On the Interference Cancellation by Reduced Channel Zero Forcing Class of Precodings in Massive MIMO Systems

Dmitry Mineev
Huawei Technologies Co Ltd
Evgeny Bobrov (eugenbobrov@ya.ru)
Lomonosov Moscow State University Faculty of Computational Mathematics and Cybernetics: Moskovskij gosudarstvennyj universitet imeni M V Lomonosova Fakul'tet vycislitel'noj matematiki i kibernetiki https://orcid.org/0000-0002-2584-6649
Viktor Kuznetsov
Huawei Technologies Co Ltd

Research Article

Keywords: Massive MIMO, Wireless Systems, Precoding, Interference, QR-MLD, MMSE-IRC, Generalized LSE

Posted Date: August 26th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1861657/v1

On the Interference Cancellation by Reduced Channel Zero Forcing Class of Precodings in Massive MIMO Systems*

Dmitry Mineev[1], Evgeny Bobrov[1,2][0000−0002−2584−6649], Viktor Kuznetsov[1]

[1] Moscow Research Center, Huawei Technologies, Russia
dmitry.mineyev@gmail.com, kuznetsov.victor@huawei.com
[2] M. V. Lomonosov Moscow State University, Russia
eugenbobrov@ya.ru

Abstract. In this research, we study the interference cancellation capabilities of receivers and transmitters in multiple-input-multiple-output (MIMO) systems using theoretical calculations and numerical simulations in Quadriga. We study so-called Reduced Channel Zero-Forcing (RCZF) class of precoding as well as Minimum MSE Interference Rejection Combiner (MMSE-IRC) and QR Maximum Likelihood Detection (QR-MLD) receivers. The precodings from the RCZF class are widely used in practice and include for example Zero-Forcing (ZF), Regularized Zero-Forcing (RZF) and Iteratively Weighted MMSE precodings asymptotically. Our theoretical and experimental results confirm that MMSE-IRC and QR-MLD receivers in combination with the RCZF precoding provide complete interference suppression asymptotically.

Keywords: Massive MIMO · Wireless Systems · Precoding · Interference · QR-MLD · MMSE-IRC · Generalized LSE

1 Introduction

Massive multi-input multi-output (MIMO) systems are considered to be a key component of fifth-generation (5G) networks [1, 2, 6, 9, 15], because they are designed to serve many more users with a much higher overall bandwidth [11]. It has been proved [17,19] that a massive MIMO system can drastically increase the spectral efficiency of the wireless channel. One of the reasons for this comes from the fact that, as the number of transmitting antennae at the base station (BS) grows, both inter-cellular and intra-cellular interference can be effectively negated [3].

The first layer of MIMO system containing scheduling procedure also addresses the phenomenon of interference, and many useful algorithms of user selection and resource allocation for the MIMO downlink have been investigated in [16] (see also references therein). Scheduling techniques try to minimize the interference taking into account geometrical alignment of users, but the residual

* Supported by Moscow Research Center, Huawei Technologies, Russia.

inter-user interference is still significant. For proper detection at each user equipment (UE), this residual interference must be suppressed by means of the second layer of MIMO system. Furthermore, the first layer of MIMO system provides the decision for rank selection of every UE, and it is up to the second layer again to devise precoding with given user ranks.

A lot of effort has been put into developing precoding and detection methods for multi-user MIMO systems [3–5, 7, 8, 10, 13, 20, 23, 24]. In this paper, we introduce the novel class of precoding called Reduced Channel Zero-Forcing (RCZF) and study its properties. RCZF precoding generalizes known constructions such as Eigen Zero-Forcing (EZF) [21]. In Section 2 we define necessary terms and provide two descriptions of MIMO system suitable for our study. In Section 3 we proceed to prove Theorem 1 which states that RCZF class precoding is capable of eliminating interference while others are not. In Section 4 we provide detailed description of two widespread detection schemes, namely Minimum MSE Interference Rejection Combiner (MMSE-IRC) [18,22] and QR Maximum Likelihood Detection (QR-MLD) [14]. The theoretical part of this paper culminates in Theorems 2 and 3 stating that these two detection schemes suppress interference combined with RCZF precoding. In Section 5 we support all the claims of the paper with Quadriga [12] based experiments and present their description and graphical evidence. All necessary algebraic notations are given in Tab 1.

2 System Model

The multi-user multi-input multi-ouput (MU-MIMO) model is described by the following linear system, where $r_k \in \mathbb{C}^{p_k}$ is a vector of detected symbols at the k-th receiver, $s \in \mathbb{C}^p$ is a common vector of sent symbols, $H_k \in \mathbb{C}^{q_k \times t}$ is a channel matrix of the k-th receiver, $W \in \mathbb{C}^{t \times p}$ is a common precoding matrix, $G_k \in \mathbb{C}^{p_k \times q_k}$ is a detection matrix at the k-th receiver, and $n_k \sim \mathcal{CN}(0, I_{p_k})$ is a k-th receiver noise:

$$r_k = G_k(H_k W s + n_k).$$

In the MIMO system, it is possible to send several information symbols to a multi-antenna user on a single physical resource. The number of such symbols is called the rank of the k-th user and is denoted by p_k, and $p_1 + \cdots + p_l = p$ means the total number of layers. The constant t means number of transmitting antennas on the base station, q_k means number of receiving antennas of the k-th user, where $q_1 + \cdots + q_l = q$. We assume they are related as follows: $p_k \leqslant q_k \leqslant t$.

