanatory* and yields an *IBE*. And for the economist it restricts the classes of economic theories that may represent crisis-type events.  $\Delta_4$  provides a foundation for such theoretical developments, based on assuming that the behavior of the agents starts from ex-ante beliefs on the overall prospects of the economy that end up being mistaken.

# 5.2 A case of manipulative abduction: exploring data

Abduction may also arise from the observation of unruly behavior that, while admitting traditional observations, may still seem open to analysis. Let us consider here the extremely high inflations in Argentina at the end of the 1980's and beginning of the



1990's. Although well-established theories of inflationary behavior exist, we will try to detect new features of this long winding process.

First of all we will commit ourselves to the idea that even if the inflation rate levels differ enormously in different points of time, the same process generated them all. This conjecture is backed up by statistical evidence: the series of observations we consider (monthly observations of the variation of prices from 1960 to 1993) is stationary in the weak sense. <sup>11</sup> This means that the assumed parameters of the generation process remained the same along the entire period of observation. In other words, statistically, the series seems to have been generated by a single process.

The conjecture of a single process is hard to accept for most economists, since institutional, international and social conditions varied substantively during the three decades of observations. Therefore, the conjectured single process should have been robust enough to persist under those different conditions.

Three basic possibilities are to be considered:

- $\Delta^r$ : the process was purely random, i.e. the values generated were uncorrelated, and therefore independent of the environmental conditions of the economy.
- $-\Delta^d$ : the process was purely deterministic, i.e. it followed an internal law absolutely independent of the external conditions. Additionally, it should be hard or impossible to predict the values generated by the process. Otherwise economic agents (the government, for example) could have predicted future values and could have taken protective measures to avoid the economic damage caused by inflation.
- Delta<sup>s-o</sup>: the process had a combination of random and deterministic elements.
   It absorbed the external influences but it reprocessed them according to its inner rules, which account for most of its behavior.

These are the hypotheses to be checked out using the database of observations. Before doing that, one has to precise the meaning of each hypothesis:

- 1. The series constitutes (in a sense which will be defined below) a white noise.
- 2. The series is the outcome of a *chaotic* dynamic system.
- 3. The series is the outcome of a *self-organized* system.

In particular, we have to make clear what we mean with self-organized. There are a number of works in applied physics where this expression is applied to describe systems of equations exhibiting bifurcations. That is, values of their parameters for which the qualitative nature of their solutions changes abruptly. Here, instead, we are concerned with a certain state of affairs that stabilizes itself, becoming robust to external perturbations. The best candidate we have in mind for this notion is *self-organized criticality*. Here, instead of jumping from a type of qualitative solution to another the system, through its internal dynamics, reaches a critical value. At this value it is capable to generate responses uncorrelated with the external shocks it receives. 13

<sup>&</sup>lt;sup>13</sup> Critically self-organized systems consist of a number of coupled components. The state of the components in a period plus the external shocks determines the state of the components in the next period. See Bak (1997).



<sup>&</sup>lt;sup>11</sup> For a lore of statistical evidence about the behavior of the Argentinean inflation during the last decades see Dabús (2000).

<sup>&</sup>lt;sup>12</sup> See, for example, Haken (1988).

Having settled the meaning of our conjectures we have to provide some ways of testing them. Then, some formal definitions are in order.

**Definition 8** Given a time series  $A = \{a_t\}_{t=0...T}$ , two functions, derived from the series provide information about the structure that generates the process:

- S<sub>A</sub> :spectral density of the series. To each frequency it associates the mean of the corresponding Fourier coefficients.
- $P_A$ : sample density of the series.
- Given the average of the series,  $\bar{a} = \sum_{t=0}^{T} \frac{a_t}{T+1}$ , the minimal and maximal "accumulated flow" of the series are, respectively:
- $min = \min_{t=0...T} \sum_{k=0}^{t} \frac{a_k \bar{a}}{t}$ .  $max = \max_{t=0...T} \sum_{k=0}^{t} \frac{a_k \bar{a}}{t}$ . Given  $R_{\mathcal{A}} = max min$  and the standard deviation of the series,  $\sigma_{\mathcal{A}} = max min$  $\sqrt[2]{\sum_{t=0}^{T} \frac{(a_t - \bar{a})^2}{T+1}}$ , the Hurd Coefficient is  $H = \frac{\log \frac{R_A}{S_A}}{\log(T+1)}$ .

