

Radio Resource Allocation Algorithms for Multi-Service OFDMA Networks: The Uniform Power Loading Scenario

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Abstract Adaptive Radio Resource Allocation is essential for guaranteeing high bandwidth and power utilization as well as satisfying heterogeneous Quality-of-Service requests regarding next generation broadband multicarrier wireless access networks like LTE and Mobile WiMAX. A downlink OFDMA single-cell scenario is considered where heterogeneous Constant-Bit-Rate and Best-Effort QoS profiles coexist and the power is uniformly spread over the system bandwidth utilizing a Uniform Power Loading (UPL) scenario. We express this particular QoS provision scenario in mathematical terms, as a variation of the well-known generalized assignment problem answered in the combinatorial optimization field. Based on this concept, we propose two heuristic search algorithms for dynamically allocating subchannels to the competing QoS classes and users which are executed under polynomially-bounded cost. We also propose an Integer Linear Programming model for optimally solving and acquiring a performance upper bound for the same problem at reasonable yet high execution times. Through extensive simulation results we show that the proposed algorithms exhibit high close-to-optimal performance, thus comprising attractive candidates for implementation in modern OFDMA-based systems.

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1 Introduction

Radio Resource Allocation (RRA) mechanisms are expected to play a key role in emerging and future OFDMA-based multiuser wireless access networks. RRA aims at simultaneously guaranteeing high utilization of the available system resources, satisfying the individual Quality-of-Service (QoS) requirements of the competing users, and maximizing overall system performance. In order to accomplish these targets, an RRA or Frequency-Domain Packet Scheduling (under the LTE terminology) technique exploits the differentiated channel conditions experienced by the various users over the available bandwidth. In particular, a complete allocation decision comprises the specific set of OFDM subchannels assigned to each user as well as the transmission format, namely the amount of power and the modulation mode, for each resource block [1].

1.1 Background and Related Work

Adaptive RRA aims at positively exploiting the rate differentiation that occurs on two levels closely related to respective physical phenomena: the single-user centric variation of the achieved rate on a subchannel basis and the multi-user centric differentiation of the rate achieved by each user on each subchannel. The former phenomenon is related to the inherent frequency selectivity of the wideband wireless medium, while the latter to the so-called multi-user diversity caused by the statistical independence of the corresponding subchannels. Therefore, by assigning to each user the subchannels that experience favorable channel (and thus rate) conditions, we expect to significantly improve system performance.

In [2] the above concepts were thoroughly presented first for single- and then for multi-antenna wireless mobile systems. According to this concept each receiver monitors the experienced SNR levels, feeds them back to the BS, while the BS schedules transmissions and adapts users' bit rates depending on the particular channel quality reports. Similar arguments were raised in [3] where it was demonstrated that for 2G/3G systems the cellular spectral efficiency may significantly improve, even double at certain conditions, when the BS utilizes the per-user channel/rate information. In [4] the ideas were extended to multi-channel OFDM wireless systems like the one we examine in this paper. The widely used term "opportunistic" bears a strong relation with our work, since we tend to allocate subchannels with high channel conditions (near to their peak) to the corresponding users (frequency-domain opportunism), while in [2] a similar policy is employed in the time-domain.

The works of [5] and [6] were the first to introduce an optimization framework for handling multi-user OFDM resource allocation problems, paving the way for an extensive utilization of concepts and methods addressed in the engineering optimization field,

towards efficiently assigning system resources to active users. The original formulations of the respective system power minimization [5] and sum-throughput maximization [6] problems indicated the hard large-scale non-linear integer nature of the underlying RRA decision problems, rendering the straightforward discovery of the optimal allocation practically impossible. Towards relaxing the complexity of the related problems, a plethora of simplifying approaches has been proposed in the literature aiming at the development of suboptimal, yet efficient and computationally tractable allocation routines [7,8]. The works in [9] and [10] provide extensive surveys on the methodological and algorithmic aspects of the particular research area. A popular assumption which was first proposed in [11] and further elaborated in [12] and [13] regarded the uniform power loading (UPL) of the system subchannels. Under a known power distribution, the achieved bit-rates for all the possible subchannel/user combinations can be pre-calculated, and thus the original multidimensional RRA problems resort to exclusive subchannel allocation problems. From a system perspective, OFDMA is considered the major transmission and multiple access technology for modern wireless networks such as 3GPP-LTE [14], Mobile WiMAX [15] and also plays a key role in the new paradigm of Cognitive Radio Networking [16]. In particular, LTE-oriented works may be found in [17,18], WiMax-oriented ones in [19,20,21] and cognitive networking related in [22, 23] correspondingly.

1.2 Contributions and Novelty

In this paper we examine a single-cell downlink OFDMA system scenario supporting multiple QoS profiles under the UPL assumption. Unlike single-profile studies, which have attracted enormous interest in the related literature, the heterogeneous problem, which is more interesting and realistic has not been given great attention. In [24] a mixed CBR/VBR scenario was introduced for the single-user case, while in our work we consider multiple users demanding mixed services. The multi-user case was studied in [25,26] where suboptimal allocation algorithms were proposed. However the UPL hypothesis have not been taken into account as in our work and the efficiency of the algorithms compared to the maximum achieved performance (that is, the optimal one) was not demonstrated. Finally, in [12] an efficient UPL-based resource allocation algorithm was proposed guaranteeing a set of minimum bit rates, while in our paper a more realistic QoS scenario is assumed, comprising mixed CBR and Best-Effort traffic profiles.

In particular, in this work we employ two optimization approaches to our problem. The former is based on Integer Linear Programming (ILP) and allows us to find: (a) the exact optimal resource allocation decision as well as the maximum sum-rate performance in reasonable, yet high execution time, and (b) a performance upper bound

in polynomial time solving the LP-relaxed version of the IP formulation. The second approach is based on our observation that the specific heterogeneous formulation bears similarities with a well-known combinatorial optimization problem, the Generalized Assignment Problem (GAP) [27]. In order to efficiently solve the GAP formulation we devise two heuristic schemes, which are executed with polynomially bounded computational cost. The development is mainly inspired by the ideas utilized in the respective GAP heuristics found in the related literature. The examined schemes are tested in an extensive set of realistic computational-based network scenarios, in terms of the achieved sum-rate performance. The simulation results show that the devised heuristic schemes: (i) outperform drastically a semi-random subchannel assignment approach, and (ii) perform close to the exact optimal allocation, while their complexity cost is significantly lower.

1.3 Paper Structure

The remaining of the paper is structured as follows. In Sec. 2 we present the adopted system model and the holding assumptions, state the multiple-QoS profile RRA problem and formulate it as a Binary Integer Linear Programming optimization problem. In Sec. 3 we discuss how we can express our problem as a variation of the GAP combinatorial format, and which are the advantages of this approach, while in Sec. 4 we devise two heuristic allocation algorithms inspired by the GAP concept and discuss computational complexity issues. In Sec. 5 we present the results of the computational experiments and comment on them, while in Sec. 6 we summarize our work and propose possible extensions of it.