For our purposes, the following equivalent description of MIMO system will be convenient (see Fig. 1). The noiseless linear MIMO system consists of three sets of complex vector spaces: S_k — *symbol spaces*, A — *antenna space*, and U_k — *user spaces*; and of three sets of linear maps between them: $W_k \colon S_k \to A$ — *precoding*, $H_k \colon A \to U_k$ — *channel*, and $G_k \colon U_k \to S_k$ — *detection*.

Note that precoding maps separate symbol spaces to the common antenna space, channel maps common antenna space to separate user spaces, and detection is completely independent for each user, mapping separate user spaces to

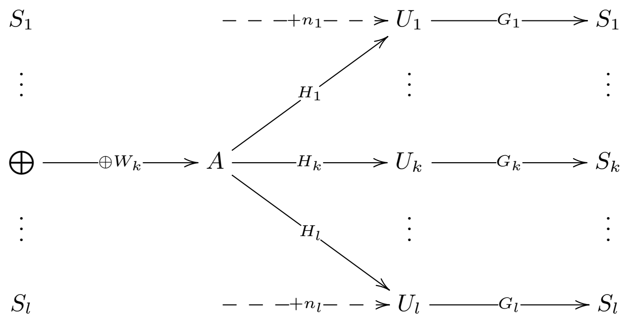

Fig. 1. Spaces and maps description of MIMO system.

separate symbol spaces. We also assume that channel maps H_k have full rank, i.e., $\operatorname{rk} H_k = \dim U_k = q_k$.

Noisy MIMO system is obtained by adding randomly distributed noise vectors $n_k \in U_k$. Roughly speaking, the goal of a MIMO system is to make $G_k H_k W_k$ as close as possible to identity map (this is an effective signal addressed to the user) and to make $G_k H_k W_j$ for $j \neq k$ as close as possible to zero map (this is interference with the signal intended for other users).

Definition 1. *We say that noiseless MIMO system* ***cancels interference*** *if* $G_k H_k W_k = \mathrm{Id}$ *and* $G_k H_k W_j = 0$ *for* $j \neq k$*.*

All vector spaces participating in the MIMO system are equipped with natural bases: for symbol spaces, the coordinates in the base are the symbols themselves; for antenna and user spaces, the coordinates in the base are values on antennae (either transmitted or received). We will denote matrices of linear maps in MIMO system in these bases by the same letters W, H, G. This notation coincides with the standard matrix description of MIMO system.

3 RCZF Precoding

Definition 2. *Set of precoding maps* W_k *is called* ***Reduced Channel Zero-Forcing*** *(abbreviated as RCZF) if there exist matrices* V_k *such that*

- $V_k = B_k H_k$ *for some* B_k*;*
- $V_k W_j = 0$ *for* $j \neq k$*;*
- $\operatorname{rk} V_k W_k = \operatorname{rk} W_k = \dim S_k = p_k$*.*

We will refer to matrices V_k as *reduced channel* matrices. In particular, usual zero-forcing is obtained if $B_k = I$, and $V_k = H_k$ are whole channel matrices. A less trivial example is given by EZF [21]), which determines B_k by the following procedure. Write SVD decomposition of channel $H_k = \mathcal{U}_k \mathcal{S}_k \mathcal{V}_k$. Then $B_k = I' \mathcal{S}_k^{-1} \mathcal{U}_k^{-1}$, where $I' = \big(I | 0\big)$, so that V_k is the upper cut of $\mathcal{V}_k$. Precoding itself is calculated as pseudo-inverse of concatenated matrices V_k, which is one of possible ways to satisfy conditions of RCZF.

Let us make another important remark on the definition of RCZF. Instead of usual zero-forcing relations such as $H_k W_k = I$ or, more realistically, $H_k W_k$ is non-zero diagonal, we make a weaker requirement on the full rank. This is done for the general case when the number of layers for the user k (i.e., $\dim S_k = p_k$), is strictly less than $\dim V_k$ (i.e., the dimension of the reduced channel). This happens, for instance, if the number of layers is strictly less than the number of user antennae (i.e., $\dim U_k = q_k$), and we do full zero-forcing with $V_k = H_k$.

H_k	$t \times q_k$ (64×4) channel matrix of user k
V_k	$t \times p_k$ (64×2) layers matrix of UE k
B_k	$p_k \times q_k$ (4×2) reducing channel to layers, i.e., $H_k B_k = V_k$
W_k	$t \times p_k$ (64×2) precoding: $V_k^* W_k = I, V_i^* W_k = 0$ for $i \neq k$
A_k	$p_k \times q_k$ (4×2) transmitting matrix, i.e. $A_k = H_k^* W_k$
n_k	q_k (4) external noise
n', n'', n'''	q_k (4) normalized independent noise vectors
x_k	p_k (2)-symbol vector sent to UE k
y_k	q_k (4)-symbol vector received by UE k
G_k	$p_k \times q_k$ (2×4) detection matrix
R_k	$q_k \times q_k$ (4×4) noise covariance matrix
L_k	$q_k \times q_k$ (4×4) external noise power matrix, i.e., $n_k = L_k n'_k$
Ω	$q_k \times q_k$ (4×4) noiseless covariance matrix

Table 1. The algebraic notations used in the work and the typical matrix dimensions.