Here  $S_A$  indicates the degree of dependence among values of the series while  $P_A$ approximates the theoretical density function of the generating mechanism. A taxonomy of possible cases is the following, which represent possible  $\lambda_R$ s drawn from the database:

- $-S_A \approx \lambda^0$  (where  $\lambda \in [0, \frac{1}{T+1}]$  is the frequency) the series is a *white noise*.
- $S_A \approx \lambda^{-2}$  it is a *brownian noise*.  $S_A \approx \lambda^{-\alpha}$  with  $0 < \alpha < 2$ , it is a *fractional brownian noise* ( $\frac{1}{f}$  noise).

The last case is an intermediate case between the first two. A white noise is a series in which observations are uncorrelated. A brownian noise, instead, is a series in which observations are highly correlated. A  $\frac{1}{f}$  noise shows a certain correlation among data but diminished by the effect of random movements.

Other evidence can be obtained from the properties of  $P_A$ , in particular if it has a variance that changes with the size of the sample (infinite variance property). Among the distributions that verify this property the most important is the Pareto-Levy distribution, where  $P_A = |a|^{-\alpha}$  with  $0 < \alpha < 2$  (where |a| represents the magnitude of an element  $a \in \mathcal{A}$ ).

On the other hand, additional evidence for  $\frac{1}{f}$  noise in a time series can be detected by means Hurd's Coefficient. This number provides another form to determine the degree of correlation among data in the time series. In particular if H > 0.5, it indicates that an increase of magnitude in an earlier stage implies an increase in a later stage.

Having defined the statistical information to be used, we will define the "tests" according to which we will accept or discard the alternative hypotheses:

**Definition 9** The following are necessary conditions for the respective hypotheses:

- Random process:  $S_A$  is a white noise.
- Chaotic system:  $0 < H < \frac{1}{2}$ . 14

<sup>14</sup> A chaotic process (different from an ergodic process) converges to what is called a strange attractor even if trajectories that began being close to each other tend to diverge. In the average, great increases in a given stage of the evolution of the system have to be compensated by decreases in the future.



- Critically self-organized system:  $S_A$  is a  $\frac{1}{f}$  noise,  $P_A$  approximates a Pareto-Levy distribution and  $H \neq 0$  and  $H \neq \frac{1}{2}$ .

Without going into the discussion of numerical values <sup>15</sup>, we can say that the series of monthly inflation for Argentina, from 1960 to 1993 constitutes a  $\frac{1}{f}$  noise. It approximates fairly well a Pareto–Levy distribution and its Hurd's Coefficient is over  $\frac{1}{2}$ .

That indicates that the series is not generated by a random process or by a chaotic system (since it does not verify the necessary conditions for those processes). The hypothesis of self-organized criticality  $\Delta^{s-o}$  cannot be discarded.

That is, the process of abduction does not yield a *IBE* and not even an *explanatory hypothesis*, but the *manipulative* reasoning yields a new feature of the data on which the abduction has been performed.

## 5.3 A case of instrumental abduction: betting on the wrong sequence

It is relatively easy to represent any anomaly in the behavior of a rational agents as a consequence of her information structure. Consider for instance the Gambler's Fallacy, which can be illustrated with the usual coin toss example. Assume that the coin is fair and therefore either tails (T) or heads (H) has a probability of  $\frac{1}{2}$  and this is the only data  $(\Lambda)$  the agent has about the situation. Then,  $\Delta_0$  will be such that it maximizes  $\leq$  under some constraints. Assume that  $\mathcal{C} = \{\mathbf{c}^{fair}, \mathbf{c}^{min}\}$ , where  $\mathbf{c}^{fair}$  is a condition that requires that the alternatives should keep a balance:

-  $\mathbf{c}^{fair}$  (Fairness):a given structure  $\Delta$  is selected if is such that  $Prob(T \in \mathcal{S}) = Prob(H \in \mathcal{S})$ , where  $\mathcal{S}$  is the set-theoretic union of all feasible (i.e. with positive probability) sequences of tosses, for an outcome O(H or T).