2 System Model and Mathematical Formulation

2.1 System Description and Assumptions

The downlink (DL) of an OFDMA cell is considered in the context of this work, where a Base Station (BS) located at the center of a 2D area is fully responsible for allocating system subchannels (or equivalently physical resource blocks according to the LTE terminology) to the competing users. The available bandwidth comprises N mutually-orthogonal subchannels (or resource blocks) forming the set $\mathcal{S} = \{1, 2, \dots, n, \dots, N\}$. The available transmission power P_{bs} is uniformly spread over the bandwidth, namely, each subchannel n is loaded with an equal amount of power given by $P_n = P_{bs}/N$. We assume that K users are present in the cell, forming the corresponding set $\mathcal{U} = \{1, 2, \dots, k, \dots, K\}$, and that each user is assigned one of the two available QoS profiles/classes, the Constant-Bit-Rate (CBR) or the Best-Effort (BE) one. Users belong-

ing to the CBR-class subset (\mathcal{U}_{CBR}) demand a specific constant service rate denoted by $R_k^{\min} (\forall k \in \mathcal{U}_{\text{CBR}})$, representing Voice/Video-like services. On the other hand, users belonging to the BE-class subset (\mathcal{U}_{BE}) have infinitely backlogged queues and no strict QoS guarantee, reflecting FTP-downloading services.

In order to perform adaptive resource allocation in the above typical multi-user OFDMA system setup the following key operations/assumptions are supported:

- Each user estimates the BS-user link channel quality over the available subchannels and feeds back this information to the BS through an error-free and delay-less uplink signaling channel.
- Due to the wideband and multi-user nature of the transmission medium [28] each user k achieves different rate performance over each subchannel n , given by $r_{n,k}$ bits/symbol. Given that the BS exactly knows the wideband and multi-user channel response as well as the allocated power per subchannel P_n , the $r_{n,k}$ bit rates are pre-calculated (please refer to the discussion in Appedix A).
- Based on the above 2D rate matrix and the QoS targets, the BS decides the exact subchannel set assignments for each user. This information is also transmitted to each user in order for the useful data to be correctly decoded.
- Each subchannel should be allocated to a single user, thus avoiding intra-cell inter-user interference [1].

2.2 Mathematical Formulation and Optimal Solution

We introduce $N \cdot K$ binary integer variables notated by $\rho_{n,k}$ where $\rho_{n,k} = 1$ if the n^{th} arbitrary subchannel is allocated to the k^{th} arbitrary user, else equals zero. The data-rate $r_{n,k}$ supported on each subchannel/user combination can be pre-calculated based on the abstraction modeling function given in Appedix A. We are now able to formally define the RRA problem, which constitutes the identification of the allocation decision that maximizes the cell sum-rate (R_{tot}), satisfies the individual data-rate constraints for the CBR-class users and preserves subchannels orthogonality. Eq.(1a) express the system-wise objective function, Eq.(1b) the $K_1(|\mathcal{U}_{\text{CBR}}|)$ QoS constraints, and Eq.(1c) the N subchannel-sharing-avoidance system constraints. Power availability and minimum BER constraints are handled implicitly as explained in Appendix A. Although the generalized bandwidth and power allocation problem is non-linear [5], the UPL hypothesis allows us to recast RRA in a linear form, as shown in Eq.(1). Actually, due to the exclusive assignment of each subchannel to one user, the UPL variation of RRA is actually an Integer Linear Programming (ILP) optimization problem. In fact we are dealing with an ILP problem, including $N \cdot K$ binary variables and $|\mathcal{U}_{\text{CBR}}| + N$ equality constraints, where $N \sim 100$ and $K \sim 10-20$ under a typical network setup [14]. Despite the fact that ILP problems are still hard to solve due to their NP-hard nature,

efficient methods and solvers like CPLEX, SCIP or GUROBI are available and may be employed for problems of such scale.

Finally, an upper bound on the system performance could be estimated by relaxing the integrality constraints or equivalently allowing for one or more subchannels to be shared among users. For this case an LP solver could solve the specific problem with 3rd order polynomial complexity cost. Note however that the LP relaxation approach is able to provide us only with the system performance (in fact the theoretical achieved upper bound) and not with an implementable allocation decision due to the violation of the subchannel-sharing constraints set. We have to emphasize the implementation limitations of both IP and LP approaches, underlying that their value is mostly theoretical. For this reason we proceed with developing simpler allocation schemes in the following two sections.

$$\max_{\boldsymbol{\rho}} R_{tot} = \sum_{n \in \mathcal{S}} \sum_{k \in \mathcal{U}} r_{n,k} \cdot \rho_{n,k} = \sum_{k \in \mathcal{U}_{\text{CBR}}} R_{\text{min}}^k + \sum_{n \in \mathcal{S}} \sum_{k \in \mathcal{U}_{\text{BE}}} r_{n,k} \cdot \rho_{n,k} \quad (1a)$$

subject to

$$\sum_{n \in \mathcal{S}} r_{n,k} \cdot \rho_{n,k} = R_{\text{min}}^k, \forall k \in \mathcal{U}_{\text{CBR}} \quad (1b)$$

$$\sum_{n \in \mathcal{S}} \rho_{n,k} = 1, \forall n \in \mathcal{S}, \boldsymbol{\rho} \in \mathbb{I}^{N \times K}, \mathbb{I} = \{0, 1\} \quad (1c)$$

3 Radio Resource Allocation as a Variation of the Generalized Assignment Problem

The Generalized Assignment Problem (GAP) along with its variations is a well-studied special formulation of Combinatorial Optimization. Although GAP is still an NP-hard problem, several efficient approximate heuristic algorithms have been proposed for solving it [27, 29] due to its wide application in real-world problems. A Knapsack-based definition of GAP proposed in [27] constitutes the assignment of items to knapsacks in order to maximize the overall profit while not exceeding the capacity constraints of each knapsack. The assignment of each item to a knapsack incurs different profit and induces different cost. Note that an item can be obviously assigned to a single knapsack. A slightly different definition is given in [29], where one searches for the best scheduling of tasks or jobs to agents, in order to minimize the overall processing cost and do not violate the available resource budget of each agent. Again a task cannot be split into multiple agents. Finally, a variation of the GAP, known as the Covering Assignment Problem (CAP) is given in [30] which applies to a dairy industry distribution scenario.

Specifically, CAP regards the optimal distribution procedure of the milk produced by several farms to a set of factories, in order to minimize the cost of processing and simultaneously satisfy the minimum resource demand of each factory.

Towards expressing our RRA problem as a GAP, we first define the correspondence of an item/task/farm to a subchannel and a knapsack/agent/factory to a user of the system. RRA may then be seen as the optimal exclusive allocation of each subchannel to a single user in order to maximize the sum-rate (profit) function and provide minimum rate (resource) assignments for a user subset. We also have to mention that our formulation possesses several distinctive features compared to the aforementioned classical GAP/CAP approaches:

- The constraint expressions are tight whereas in GAP and CAP they correspond to left-hand-side or right-hand-side inequalities
- The constraint expressions are imposed on a subset of users and not over the complete users' set
- the profit/weight/cost factors are equal ($r_{n,k}$) and depend on both subchannel and user indexes through the experienced rate/channel conditions.