To avoid confusion, we will choose B_k (and, respectively, V_k) as *maximal* reduced channels corresponding to some RCZF precoding. For example, a usual set of zero-forcing matrices W_k is obviously a zero-forcing some non-trivial reduced channels V'_k, but we still choose V_k corresponding to this RCZF to be H_k.

We are now ready to formulate the main result.

Theorem 1. ***Interference Cancellation Condition.*** *1) In the noiseless MIMO system interference cancellation can be achieved only if the set of precoding matrices W_k forms RCZF; 2) In this case, detection maps G_k such that interference is cancelled, exist; 3) These maps are unique, provided that* $\operatorname{rk} B_k = \operatorname{rk} W_k$.

The last condition is indeed practical, since RCZF condition $\operatorname{rk} V_k W_k = \operatorname{rk} W_k$ implies $\operatorname{rk} B_k \geqslant \operatorname{rk} W_k$, and we would usually like to choose B_k as small as possible to satisfy fewer restrictions posed by $V_k W_j = 0$ for $j \neq k$ (these restrictions decrease effective transmission power).

We will now see that Theorem 1 is surprisingly tautological. For the first statement, suppose that a noiseless MIMO system cancels interference, and take $B_k = G_k$. Then RCZF conditions follow immediately from interference cancellation.

For the second statement, denote $\ker B_k = T_k \subset U_k$. Then, as $V_k W_j = B_k H_k W_j = 0$ for $j \neq k$, we have $\operatorname{Im} H_k W_j \subset T_k$. Denote $\operatorname{Im} H_k W_k = P_k \subset U_k$.

Since $\operatorname{rk} V_k W_k = \operatorname{rk} B_k H_k W_k = \operatorname{rk} W_k$, image of $H_k W_k$ is transverse to the kernel of B_k, i.e., $P_k \cap T_k = 0$. Now choose $G_k|_{T_k} = 0, G_k|_{P_k} = (H_k W_k)^{-1}$ (this inverse is defined properly on P_k), and the rest of detection map arbitrarily in case P_k and T_k do not generate U_k.

For the third statement, observe that subspaces $\operatorname{Im} H_k W_j \subset U_k$ generate the whole T_k. This is due to B_k chosen in the maximal way — otherwise we could replace it by $B'_k \supset B_k$ such that $T'_k = \ker B'_k \subset T_k$ is generated by $\operatorname{Im} H_k W_j$. Now, since $\operatorname{rk} B_k = \operatorname{rk} W_k$, we have $\dim T_k + \dim P_k = \dim U_k = q_k$, so $U_k = T_k \oplus P_k$, and G_k is uniquely determined on both P_k and T_k. Therefore, it is uniquely determined on U_k.

4 Detection options

In this section, we assume that precoding is RCZF and condition from the third statement of Theorem 1 is satisfied, i.e., $\operatorname{rk} B_k = \operatorname{rk} W_k$. Furthermore, we assume that $V_k W_k = I$ (which means that after establishing reduced channels V_k precoding is obtained by taking pseudo-inverse, like in EZF).

Existence and uniqueness of suitable detection maps G_k established in the previous section is purely theoretical for now. In real-time MIMO systems, the user must be able to calculate G_k from the information it is given (this information is provided by reference signals). To be more precise, we know neither all the maps participating in MIMO system, nor even those concerning us directly — H_k and all of W_j. What the k-th user does know (with decent precision), though, is the transmitting matrix $A_k = H_k W_k$ and *total covariance of the noise* given by the formula

$$R_k = H_k \left(\sum_{j \neq k} W_j W_j^* \right) H_k^* + L_k L_k^*,$$

where noise $n_k = L_k n'_k$ with $n'_k = \mathcal{N}(0,1)$. Sometimes L_k is far from I because this "white" noise may also contain inter-cell interference. Nevertheless, when we say that external noise tends to zero, we mean that $||L_k|| \to 0$.

In this notation, one of the widely used receivers, namely, MMSE-IRC detector, is given by the formula:

$$G_k = A_k^* (A_k A_k^* + R_k)^{-1}.$$

Theorem 2. ***Interference Cancellation of MMSE-IRC.*** *In the noiseless MIMO system with RCZF precoding, MMSE-IRC detector coincides with the unique interference cancellation detector. In a noisy MIMO system if $||L_k|| \to 0$ the interference is also cancelled.*

Proof. For the first statement, we observe that $A_k A_k^* + R_k = H_k (\sum W_j W_j^*) H_k^*$. Denote this matrix by Ω, so that $G_k = A_k^* \Omega^{-1}$. Though Ω depends on H_k, we do not add index to emphasize that inner sum is taken by all k. It is important to remark that, for Ω to be invertible, or, equivalently, $\operatorname{rk} \Omega = \dim U_k = q_k$, we require at least q_k layers in total. Otherwise, as we can see from the formula, Ω is decomposed as a sequence of linear transformations passing through $<q_k$-dimensional vector space. Of course, this condition is almost always satisfied (for example, it is so whenever we have at least 2 users with 4 antennae and 2 layers each).