If follows that  $\Delta_0$  will verify:

$$\textbf{WFF}(\Delta_0)\bar{\cap}\Lambda \ = \ \{\overline{\textit{Prob}}(\bar{T},\frac{\overline{1}}{2}),\overline{\textit{Prob}}(\bar{H},\frac{\overline{1}}{2})\}$$

Now assume that the agent observes that H came out in two consecutive tosses of the coin, a fact which together with the fairness of the coin constitutes  $\Lambda'$ .  $\Delta_1$  has to maximize  $\leq$  subject to  $\mathcal{C} = \{\mathbf{c}^{fair}, \mathbf{c}^{min}\}$ . That is:

$$\begin{aligned} \mathbf{WFF}(\Delta_{1})\bar{\cap}\Lambda^{'} &= \\ &\{\overline{Prob}(\{\bar{T},\bar{T},\bar{T}\},\bar{0}),\overline{Prob}(\{\bar{T},\bar{T},\bar{H}\},\bar{0}),\overline{Prob}(\{\bar{T},\bar{H},\bar{H}\},\bar{0}),\\ &\overline{Prob}(\{\bar{T},\bar{H},\bar{T}\},\bar{0}),\overline{Prob}(\{\bar{H},\bar{T},\bar{H}\},\bar{0}),\overline{Prob}(\{\bar{H},\bar{T},\bar{T}\},\bar{0}),\\ &\overline{Prob}(\{\bar{H},\bar{H},\bar{T}\},\overline{X}),\overline{Prob}(\{\bar{H},\bar{H},\bar{H}\},\overline{Y})\} \end{aligned}$$

where X + Y = 1, but X > Y. To see why this is so, first notice that  $\mathbf{c}^{min}$  forces to select those  $\Delta s$  in which only the series of tosses that are consistent with the observations have

<sup>&</sup>lt;sup>15</sup> Which can be checked out in Tohmé et al. (2005).



positive probability. That is, structures in which the first two consecutive outcomes are H. On the other hand,  $\mathbf{c}^{fair}$  requires that  $Prob(T \in \mathcal{S})$ , should be equal to  $Prob(H \in \mathcal{S})$ . But then,  $\mathcal{S} = \{H, H, H, H, H, T\}$  since the only feasible sequences are  $\{H, H, T\}$  and  $\{H, H, H\}$ . Therefore, according to  $\mathbf{c}^{min}$ ,  $Prob(T \in \mathcal{S})$  must be higher than the frequency of T in  $\mathcal{S}$ . Using this fact, it follows that  $Prob(\{H, H, T\}) > \frac{1}{2}$ .

We can see that once accepted  $\Delta^*$  it becomes rational (i.e. consistent with the preferences) to behave in forms that are otherwise not deemed as "rational". The agent, faced with the opportunity to bet either for H or T in the third toss of the coin, would be willing to put her money on T, since she wants to maximize her earnings and believes that the probability of T exceeds that of H. There is nothing irrational in this behavior although constraints like  $\mathbf{c}^{min}$  or  $\mathbf{c}^{fair}$  would not be applied by someone familiar with Probability Theory, but mathematical proficiency has never been an assumption in Economic Theory.

Since the goal of this abduction was to determine how to behave in the face of a challenge, it is not *explanatory* but *instrumental*, and according to the constraints it is also a *IBE*. Nevertheless, it cannot be expected to yield a good result, showing that abduction does not imply a convergence to truth or to the right behavior.

## 5.4 Several kinds of abduction in a framework: lemons

For an example closer to the mainstream in Economic Theory, let us consider a situation of *asymmetric information*, that is, one in which several agents interact and some are more informed than the others. This asymmetry helps to explain why certain markets are not complete. The signature of this incompleteness is the absence of prices for certain goods, indicating that they are not traded, even if there exist agents that could be interested in purchasing them.

The classical example of an incomplete market (due to informational asymmetries) is the market for used cars (*lemons*) (Akerlof 1970). The casual evidence shows that in the parking lots of the sellers only medium to low quality cars are offered. There is no organized market for high quality used cars. The only trades of this kind of vehicles are individual owner-to-buyer transactions that do not define a market price. Akerlof's explanation can be put in simple terms. It begins with the fact that, since sellers know better about a car's quality, they can hide this information from the potential buyers and claim that the cars they sell are of high quality. Buyers, in turn, know that, and are willing to pay only the price of low quality cars. The sellers, being aware of this, sell only cars for which the prices accepted by the buyers yield a profit. This means that no high quality cars will be traded.

This story can be easily represented in our framework. Consider that the data received by the buyer during her negotiation with the seller is summarized in  $\Lambda^b = \{p_0, \mathbf{a}\}$ , where  $p_0$  is the price asked for a potential car and  $\mathbf{a}$  an "attitude" variable, summarizing the flamboyancy and self-confidence of the seller. Assume that the constraints for the buyer are  $\mathcal{C}^b = \{\mathbf{c}^{hed}, \mathbf{c}^{min}\}$ , where:

-  $\mathbf{c}^{hed}$  (**Hedging**): a given structure  $\Delta$  is selected if its accepted claims ensure the minimization of possible losses.