As far as GAP and its variations, several efficient heuristic algorithms could be found in the literature. In [27] the authors proposed a scheme which was based on the construction of an initial effective feasible solution and its subsequent improvement through item reallocations. In [31] the author proposed a “dual” algorithm, according to which the optimal solution was approached from the exterior of the feasible region. Finally, in [30] the authors devised similar schemes for the CAP variation. In the following section we devise two algorithms, inspired by the previous GAP/CAP works. We remark that due to the differences between our RRA problem and the classical formulations, none of the existing algorithms could be employed as-is. Moreover, although other solution approaches may be found in the literature, such as the Lagrangean Relaxation, Branch-and-Bound, Metaheuristics, etc. (see for example a survey in [32]) we focus on the heuristic solution approach since it is appealing from an implementation point of view. One should bear in mind that in a practical network scenario the complete resource assignment must be updated every transmission frame. For example, regarding contemporary wireless access systems like LTE and WiMAX, the update rate should be of the order of 0.5–1 msec [14].

4 Heuristics for Solving the Resource Allocation Problem

4.1 Heuristic I: Approaching the Optimal Allocation from the Interior of the Feasible Space

Description The first heuristic is mainly inspired by the Knapsack solution approach given in [27]. Resource Allocation is performed in 2 phases, that is the construction of an initial feasible solution and the search for a gradually improved allocation. Hereafter, when we refer to a “feasible” allocation decision, we will mean that the $|\mathcal{U}_{\text{CBR}}|$ data rate constraints are satisfied. As far as the remaining N orthogonality constraints, these will be implicitly met, since at each subchannel allocation step, the subchannel which is assigned to a user will be removed from the available bandwidth “pool”. Hence, we should not confuse the use of the feasibility term with the one regarding the LP relaxation given in Sec.2. The complete algorithm including all the intermediate steps for each phase is given after the description of its key features.

During the 1st phase, our primary objective is to construct a feasible RRA solution, that is, satisfy the strict data-rate constraints of all CBR users, while secondarily the respective allocation has to be as efficient as possible in terms of the overall system performance. Towards the 1st point we prioritize the CBR-class subchannel allocation procedure over BE-class, thus decoupling the inter-class problem to two consecutive intra-class sub-problems. Concerning the secondary target, we state that if the above QoS constraints are satisfied by utilizing the minimum amount of the available Tx power as well as the “best” subchannels in terms of the achieved channel/rate conditions, then plenty of resources will be available for BE-class users. Such a policy would enhance system performance, since BE users are the ones that contribute to the objective function, due to the predetermined static nature of the CBR-class users’ data-rates. Hence, after the necessary initializations and declarations (**Step 1**), an iterative joint subchannel/user allocation is applied over the CBR users subset ignoring BE users (**Step 2**). The selection criterion for picking the best CBR subchannel-user pair at each iteration is dual: on the one hand the user experiencing the minimum averaged achieved data-rate over the remaining available subchannels set is prioritized and on the other hand the “best” subchannel in terms of rate performance is promoted. The user-selection criterion prioritizes the users with the worse rate conditions, which are expected to consume the largest portion of resources, and by allocating to them their most efficient subchannels we succeed into constraining the overall amount of CBR-class resources consumption.

Subsequently, we finalize the initial feasible RRA solution by assigning the unallocated resources to the BE users (**Step 3**). Since no QoS requirements are imposed for the particular class, the simple policy of best-rate user allocation is also the optimal

one (see also [11]). Up to now, we have provided a feasible allocation decision, however we argue that we could further improve it since the previous steps may induce suboptimality to the sum-rate performance. Note that this is caused by the “greedy” (local) nature of the heuristic allocations during Step 2. Towards the above purpose we employ a series of subchannels swaps (**Step 4**). Specifically, we perform a subchannel interchange between 2 users if and only if the system performance is enhanced and simultaneously the feasibility for any CBR user is not violated. Due to the presence of multiple QoS classes, we have to discriminate between $2^2 = 4$ possible swap scenarios (2 classes are considered in this work). Finally in order to limit the number of interchanges (and thus the execution complexity) we perform only one round of comparisons and possible reallocations by a single sweep of the set of system users. We remark that by increasing the comparison rounds, a marginal additional performance gain is expected, since already at each user iteration the owner’s subchannels are compared with the subchannels of all the other users.

The last subprocedure given in **Step 5** comprises the release of possible redundant subchannels of CBR-class users due to the active nature of the related constraints and the reallocation of them to a BE user. We put an emphasis on the fact that for CBR users surplus allocated data-rates are ignored.

Complete Algorithm

Step 1 *Initializations-Declarations*

Let $\{r_{n,k}\}_{n \in \mathcal{S}, k \in \mathcal{U}}$ the known achieved data-rates for all subchannel/user combinations and $\{\rho_{n,k}\}_{n \in \mathcal{S}, k \in \mathcal{U}}$ the set of the allocation variables.

Define: $lhs_k = \sum_{n=1}^N r_{n,k} \cdot \rho_{n,k}$ and $b_k = R_{\min}^k, \forall k \in \mathcal{U}_{\text{CBR}}$.

Step 2 *Subchannel Assignment to the CBR-class users – Feasible Allocation*

Define the available subchannels pool \mathcal{N} and the subset of the unsatisfied CBR users \mathcal{K} as:

$$\mathcal{N} = \left\{ n \in \mathcal{S} : \sum_{k=1}^K \rho_{n,k} = 0 \right\}, \mathcal{K} = \{k \in \mathcal{U}_{\text{CBR}} : lhs_k - b_k < 0\}.$$

Update $lhs_k = \sum_{n=1}^N r_{n,k} \cdot \rho_{n,k}$ and $b_k = R_{\min}^k, \forall k \in \mathcal{U}_{\text{CBR}}$.

Pick the subchannel/user combination according to:

$$k^* = \arg \min_{k \in \mathcal{K}} \left\{ (1/|\mathcal{N}|) \cdot \sum_{n \in \mathcal{N}} r_{n,k} \right\}, n^* = \arg \max_{n \in \mathcal{N}} \{r_{n,k^*}\} \text{ and}$$

Perform the subchannel allocation: $\rho_{n^*,k^*} = 1$

If $\mathcal{K} \neq \emptyset$ Repeat Step 2 else Break the Loop and Go To Step 3

Step 3 *Subchannel Allocation to BE-class users – Finalization of the Feasible Solution*

Assign each unallocated subchannel to the “best” users in terms of the achieved rate:

For each $n \in \mathcal{N}$, set $\rho_{n,k^*} = 1$, where $k^* = \arg \max_{k \in \mathcal{U}_{BE}} \{r_{n,k}\}$

Step 4 *Subchannel Swapping – Improved Solution*

For every user $k \in \mathcal{U}$ (either CBR or BE) repeat the following procedure serially:

- (a) Find his/her allocated subchannels set: $\mathcal{I}_k = \{n \in \mathcal{S} : \rho_{n,k} = 1\}$
- (b) Find the complementary users/subchan. sets: $\mathcal{U}'_k = \mathcal{U} \setminus \{k\}$, $\mathcal{I}'_k = \mathcal{S} \setminus \mathcal{I}_k$
- (c) For all possible combinations $\langle (n, k), (n', k') \rangle$, where $n \in \mathcal{I}_k$, $n' \in \mathcal{I}'_k$, $k \in \mathcal{U}'_k$ check the inner conditional expression depending on the active scenario:
 - (c.1) If $k \in \mathcal{U}_{CBR}$ and $k' \in \mathcal{U}_{CBR}$: $r_{n',k} > r_{n,k}$ && $lhs_{k'} + r_{n,k'} - r_{n',k'} \geq b_{k'}$
or $r_{n,k'} > r_{n',k'}$ && $lhs_k + r_{n',k} - r_{n,k} \geq b_k$
 - (c.2) If $k \in \mathcal{U}_{BE}$ and $k' \in \mathcal{U}_{BE}$: $r_{n',k} - r_{n,k} + r_{n,k'} - r_{n',k'} > 0$
 - (c.3) If $k \in \mathcal{U}_{CBR}$ and $k' \in \mathcal{U}_{BE}$: $r_{n,k'} > r_{n',k'}$ && $lhs_k + r_{n',k} - r_{n,k} \geq b_k$
 - (c.4) If $k \in \mathcal{U}_{BE}$ and $k' \in \mathcal{U}_{CBR}$: $r_{n',k} > r_{n,k}$ && $lhs_{k'} + r_{n,k'} - r_{n',k'} \geq b_{k'}$

If any of the conditions is TRUE employ the corresponding resource swapping:

$$\rho_{n,k} = 0, \rho_{n',k'} = 0, \rho_{n',k} = 1, \rho_{n,k'} = 1.$$

Step 5 *Release of CBR-class Subchannels for BE-class*

For every CBR user $k \in \mathcal{U}_{CBR}$:

- (a) Find his/her allocated subchannels set: $\mathcal{I}_k = \{n \in \mathcal{S} : \rho_{n,k} = 1\}$
- (b) For each $n \in \mathcal{I}_k$ if $lhs_k - r_{n,k} \geq b_k$ release it from this user and allocated it to the best BE-user: $\rho_{n,k} = 0, \rho_{n,k^*} = 1$, where $k^* = \arg \max_{k \in \mathcal{U}_{BE}} \{r_{n,k}\}$

4.2 Heuristic II: A Dual Approach of the Optimal Allocation

Description An alternative heuristic algorithm inspired by the works of [31] and [30], where the optimal allocation decision is approached from the exterior of the feasible space, is proposed in the present subsection. The “dual” approach shares also several similarities with the Lagrangean Relaxation technique, which dualizes the hard constraints. Specifically, the solution is approached on 2 phases: the construction of the optimal unconstrained subchannel allocation and the feasibility transformation of the initial decision through a series of subchannel reallocations. All the intermediate allocations generated by the dual approach are infeasible except for the last one. Note, that a similar algorithm for a single-QoS profile RRA problem has been proposed in [12]. We

now provide a summary of the algorithm as well as a complete step-by-step description of the dual approach in the context of our RRA problem.

During the 1st phase, the necessary initializations and declarations (**Step 1**) as well as the optimal unconstrained subchannel allocation are performed (**Step 2**). The optimal solution to the corresponding unconstrained problem is extracted by simply allocating each subchannel to the user that experiences the maximum data-rate on it. Obviously, the sum-rate performance of the best-user allocation policy is an upper bound on any QoS-constrained scenario. The 2nd phase on the other hand comprises a series of subchannel reallocations aiming at satisfying the minimum QoS targets or equivalently rendering the solution feasible (**Step 3**)¹. The selection of user/subchannel pairs participating in each reallocation cycle is dictated by the efficiency metric shown in Eq.(2), where n stands for a subchannel candidate for reassignment, $k^*(n)$ for the current owner of the subchannel (as determined by Step 2), and k the index of the candidate reassigned user.

$$a_{n,k} = \frac{r_{n,k^*(n)} - r_{n,k}}{r_{n,k}} \Big|_{n \in \mathcal{N}, k \in \mathcal{J}'} \quad (2)$$

The nominator of the metric expresses the decrease in sum-rate performance due to the reallocation, whereas the denominator the increase of the the infeasible user data-rate or equivalently the decrease of the distance from feasibility. Thus, by minimizing this metric we simultaneously harm as less as possible the original objective function value and approach as fast as possible the feasibility region. Notice, however that a specific resource-block reallocation is possible if the minimum QoS target for the owner-user is not violated. The particular iterative procedure ends when all CBR users' demands are satisfied. Finally, the last step of Phase 2 (**Step 4**) constitutes the release of redundant subchannels (if any) from the CBR-class users, similarly to the procedure followed in the 1st Heuristic.

Complete Algorithm

Step 1 *Initializations – Declarations*

Let $\{r_{n,k}\}_{n \in \mathcal{S}, k \in \mathcal{U}}$, $\{\rho_{n,k}\}_{n \in \mathcal{S}, k \in \mathcal{U}}$, $\{lhs_k\}_{k \in \mathcal{U}_{\text{CBR}}}$, $\{b_k\}_{k \in \mathcal{U}_{\text{CBR}}}$ as in Heuristic I.

Step 2 *Optimum Unconstrained (QoS-unaware) Allocation*

Best-Rate Allocation: For each $n \in \mathcal{S}$, set $\rho_{n,k^*} = 1$, where $k^* = \arg \max_{k \in \mathcal{U}} \{r_{n,k}\}$

Minimum QoS satisfaction check: If for all $k \in \mathcal{U}_{\text{CBR}}$: $lhs_k \geq b_k$ Then Go To Step 4

Else Go To Step 3.

¹ During the reallocation subprocedure followed here, subchannels are removed from one user and given to another, whereas in the 1st heuristic subchannels interchanges occur

Step 3 Subchannels Reallocation – QoS Satisfaction

Define the following subchannels and users subsets:

$\mathcal{J}_{\text{CBR}} = \{k \in \mathcal{U}_{\text{CBR}} : lhs_k \geq b_k\} \subseteq \mathcal{U}_{\text{CBR}}$: the subset of CBR users for which minimum QoS is met.

$\mathcal{J}'_{\text{CBR}} = \mathcal{U}_{\text{CBR}} \setminus \mathcal{J}_{\text{CBR}}$: the complementary subset of undersatisfied CBR users.

$\mathcal{J}_{\text{BE}} = \{k \in \mathcal{U}_{\text{BE}} : lhs_k > 0\} \subseteq \mathcal{U}_{\text{BE}}$: the subset of BE users allocated at least one subchannel or equivalently possess non-zero data rate.

$\mathcal{J} = \mathcal{J}_{\text{CBR}} \cup \mathcal{J}_{\text{BE}}$: the subset of users from which we can remove subchannels

$\mathcal{J}' = \mathcal{J}'_{\text{CBR}}$: the subset of users to which we must add subchannels in order to meet the minimum QoS requirements. $\mathcal{N} = \{n \in \mathcal{S}, k \in \mathcal{J} : \rho_{n,k} = 1\}$: the subchannels pool from which we can extract subchannels and reallocate to the users subset \mathcal{J}' .

Repeat the following procedure while there exist unsatisfied CBR users, namely as long as $\mathcal{J}' \neq \emptyset$:

(a) For each subchannel $n \in \mathcal{N}$ identify the owner $k^*(n) = \{k \in \mathcal{J} : \rho_{n,k} = 1\}$.