We need to prove that, in the notation of previous section, $G_k = B_k$. Indeed,

$$A_k = H_k W_k = H_k \left(\sum W_k W_k^*\right) V_k^* = \Omega B_k^*,$$

since $W_k = W_k W_k^* V_k^*$, while $W_j W_j^* V_k = 0$ for $j \neq k$. Therefore,

$$G_k = (\Omega B_k^*)^* (\Omega)^{-1} = B_k,$$

since Ω is self-conjugate.

For the second statement, the final formula changes to $B_k \Omega(\Omega + L_k L_k^*)^{-1} = B_k(I + L_k L_k^* \Omega^{-1})^{-1}$, and the limit equals B_k again.

We proceed to show that other widespread detection methods, namely, generalised LSE and QR-MLD, also cancel interference in MIMO system with RCZF precoding when the external noise power tends to zero. In particular, the limit of all these detecting techniques is the unique interference cancellation obtained in Theorem 1. In the following formulas, we omit the index k for convenience.

Proposition 1. *Let F and G be two self-conjugate operators on Hermitian space V satisfying $\operatorname{Im} F \cap \operatorname{Im} G = 0$ and $\operatorname{Im} F \oplus \operatorname{Im} G = V$. Then $(F + \lambda G)^{-1}|_{\operatorname{Im} F}$ does not depend on $\lambda \neq 0$.*

Proof. Observe that $(F + \lambda G)(\ker G) = F(\ker G) = \operatorname{Im} F$. It follows from the fact that $\ker F \cap \ker G = 0$ and $\ker F \oplus \ker G = V$ which, in its turn, is the consequence of $\operatorname{Im} F \perp \ker F$ and $\operatorname{Im} G \perp \ker G$. This bijection $\varphi\colon \ker G \to \operatorname{Im} F$ clearly does not depend on λ. It follows that $(F + \lambda G)^{-1}|_{\operatorname{Im} F} = \varphi^{-1}$ also does not depend on λ.

Proposition 2. *In the noiseless MIMO system with RCZF precoding, the generalized LSE detector $A^*(AA^* + \lambda R)^{-1}$ does not depend on $\lambda \neq 0$.*

Proof. We have $\operatorname{Im} AA^* = \operatorname{Im} A = P_k \subset U_k$ as in Theorem 1. Similarly, $\operatorname{Im} R = T_k \subset U_k$, as we saw in the proof of the third statement of Theorem 1. Since $P_k \cap T_k = 0$ and $U_k = P_k \oplus T_k$, we are in conditions of Prop. 1, and $(AA^* + \lambda R)^{-1}|_{\operatorname{Im} AA^*}$ does not depend on λ. Observe that $\ker AA^* \oplus \operatorname{Im} AA^* = U_k$ and $\ker AA^* = \ker A^*$. We have two projections $\pi_1\colon U_k \to \ker AA^*$ and $\pi_2\colon V \to \operatorname{Im} AA^*$, and for any $v \in U_k$ we have $\pi_1(v) + \pi_2(v) = v$. Decomposing

$$\begin{aligned} A^*(AA^* + \lambda R)^{-1}(v) &= A^*(\pi_1 + \pi_2)(AA^* + \lambda R)^{-1}(v) = \\ &= A^*\pi_2(AA^* + \lambda R)^{-1}(v) = A^*((AA^* + \lambda R)^{-1}\iota_2)^*(v) = \\ &= A^*((AA^* + \lambda R)^{-1}|_{\operatorname{Im} AA^*})^*(v) \end{aligned}$$

— does not depend on λ. The third equality is true due to $(XY)^* = Y^* X^*$ and $\pi_2^* = \iota_2$, where $\iota_2\colon \operatorname{Im} AA^* \to U_k$ is the embedding.

Thus, we can see that generalized LSE detector also cancels interference in the limit.

We will now describe QR-MLD process. Its main feature is being symbol-wise, adjusting itself during intermediate steps (and therefore non-linear), but we will see that it is irrelevant in the consideration of interference.

First, the QR-MLD finds a square matrix $\mathcal{L}_k$ such that $\mathcal{L}\mathcal{L}^* = R_k$ (Cholesky decomposition, for instance, allows doing just that, though it is worth noting that the matrix $\mathcal{L}$ with this property is not unique and may be replaced by any $\mathcal{L}U$ with U unitary). Then the received vector in U_k may be rewritten as $H_k W_k s_k + \mathcal{L}n''$, where n'' is normalized and independent noise.