In our case  $\mathbf{c}^{hed}$  amounts to choose the possible scenarios in which the quality of the car ensures a minimal loss. This amounts to a  $\Delta^*$  in which the assumed quality of the car is low,  $q_0 = q^L$ . Then, the price that the buyer is willing to pay for it,  $p(q_0)$ , verifies that  $p(q_0) \leq p_0$ . If  $p(q_0) = p_0$  the buyer accepts and purchases the car. Otherwise it is rational for the buyer to propose a counter-offer,  $p_1 < p(q_0)$ . The seller will accept the offer only if  $p_1 \geq p(q^*)$ , where  $q^*$  is the real quality of the car (perfectly known to her). If that's not the case, she can ask a price  $p_2$ , such that  $p_1 < p_2 \leq p_0$ .

The buyer, again because of  $\mathbf{c}^{hed}$ , must ask for a price  $p_3$  such that  $p_1 \leq p_3 \leq p(q_0)$ . But, since the other criterion used in the selection of  $\Delta^*$  is  $\mathbf{c}^{min}$ , it follows that  $p_1 = p_3 = p(q_0)$ . If that is the case, the negotiation breaks down and the car is not traded.

On the other hand, the evidence amassed by the used cars seller is  $\Lambda^s = \{\{\langle p, \mathbf{s}old \rangle\}_{p \leq \bar{p}}, \{\langle p, \neg \mathbf{s}old \rangle\}_{p > \bar{p}}\}$ , i.e. that cars are sold if the asked price is low  $(p \leq \bar{p})$ . Then, her  $\Delta^*$  obtained according to  $\mathcal{C}^s = \{\mathbf{c}^{hed}, \mathbf{c}^{min}\}$  (the same constraints as the buyer) will support the claim that a car will be sold and will yield a profit if its quality (and consequently its price) is low. This will lead her to put on sale only low quality cars.

While this shows that traditional arguments in Information Economics can be recasted in our framework, it allows to depict alternative scenarios. For example consider a rather naive buyer, who uses as a constraint  $\mathbf{c}^{con}$  instead of  $\mathbf{c}^{hed}$ , where:

-  $\mathbf{c}^{con}$  (**Confidence**): a given structure  $\Delta$  is selected if its accepted claims are strongly endorsed by a more informed individual.

In our case this means that, since the high price  $p_0$  is accompanied by a strong endorsement (the attitude variable **a**),  $\Delta^*$  will include the claim that the quality of the car is high, i.e.,  $q_0 = q^H$ . This will lead the buyer to buy the car at the asked price.

A seller, will then face diverse responses to his asking high prices, i.e.,  $\Lambda^s = \{\langle Prob, p, sold \rangle : p > \bar{p} \}$  (which means that with probability Prob she will sell a car for which she asked a non-low price). If her constraints are again  $C^s = \{\mathbf{c}^{hed}, \mathbf{c}^{min}\}$ , she will put on sale quality cars if  $Prob > \frac{1}{2}$  because this ensure her to more than break even.

We have shown that in the same framework we can have both *explanatory* and *instrumental* abductions because the information obtained can be used at the same time as an explanation of why no market prices for near-new cars exist and as a guide for behavior in a car-sale transaction. Besides, we see that those results are obtained through a *manipulative* abduction, playing with possible proposals and counter-proposals.

#### 6 Conclusions

In this paper we introduced several notions of abduction and the related concept of Inference to the Best Explanation. Then we have explained why this stage of scientific development is relevant for Economics. The complexity of economic affairs, confronted with the limitations of modeling and measuring, leaves ample room for the

<sup>&</sup>lt;sup>16</sup> Because a model in which these prices differ involves a larger number of true formulas.



existence of competing hypotheses. Economists have developed the ability to make conjectures, construct new tools, find insights and even guides for behavior given their background knowledge and specific querying situations. Furthermore, they have designed ways of testing those hypotheses and choosing the best among them.

The economy of research, the lack of time facing urgent decisions, the difficulties to obtain trustable data or to design accurate models representing complex situations often lead economists to accept, by intuition, hypotheses. To capture the many ways in which this process may abduct them we introduced a formal model and have shown in examples how it works.

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