(b) Compose the 2D Reallocations Array $\mathbf{A}_{|\mathcal{N}| \times |\mathcal{J}'|} = [a_{n,k}]$, where:

$$a_{n,k} = \begin{cases} \frac{r_{n,k^*(n)} - r_{n,k}}{r_{n,k}}, & \text{if } lhs_{k^*(n)} - r_{n,k^*(n)} \geq b_{k^*(n)} \text{ and } r_{n,k} > 0 \\ +\infty & \text{, elsewhere} \end{cases}$$

(c) Check all the elements of the array:

If for all elements $a_{n,k} = +\infty$, Break the Loop and Go To Step 4.

Else pick the subch./user combination according to $\{n', k'\} = \arg \min_{n \in |\mathcal{N}|, k \in \mathcal{J}'} \mathbf{A}$ and perform the reallocation: $\rho_{n', k^*(n')} = 0, \rho_{n', k'} = 1$.

(d) Update the related quantities/subsets: $\mathcal{J}, \mathcal{J}', \mathcal{N}, lhs_k, k \in \mathcal{U}_{\text{CBR}}$.

Step 4 Redundant CBR-class Subchannels Reallocation

Define the following subchannels and users subsets:

$\mathcal{J} = \{k \in \mathcal{U}_{\text{CBR}} : lhs_k > b_k\}$: the subset of CBR users assigned redundant data-rate (over-satisfied users).

$\mathcal{N} = \{n \in \mathcal{S}, k \in \mathcal{J} : \rho_{n,k} = 1\}$: possible redundant subchannels pool.

Repeat the following procedure while there exist over-satisfied users, namely as long as $\mathcal{J} \neq \emptyset$:

For each $n \in \mathcal{N}$:

(a) find the owner $k^*(n) = \{k \in \mathcal{J} : \rho_{n,k} = 1\}$

(b) If $lhs_{k^*} - r_{n,k^*} \geq b_{k^*}$ remove the subchannel n from the owner $k^*(n)$ and reallocate it to the “best” BE user in terms of achieved data-rate k' :

$\rho_{n',k^*(n')} = 0, \rho_{n',k'} = 1$, where $k'(n) = \arg \max_{k \in \mathcal{U}_{\text{BE}}} \{r_{n,k}\}$, Else Go To the next subchannel.

4.3 Computational Complexity Estimation

An estimation of the computational complexity of the proposed optimal and suboptimal schemes follows. With respect to the heuristic algorithms we take into account the involved searching, sorting and comparison operations. We assume that the order of CBR and BE users is the same that is, $|\mathcal{U}_{\text{CBR}}| \sim |\mathcal{U}_{\text{BE}}| \sim K$ and that the number of subchannels is an order greater than the number of active users, namely $N \gg K$.

Exhaustive and ILP/LP models We first consider an exhaustive search approach, where all the possible combinations of subchannels-to-users assignments are examined. The particular procedure has an exponential complexity order of $O(K^N)$, which is obviously computationally intractable. As far as the exact optimal ILP model there is no guarantee for the execution complexity, due to its NP-hard nature, however it is expected to be significantly lower than the complete enumeration. The relaxed LP model on the other hand has a provable third-order polynomial solution complexity with respect to the number of the involved variables. Hence, in order to find the performance upper bound of the RRA problem we have to spend a set of computational operations of order $O(N^3 K^3)$.

Heuristic I Since the introductory Step 1 induces no complexity, we proceed directly with the next step. We first employ a necessary pre-sorting operation regarding the achieved data-rate values over all users, which costs $KN \log N$ operations (this is an implementation issue, and this is why it was omitted from the previous algorithmic description). Regarding Step 2 we perform at most N iterations for satisfying the CBR-class QoS constraints, and at each iteration we have to select the worst user in terms of the average achieved data-rate with complexity cost K and locate its best subchannel with cost $\log N$ (due to pre-sorting), leading to a total cost of $NK + N \log N$ operations. Step 3 involves $N \log K$ operations for identifying the best-rate user for each remaining subchannel. Step 4 is the most complex one, since it entails a series of comparisons: without loss of accuracy we assume that each user is preallocated on average N/K subchannels, and then for each user the assigned N/K subchannels are compared with the N/K subchannels of the complementary $(K - 1)$ users. All users are scanned for a full cycle, hence the whole process costs $\frac{N}{K} \cdot \frac{N}{K} \cdot (K - 1) \cdot K \approx N^2$ operations. Finally, Step 5 involves at most N subchannel releases, and for each one the best-rate BE user must be located with cost $\log K$, hence $N \log K$ operations are needed. Combining all the above algorithmic steps we get $KN \log N + NK + N \log N + N^2 + N \log K$

operations and after performing several manipulations and approximations we resort to an estimated computational complexity order of $O(N^2 + NK \log N)$.

Heuristic II Similarly to the 1st heuristic we first employ a pre-sorting operation for the 2D rates array which costs $KN \log N$ operations. Step 2 needs $N \log K$ operations in order to find the optimal unconstrained allocation. Step 3 contributes significantly to the overall complexity: at most N subchannel reallocations are performed and for each one a reallocation cost matrix containing NK elements must be devised and then searched for the combination experiencing the minimal cost. Thus $N \cdot NK = N^2K$ operations are needed to fulfil the particular step. Step 4 induces an additional cost of $N \log K$ operations as in the 1st heuristic. Accounting for all the algorithmic steps we result to an overall approximate complexity order of $O(N^2 + NK \log N)$.

5 Simulation Results and Discussion

Simulation Setup The DL of a single-cell OFDMA-based packet data network is modeled and simulated in the context of this work, while the selection of system parameters reflects an LTE scenario [14]. A system bandwidth of 20 MHz is assumed, consisting of $N = 100$ orthogonal data subchannels. DL transmissions occur on frame bursts of $T_f = 1$ msec. A realistic pedestrian NLOS macro-cell urban channel from the WinnerII models family is adopted [33]. The PHY-abstraction function of Cioffi is utilized for associating each channel-to-noise ratio level with an achieved data rate (see Appendix A). Channel conditions are assumed perfectly known at the BS, allowing for an opportunistic subchannel allocation. At the receiver side transmissions are harmed due to thermal noise with power density of $N_0 = -174$ dBm/Hz.

As far as the traffic/QoS models, a CBR/BE dual-class scenario is formed. Each CBR user requires $R_k^{\min} = 36$ bits/OFDMA symbol²; the overall required CBR load is determined by varying the number of users, where $|\mathcal{U}_{\text{CBR}}| = 6-12$. The number of BE users is held constant at $|\mathcal{U}_{\text{BE}}| = 5$. System capacity may be calculated as $C_{\max} = N \cdot c_{\max} = 600$ bits, where c_{\max} is the maximum supported data-rate per subchannel, however the true achieved cell-rate is expected to be lower due to the hard CBR rate constraints and BE-class users' power shortage. The simulation parameters are summarized in Table 1.

We consider five schemes which are compared below in terms of the achieved sum-rate performance while we guarantee the CBR-class rates:

- The exact optimal scheme extracted by solving the ILP optimization problem given in in Eq. (1) (**IP**)

² For simplicity reasons we may assume that an OFDMA frame carries one data-symbol. Thus all the data-rate/throughput quantities may be identically expressed in bits/OFDMA symbol/frame or simply in bits.