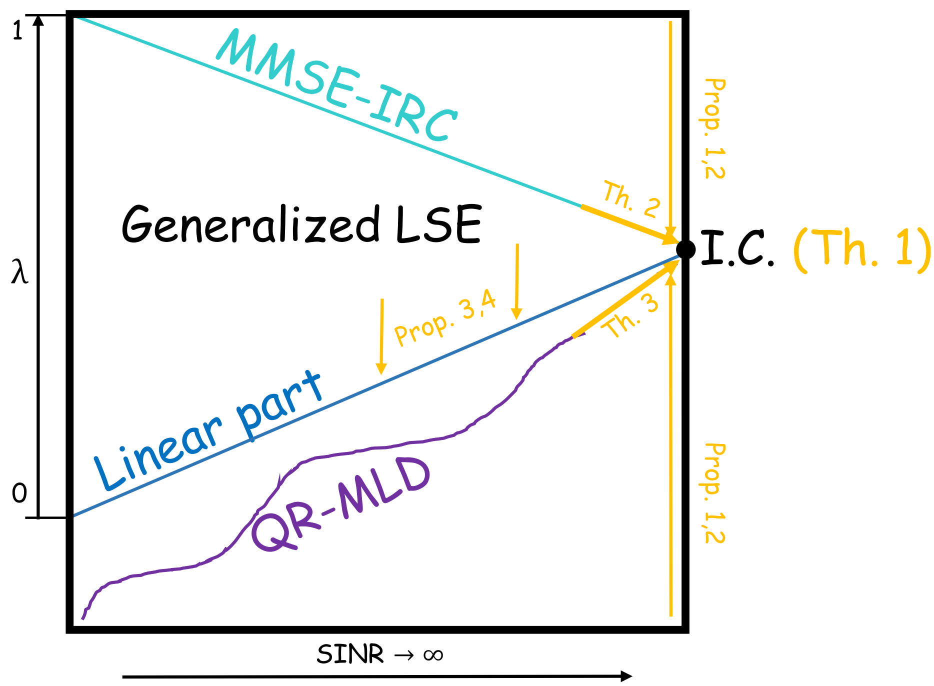

Fig. 2. Detection limits and interference cancellation.

The next and the main step of QR-MLD is to perform the QR-decomposition of matrix $\mathcal{L}^{-1}H_k W_k = Q\mathcal{R}$ (here Q is unitary, and $\mathcal{R}$ is upper-triangular). It is at this moment when we face the main complication, namely, if R_k does not contain external noise part $L_k L_k^*$ at all, then its rank is strictly less than q_k, and there is no way we can use $\mathcal{L}^{-1}$. Unable to deal with a simplified scenario, we introduce small L_k to make $\mathcal{L}$ invertible and study its behaviour when $||L_k|| \to 0$ as previously. After establishing Q and $\mathcal{R}$, QR-MLD detector extracts the diagonal part $\mathcal{R}_d$ of $\mathcal{R} = \mathcal{R}_d + \mathcal{R}_u$ and uses $\mathcal{R}_d^{-1}Q^*\mathcal{L}^{-1}$ as a first approximation of G_k. Applying this matrix to the received vector, we get:

$$\mathcal{R}_d^{-1}Q^*\mathcal{L}^{-1}(\mathcal{L}Q\mathcal{R}s_k + \mathcal{L}n'') = \mathcal{R}_d^{-1}(\mathcal{R}_d + \mathcal{R}_u)s_k + \mathcal{R}_d^{-1}n''' = \\ = s_k + \mathcal{R}_d^{-1}\mathcal{R}_u s_k + \mathcal{R}_d^{-1}n''',$$

where $n''' = Q^*n''$ is again normalized independent noise. The second summand applies *strictly* upper-triangular matrix $\mathcal{R}_d^{-1}\mathcal{R}_u$ to s_k and, thus, we may unveil s_k symbol by symbol starting from the end and subtracting the upper-triangular term. The third summand, $\mathcal{R}_d^{-1}n'''$ which represents the effective noise after detection.

To conclude, the non-linearity of QR-MLD detection comes from symbol-wise discretization of the decoded vector, which is then plugged in the formula to find the rest of the symbols. It means that we can speak of *linear part* of QR-MLD detection, which is equal to $\mathcal{R}^{-1}Q^*\mathcal{L}^{-1}$, and the difference between actual QR-MLD and its linear part tends to zero if the error itself tends to zero. We are now ready to prove Theorem 3.

Proposition 3. *In the conditions as above, when $\lambda \to 0$, generalized LSE detector $A^*(AA^* + \lambda R)^{-1} \to (A^*R^{-1}A)^{-1}A^*R^{-1}$.*

Proof. Using matrix identities, obtain that

$$A^*(AA^* + \lambda R)^{-1} = (A^*R^{-1}A + \lambda I)^{-1}A^*R^{-1},$$

and take the limit $\lambda \to 0$.

Proposition 4. *The linear part of the QR-MLD detector is equal to*

$$(A^*R^{-1}A)^{-1}A^*R^{-1},$$

provided that external noise is non-zero (so that R is invertible).

Proof. We have $Q\mathcal{R} = \mathcal{L}^{-1}A \Leftrightarrow A = \mathcal{L}Q\mathcal{R}$, where $R = \mathcal{L}\mathcal{L}^*$. Substituting this into $(A^*R^{-1}A)^{-1}A^*R^{-1}$, we get $\mathcal{R}^{-1}Q^*\mathcal{L}^{-1}$ which is equal to the linear part of QR-MLD detector.

Theorem 3. ***Interference Cancellation of QR-MLD.*** *In the noisy MIMO system with RCZF precoding, as $||L_k|| \to 0$, the limit of QR-MLD detector coincides with the unique interference cancellation detector.*

Proof. Substituting $\lambda = 1$ in formula $A^*(AA^* + \lambda R)^{-1}$, we get MMSE-IRC detector which eliminates interference with zero noise as was shown in Theorem 2. By Prop. 2 this is also true for arbitrarily small λ. By Prop. 3 and Prop. 4, this is also what QR-MLD achieves.

The structure of our detection investigations is summarized in Fig. 2.

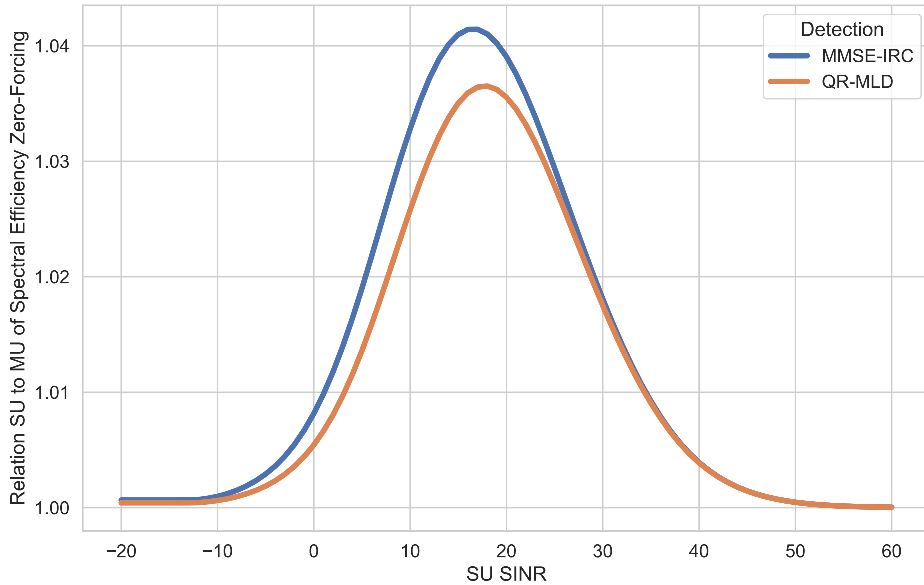

Fig. 3. Ratio of Single-User to Multi-User Spectral Efficiency for MMSE-IRC and QR-MLD detection matrices assuming Zero-Forcing precoding.

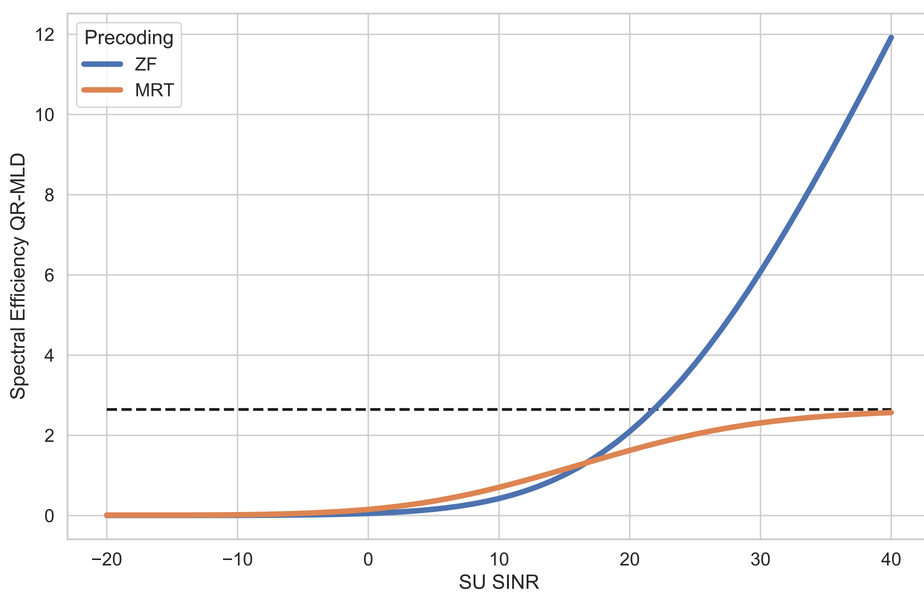

Fig. 4. Multi-User Spectral Efficiency for QR-MLD detection matrix assuming Zero-Forcing (ZF) and Maximum Ratio Transmission (MRT) precoding matrices.