Table 1 Simulation Parameters

Quantity	Symbol	Value/Comment
Carrier Frequency	f_c	2.5 GHz
System Bandwidth	BW	20 MHz
Subchannel Bandwidth	Δf	200 kHz
Number of Subchannels	N	100
Transmission Error Rate	P_e	10^{-6}
Noise Power Density	N_0	-174 dBm/Hz
Higher Tx Order per suchannel	c_{max}	6 bits/symbol
Cell Radius	R_{cell}	2 km
Channel Model	–	WinnerII C3 BU-NLOS Macro
Users Distribution	–	Uniform
Users Mobility Model	–	Pedestrian (4 km/hr)
Number of OFDMA frames per drop	–	100
CBR QoS model	$\{R_k^{\min}\}_{k \in \mathcal{U}_{CBR}}$	36 bits/OFDMA symbol
CBR users	$K_1 = \mathcal{U}_{CBR} $	6–12
BE users	$K_2 = \mathcal{U}_{BE} $	5
Max Number of drops	N_{drops}^{ulim}	1000
Min Number of drops	N_{drops}^{llim}	25
Statistics Convergence Threshold	σ_{norm}	0.02

- A scheme that provides us with a performance upper bound, obtained by solving the continuous relaxed version of the previous scheme (**LP**)
- The 1st heuristic proposed in Sec.4.1 (**HEUR1**), for which we also run a version without using the swapping sub-procedure (HEUR1 no swap)
- The 2nd heuristic proposed in Sec.4.2 (**HEUR2**)
- A semi-random allocation algorithm (**RANDOM**) which: (a) satisfies the target rates for the CBR-class users, by picking them one by one and assigning their best available channel until all requirements are met and, (b) assigns the remaining subchannels to the BE users randomly

All the algorithms are implemented in MATLAB and for the ILP/LP problems we utilize the CPLEX solver [34], calling it through the TOMLAB interface [35]. In order to capture the effect of different system parameters to the achieved performance, we simulate 20 realistic system scenarios by varying:

- (i) the CBR-loading/QoS levels in the service area, by considering a different number of active CBR users ($K_1 = |\mathcal{U}_{CBR}| = 6, 8, 10, 12$)
- (ii) the average experienced SNR conditions in the cell, by tuning the ratio of the BS transmission power (P_{bs}) to the minimum required Tx power for guaranteeing feasibility regarding the K_1 data-rate constraints ($P_{CBR}^{feas}(K_1)$) in each scenario ($P_{bs}/P_{CBR}^{feas} = 2.0, 2.5, \dots, 4.0$ ³)

³ Note that lower values for the power availability metric (e.g. 1.0 or 1.5) are not examined. This is justified by the fact that at lower SNR conditions, one or more CBR rate constraints can not be satisfied (“outage” conditions), rendering the QoS constrained problem infeasible.

We define an arbitrary realization or “drop” as a system setup consisting of a set of users randomly placed in the cell, for which their large-scale channel conditions are held constant, whilst the small-scale conditions vary in time. Each drop consists of 100 consecutive OFDMA frames, spanning $100 \cdot T_f = 100$ msec. For each scenario we simulate multiple statistically independent drops and record the average sum-rate performance of each scheme. The simulation is terminated if the normalized (to the mean) variance of the performance statistics drops below the convergence target threshold (which is set to 0.02) or if the maximum number of simulated drops (which is set to 1000) has been reached [37]. Moreover, a minimum number of 25 drops are executed in any scenario, in order to avoid transient effects. Thus, at least $25 \cdot 100 = 2,500$ and at most $1000 \cdot 100 = 100,000$ optimization problems are formulated and solved for each scenario.

Results and Discussion In Figs.1 and 2 we depict the sum-rate performance for all schemes as a function of the required CBR loading and the average experienced SNR conditions. The effect of the subchannel swapping procedure to the performance of the HEUR1 algorithm is illustrated in Fig.3 (HEUR1 with/no swap). In Figs.4(a) and 4(b) we provide the optimality gaps of our heuristic schemes (HEUR1,HEUR2) for two representative power availability scenarios. Finally in Table 2 we present the achieved performance of the heuristic schemes compared to the exact optimal (IP) as well as the performance difference between the IP and the relaxed LP approach. The main observations/comments regarding the behavior of the various schemes/algorithms are the following.

- (i) Both heuristic approaches perform close to the optimal allocation scheme and follow its performance trend for different system conditions (Figs.1,2) while their complexity cost is significantly lower.
- (ii) The proposed heuristics outperform the RANDOM assignment scheme significantly. This means that exploiting the inherent 2D rate selectivity/diversity of the system leads to a dramatic increase on the cell performance. The performance gain of the 1st heuristic is 60.6% on average. In particular in low SNR conditions the gain is approximately 80% and at higher SNR conditions drops to 48.64%, which is still high. This is reasonable, since at deteriorating channel conditions, the selection of the “best” subchannels-set for each user becomes more critical. On the other hand the 2nd heuristic provides us with an average gain of 52.8%, which is still remarkable.
- (iii) The optimality gap (defined as the percentage sum-rate loss from the optimal scheme) for each algorithm depends on the CBR-loading and the received SNR

In order to cope with such situations, an adaptive power allocation strategy must be employed, like the one we proposed in our recently published work [36].

conditions experienced in the cell. The gap seems to narrow as SNR conditions are improved through the increase in the BS power. This is justified by the fact that for higher amounts of BS power, larger data-rates per subchannel are supported (close to the upper bound of 6 bits), leading to the vast majority of available bandwidth resources experiencing high performance, and thus the selection of the optimal subchannel set for each user is not so critical anymore.

- (iv) The 1st heuristic has an average optimality gap of 6.7% for the lowest BS-power scenario which decreases to 2.3% for the highest BS-power scenario. On the other hand the alternative dual heuristic performs worse than the first one. Its recorded optimality gap is 15.4% for the lowest SNR scenario and 4.6% for the highest on average.
- (v) The worst-case performance for the 2nd heuristic is observed at the lowest SNR and highest CBR-loading scenario, where the performance loss compared to the optimal scheme is 23.87% (Fig.4(a)–rightmost group of bars). Recall that the particular algorithm first allocates the available resources ignoring the demanded QoS levels. Under such conditions, the CBR-class sum-rate heavily dominates the overall dual-class sum-rate, and thus the subsequent resources reallocation phase finds great difficulty in leading to a feasible allocation. In other words when the propagation conditions are harsh and the QoS targets demanding, it is better to first guarantee the strict CBR rate constraints and then look for an improved allocation. On the contrary at higher SNR/lower CBR-loading conditions (Fig.4(b)–leftmost group of bars) the achieved performance is quite high (or equivalently the associated optimality gap is quite low) since the distance between the original solution and the feasibility is significantly smaller.
- (vi) The performance of the 1st heuristic is not significantly affected by the CBR-loading conditions, contrary to the 2nd heuristic behavior. For the lowest power scenario the optimality gap of the 1st heuristic increases by 2.3% (from 5.9% goes to 8.2%) as the number of CBR users increases from 6 to 12, whereas for the 2nd heuristic the corresponding increase is 14.3% (from 9.6% goes to 23.9%). This is justified by the fact that the 1st heuristic focuses on finding a feasible solution by prioritizing CBR-class users at the initial allocation phase. Similar conclusions may be drawn for higher SNR scenarios as well.
- (vii) As far as the 1st heuristic the importance of the subchannel swapping step is demonstrated through Fig.3, where one may observe the improvement level of the sum-rate performance for all the simulation experiments. The performance gain varies from 1.2–7.7% and it is more noticeable in lower CBR-loading scenarios. In such cases a lower number of subchannels is required for the CBR-class and thus a larger number of re-assignments is expected to occur.