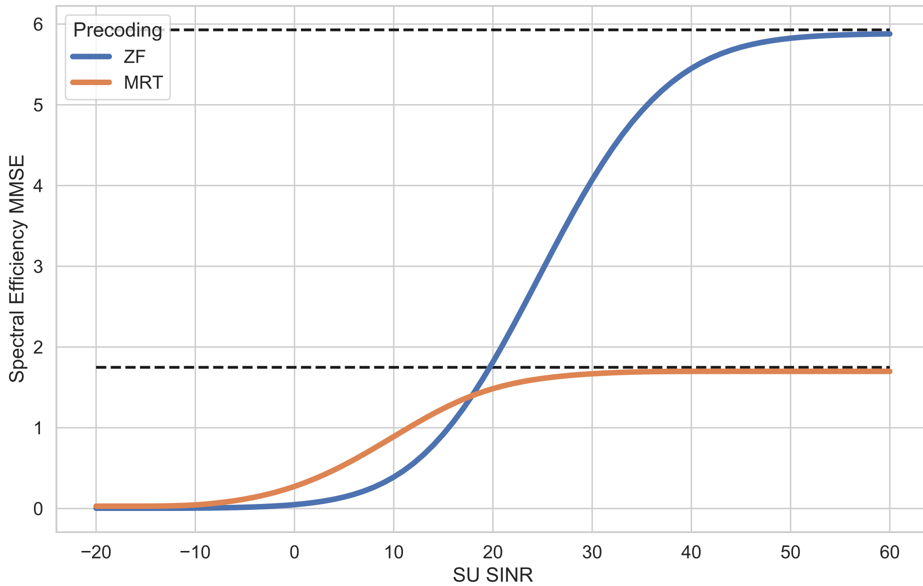

Fig. 5. Multi-User Spectral Efficiency for MMSE detection matrix assuming Zero-Forcing (ZF) and Maximum Ratio Transmission (MRT) precoding matrices.

5 Experiments

To support theoretical research, we conduct several important experiments. The goal of these experiments is to show the sufficiency and necessity of Theorem 1 on practical examples. We used the Quadriga simulator [12] to create channel matrices. We draw all figures depending on average user Single-User (SU) SINR (db).

Firstly, we study interference cancellation property of the QR-MLD and MMSE-IRC detection matrices and Zero-Foring (ZF) precoding matrix in Fig. 3. This figure plainly demonstrates interference cancellation for precoding of Reduced Channel Zero-Forcing (RCZF) class (2). It shows the ratio between Single-User (SU) and Multi-User (MU) Spectral Efficiency. And therefore, by definition, when this ratio approaches unity, interference between users is cancelled. The successful inter-user interference cancellation using the proper precoding and detection matrices experimentally proves the sufficiency of Theorem 1.

To show experimental evidence of the necessity, we investigate the case when the conditions of Theorem 1 are violated. In Fig. 4 we take the Maximum Ratio Transmission (MRT) as a precoding matrix out of the RCZF class (2). We added ZF precoding from the RCZF class for comparison with MRT. Additionally, QR-MLD was chosen as a detection matrix. In Fig. 4 we study the dependence of MU SE on SU SINR value. The value of MU SE grows using the ZF precoding when average SU SINR tends to infinity. In this case, the system only encounters negligible white noise. Unlike it, precoding that does not belong to the RCZF (2) class (we took MRT for instance), does not eliminate interference. The power of interference increases in this case. In Fig. 4, we can see that MU SINR for MRT precoding approximates its limit denoted by black dotted asymptote.

Limit of spectral efficiency also appears in the last comparison in Fig 5. In this experiment, the usual MMSE receiver is used instead of its MMSE-IRC counterpart. In conclusion, we would like to add that these experiments support the statement that only the combination of both precoding from the RCZF class (2) and generalized LSE detection (cf. Prop. 3 and Fig. 2) class yields complete interference cancellation.

6 Conclusion and Future Direction

The bulk of results discussed in this paper is demonstrated in Fig. 2 and can be summarized as follows. As SU SINR tends to infinity, there exist a unique linear detection cancelling inter-user interference in noisy MIMO scenario. Interference cancellation property can be achieved by combining RCZF precoding W with any detections G belonging to the family of Generalized LSE as well as MMSE-IRC and QR-MLD. These techniques exhaust all the possibilities for interference cancellation in MIMO systems, which is crucial for constructing new precoding and detection methods. As the main focus of this work is analytical study of interference suppression in Massive MIMO system using RCZF class of precoding, we assume the base station has perfect channel measurements and neglect all other potential hardware impairments. Nevertheless, noise robustness to given measurements keeps the current results asymptotically correct and will be carefully considered in future work.

References

1. Bai, Z., Badic, B., Iwelski, S., Scholand, T., Balraj, R., Bruck, G., Jung, P.: On the equivalence of MMSE and IRC receiver in MU-MIMO systems. IEEE communications letters **15**(12), 1288–1290 (2011)
2. Björnson, E., Bengtsson, M., Ottersten, B.: Optimal multiuser transmit beamforming: A difficult problem with a simple solution structure [lecture notes]. IEEE Signal Processing Magazine **31**(4), 142–148 (2014)
3. Björnson, E., Hoydis, J., Sanguinetti, L.: Massive MIMO networks: Spectral, energy, and hardware efficiency. Foundations and Trends in Signal Processing **11**(3-4), 154–655 (2017)
4. Bobrov, E., Chinyaev, B., Kuznetsov, V., Lu, H., Minenkov, D., Troshin, S., Yudakov, D., Zaev, D.: Adaptive regularized zero-forcing beamforming in massive MIMO with multi-antenna users. arXiv preprint arXiv:2107.00853 (2021)
5. Bobrov, E., Chinyaev, B., Kuznetsov, V., Minenkov, D., Yudakov, D.: Power allocation algorithms for massive mimo systems with multi-antenna users. arXiv preprint arXiv:2201.08068 (2022)
6. Bobrov, E., Kropotov, D., Lu, H., Zaev, D.: Massive mimo adaptive modulation and coding using online deep learning algorithm. IEEE Communications Letters **26**(4), 818–822 (2022). https://doi.org/10.1109/LCOMM.2021.3132947

7. Bobrov, E., Kropotov, D., Troshin, S., Zaev, D.: L-BFGS precoding optimization algorithm for massive MIMO systems with multi-antenna users (2022)
8. Bobrov, E., Markov, A., Vetrov, D.: Variational autoencoders for studying the manifold of precoding matrices with high spectral efficiency. arXiv preprint arXiv:2111.15626 (2021)
9. Davydov, A., Sergeev, V., Mondal, B., Papathanassiou, A., Sengupta, A.: Robust MMSE-IRC for uplink massive MIMO aided C network. In: 2020 IEEE Globecom Workshops (GC Wkshps. pp. 1–5. IEEE (2020)
10. El Chall, R., Nouvel, F., Hélard, M., Liu, M.: Iterative receivers combining MIMO detection with turbo decoding: performance-complexity trade-offs. EURASIP Journal on Wireless Communications and Networking **2015**(1), 1–19 (2015)
11. Ivanov, A., Osinsky, A., Lakontsev, D., Yarotsky, D.: High performance interference suppression in multi-user massive MIMO detector. In: 2020 IEEE 91st Vehicular Technology Conference (VTC2020-Spring). pp. 1–5. IEEE (2020)
12. Jaeckel, S., Raschkowski, L., Börner, K., Thiele, L.: QuaDRiGa: A 3-D multi-cell channel model with time evolution for enabling virtual field trials. IEEE Transactions on Antennas and Propagation **62**(6), 3242–3256 (2014)
13. Joham, M., Utschick, W., Nossek, J.A.: Linear transmit processing in mimo communications systems. IEEE Transactions on signal Processing **53**(8), 2700–2712 (2005)
14. Kim, Y., Park, J.H., Kim, J.W.: Hybrid MIMO receiver using QR-MLD and QE-MMSE. In: GLOBECOM 2009-2009 IEEE Global Telecommunications Conference. pp. 1–5. IEEE (2009)
15. Larsson, E.G., Edfors, O., Tufvesson, F., Marzetta, T.L.: Massive MIMO for next generation wireless systems. IEEE communications magazine **52**(2), 186–195 (2014)
16. Papoutsis, V.D., Fraimis, I.G., Kotsopoulos, S.A.: User selection and resource allocation algorithm with fairness in MISO-OFDMA. IEEE Communications Letters **14**(5), 411–413 (2010)
17. Redana, S., Bulakci, Ö., Zafeiropoulos, A., Gavras, A., Tzanakaki, A., Albanese, A., Kousaridas, A., Weit, A., Sayadi, B., Jou, B.T., et al.: 5G PPP architecture working group: View on 5G architecture (2019)
18. Ren, B., Wang, Y., Sun, S., Zhang, Y., Dai, X., Niu, K.: Low-complexity MMSE-IRC algorithm for uplink massive MIMO systems. Electronics Letters **53**(14), 972–974 (2017)
19. Shafi, M., Molisch, A.F., Smith, P.J., Haustein, T., Zhu, P., De Silva, P., Tufvesson, F., Benjebbour, A., Wunder, G.: 5G: A tutorial overview of standards, trials, challenges, deployment, and practice. IEEE journal on selected areas in communications **35**(6), 1201–1221 (2017)
20. Shi, Q., Razaviyayn, M., Luo, Z.Q., He, C.: An iteratively weighted mmse approach to distributed sum-utility maximization for a mimo interfering broadcast channel. IEEE Transactions on Signal Processing **59**(9), 4331–4340 (2011)
21. Sun, L., McKay, M.R.: Eigen-based transceivers for the MIMO broadcast channel with semi-orthogonal user selection. IEEE Transactions on Signal Processing **58**(10), 5246–5261 (2010)
22. Tavares, F.M., Berardinelli, G., Mahmood, N.H., Sorensen, T.B., Mogensen, P.: On the potential of interference rejection combining in B4G networks. In: 2013 IEEE 78th Vehicular Technology Conference (VTC Fall). pp. 1–5. IEEE (2013)
23. Verdu, S., et al.: Multiuser detection. Cambridge university press (1998)
24. Yang, S., Hanzo, L.: Fifty years of MIMO detection: The road to large-scale MIMOs. IEEE Communications Surveys & Tutorials **17**(4), 1941–1988 (2015)

Statements and Declarations

Funding

Authors have received research support from Huawei Technologies.

Competing Interests

The authors have no relevant financial or non-financial interests to disclose.