- (viii) Averaged over all conditions, the 1st heuristic achieves 96.21% of the optimal sum-rate performance while the 2nd 91.63% of it.
- (ix) The performance gap between the actual optimal solution (IP) and the upper bound (LP) is not negligible. For the lowest power scenario (Fig.4(a)) this gap ranges between 4.4% and 17.9% whereas for higher BS power (Fig.4(b)) varies between 2.8% and 11.9%. Therefore, although the LP solution is extracted very efficiently compared to the IP, it often fails on providing a tight bound on the exact optimal performance. Consequently, if the LP bound is used as a performance benchmark for the evaluation of a suboptimal scheme, then the actual efficiency of the latter would be underestimated.

6 Conclusions - Future Work

The resource allocation problem of maximizing the sum-rate performance for a down-link OFDMA single-cell network (like LTE) assuming heterogeneous traffic requests and uniform power loading over the system subchannels was studied in this work. The problem was first mathematically modeled as an ILP optimization problem, allowing us to extract the actual optimal allocation decision and the associated maximum achieved sum-rate performance in reasonable execution time. By relaxing the integrality constraints on the above model, a performance upper bound may also be extracted with 3rd order polynomial complexity cost. In the second part of this work, motivated by the resemblance of the specific problem with a well-known combinatorial optimization problem that is, the Generalized Assignment, we developed two heuristic algorithms for efficiently allocating system subchannels to the competing classes and users. We finally demonstrated through extensive simulation experiments that the performance loss of the heuristic schemes compared to the optimal is rather low, especially for the 1st heuristic, and that our schemes heavily outperform semi-random subchannel assignments. Possible extensions of this work may involve the cooperation of the proposed schemes with time-domain scheduling algorithms handling packet delay and fairness QoS objectives, the employment of practical channel state reporting schemes and the consideration of interference in multi-cell deployments.

A The Data-Rate Abstraction Model

Let $\langle n, k \rangle$ an arbitrary subchannel/user combination and assume that a bit-stream is transmitted from the BS to the k^{th} user over the n^{th} subchannel. We denote by $P_{n,k}$ the allocated transmitted power, $|h_{n,k}|^2$ the propagation channel power gain (which is known at both transceiver ends), N_0 the noise power density and Δf the bandwidth of each subchannel.

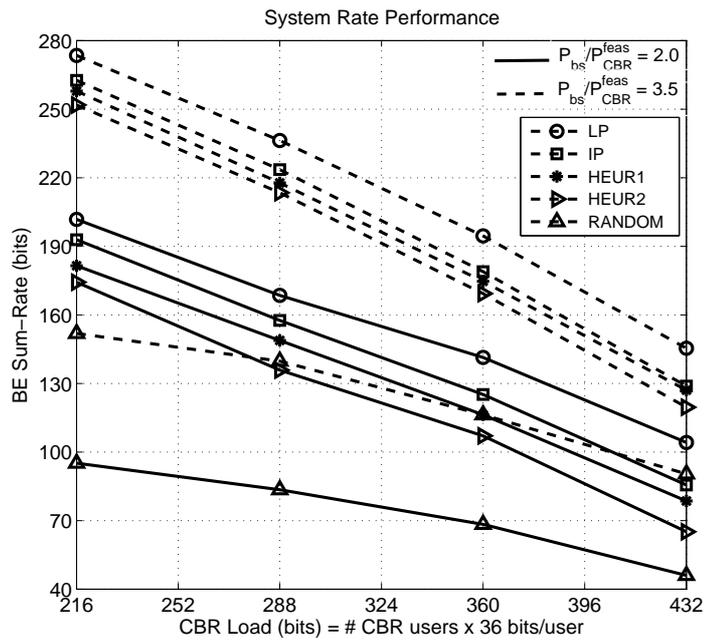


Fig. 1 System Rate performance vs CBR-Loading conditions

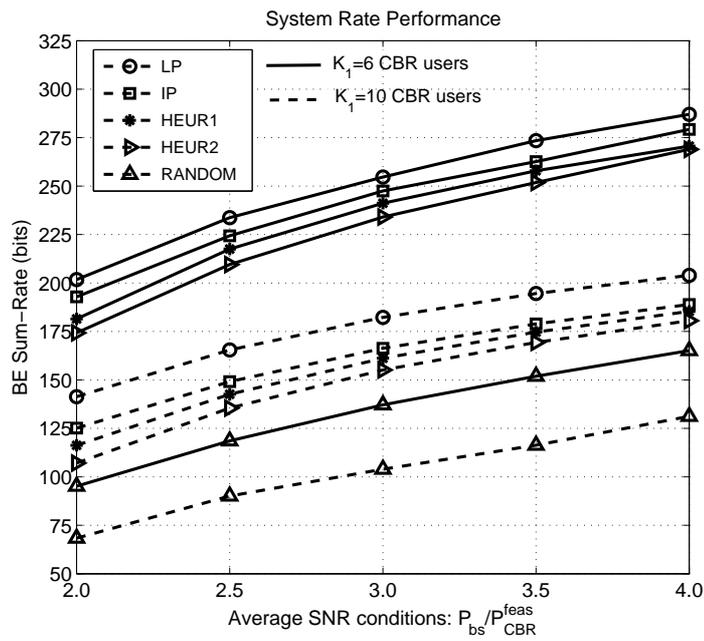


Fig. 2 System Rate performance vs average received SNR conditions

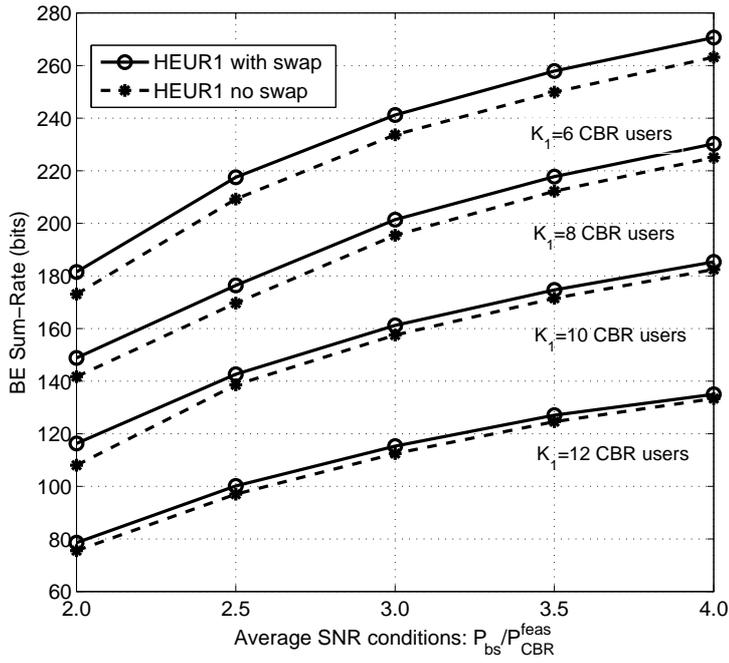
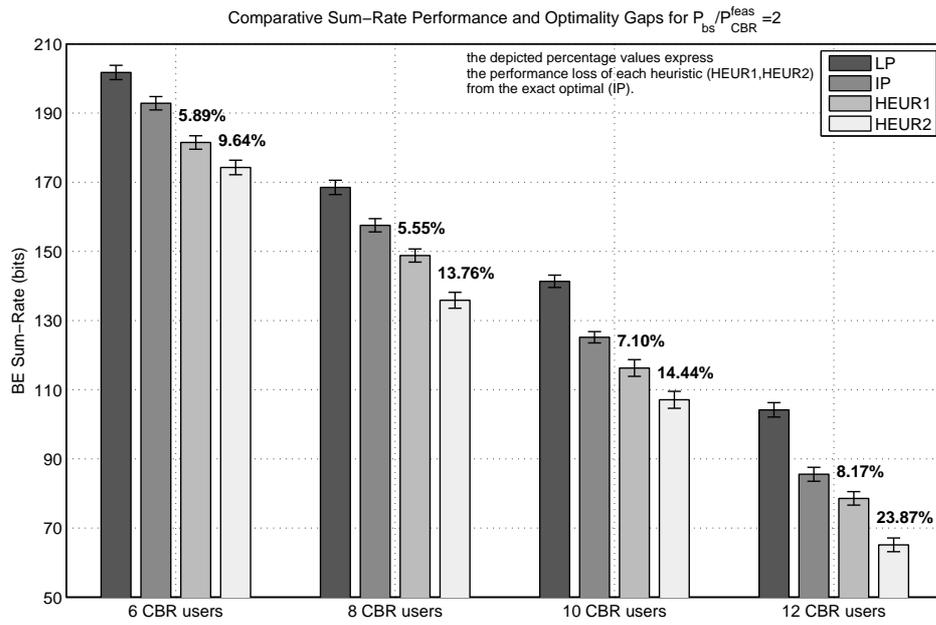


Fig. 3 The effect of the subchannels swap procedure to the HEUR1 performance

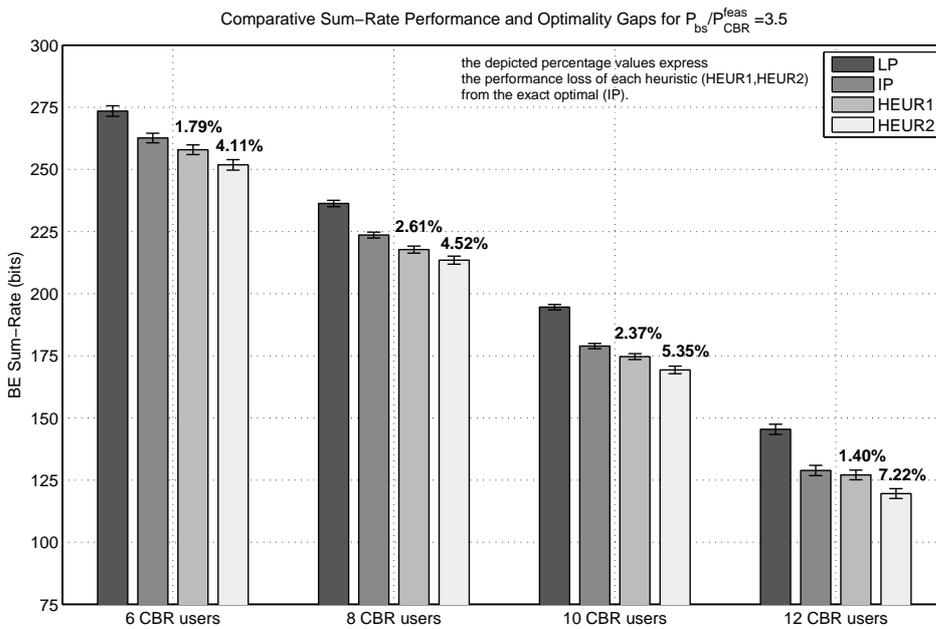
Table 2 Achieved Optimality Gap values for all Scenarios

Scenario $\{K_1, P_{bs}/P_{CBR}^{feas}\}$	IP/LP %	HEUR1/IP %	HEUR2/IP %
{6, 2.0}	95.59	94.11	90.36
{6, 2.5}	95.99	96.94	93.41
{6, 3.0}	97.18	97.46	94.58
{6, 3.5}	96.03	98.21	95.89
{6, 4.0}	97.29	96.92	96.32
{8, 2.0}	93.50	94.45	86.24
{8, 2.5}	93.60	93.77	92.16
{8, 3.0}	94.11	96.74	94.31
{8, 3.5}	94.62	97.39	95.48
{8, 4.0}	94.91	97.80	96.23
{10, 2.0}	88.55	92.90	85.56
{10, 2.5}	90.12	95.61	90.83
{10, 3.0}	91.24	96.93	93.28
{10, 3.5}	91.94	97.63	94.65
{10, 4.0}	92.62	98.11	95.59
{12, 2.0}	82.12	91.84	76.12
{12, 2.5}	85.54	94.51	85.27
{12, 3.0}	88.09	96.44	90.05
{12, 3.5}	88.62	98.60	92.78
{12, 4.0}	91.11	97.97	95.40

Then, by applying the closed-form approximation model of [1], the achieved data-rate (chan-



(a) Lower SNR-conditions



(b) Higher SNR-conditions

Fig. 4 Sum-Rate and Optimality Gap Performance

nel capacity) for preserving a minimum transmission error rate of P_e will be given by Eq.(3), where f stands for the abstraction function.

$$r_{n,k} = f(P_{n,k}, |h_{n,k}|, P_e) = \log_2 \left(1 + \frac{P_{n,k} \cdot |h_{n,k}|^2}{\frac{1}{3} [Q^{-1}(\frac{P_e}{4})]^2 \cdot N_0 \cdot \Delta f} \right) \quad (3)$$

We further define the normalized channel-to-noise ratio as in Eq.(4), and by employing the Uniform Power Loading assumption, we resort to the expression of Eq.(5) where P_{bs} is the total available BS power. As also seen in Eq.(5) the achieved data rate on each subchannel is hard-limited by the highest available transmission order denoted by c_{max} bits. The latter comprises a system constraint similar to the BS power. Finally, notice that if power loading is not a-priori known then the achieved data-rates can not be precalculated, since they depend on the allocated amount of power. In such scenarios joint power and subchannel allocation must be employed (see [36] for example).

$$\gamma_{n,k} = |h_{n,k}|^2 / \left[(1/3) \cdot [Q^{-1}(P_e/4)]^2 \cdot N_0 \cdot \Delta f \right] \quad (4)$$

$$r_{n,k} = \min \left\{ \log_2 \left(1 + \frac{P_{bs}}{N} \cdot \gamma_{n,k} \right), c_{max} \right\} \quad (5)$$

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